Selected problems in decays of heavy quarkonia

N.N. Achasov, A.V. Kiselev, A.A. Kozhevnikov, G.N. Shestakov

Laboratory of Theoretical Physics,

Sobolev Institute for Mathematics,

**Academician Koptiug Prospekt, 4,
Novosibirsk, 630090, Russia**

Electronic Address: achasov@math.nsc.ru

ABSTACT

The problems suggested by authors for studying at the c-τ, b and super-c-τ, b factories are stated.

- Comparison the production mechanism of the light scalar mesons in $D_s^+ \to s\bar{s}\, e^+\nu \to [\sigma(600) + f_0(980)]\, e^+\nu \to \pi^+\pi^-\, e^+\nu$ with the production mechanism of the light pseudoscalar mesons in $D_s^+ \to s\bar{s}\, e^+\nu \to (\eta/\eta')\, e^+\nu$ shows that $s\bar{s} \to \sigma(600)$ is negligibly small in comparison with $s\bar{s} \to f_0(980)$. As for $f_0(980)$, $s\bar{s} \to f_0(980)$ is not more 30% of $s\bar{s} \to \eta_s$ ($\eta_s = s\bar{s}$).

The study of the light scalar mesons in semileptonic decays of the $D^+(D^-)$, $D^0(\bar{D}^0)$, $B^+(B^-)$, $B^0(\bar{B}^0)$ mesons is suggested.

ABSTACT

- Interference phenomenon observed in the $\psi(3770)$ resonance region the $e^+e^- \to D\bar{D}$ reactions is described with models satisfying the elastic unitarity requirement. As a candidate, a model with the mixing $\psi(3770)$ and $\psi(2S)$ resonances is proposed. The selection of theoretical models in the non-$D\bar{D}$ channels $e+e- \to \psi(3770) \to \gamma\chi_{c0}$, $J/\psi\eta$, $\phi\eta$, etc is suggested.

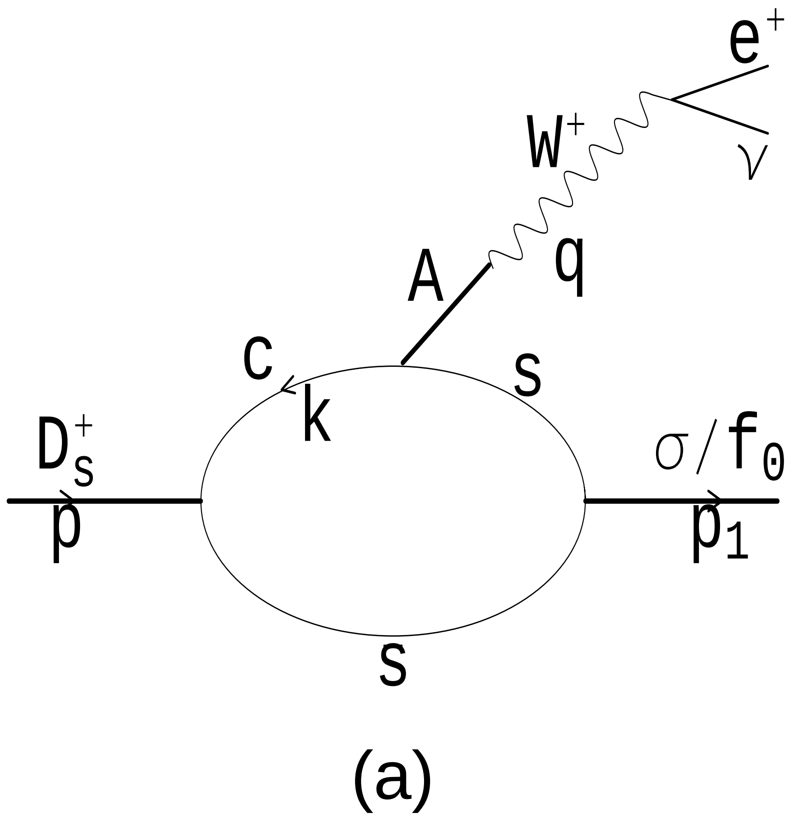
- Branching ratios of decays $\psi(3770)$, $\psi(4040)$ and $\Upsilon(10580)$ into light hadrons caused by the intermediate real $D\bar{D}$, $D_s\bar{D}_s$ and $B\bar{B}$ states are calculated. Their total value is 10 times as large as the branching ratio of annihilation into three gluons, but nevertheless is small (at the level of 1%).

