Gluon propagator in the SU(2) gauge theory near critical temperature

V. G. Bornyakov[1], V. K. Mitrjushkin[2], and R. N. Rogalyov[1]

1. Institute for High-Energy Physics
2. Joint Insitute of Nuclear Research

07.11.2013

- Gluon propagator: definition, momentum dependence, Gribov-Stingl fit
- Gauge fixing
- Gribov-copy problem
- Electric mass behavior

We compute the “gluon” propagator in the $SU(2)$ theory at the temperature $T = \frac{1}{\beta}$, defined as follows:

$$D_{\mu\nu}(\tau, \vec{x}) = \frac{1}{\mathcal{Z}} \int_{A_\mu(0,\vec{x}) = A_\mu(\beta,\vec{x})} DA^a_\mu(x) A_\mu(\tau, \vec{x}) A_\mu(0,0) e^{-S_E[A]} |\det M_{FP}(A)| \quad (1)$$

$$S_E[A] = \int_0^\beta d\tau \int_V d^3\vec{x} \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\alpha} (\partial_\mu A^a_\mu)^2 \right) \quad (2)$$

$$\mathcal{Z} = \int DA^a_\mu(x) e^{-S_E[A]} |\det M_{FP}(A)| \quad (3)$$

$$(D(p))^{-1}{}_{\mu\nu} = (D^0(p))^{-1}{}_{\mu\nu}(p) - \Pi_{\mu\nu}(p)$$

$$\begin{aligned} p_\mu \, \Pi_{\mu\nu}(p) &= 0 \\ p_\mu p_\nu D_{\mu\nu}(p) &= \alpha \end{aligned} \tag{4}$$

$$\Pi_{\mu\nu} = G(p)P^T_{\mu\nu} + F(p)P^L_{\mu\nu} \tag{5}$$

$$P^T_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \frac{p_i p_j}{|\vec{p}|^2} \end{pmatrix} \qquad P^L_{\mu\nu} = \frac{1}{p^2} \begin{pmatrix} |\vec{p}|^2 & p_4 \vec{p} \\ p_4 \vec{p} & \frac{p_4^2}{|\vec{p}|^2} p_i p_j \end{pmatrix}$$

$$p_\mu P^L_{\mu\nu} = p_\mu P^T_{\mu\nu} = 0$$

$$P^{L,T}_{\mu\nu} P^{L,T}_{\nu\lambda} = P^{L,T}_{\mu\lambda}, \qquad P^L_{\mu\nu} P^T_{\nu\lambda} = 0$$

$$P^L_{\mu\mu} = 1, \qquad P^T_{\mu\mu} = 2$$

$$D_{\mu\nu}(p) = D_L(p)P^L_{\mu\nu} + D_T(p)P^T_{\mu\nu} + \alpha\frac{p_\mu p_\nu}{p^4}$$

$$D_L(p) = \frac{1}{p^2 - F(p)}, \qquad D_T(p) = \frac{1}{p^2 - G(p)}$$

$$\Pi_{\mu\nu} = F(p)P^L_{\mu\nu} + G(p)P^T_{\mu\nu}$$

$$D_{ii}(|\vec{p}|^2) = 2D_T(0,\vec{p}), \qquad D_{44}(|\vec{p}|^2) = D_L(0,\vec{p}),$$

$$S = \frac{4}{g^2} \sum_{P=x,\mu,\nu} \left(1 - \frac{1}{2} \mathrm{Tr}\; U_P \right)$$

