Mesons in strong magnetic field

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Plan

- Green's function for qq-system in magnetic field
- Hamiltonian for neutral and charged mesons, pseudomomentum and separation of variables
- Wave function, eigenvalues and corrections
  - ✓ One-gluon exchange: screening
  - ✓ Self-energy contribution
  - ✓ Spin-spin interaction
- Results

Green’s function for qq-system.

Quark propagator $$S_i(x,y) = (m_i + \hat\partial - ig\hat A - ie_i \hat A^{(e)})^{-1}_{xy} \equiv (m_i + \hat D^{(i)})^{-1}_{xy}$$

in Feynman-Fock-Schwinger representation

$$S_i(x,y) = (m_i - \hat D^{(i)}) \int_0^\infty ds_i (D^4 z)_{xy} e^{-K_i} \Phi^{(i)}_\sigma (x,y) \equiv (m_i - \hat D^{(i)}) G_i(x,y)$$

m_i – quark mass, s_i – proper time

$$K_i = m_i^2 s_i + \frac{1}{4}\int_0^{s_i} d\tau_i \left(\frac{dz^{(i)}_\mu}{d\tau_i}\right)^2$$

$$\Phi^{(i)}_\sigma(x,y) = P_A P_F \exp\left(ig\int_y^x A_\mu dz^{(i)}_\mu + ie_i \int_y^x A^{(e)}_\mu dz^{(i)}_\mu\right) \exp\left(\int_0^{s_i} d\tau_i \sigma_{\mu\nu}(gF_{\mu\nu} + e_i B_{\mu\nu})\right)$$

A_μ, $F_{\mu\nu}$ – gluon fields, $A_\mu^{(e)}$, $B_{\mu\nu}$ – external magnetic field

$$\sigma_{\mu\nu}F_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma}\mathbf{H} & \boldsymbol{\sigma}\mathbf{E} \\ \boldsymbol{\sigma}\mathbf{E} & \boldsymbol{\sigma}\mathbf{H} \end{pmatrix}, \quad \sigma_{\mu\nu}B_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma}\mathbf{B} & 0 \\ 0 & \boldsymbol{\sigma}\mathbf{B} \end{pmatrix}$$

Wilson loop

For qq-system

$$\langle W_\sigma(A)\rangle_A = \exp\left(-\frac{g^2}{2}\int d\pi_{\mu\nu}(1)d\pi_{\lambda\sigma}(2)\langle F_{\mu\nu}(1)F_{\lambda\sigma}(2)\rangle + \mathcal{O}(\langle FFF\rangle)\right)$$

$$d\pi_{\mu\nu} \equiv ds_{\mu\nu}+\sigma^{(1)}_{\mu\nu}d\tau_1-\sigma^{(2)}_{\mu\nu}d\tau_2$$

$$G_{q_1\bar{q}_2}(x,y) = \int_0^\infty ds_1\int_0^\infty ds_2(D^4z^{(1)})_{xy}(D^4z^{(2)})_{xy}e^{-K_1-K_2}\mathrm{tr}\langle\hat{T}W_\sigma(A)\rangle_A\times$$
$$\times\exp\left(ie_1\int_y^x A^{(e)}_\mu dz^{(1)}_\mu - ie_2\int_y^x A^{(e)}_\mu dz^{(2)}_\mu + e_1\int_0^{s_1}d\tau_1(\boldsymbol{\sigma}\mathbf{B}) - e_2\int_0^{s_2}d\tau_2(\boldsymbol{\sigma}\mathbf{B})\right)$$

correspond to external field

$$\hat{T} = \Gamma_1(m_1-\hat{D}_1)\Gamma_2(m_2-\hat{D}_2)$$

Introducing monotonous time $t_E(\tau) = x_4+\frac{\tau}{s}T$ $\quad T\equiv|x_4-y_4|$

so that $z_4(\tau) = t_E(\tau)+\Delta z_4(\tau)$

Change of variables $\omega_i\equiv\frac{T}{2s_i}$

Green's function in terms of new variables

$$G_{q_1\bar{q}_2}(x,y) = \frac{T}{8\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\frac{d\omega_2}{\omega_2^{3/2}}(D^3z^{(1)}D^3z^{(2)})_{\mathbf{xy}}e^{-K_1(\omega_1)-K_2(\omega_2)}\langle\langle\hat{T}W_F\rangle\rangle_{\Delta z_4}$$