Light scalars in semi-leptonic decays of heavy quarkonia

Based on N.N.Achasov and A.V. Kiselev,
Physical Review D 86, 114010 (2012)

It is time to explore the light scalar mesons in the decays of heavy quarkonia. The semi-leptonic decays are of prime interest because they have the clear mechanisms.

The $D_s^+ \to (\sigma/f_0)\, e^+\nu$ and $D_s^+ \to (\eta/\eta')\, e^+\nu$ decays



Model of the $D_s^+ \to (\sigma/f_0)\, e^+\nu$ and $D_s^+ \to (\eta/\eta')\, e^+\nu$ decays

The $D_s^+ \to (\sigma / f_0)\, e^+\nu$ and $D_s^+ \to (\eta/\eta')\, e^+\nu$ decays

Below we study the mechanism of production of the light scalar mesons in the $D_s^+ \to \pi^+\pi^-\, e^+\nu$ decays:

$D_s^+ \to s\bar{s}\, e^+\nu \to [\sigma(600) + f_0(980)]\, e^+\nu \to \pi^+\pi^-\, e^+\nu$,

and compare it with the mechanism of production of the light pseudoscalar mesons in the $D_s^+ \to (\eta/\eta')\, e^+\nu$ decays:

$D_s^+ \to s\bar{s}\, e^+\nu \to (\eta/\eta')\, e^+\nu$, in a model of the NJL type.

$$M[D_s^+(p) \to P(p_1)W^+(q) \to P(p_1)\, e^+\nu] = \frac{G_F}{\sqrt{2}} V_{cs} V_\alpha L^\alpha\,,$$

$$M[D_s^+(p) \to S(p_1)W^+(q) \to S(p_1)\, e^+\nu] = \frac{G_F}{\sqrt{2}} V_{cs} A_\alpha L^\alpha\,,$$

$$\mathrm{V}_\alpha = \mathrm{f}_+^{\mathrm{P}}(\mathrm{q}^2)(\mathrm{p}+\mathrm{p}_1)_\alpha + \mathrm{f}_-^{\mathrm{P}}(\mathrm{q}^2)(\mathrm{p}-\mathrm{p}_1)_\alpha\,,$$

$$\mathrm{A}_\alpha = \mathrm{f}_+^{\mathrm{S}}(\mathrm{q}^2)(\mathrm{p}+\mathrm{p}_1)_\alpha + \mathrm{f}_-^{\mathrm{S}}(\mathrm{q}^2)(\mathrm{p}-\mathrm{p}_1)_\alpha\,,$$

$$\mathrm{L}_\alpha = \bar{\nu}\gamma_\alpha(1+\gamma_5)\mathrm{e}\,, \qquad \mathrm{q} = (\mathrm{p}-\mathrm{p}_1)\,.$$

The influence of $f_-^P(q^2)$ and $f_-^S(q^2)$ are negligible for m_{e^+}.

The decay rates in the stable P and S states

$$\frac{d\Gamma(D_s^+ \to P\, e^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2}{24\pi^3} p_1^3(q^2)|f_+^P(q^2)|^2,$$

$$\frac{d\Gamma(D_s^+ \to S\, e^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2}{24\pi^3} p_1^3(q^2)|f_+^S(q^2)|^2.$$

For the $f_+^P(q^2)$ and $f_+^S(q^2)$ form factors we use the vector dominance model

$$f_+^P(q^2) = f_+^P(0)\frac{m_V^2}{m_V^2 - q^2} = f_+^P(0) f_V(q^2)\,,$$

$$f_+^S(q^2) = f_+^S(0)\frac{m_A^2}{m_A^2 - q^2} = f_+^S(0) f_A(q^2)\,,$$

where $V = D_s^*(2112)^\pm$, $A = D_{s1}(2460)^\pm$.

