Accurate bottom quark mass from sum rules for decay constants of B – mesons

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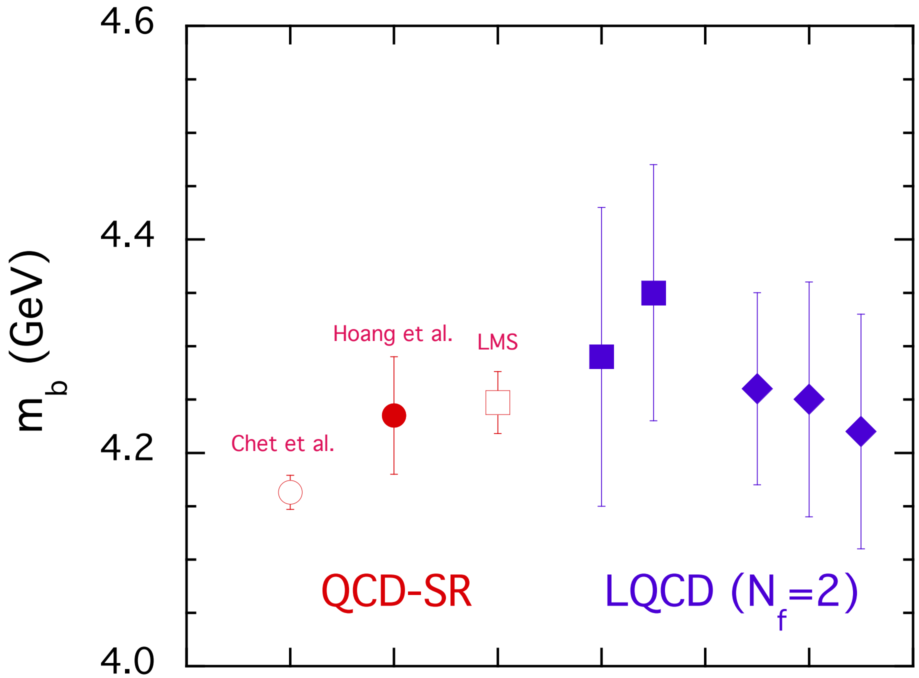
We show that Borel QCD sum rules for heavy–light currents yield very strong correlations between the b-quark mass m_b and the B-meson decay constant f_B:

$$\delta f_B/f_B \approx -8\,\delta m_b/m_b.$$

This fact opens the possibility of an accurate extraction of m_b from QCD sum rules using f_B as input. Combining precise lattice QCD determinations of f_B with our sum-rule analysis based on heavy–light correlation function leads to

$$\overline{m}_b(\overline{m}_b) = (4.247 \pm 0.034)\ \mathbf{GeV}$$

Precise knowledge of the b-quark mass is highly desirable [$m_b \equiv \overline{m}_b(\overline{m}_b)$]



Moment SRs for $\bar{b}b$ two-point functions in pQCD with 4-loop accuracy vs experimental data:

low-n moments (Chetyrkin et al): $m_b = 4.163 \pm 0.016$ GeV

large-n moments (Hoang et al): $m_b = 4.235 \pm 0.055_{(\mathrm{pert})} \pm 0.03_{(\mathrm{exp})}$ GeV

We report that Borel QCD sum rules for heavy–light correlators provide the possibility to extract m_b with comparable accuracy if a precise value for f_B is used as input.

Explore the sensitivity of f_B to the precise value of the b-quark mass:

In a nonrelativistic potential model

$$|\psi(r=0)| \propto \varepsilon^{3/2}.$$

The decay constant $f_B \sim \psi(r=0)$;

in the heavy-quark limit $f_B \propto 1/\sqrt{m_Q}$:

$$f_B\sqrt{M_B} = \kappa(M_B - m_Q)^{3/2}.$$

Dependence of f_B on small variations δm_Q of the heavy-quark (pole) mass near some m_Q:

$M_B = 5.27$ GeV**;** $f_B \approx 200$ MeV **for** $m_Q \approx 4.6 \div 4.7$ GeV $\to \kappa \approx 0.9 \div 1.0$ **and** $\delta f_B \approx -0.5\,\delta m_Q$**.**

The sensitivity of f_B to the precise value of the heavy-quark mass should be very high:

$$\frac{\delta f_B}{f_B} \approx -(11 \div 12)\,\frac{\delta m_Q}{m_Q}.$$

QCD: for fixed inputs of the correlator (condensates, α_s, etc.) we find

$$f_B(m_b) = \left(192.0 - 37\,\frac{m_b - 4.247\ \textbf{GeV}}{0.1\ \text{GeV}} \pm 3_{(\text{syst})}\right)\text{MeV}.$$

