Scalar Graviton as Dark Matter

Yu. F. Pirogov

Institute for High Energy Physics, Protvino, Russia

NP Session RAS, Protvino, 05–08 November 2013

Yu. F. Pirogov, Eur. Phys. J. C **72** (2012) 2017;
arXiv:1111.1437 [gr-qc]

Content

Unimodular relativity

- Local scale invariance: $x^\mu \to x'^\mu = e^{\lambda(x)} x^\mu$,
  $\sqrt{-g} \to e^{-4\lambda}\sqrt{-g}$.
- Diffeomorphism invariance/relativity:

$$\mathrm{GDiff}: x^\mu \to x^\mu - \xi^\mu \ \ \forall \xi^\mu,$$

$$\delta_\xi \sqrt{-g} = \partial_\mu(\sqrt{-g}\xi^\mu) \quad \Rightarrow \mathrm{GR}.$$

$$\mathrm{TDiff}: \exists \sqrt{-g} = 1, \ \ \partial_\mu \xi^\mu = 0$$

Necessary and sufficient for $m_g = 0 \Rightarrow \mathrm{GC\ violation} \Rightarrow$

$$\mathrm{UDiff}: \bar{\mu} \sim \sqrt{-g}, \ \ \delta_\xi \bar{\mu} = \partial_\mu(\bar{\mu}\xi^\mu) = 0$$

$$\Rightarrow \mathrm{UR} \Rightarrow \mathrm{UG}$$

UR (cont'd)

- Weak-field decomposition: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- $SO(3)$ decomposition:

$$h_{\mu\nu} = \left\{ (h_{mn} - 1/3\, h_{kk}\delta_{mn}, h_{kk}\delta_{mn}), h_{m0}, h_{00} - h_{kk} \right\}$$

- Gravity spin/helicity content

$$10 = (5 \oplus 1) \oplus (3 \oplus \mathbf{1}'),$$

$$5 = (\pm\mathbf{2}, \pm 1, 0)$$

- Massive gravity: $(5 \oplus 1)$
- GR/UMG: $(\pm\mathbf{2})$
- UBG: $(\pm\mathbf{2}) \oplus \mathbf{1}'$
  Transverse deformation (4-volume preserving) mode: (tensor) graviton.
  Compression (form preserving) mode: scalar graviton/*systolon*.

Unimodular Gravity

- UG action:

$$S = \int \mathcal{L}(g_{\mu\nu}, \bar{\mu}) d^4x$$

- UG field equations (FEs): $\delta\bar{\mu} = 0,$

$$\mathcal{G}^{\mu\nu} \equiv \delta\mathcal{L}/\delta g_{\mu\nu} = 0.$$

- General Relativity (GR): no $\bar{\mu}$.

$$\mathcal{L} = -\frac{\kappa_g^2}{2} R\sqrt{-g}$$

$$\mathcal{G}^{\mu\nu} = \left(R^{\mu\nu} - R/2 \;\; g^{\mu\nu}\right)\sqrt{-g} \;\; \Rightarrow R = 0 \text{ vacuum}$$

- Unimodular Monomode Gravity (UMG): measure $\bar{\mu}$.

$$\mathcal{L} = -\frac{\kappa_g^2}{2} R\bar{\mu}$$

$$\mathcal{G}^{\mu\nu} = \left(R^{\mu\nu} - 1/4\, R g^{\mu\nu}\right)\bar{\mu}$$

$$\Rightarrow \partial_\mu R = 0 \Rightarrow R = \Lambda \Rightarrow \text{GR}\Lambda.$$

UG (cont'd 1)

- Unimodular Bimode Gravity (UBG): derivatives of $\bar{\mu}$. Bimode, biscale ($\kappa_s < \kappa_g$):

$$\mathcal{L} = \Big(-\frac{\kappa_g^2}{2}R + \frac{\kappa_s^2}{2}g^{\mu\nu}\partial_\mu\varsigma\partial_\nu\varsigma\Big)\sqrt{-g},$$
$$\varsigma = \ln\sqrt{-g}/\bar{\mu}, \quad \sigma \equiv \kappa_s\varsigma.$$

- Unique:
  - Global shift invariance $\sigma \to \sigma + \sigma_0 \Rightarrow$ No derivativeless terms with σ.
  - Suppression by powers of $\partial_\mu/\kappa_s \Rightarrow$ No higher derivatives of σ.

$$\mathcal{G}^{\mu\nu} = \Big(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}\Big)\sqrt{-g} - \kappa_g^{-2}T_s^{\mu\nu}\sqrt{-g},$$

