Model-Independent Analysis of $B \to \pi \ell^+ \ell^-$ Decay

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Outline

- Introduction
- $B^+ \to \pi^+ \ell^+ \ell^-$ differential branching fraction
- Form factors at large hadronic recoil
- Dilepton invariant mass spectrum
- Form factors at low hadronic recoil
- $B^+ \to \pi^+ \mu^+ \mu^-$ decay in entire range of q^2
- Comparing of theoretical estimates and experimental data
- Summary and outlook

Introduction

- $B \to K^{(*)}\ell^+\ell^-$ decays are measured both on B-factories and hadron colliders
- FCNC processes induced by $b \to d\ell^+\ell^-$ transitions are more interesting than $b \to s\ell^+\ell^-$ ones as they contain additional information on the CKM sector
- The BaBar Collab. obtained the upper bound:

$$\mathrm{Br}_{\mathrm{exp}}(B^+ \to \pi^+\mu^+\mu^-) < 5.5 \times 10^{-8}, \quad \mathrm{CL} = 90\%$$

- Experimental value of the branching fraction was obtained by the LHCb Collab. [arXiv:1210.2645v1 [hep-ex]]:

$$\mathrm{Br}_{\mathrm{exp}}(B^+ \to \pi^+\mu^+\mu^-) = (2.3 \pm 0.6(\mathrm{stat}) \pm 0.1(\mathrm{syst})) \times 10^{-8}$$

- Theoretical analysis exist but not detail enough

Effective Weak Hamiltonian

Effective Hamiltonian

$$H_{\text{eff}}^{(b\to d)} = -\frac{4G_F}{\sqrt{2}}\Big[V_{tb}^* V_{td}\sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

$$+V_{ub}^* V_{ud}\sum_{i=1}^{2} C_i(\mu)\Big(\mathcal{O}_i(\mu) - \mathcal{O}_i^{(u)}(\mu)\Big)\Big] + \text{h.c.}$$

G_F is the Fermi constant
$C_i(\mu)$ are Wilson coefficients
$\mathcal{O}_i(\mu)$ are the dimension-six operators
V_{ij} are Cabibbo-Kobayashi-Maskawa (CKM) matrix elements

$V_{tb}^* V_{td} \sim V_{ub}^* V_{ud} \sim \lambda^3$ are of the same order in $\lambda = \sin\theta_{12}$

Operator Basis

- Tree operators

$$\mathcal{O}_1 = \left(\bar{d}_L \gamma_\mu T^A c_L\right)\left(\bar{c}_L \gamma^\mu T^A b_L\right), \quad \mathcal{O}_2 = \left(\bar{d}_L \gamma_\mu c_L\right)\left(\bar{c}_L \gamma^\mu b_L\right)$$

$$\mathcal{O}_1^{(u)} = \left(\bar{d}_L \gamma_\mu T^A u_L\right)\left(\bar{u}_L \gamma^\mu T^A b_L\right), \quad \mathcal{O}_2^{(u)} = \left(\bar{d}_L \gamma_\mu u_L\right)\left(\bar{u}_L \gamma^\mu b_L\right)$$

- Dipole operators

$$\mathcal{O}_7 = \frac{e\, m_b}{g_{\mathrm{st}}^2}\left(\bar{d}_L \sigma^{\mu\nu} b_R\right) F_{\mu\nu}, \quad \mathcal{O}_8 = \frac{m_b}{g_{\mathrm{st}}}\left(\bar{d}_L \sigma^{\mu\nu} T^A b_R\right) G_{\mu\nu}^A$$

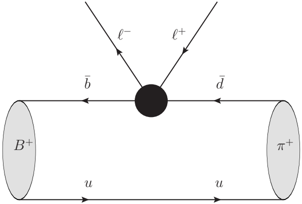
- Semileptonic operators

$$\mathcal{O}_9 = \frac{e^2}{g_{\mathrm{st}}^2}\left(\bar{d}_L \gamma^\mu b_L\right)\sum_\ell \left(\bar{\ell}\gamma_\mu \ell\right), \quad \mathcal{O}_{10} = \frac{e^2}{g_{\mathrm{st}}^2}\left(\bar{d}_L \gamma^\mu b_L\right)\sum_\ell \left(\bar{\ell}\gamma_\mu \gamma_5 \ell\right)$$