Correlation function , OPE, and heavy – quark mass

The basic object is T-product of 2 pseudoscalar currents, $j_5(x) = (m_b + m)\,\bar{q}(x) i\gamma_5 b(x)$,

$$\Pi(p^2) = i \int d^4x\, e^{ipx} \left\langle 0 \left| T\left(j_5(x) j_5^\dagger(0) \right) \right| 0 \right\rangle$$

and its Borel image

$$\Pi(\tau) = f_B^2 M_B^4 e^{-M_B^2\tau} + \int_{s_{\rm phys}}^{\infty} ds\, e^{-s\tau} \rho_{\rm hadr}(s) = \int_{(m_b+m)^2}^{\infty} ds\, e^{-s\tau} \rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu).$$

here $s_{\rm phys} = (M_{B^*} + M_P)^2$, and f_B is the decay constant.

Duality Ansatz:

hadron continuum dual to perturbative contributions above *effective continuum threshold* $s_{\rm eff}(\tau)$:

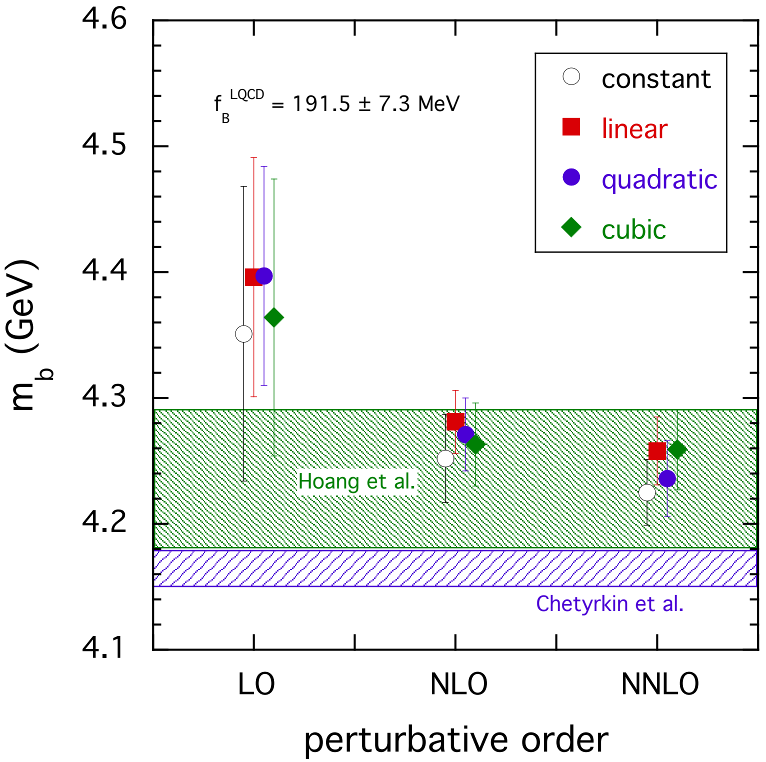
$$f_B^2 M_B^4 e^{-M_B^2\tau} = \int_{(m_b+m)^2}^{s_{\rm eff}(\tau)} ds\, e^{-s\tau} \rho_{\rm pert}(s,\mu) + \Pi_{\rm power}(\tau,\mu).$$

Even if the QCD inputs $\rho_{\rm pert}(s,\mu)$ and $\Pi_{\rm power}(\tau,\mu)$ are known, one requires $s_{\rm eff}(\tau)$ which can be determined only with some accuracy, yielding the systematic error of the extracted f_B.

Consider polynomial Ansätze for $s_{\rm eff}(\tau)$;
For any Ansatz our algorithm determines it paramaters and thus f_B;
Results for f_B for linear, quadratic, and cubic Ansätze form a band which encompasses the true value.
Thus, the half-width of this band provides the systematic error of f_B.