where

$$U_P = U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu}$$

$U_{x,\mu} \in SU(2)$,

$$U_{x,\mu} = u_0 + i \sum_{a=1}^{3} u_a \sigma_a, \quad (6)$$

$$A^a_\mu = -\frac{2u^a_\mu}{ga}, \quad (7)$$

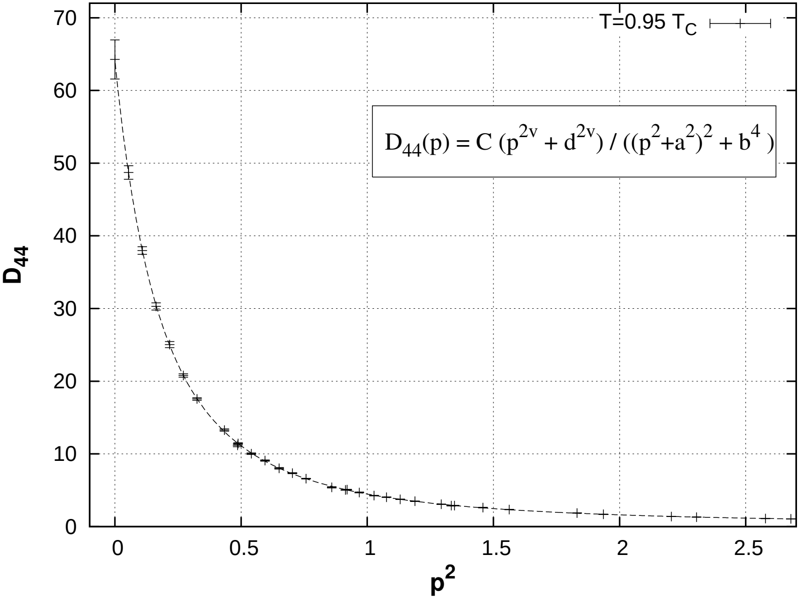
$$\Lambda : \quad U_{x,\mu} \to \Lambda^{\dagger}_x U_{x,\mu} \Lambda_{x+\hat{\mu}},$$

We fix the absolute Landau gauge by finding the global maximum of the functional

$$\mathcal{F}[\,\mathcal{U}\,] = \frac{1}{2} \sum_{x,\mu} \mathrm{Tr}\, U_{x,\mu}, \quad (8)$$

Stationarity condition:

$$\partial_\nu A^a_\nu = 0.$$



T=0.95 T_C
$D_{44}(p) = C\,(p^{2v} + d^{2v}) / ((p^2+a^2)^2 + b^4)$
D_{44}
p^2
0
10
20
30
40
50
60
70
0
0.5
1
1.5
2
2.5

Definition of the electric gluon (color-screening) mass:

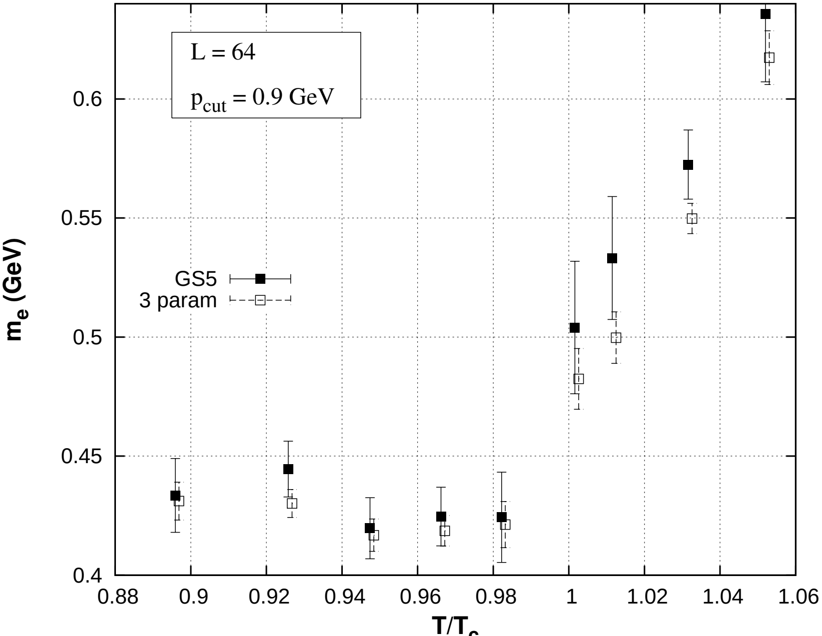
$$\frac{1}{D_{44}(p)} = Z\ (m_e^2 + p^2 + O(p^4))$$