UG (cont'd 2)

Remarkable property:

$$T_{s\mu\nu} = \left(\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}\partial\sigma\cdot\partial\sigma g_{\mu\nu}\right) + \kappa_s\nabla\cdot\nabla\sigma g_{\mu\nu}$$

- UBG signature: specific non-harmonic term $\kappa_s\nabla\cdot\nabla\sigma$.
- Unique:
  - GR $\oplus$ free scalar field (SF).
  - Brans-Dicke theory in Einstein frame $\Rightarrow$ GR $\oplus$ SF.
  - UBG with measure $\sqrt{-g}\to\bar{\mu}$.
    FEs $\Rightarrow \nabla\cdot\nabla\sigma = 0 \Rightarrow$ GR$\Lambda \oplus$ SF.

Dark holes

- Harmonic field: $\nabla \cdot \nabla\sigma = 0$.
- Dark holes: scalar field $\oplus$ matter
  - Black holes: no scalar field
  - Vacuum holes: no matter (UBG)

Dark halos

- Non-harmonic field: $\nabla \cdot \nabla\sigma \neq 0$

$$\varsigma_h = \begin{cases} \tau^2 - \frac{3}{10}\tau^4 + \mathcal{O}(\tau^6), & \tau \leq 1, \\ \ln 3\tau^2, & \tau \gg 1. \end{cases}$$

Scaled distance: $\tau = r/R_0$.

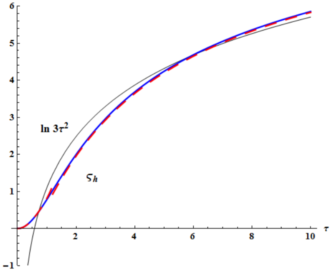

Figure: Normalized regular solution $\sigma(\tau)/\kappa_s$.

Dark halos (cont'd 1)

- Halo energy density profile:

$$\rho_h/\rho_0 = \begin{cases} 1 - \tau^2 + \frac{4}{5}\tau^4 + \mathcal{O}(\tau^6), & \tau \leq 1, \\ 1/(3\tau^2) + \mathcal{O}(1/\tau^{5/2}), & \tau \gg 1. \end{cases}$$

Central energy density: $\rho_0 = 3\kappa_s^2/R_0^2$.

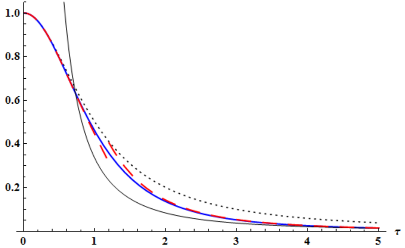

Figure: Normalized energy density profile $\rho(\tau)/\rho_0$.

Dark halos (cont'd 2)

- Monte Carlo
  - Pseudo-isothermal sphere: $\rho_{ref}/\rho_0 = 1/(1+\tau^2)$.
  - Isothermal sphere: $\bar{\rho}/\rho_0 = 1/(3\tau^2)$
- Halo core corrections:
  - Luminous matter ρ_{lum}.
  - Particle DM ρ_{pdm}.

Dark halos (cont'd 3)

- Rotation curve profile:

$$v_h^2/v_\infty^2 = \begin{cases} \tau^2 - \frac{3}{5}\tau^4 + \mathcal{O}(\tau^6), & \tau \le 1, \\ 1 + \mathcal{O}(1/\sqrt{\tau}), & \tau \gg 1. \end{cases}$$

Asymptotic rotation velocity:

$$v_\infty = \kappa_s/\sqrt{2}\kappa_g \sim 10^{-3}.$$

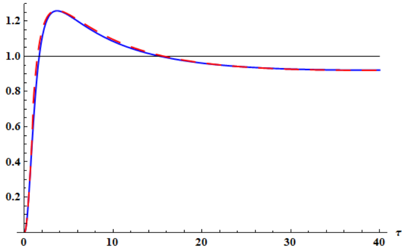

Figure: Normalized RC profile $v(\tau)/v_\infty$.

Conclusion

Coherent scalar-graviton halos:

- In the vacuum, the dark halos are not only possible but are in fact "obligatory". The signature of the theory.
- In reality, they may reflect only the universal long-range tail, with the specific short-range core strongly influenced by matter (including possibly the particle DM).
- The two-component DM (particle continuous medium $\oplus$ coherent scalar-graviton field) may give the answer to the DM puzzle.