Green's function after averaging:

$$G_{q_1\bar{q}_2}(x,y) = \frac{T}{8\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\int_0^\infty \frac{d\omega_2}{\omega_2^{3/2}}\left\langle \mathbf{x}\left|\mathrm{tr}(\hat{T}e^{-H_{q_1\bar{q}_2}T})\right|\mathbf{y}\right\rangle$$

Simonov Yu.A., arXiv:1303.4952

P=0 component:

$$\int G_{q_1\bar{q}_2}(x,y)d^3(x-y) = \frac{T}{8\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\int_0^\infty \frac{d\omega_2}{\omega_2^{3/2}}\sum_{n=0}^\infty \varphi_n^2(0)\langle\mathrm{tr}(\hat{T})\rangle e^{-M_n(\omega_1,\omega_2)T}$$

If we take the integral by the saddle point method in large T limit, ω_1, ω_2 are determined by the minimization condition for $M_n(\omega_1,\omega_2)$.

$$\hat{H}\psi = M_n(\omega_1,\omega_2)\psi, \quad \frac{\partial M_n(\omega_1,\omega_2)}{\partial\omega_i} = 0$$

Hamiltonian

Neutral mesons e_1=-e_2=e

$$H_{q_1\bar{q}_2} = H_0 + H_\sigma + W$$

- $$H_0 = \frac{1}{2\tilde{\omega}}\left(-\frac{\partial^2}{\partial \boldsymbol{\eta}^2} + \frac{e^2}{4}(\mathbf{B}\times\boldsymbol{\eta})^2\right)$$ $$\tilde{\omega} = \frac{\omega_1\omega_2}{\omega_1+\omega_2}$$

oscillator in transverse (with respect to **B**) direction

ω_1, ω_2 – effective quark energies,

η – relative coordinate

ω_1, ω_2 are defined by minimization condition for eigenvalues of the Hamiltonian.

- $$H_\sigma = \frac{m_1^2+\omega_1^2 - e\boldsymbol{\sigma}_1\mathbf{B}}{2\omega_1} + \frac{m_2^2+\omega_2^2 + e\boldsymbol{\sigma}_2\mathbf{B}}{2\omega_2}$$

contains interaction of quarks magnetic moments with the magnetic field

m_1, m_2 – quark current mass

- $$W = V_{\text{conf}} + V_{\text{Coul}} + V_{SS} + \Delta M_{SE}$$

Confinement: $$V_{\text{conf}} = \sigma\eta \to \tilde{V}_{\text{conf}} = \frac{\sigma}{2}\left(\frac{\eta^2}{\gamma} + \gamma\right)$$

3d oscillator potential

γ is defined by the minimization condition for eigenvalues of the Hamiltonian (accuracy ~5%)

<u>Charged mesons</u> $e_1=e_2=e$, $m_1=m_2=m$, $\omega_1= \omega_2=\omega$

$$H_{q_1\bar{q}_2} = \frac{\mathbf{P}^2}{4\omega} + \frac{e^2}{4\omega}(\mathbf{B}\times\mathbf{R})^2 + \frac{\boldsymbol{\pi}^2}{\omega} + \frac{e^2}{16\omega}(\mathbf{B}\times\boldsymbol{\eta})^2 + \\ + \frac{2m^2 + 2\omega^2 - e(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\mathbf{B}}{2\omega} + \frac{\sigma}{2}\left(\frac{\eta^2}{\gamma} + \gamma\right) + \\ + V_{\text{OGE}} + V_{SS} + \Delta M_{SE}.$$

$$(H_0 + H_\sigma + W)\Psi_n(\eta) = M_n(\omega_1, \omega_2, \gamma)\Psi_n(\eta)$$

Eigenvalue equation

Contributions from V_{Coul}, V_{SS}, ΔM_{SE} are treated as corrections.