Extraction of the bottom – quark mass

Using lattice average $f_B = (191.5 \pm 7.3)$ MeV **and applying our algorithms yields:**



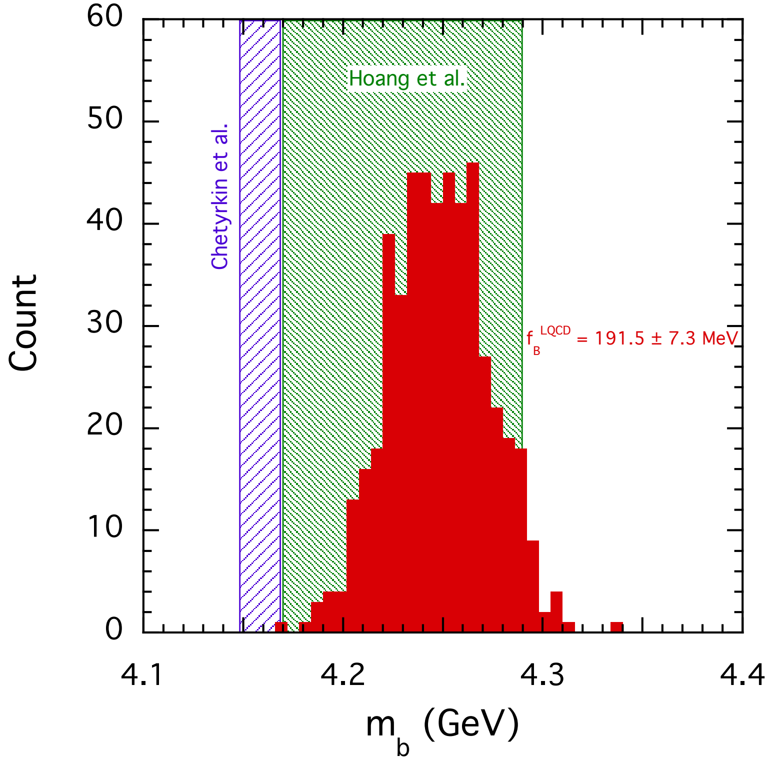
$m_b^{LO} = (4.38 \pm 0.1 \pm 0.020_{syst})$ GeV

$m_b^{NLO} = (4.27 \pm 0.04 \pm 0.015_{syst})$ GeV

$m_b^{NNLO} = (4.247 \pm 0.027 \pm 0.011_{syst})$ GeV

Moving from LO to NLO of the perturbative expansion (i) decreases sizeably m_b and reduces OPE-error. The extracted values of m_b exhibit a nice "convergence" depending on the accuracy of the perturbative correlation function.

The N^3LO correction is not known. Nevertheless, we do not expect a sizeable shift of the central value of m_b, but expect a reduction of the OPE-error.



Distribution of m_b as obtained by the bootstrap analysis:

Gaussian distributions for f_B in the range $f_B = (191.5 \pm 7.3)$ MeV and

for OPE parameters: $m_d(2\ \mathbf{GeV}) = (3.5 \pm 0.5)$ **MeV,** $m_s(2\ \mathbf{GeV}) = (95 \pm 5)$ **MeV,**

$\alpha_s(M_Z) = 0.1184 \pm 0.0007$, $\langle \bar{q}q \rangle(2\ \mathbf{GeV}) = -((269 \pm 17)\ \mathbf{MeV})^3$, $\langle \bar{s}s \rangle(2\ \mathbf{GeV})/\langle \bar{q}q \rangle(2\ \mathbf{GeV}) = 0.8 \pm 0.3$, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle = (0.024 \pm 0.012)\ \mathbf{GeV}^4$.

Uniform distributions for the renormalization scales μ and ν in the range 3 GeV $< \mu, \nu < 6$ GeV.

Summary

We presented our extraction of the bottom-quark mass $m_b(m_b)$ from QCD sum rules using the lattice data on f_B and f_{Bs} as data input.

Particular emphasis was laid on the errors:

(i) OPE uncertainty due to the errors of the QCD parameters

(ii) intrinsic error due to the limited accuracy of the extraction procedure of the method

• For beauty mesons, QCD sum rule yield a strong correlation between m_b and the sum-rule result for f_B:

$$\frac{\delta f_B}{f_B} \approx -8\,\frac{\delta m_b}{m_b}.$$

Combining our sum-rule analysis with the latest results for f_B and f_{Bs} from lattice QCD yields

$$m_b = 4.247 \pm 0.027_{(OPE)} \pm 0.018_{(exp)} \pm 0.011_{syst}\ \text{GeV}$$

OPE error:

14 MeV (μ, ν), 20 MeV (quark cond.), 7 MeV (gluon cond.), 8 MeV (α_s), 4 MeV (light-quark mass).

Good news is that the systematic uncertainty is relatively small.

Our m_b from $O(\alpha_s^2)$ heavy–light correlator, and we expect the influence of $O(\alpha_s^3)$ to be small.

Compared with the results from heavy-heavy correlator:

Good agreement with $O(\alpha_s^2)$ correlator:

$m_b = (4.209 \pm 0.050)$ **GeV**.