- Strong penguin operators are not presented

$B \to \pi$ transition matrix elements

Momentum transfered
$q = p_B - p_\pi = p_{\ell^+} + p_{\ell^-}$



$$\langle \pi(p_\pi)|\bar{b}\gamma^\mu d|B(p_B)\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu\right) + \left[f_0(q^2) - f_+(q^2)\right]\frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

$$\langle \pi(p_\pi)|\bar{b}\sigma^{\mu\nu}q_\nu d|B(p_B)\rangle = \frac{i f_T(q^2)}{m_B + m_\pi}\left[\left(p_B^\mu + p_\pi^\mu\right) q^2 - q^\mu \left(m_B^2 - m_\pi^2\right)\right]$$

Form factors $f_+(q^2)$, $f_0(q^2)$, $f_T(q^2)$ are non-pert. scalar functions

Heavy-Quark Symmetry (HQS) relations

Consider the large-recoil limit (small q^2-values)
Relations to NLO order worked out by Beneke & Feldmann (2000)

$$f_0(q^2) = \left(\frac{m_B^2 + m_\pi^2 - q^2}{m_B^2}\right)\left[\left(1 + \frac{\alpha_s(\mu)C_F}{4\pi}\left(2 - 2L(q^2)\right)\right) f_+(q^2) + \frac{\alpha_s(\mu)C_F}{4\pi}\frac{m_B^2(q^2 - m_\pi^2)}{(m_B^2 + m_\pi^2 - q^2)^2}\Delta F_\pi\right],$$

$$f_T(q^2) = \left(\frac{m_B + m_\pi}{m_B}\right)\left[\left(1 + \frac{\alpha_s(\mu)C_F}{4\pi}\left(\ln\frac{m_b^2}{\mu^2} + 2L(q^2)\right)\right) f_+(q^2) - \frac{\alpha_s(\mu)C_F}{4\pi}\frac{m_B^2}{m_B^2 + m_\pi^2 - q^2}\Delta F_\pi\right],$$

$$L(q^2) = \left(1 + \frac{m_B^2}{m_\pi^2 - q^2}\right)\ln\left(1 + \frac{m_\pi^2 - q^2}{m_B^2}\right), \quad \Delta F_\pi = \frac{8\pi^2 f_B f_\pi}{N_c m_B}\langle l_+^{-1}\rangle_+ \langle \bar{u}^{-1}\rangle_\pi$$

Only one form factor $f_+(q^2)$ is required
Fitted from the data on the $B \to \pi \ell^+ \nu_\ell$ decays

$B \to \pi \ell^+ \nu_\ell$ decay

$$\langle \pi | \bar{u} \gamma^\mu b | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

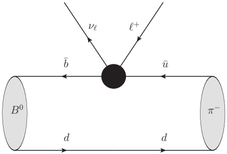
- Isospin symmetry assumed $\Longrightarrow$ the same $f_+(q^2)$ and $f_0(q^2)$ in charged and neutral semileptonic decays
- Differential decay width

$$\frac{d\Gamma}{dq^2}(B^0 \to \pi^- \ell^+ \nu_\ell) = \frac{G_F^2 |V_{ub}|^2}{192 \pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$

Kinematical function $\lambda(q^2) = \left(m_B^2 + m_\pi^2 - q^2\right)^2 - 4 m_B^2 m_\pi^2$

- The $f_0(q^2)$ contribution is suppressed by m_ℓ^2 / m_B^2 for $\ell = e, \mu$
- Global fit on the CKM matrix elements [PDG, 2012]

$$|V_{ub}| = (3.51 \pm 0.15) \times 10^{-3}$$



Parametrizations of $f_+(q^2)$

- Becirevic-Kaidalov (BK)
- Ball-Zwicky (BZ)
- Boyd-Grinstein-Lebed (BGL)

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2, q_0^2)} \sum_{k=0}^{k_{\max}} a_k \left[z(q^2, q_0^2)\right]^k$$

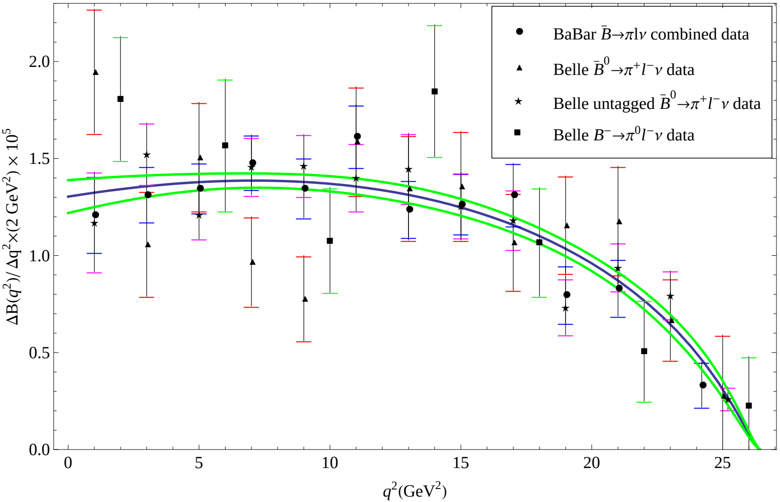
Blaschke factor $P(q^2) = z(q^2, m_{B^*}^2) \implies$ pole at $q^2 = m_{B^*}^2$