Solution of eigenvalue equation without corrections

$$(H_0 + \tilde{V}_{\rm conf})\psi = M(\omega_1, \omega_2, \gamma)\psi$$

gives the **wave function**

$$\psi(\boldsymbol{\eta}) = \frac{1}{\sqrt{\pi^{3/2} r_\perp^2 r_0}} \exp\left(-\frac{\eta_\perp^2}{2r_\perp^2} - \frac{\eta_z^2}{2r_0^2}\right)$$

Transverse and longitudinal size of meson:

$$r_\perp = \sqrt{\frac{2}{eB}}\left(1 + \frac{4\sigma\tilde{\omega}}{\gamma e^2 B^2}\right)^{-1/4} \qquad r_0 = \left(\frac{\gamma}{\sigma\tilde{\omega}}\right)^{1/4}$$

At large fields $$r_\perp \propto \frac{1}{\sqrt{eB}}, \quad r_0 \propto \frac{1}{\sqrt{\sigma}}, \quad B \to \infty$$

Eigenvalues of unperturbed Hamiltonian:

Neutral mesons ($e_1=-e_2=e$)

$$M_n(\omega_1,\omega_2,\gamma)=\varepsilon_{n_\perp,n_z}+\frac{m_1^2+\omega_1^2-e\mathbf{B}\boldsymbol{\sigma}_1}{2\omega_1}+\frac{m_2^2+\omega_2^2+e\mathbf{B}\boldsymbol{\sigma}_2}{2\omega_2}$$

$$\varepsilon_{n_\perp,n_z}=\frac{1}{2\tilde{\omega}}\left[\sqrt{e^2B^2+\frac{4\sigma\tilde{\omega}}{\gamma}}(2n_\perp+1)+\sqrt{\frac{4\sigma\tilde{\omega}}{\gamma}}\left(n_z+\frac{1}{2}\right)\right]+\frac{\gamma\sigma}{2}$$

4 states

$$M_n^{++},\quad M_n^{--},\quad M_n^{+-},\quad M_n^{-+}$$

Charged mesons ($e_1=e_2=e$, $m_1=m_2=m$, $\omega_1=\omega_2=\omega$)

$$M_n(\omega,\gamma)=\frac{eB}{2\omega}(2N_\perp+1)+\sqrt{\left(\frac{eB}{2\omega}\right)^2+\frac{2\sigma}{\omega\gamma}}(2n_\perp+1)+\sqrt{\frac{2\sigma}{\omega\gamma}}\left(n_\parallel+\frac{1}{2}\right)-\frac{eB}{\omega}+\frac{\sigma\gamma}{2}+\frac{m^2+\omega^2}{\omega}$$

One-gluon exchange potential contribution

$$V_{OGE}(q) = -\frac{16\pi}{3Q^2}\alpha_s(q)$$

Running coupling constant

$$\alpha_s(q) = \frac{4\pi}{\beta_0 \ln\left(\frac{q^2+M_B^2}{\Lambda_{QCD}^2}\right)}$$

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$$

Parameter of IR freezing M_B~1 GeV

Simonov Yu.A., arXiv:1011.5386

$$\Delta M_{\mathrm{OGE}} \equiv \int V_{\mathrm{OGE}}(q)\tilde{\psi}^2(\mathbf{q})\frac{d^3q}{(2\pi)^3} \propto -\sqrt{\sigma}\ln\ln\frac{eB}{\sigma}$$

Diverges logarithmically at large fields.

Accounting quark loops

Self-energy part of gluon propagator

$$D(q) = \frac{4\pi}{q^2 - \frac{g^2(\mu_0^2)}{16\pi^2}\tilde{\Pi}(q)}$$

$$\tilde{\Pi}(q) = \tilde{\Pi}_{gl}(q) - \tilde{\Pi}_{q\bar{q}}(q)$$

In absence of magnetic field

$$\tilde{\Pi}_{gl}(q) = -\frac{11}{3}N_c q^2 \ln\frac{|q^2|}{\mu_0^2}, \quad \tilde{\Pi}_{q\bar{q}}(q) = -\frac{2}{3}n_f q^2 \ln\frac{|q^2|}{\mu_0^2}$$