Definitions

Following the NJL type model we write $f_+^P(0)$ and $f_+^S(0)$ in the form

$$f_+^P(0) = g_{D_s^+ c\bar{s}} F_P g_{s\bar{s}P} \,, \quad f_+^S(0) = g_{D_s^+ c\bar{s}} F_S g_{s\bar{s}S} \,.$$

We know the structure of η and η'

$$\eta = \eta_q \cos\phi - \eta_s \sin\phi \,, \quad \eta' = \eta_q \sin\phi + \eta_s \cos\phi \,,$$

where $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$.

The angle $\phi = \theta_i + \theta_P$, where θ_i is the ideal mixing angle with $\cos\theta_i = \sqrt{1/3}$ and $\sin\theta_i = \sqrt{2/3}$, i.e., $\theta_i = 54.7^\circ$, and θ_P is the angle between the flavor-singlet state η_1 and the flavor-octet state η_8.

Definitions

Particle Data Group give the θ_P band $-20^\circ \lesssim \theta_P \lesssim -10^\circ$ that gives us the opportunity to extract information about the $s\bar{s} \to \eta_s$ coupling constant, $g_{s\bar{s}\eta_s}$, from experiment and to compare with the $s\bar{s} \to f_0$ coupling constant, $g_{s\bar{s}f_0}$, extracted from experiment also.

We consider the next set of θ_P.

$$\theta_P = -11^\circ : \quad \eta = 0.72\eta_0 - 0.69\eta_s\,, \quad \eta' = 0.69\eta_0 + 0.72\eta_s$$
$$\theta_P = -14^\circ : \quad \eta = 0.76\eta_0 - 0.65\eta_s\,, \quad \eta' = 0.65\eta_0 + 0.76\eta_s$$
$$\theta_P = -18^\circ : \quad \eta = 0.8\eta_0 - 0.6\eta_s\,, \quad \eta' = 0.6\eta_0 + 0.8\eta_s\,.$$

$$BR(D_s^+ \to s\bar{s}\, e^+\nu \to \eta\, e^+\nu) = (2.67 \pm 0.29)\%,$$

$$BR(D_s^+ \to s\bar{s}\, e^+\nu \to \eta'\, e^+\nu) = (9.9 \pm 2.3) \times 10^{-3}.$$

$D_s^+ \to s\bar{s}\,e^+\nu \to \pi^+\pi^-\,e^+\nu$

$$M(D_s^+ \to s\bar{s}\,e^+\nu \to [\sigma(600) + f_0(980)]\,e^+\nu \to \pi^+\pi^-\,e^+\nu)$$

$$= \frac{G_F}{\sqrt{2}} V_{cs}\, L^{\alpha}\,(p + p_1)_{\alpha}\, g_{D_s^+ c\bar{s}}\, f_A(q^2) \times$$

$$e^{i\delta_B^{\pi\pi}} \frac{1}{\Delta(m)} \Big(F_{\sigma} g_{s\bar{s}\sigma} D_{f_0}(m) g_{\sigma\pi^+\pi^-} + F_{\sigma} g_{s\bar{s}\sigma} \Pi_{\sigma f_0}(m) g_{f_0\pi^+\pi^-}$$

$$+ F_{f_0} g_{s\bar{s}f_0} \Pi_{f_0\sigma}(m) g_{\sigma\pi^+\pi^-} + F_{f_0} g_{s\bar{s}f_0} D_{\sigma}(m) g_{f_0\pi^+\pi^-} \Big),$$

where m is the invariant mass of the $\pi\pi$ system, $\Delta(m) = D_{f_0}(m)D_{\sigma}(m) - \Pi_{f_0\sigma}(m)\Pi_{\sigma f_0}(m)$, $D_{\sigma}(m)$ and $D_{f_0}(m)$ are the inverted propagators of the σ and f_0 mesons, $\Pi_{\sigma f_0}(m) = \Pi_{f_0\sigma}(m)$ is the off-diagonal element of the polarization operator, which mixes the σ and f_0 mesons.