Excellent agreement with Υ sum rule

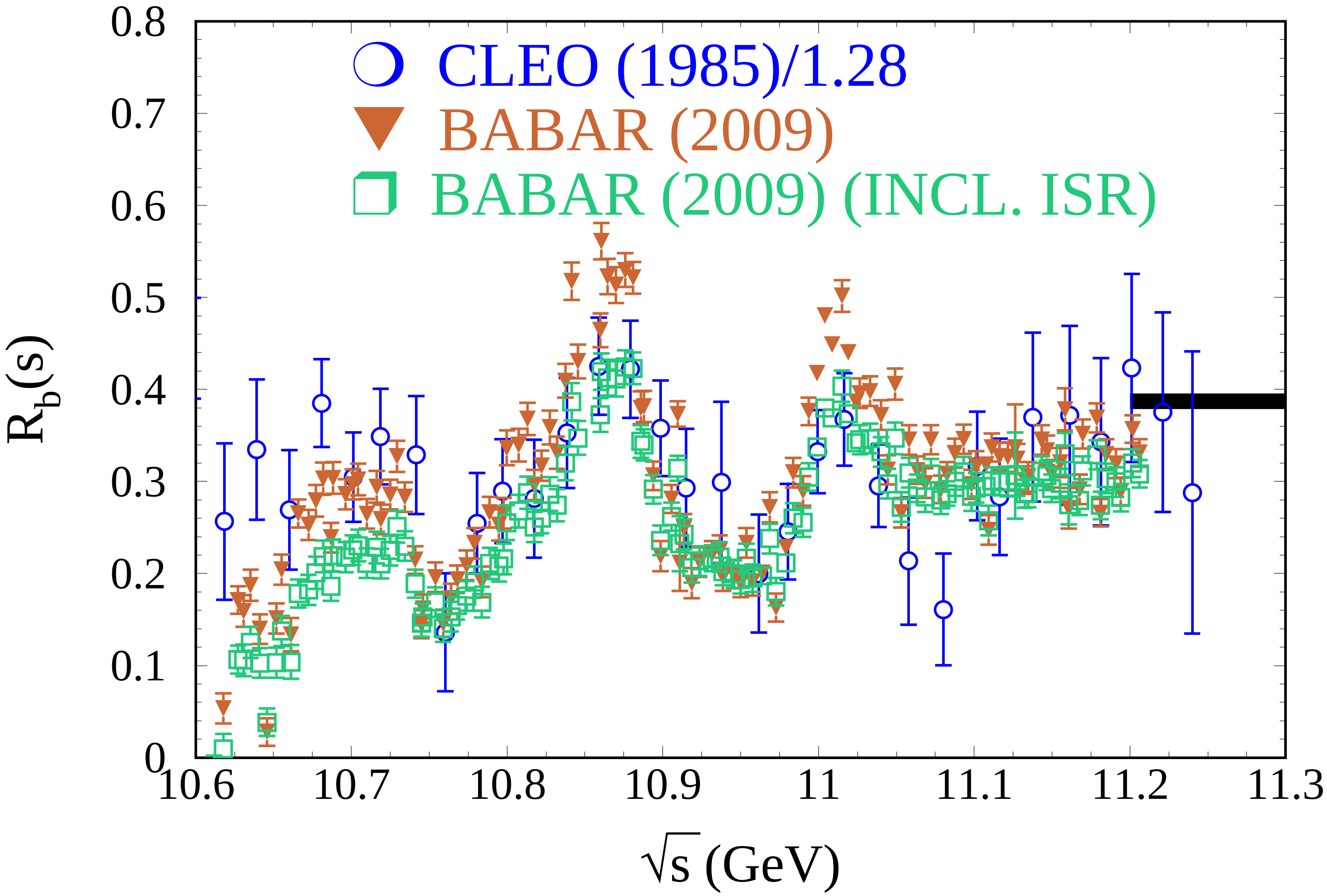
$m_b = (4.235 \pm 0.055_{(\text{pert})} \pm 0.003_{(\text{exp})})$ **GeV**.

Pronounced tension with $O(\alpha_s^3)$ correlator:

$m_b = (4.163 \pm 0.016)$ **GeV**,

The origin of this disagreement requires further considerations.

Properly formulated Borel QCD sum rules for heavy–light correlators provide a competitive tool for the reliable calculation of heavy-meson properties and for the extraction of basic QCD parameters by making use of the results from lattice QCD and/or the experimental data.



CLEO (1985)/1.28
BABAR (2009)
BABAR (2009) (INCL. ISR)
$R_b(s)$
$\sqrt{s}$ (GeV)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
10.6
10.7
10.8
10.9
11
11.1
11.2
11.3