In strong magnetic fields (lowest Landau level contribution):

$$\frac{\alpha_s^{(0)}}{4\pi}\tilde{\pi}_{q\bar{q}}(q) = -\frac{\alpha_s^{(0)} n_f |e_q B|}{\pi} \exp\left(-\frac{q_\perp^2}{2|e_q B|}\right) T\left(\frac{q_3^2}{4m^2}\right)$$

$$T(z) = -\frac{\ln\left(\sqrt{1+z}+\sqrt{z}\right)}{\sqrt{z(z+1)}} + 1$$

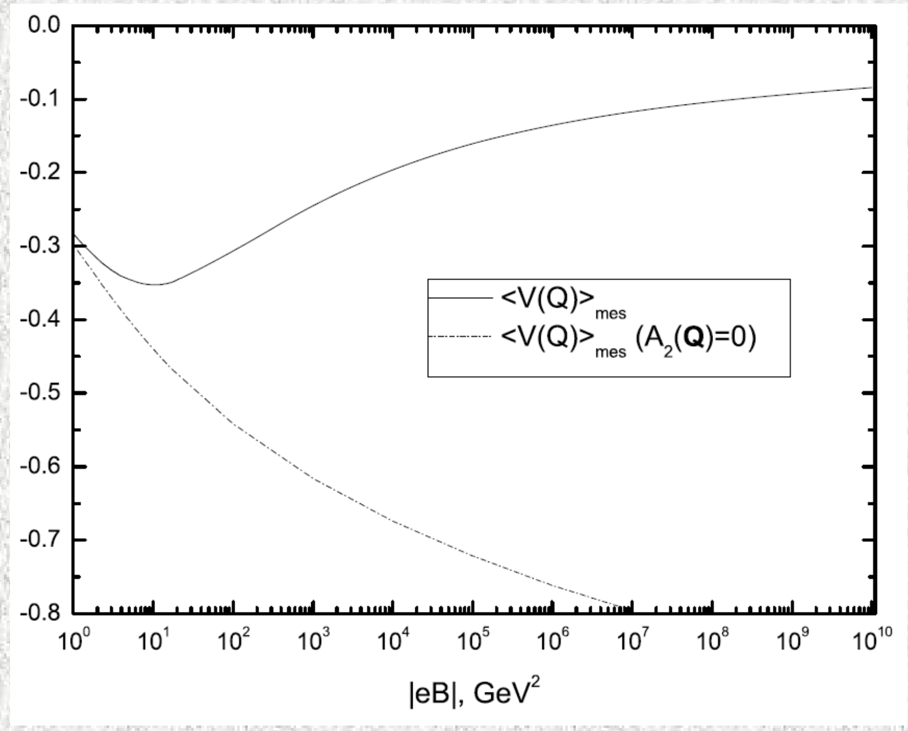
parameter $m^2 \sim \sigma$

Potential with account of screening

$$V(Q) = -\frac{16\pi\alpha_s^{(0)}}{3\left[Q^2\left(1+\frac{\alpha_s^{(0)}}{4\pi}\frac{11}{3}N_c\ln\frac{Q^2+M_B^2}{\mu_0^2}\right)+\frac{\alpha_s^{(0)}n_f|e_qB|}{\pi}\exp\left(\frac{-q_\perp^2}{2|e_qB|}\right)T\left(\frac{q_3^2}{4\sigma}\right)\right]}$$

Correction to the mass

$$\langle V(Q)\rangle_{mes} = \int V(Q)\psi^2(q_\perp, q_3)\frac{d^2q_\perp dq_3}{(2\pi)^3}$$



Spin-dependent corrections

Contributions are defined by correlators $(\sigma^{(i)} F)(\sigma^{(k)} F)$

i=k gives self-energy correction ΔM_{SE},
i≠k defines spin-spin interaction V_{SS}.