$D_s^+ \to \pi^+\pi^- e^+\nu$

$$\frac{d^2\Gamma(D_s^+ \to \pi^+\pi^- e^+\nu)}{dq^2 dm} = \frac{G_F^2\,|V_{cs}|^2}{24\,\pi^3}\, g^2_{D_s^+ c\bar{s}}\, |f_A(q^2)|^2\, p_1^3(q^2, m)$$

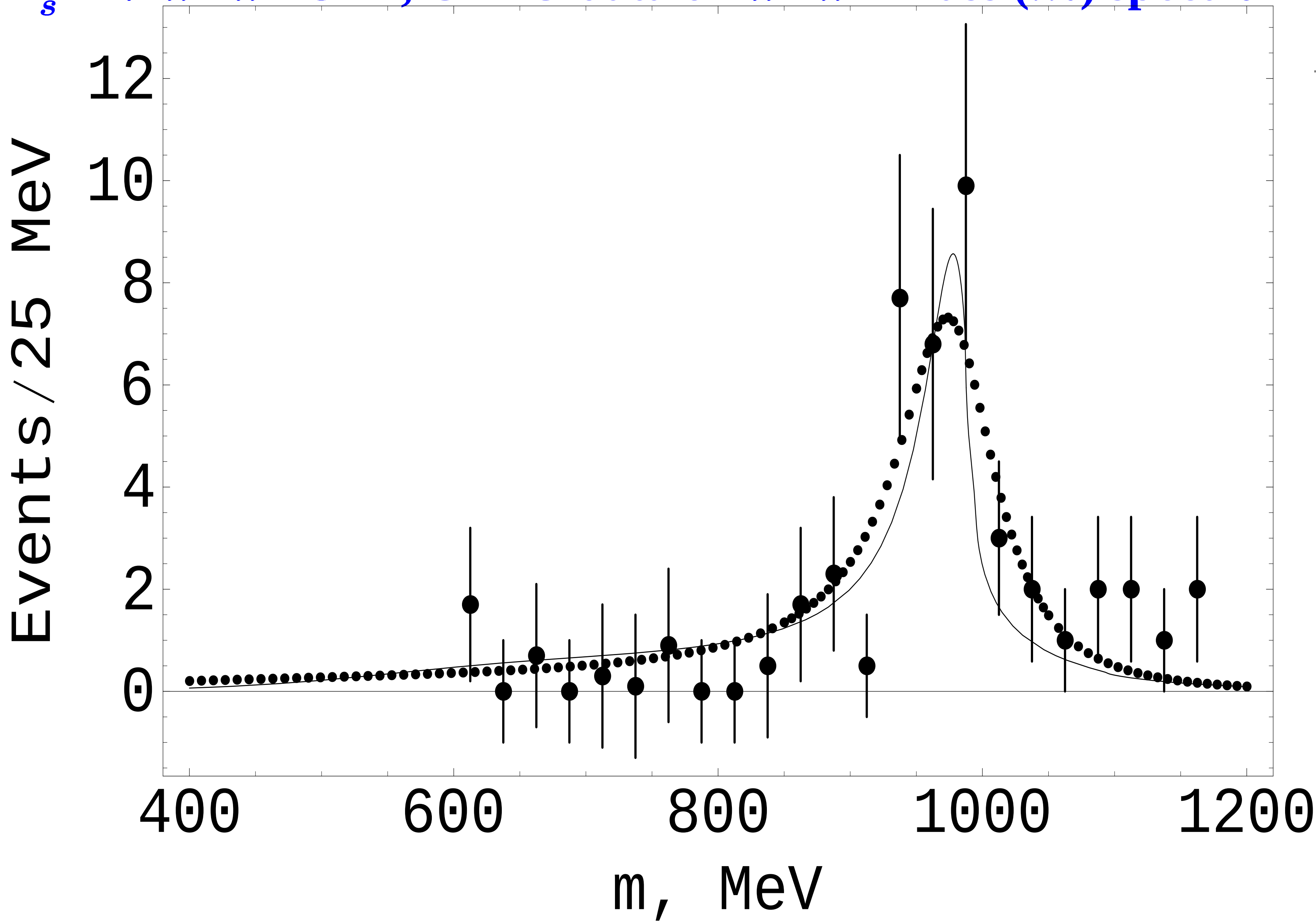
$$\times\, \frac{1}{8\pi^2}\, m\, \rho_{\pi\pi}(m) \left|\frac{1}{\Delta(m)}\right|^2$$