It should be noted that, at $p \leq 1.5 GeV$,

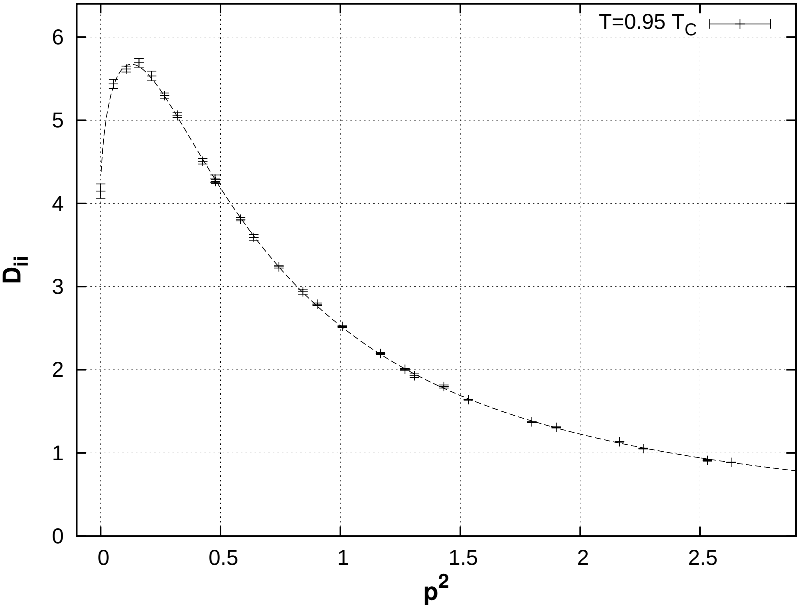
$D_{44}(p)$ can well be fitted by

$$D_{44}(p) = \frac{C}{(\mu^2 + p^2)^2} \qquad (\mu = m_e\sqrt{2})$$

Perturbation theory for SU(2): $m_e = \sqrt{\frac{2}{3}}gT$



L = 64
p_{cut} = 0.9 GeV
GS5
3 param
0.4
0.45
0.5
0.55
0.6
0.88
0.9
0.92
0.94
0.96
0.98
1
1.02
1.04
1.06
m_e (GeV)
T/T_c