$$z(q^2, q_0^2) = \frac{\sqrt{m_+^2 - q^2} - \sqrt{m_+^2 - q_0^2}}{\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2}}, \quad m_+ = m_B + m_\pi$$

q_0^2 and $\phi(q^2, q_0^2)$ are taken to ensure a fast convergence

- Bourrely-Caprini-Lellouch (BCL)

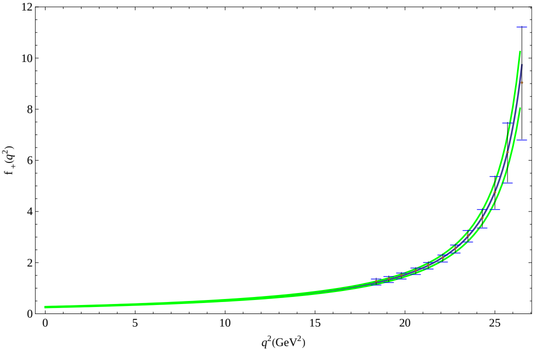
Fitting of the form-factor shape



Fitting of the form factor shape

BGL parametrization ($k_{max} = 2$):

$$a_0 = 0.0209 \pm 0.0004$$
$$a_1 = -0.0306 \pm 0.0031$$
$$a_2 = -0.0473 \pm 0.0189$$



Lattice data by the HPQCD Collab. are shown as vertical bars

The $B^+ \to \pi^+\ell^+\ell^-$ differential branching fraction

Differential branching fraction

$$\frac{d\mathrm{Br}\left(B^+ \to \pi^+\ell^+\ell^-\right)}{dq^2} = \frac{G_F^2 \alpha_{\mathrm{em}}^2 \tau_B}{1024\pi^5 m_B^3} |V_{tb} V_{td}^*|^2 \sqrt{\lambda(q^2)} \sqrt{1 - \frac{4m_\ell^2}{q^2}} F(q^2)$$

Dynamical function

$$F(q^2) = \frac{2}{3}\lambda(q^2)\left(1 + \frac{2m_\ell^2}{q^2}\right)\left|C_9^{\mathrm{eff}} f_+(q^2) + \frac{2m_b}{m_B + m_\pi} C_7^{\mathrm{eff}} f_T(q^2)\right|^2$$

$$+\frac{2}{3}\lambda(q^2)\left(1 - \frac{4m_\ell^2}{q^2}\right)|C_{10}^{\mathrm{eff}}|^2 f_+^2(q^2) + \frac{4m_\ell^2}{q^2}\left(m_B^2 - m_\pi^2\right)^2 |C_{10}^{\mathrm{eff}}|^2 f_0^2(q^2)$$

C_i^{eff} are effective Wilson coefficients:
specific combinations of Wilson coefficients

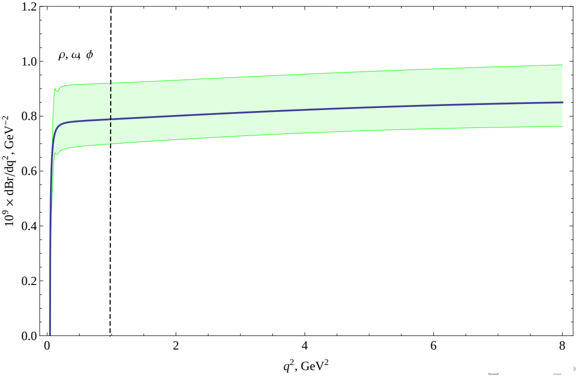
The $B^+ \to \pi^+\ell^+\ell^-$ at large hadronic recoil

- Integrated branching fractions

$$\mathrm{Br}_{\mathrm{th}}(B^+ \to \pi^+\mu^+\mu^-; 0.05\ \mathrm{GeV}^2 \le q^2 \le 8\ \mathrm{GeV}^2) = \left(0.65^{+0.08}_{-0.06}\right) \times 10^{-8}$$

$$\mathrm{Br}_{\mathrm{th}}(B^+ \to \pi^+\mu^+\mu^-; 1\ \mathrm{GeV}^2 \le q^2 \le 8\ \mathrm{GeV}^2) = \left(0.57^{+0.07}_{-0.05}\right) \times 10^{-8}$$