Simonov Yu.A., arXiv:1304.0365

Self-energy correction

$$\Delta M_{SE} = -\frac{3\sigma}{4\pi\omega_1}\left(1+\eta\left(\lambda\sqrt{2eB+m_1^2}\right)\right) - \frac{3\sigma}{4\pi\omega_2}\left(1+\eta\left(\lambda\sqrt{2eB+m_2^2}\right)\right)$$

$$\eta(t) = t\int_0^\infty z^2 K_1(tz) e^{-z} dz$$

takes into account additional suppression in case of large effective quark mass (ω_i)

λ – vacuum correlation length ($\lambda \sim 1$ GeV^{-1})

Spin-spin interaction:

$$V_{SS} = \frac{8\pi\alpha_s}{9\omega_1\omega_2}\delta^{(3)}(\mathbf{r})\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 \equiv a_{SS}\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2$$

$$\delta^{(3)}(\mathbf{r}) \to \psi^2(0) \sim eB$$

Unbounded growth at large fields

Smearing of δ-function:

$$\delta^{(3)}(\mathbf{r}) \to \tilde{\delta}^{(3)}(\mathbf{r}) = \left(\frac{1}{\lambda\sqrt{\pi}}\right)^3 e^{-\mathbf{r}^2/\lambda^2}, \quad \lambda \sim 1 \text{ GeV}^{-1}$$

$$\langle a_{SS}\rangle = \frac{c}{\pi^{3/2}\sqrt{\lambda^2 + r_0^2}(\lambda^2 + r_\perp^2)}, \quad c = \frac{8\pi\alpha_s}{9\omega_1\omega_2}$$

Nondiagonal structure of spin-spin interaction

Hamiltonian diagonalization

$$E_{1,2} = \frac{1}{2}(M_{11} + M_{22}) \pm \sqrt{\left(\frac{M_{22} - M_{11}}{2}\right)^2 + 4a_{12}a_{21}}$$

$$M_{11} = (M_0^{+-} + \Delta M_{SE} - \langle a_{SS} \rangle)\big|_{\omega_1^{(0)}=\omega_2^{(0)}=\omega_\pm}$$

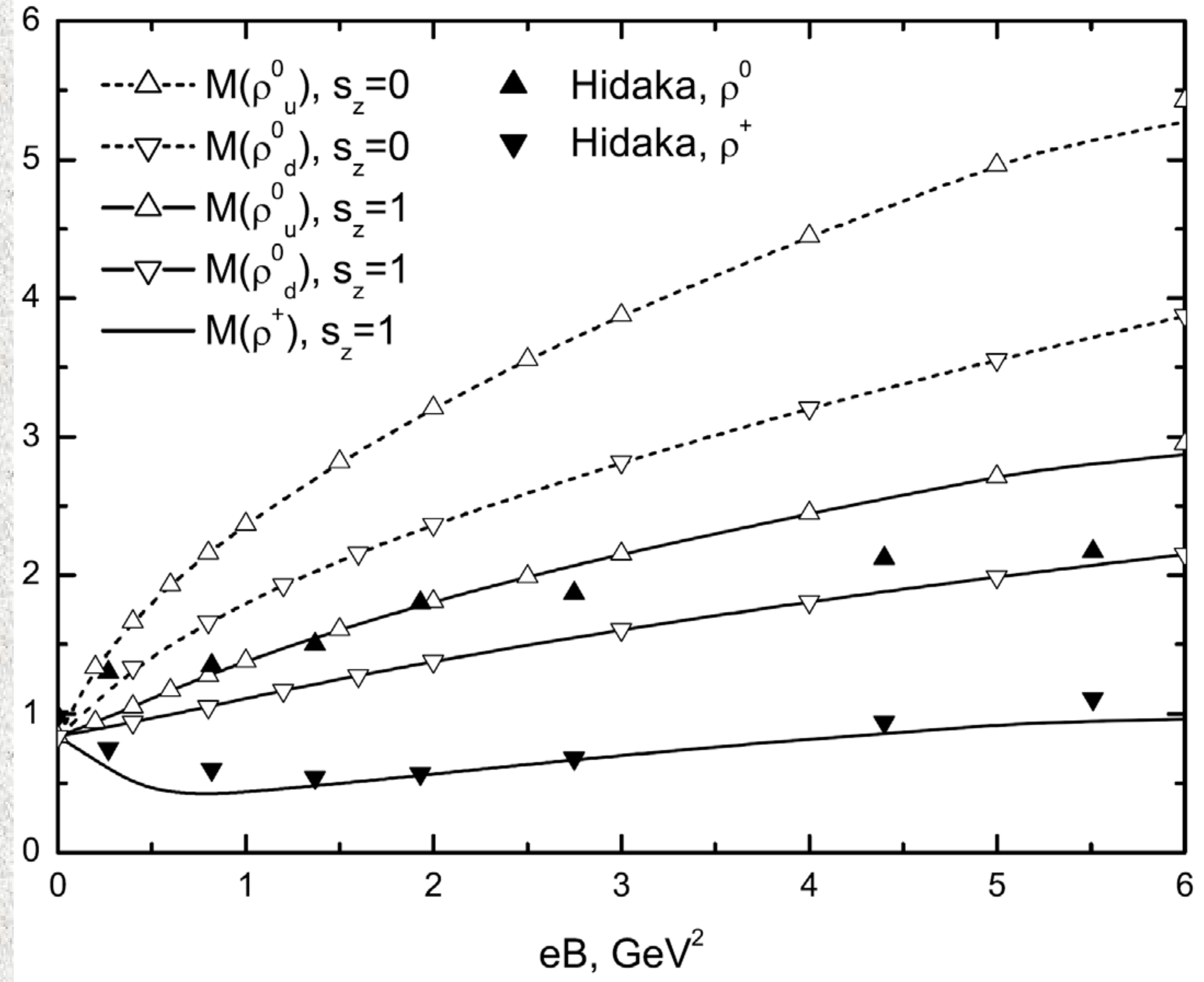
$$a_{12} = a_{21} = \langle a_{SS} \rangle\big|_{\omega_1^{(0)}=\omega_\pm,\, \omega_2^{(0)}=\omega_\mp}$$