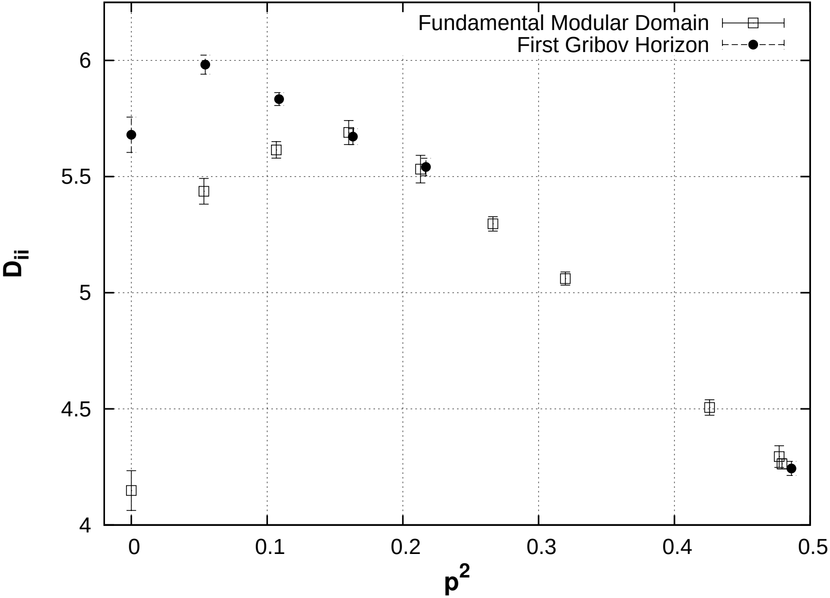


T=0.95 T_C
D_{ii}
p^2
0
1
2
3
4
5
6
0.5
1.5
2.5

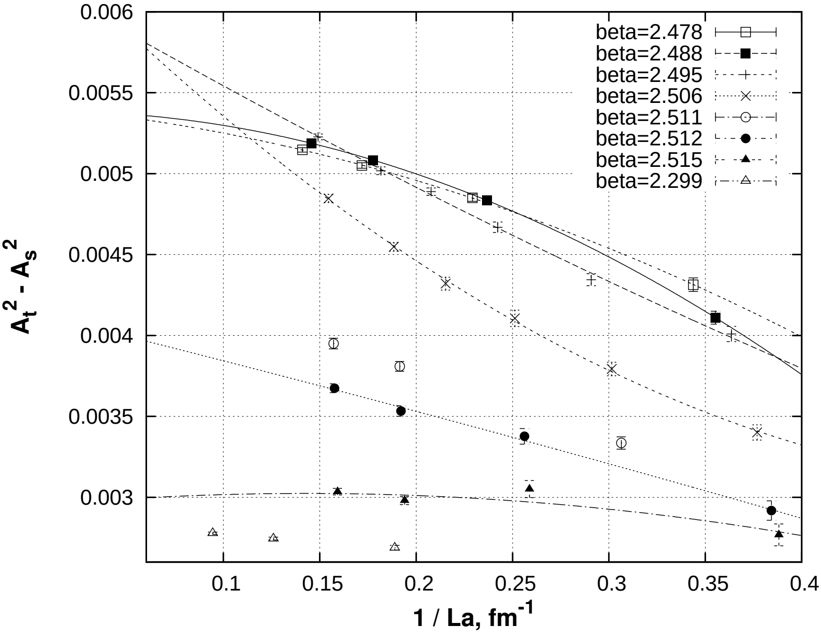
Magnetic screening mass:

$$m_M = G(0, \vec{p} \to 0) = \frac{1}{2}\Pi_{ii}(0)$$

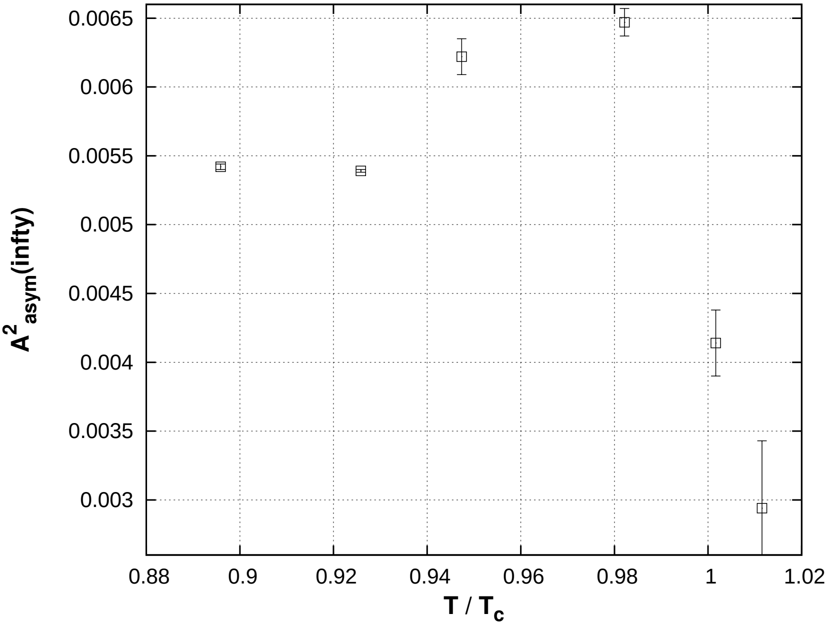
- Perturbation theory: $m_M = 0$
- Linde proposal: $m_M \simeq g^2 T$
  (to provide perturbative calculability of various quantities)
- Our conclusion: $D_{ii}(|\vec{x}|) \sim \frac{1}{|\vec{x}|^b}$,
  NOT $D_{ii}(|\vec{x}|) \sim \exp(-m_M |\vec{x}|)$



Fundamental Modular Domain
First Gribov Horizon
6
5.5
5
4.5
4
0
0.1
0.2
0.3
0.4
0.5
D_{ii}
p^2



0.006
0.0055
0.005
0.0045
0.004
0.0035
0.003
0.1
0.15
0.2
0.25
0.3
0.35
0.4
$A_t^2 - A_s^2$
1 / La, fm^{-1}
beta=2.478
beta=2.488
beta=2.495
beta=2.506
beta=2.511
beta=2.512
beta=2.515
beta=2.299



0.0065
0.006
0.0055
0.005
0.0045
0.004
0.0035
0.003
0.88
0.9
0.92
0.94
0.96
0.98
1
1.02
A^2_{asym}(infty)
T / T_c