$$\times \Big| F_\sigma g_{s\bar{s}\sigma} D_{f_0}(m) g_{\sigma\pi^+\pi^-} + F_\sigma g_{s\bar{s}\sigma} \Pi_{\sigma f_0}(m) g_{f_0\pi^+\pi^-}$$

$$+ F_{f_0} g_{s\bar{s}f_0} \Pi_{f_0\sigma}(m) g_{\sigma\pi^+\pi^-} + F_{f_0} g_{s\bar{s}f_0} D_\sigma(m) g_{f_0\pi^+\pi^-} \Big|^2,$$

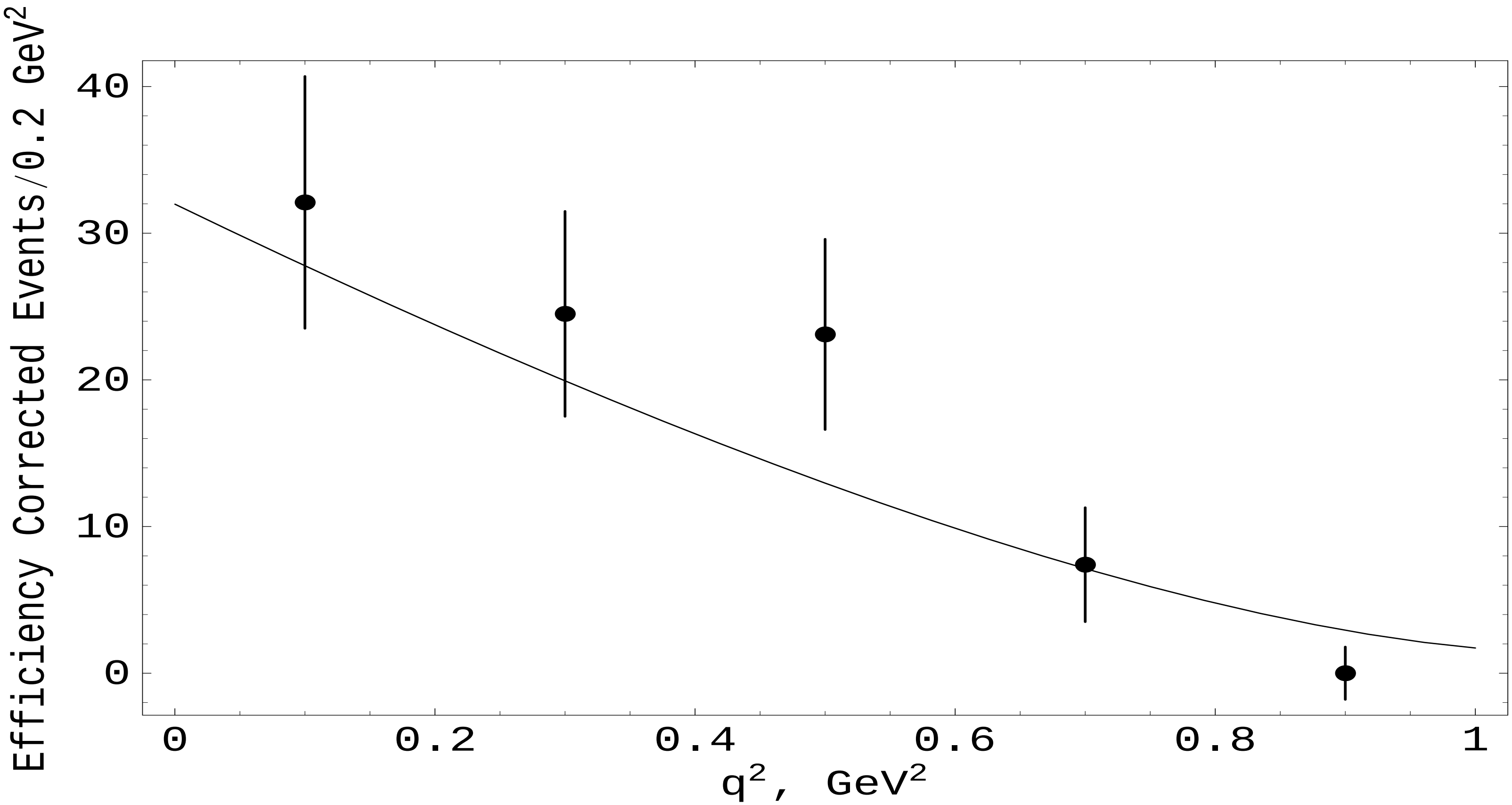
where $\rho_{\pi\pi}(m) = \sqrt{1 - 4m_\pi^2/m^2}$.