$$M_{22} = (M_0^{-+} + \Delta M_{SE} - \langle a_{SS} \rangle)\big|_{\omega_1^{(0)}=\omega_2^{(0)}=\omega_\mp}$$

$$E_3 = M_0^{++} + \Delta M_{SE} + \langle a_{SS} \rangle,\ E_4 = M_0^{--} + \Delta M_{SE} + \langle a_{SS} \rangle$$

qq-systems with different quark charges behave differently in strong magnetic field

Meson masses



Nambu-Goldstone mesons

Gell-Mann-Oakes-Renner relation is violated for charged NG mesons, but remains valid for neutral mesons

$$m_\pi^2 f_\pi^2 = \frac{\bar{m} M(0)}{M(0)+\bar{m}} |\langle \bar{u}u \rangle + \langle \bar{d}d \rangle|, \quad \bar{m} = \frac{m_u + m_d}{2}$$

Quark condensate and f_π are expressed in terms of Hamiltonian eigenfunctions and eigenvalues:

$$|\langle \bar{q}q \rangle_i| = N_c (M(0) + m_i) \sum_{n=0}^{\infty} \left(\frac{\frac{1}{2}|\psi_{n,i}^{(+-)}(0)|^2}{m_{n,i}^{(+-)}} + \frac{\frac{1}{2}|\psi_{n,i}^{(-+)}(0)|^2}{m_{n,i}^{(-+)}} \right)$$

$$f_\pi^2 = N_c M^2(0) \sum_{n=0}^{\infty} \left(\frac{\frac{1}{2}|\psi_{n,i}^{(+-)}(0)|^2}{(m_{n,i}^{(+-)})^3} + \frac{\frac{1}{2}|\psi_{n,i}^{(-+)}(0)|^2}{(m_{n,i}^{(-+)})^3} \right)$$

M(0)≈150MeV

For charged mesons

$$m_{as}(\rho^+(S_z=0)) = M_{-+}(B) \approx \sqrt{\frac{4}{3} eB}$$

$$m_{as}(\pi^+) = M_{+-}(B) \approx \sqrt{\frac{2}{3} eB}$$

Masses of Nambu-Goldstone mesons