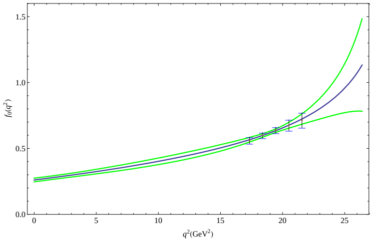
- Dimuon invariant mass spectrum at large hadronic recoil

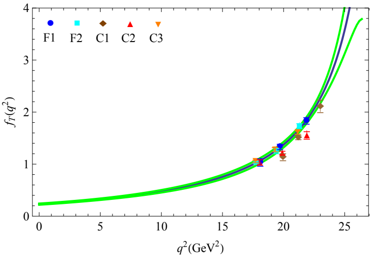


Extraction of $f_0(q^2)$ and $f_T(q^2)$ Form Factors

- In the low hadronic recoil region (large-q^2) there is no relations among form factors any more
- The form factors $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$ should be considered as independent quantities
- The Lattice QCD data on $f_T^{B\pi}(q^2)$ are absent
- Can be obtained from Lattice QCD data on $B \to K$ transition form factors and suggestion of the similarity of $SU(3)$-symmetry breaking corrections in all form factors
- z-expansion is used for $f_0^{B\pi}(q^2)$ and $f_T^{B\pi}(q^2)$

Results for $f_0(q^2)$ and $f_T(q^2)$ Form Factors





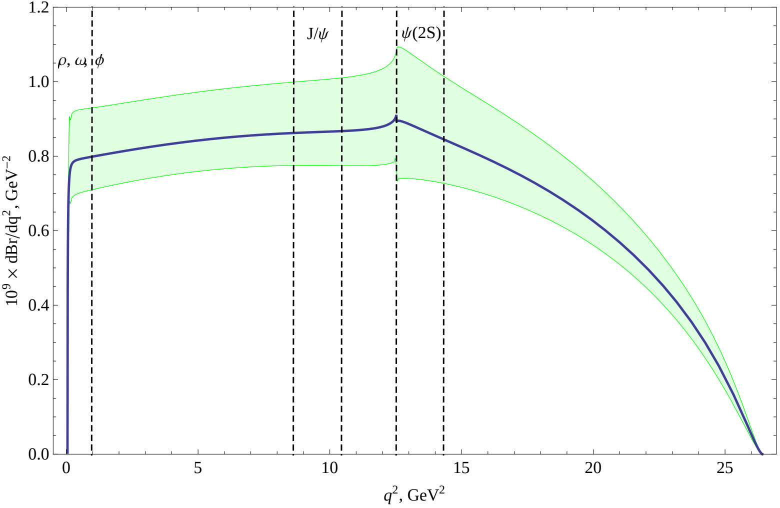
Lattice data obtained by the HPQCD Collab. are shown:

- for $f_0(q^2)$ from [arXiv:hep-lat/0601021]
- for $f_T(q^2)$ from [arXiv:1310.3207] (preliminary)

Let us wait an official Lattice data for $B \to \pi$ transition

$B^+ \to \pi^+\mu^+\mu^-$ in the entire range of q^2

The dimuon invariant mass spectrum at entire range of q^2



Theoretical estimate vs. experimental data

- SM theoretical estimate of the total branching fraction:

$$\mathrm{Br}_{\mathrm{th}}(B^+ \to \pi^+\mu^+\mu^-) = \left(1.88^{+0.32}_{-0.21}\right) \times 10^{-8}$$

- The theoretical uncertainty is mainly from the scale dependence, CKM matrix elements and form factors
- Below upper bounds from BaBar and Belle Collab.
- Experimental value of the branching fraction was obtained by LHCb Collaboration [arXiv:1210.2645v1 [hep-ex]]:

$$\mathrm{Br}_{\mathrm{exp}}(B^+ \to \pi^+\mu^+\mu^-) = (2.3 \pm 0.6(\mathrm{stat}) \pm 0.1(\mathrm{syst})) \times 10^{-8}$$

- Good agreement with theory within large experimental error
- Phenomenological dimuon invariant mass distribution is expected to be measured by the LHCb Collab.

Summary and outlook

- $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$ form-factor shapes are determined in entire range of q^2
- Invariant dimuon mass distribution of the decay $B^+ \to \pi^+\mu^+\mu^-$ at entire range of q^2 is obtained
- Numerical value of the total branching fraction of the $B^+ \to \pi^+\mu^+\mu^-$ decay is calculated and compared with experimental data

Backup Slides

Simple parametrizations of $f_+(q^2)$ form factor

- The Becirevic-Kaidalov (BK) parametrization

$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha_{BK}\, q^2/m_{B^*}^2)}$$

- The Ball-Zwicky (BZ) parametrization

$$f_+(q^2) = f_+(0)\left[\frac{1}{1 - q^2/m_{B^*}^2} + \frac{r_{BZ}\, q^2/m_{B^*}^2}{(1 - q^2/m_{B^*}^2)(1 - \alpha_{BZ}\, q^2/m_{B^*}^2)}\right]$$

Shapes are motivated by the QCD Sum Rules analysis

Both include explicitly the B^*-meson pole ($m_{B^*} = 5.325$ GeV)

Parametrizations of $f_+(q^2)$ based on z-expansion

- The Boyd-Grinstein-Lebed (BGL) parametrization

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2, q_0^2)} \sum_{k=0}^{k_{\rm max}} a_k \left[z(q^2, q_0^2)\right]^k$$

Blaschke factor $P(q^2) = z(q^2, m_{B^*}^2) \implies$ pole at $q^2 = m_{B^*}^2$

$$z(q^2, q_0^2) = \frac{\sqrt{m_+^2 - q^2} - \sqrt{m_+^2 - q_0^2}}{\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2}}, \quad m_+ = m_B + m_\pi$$

q_0^2 and $\phi(q^2, q_0^2)$ are taken to ensure a fast convergence

- The Bourrely-Caprini-Lellouch (BCL) parametrization

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{k=0}^{k_{\rm max}} b_k \times \left(\left[z(q^2, q_0^2)\right]^k - (-1)^{k-k_{\rm max}-1} \frac{k}{k_{\rm max}+1} \left[z(q^2, q_0^2)\right]^{k_{\rm max}+1} \right)$$

Calculation of $f_T(q^2)$ Form Factor

Introduce the form-factors' ratio

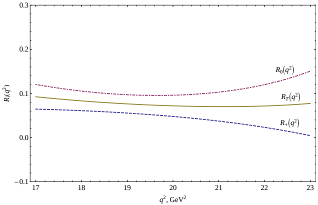
$$R_i(q^2) \equiv \frac{f_i^{BK}(q^2)}{f_i^{B\pi}(q^2)} - 1, \quad i = +, 0, T$$

$SU(3)_F$-symmetry breaking corrections in form factors are approximately the same $\quad R_+(q^2) \simeq R_0(q^2) \simeq R_T(q^2)$

Lattice data: $\quad f_+^{BK}(q^2),\ f_0^{BK}(q^2),\ f_T^{BK}(q^2),\ f_+^{B\pi}(q^2),\ f_0^{B\pi}(q^2)$

$$R_T(q^2) = \frac{1}{2}\left[R_+(q^2) + R_0(q^2)\right]$$

$$f_T^{B\pi}(q^2) = \frac{f_T^{BK}(q^2)}{1 + R_T(q^2)}$$



Comparison with existing theoretical estimates

- Our prediction:

$$\mathrm{Br}_{\mathrm{th}}(B^+ \to \pi^+\mu^+\mu^-) = \left(1.88^{+0.32}_{-0.21}\right) \times 10^{-8}$$

- [J.-J. Wang et al., Phys. Rev. D77 (2008) 014017]:

$$\mathrm{Br}_{\mathrm{th}}(B^+ \to \pi^+\mu^+\mu^-) = (2.03 \pm 0.23) \times 10^{-8}$$

- [H.-Z. Song et al., Comm. Theor. Phys. 50 (2008) 696]:

$$\mathrm{Br}_{\mathrm{th}}(B^+ \to \pi^+\mu^+\mu^-) = (1.96 \pm 0.21) \times 10^{-8}$$

- [Q. Chang & Y.-H. Gao, Comm. Theor. Phys. 57 (2012) 234]:

$$\mathrm{Br}_{\mathrm{th}}(B^+ \to \pi^+\mu^+\mu^-) = \left(2.0^{+0.18}_{-0.17}\right) \times 10^{-8}$$

- [W.-F. Wang et al., Phys. Rev. D86 (2012) 114025]:

$$\mathrm{Br}_{\mathrm{th}}(B^+ \to \pi^+\mu^+\mu^-) = \left(1.95^{+0.55+0.39+0.21+0.17}_{-0.41-0.35-0.20-0.16}\right) \times 10^{-8}$$