$D_s^+ \to \pi^+\pi^- e^+\nu$, CLEO data on $\pi^+\pi^-$ mass (m) spectrum



CLEO dotted line: $BR(D_s^+ \to f_0(980)\, e^+\nu \to \pi^+\pi^- e^+\nu) = 0.20\%$. Our solid line: 0.17%

The q^2 distribution, the CLEO data



The q^2 distribution for $BR(D_s^+ \to f_0(980)\, e^+\nu)$. The axial-vector dominance model (the theoretical curve) describes the data quite satisfactorily.

Results of the analysis of the CLEO data

$Br(D_s^+ \to f_0 e^+ \nu \to \pi^+\pi^- e^+\nu) = 0.17\%$			
$\frac{F_\sigma g_{s\bar{s}\sigma}}{F_{f_0} g_{s\bar{s}f_0}}$	$\frac{F^2_{f_0} g^2_{s\bar{s}f_0}}{F^2_\eta g^2_{s\bar{s}\eta}}$	$\frac{F^2_{f_0} g^2_{s\bar{s}f_0}}{F^2_{\eta'} g^2_{s\bar{s}\eta'}}$	$\frac{F^2_\eta g^2_{s\bar{s}\eta}}{F^2_{\eta'} g^2_{s\bar{s}\eta'}}$
0.039	0.67	0.49	0.73
The $\eta - \eta'$ mixing			
θ_P	-11°	-14°	-18°
$\frac{F^2_{f_0} g^2_{s\bar{s}f_0}}{F^2_\eta g^2_{s\bar{s}\eta_s}}$	0.32	0.29	0.24
$\frac{F^2_{f_0} g^2_{s\bar{s}f_0}}{F^2_{\eta'} g^2_{s\bar{s}\eta_s}}$	0.27	0.28	0.31

Discussion and conclusion

When fitting the CLEO data, we use the parameters of the resonances obtained by us in PRD 85, 094016 (2012) in the analysis of the $\pi\pi$ scattering and the $\phi \to \gamma(\sigma + f_0) \to \gamma\pi^0\pi^0$ decay. So the 44 events in Fig. on page 14 determine only one parameter $f_+^\sigma(0)/f_+^{f_0}(0) = F_\sigma g_{s\bar{s}\sigma}/F_{f_0} g_{s\bar{s}f_0}$.

The Adler zero at m^2 near $(m_\pi^2)/2$ determines

$f_+^\sigma(0)/f_+^{f_0}(0)$=0.039, 0.014, 0.055, 0.058, 0.032, 0.055

for six fits from PRD 85, 094016 (2012).

So the intensity of the $\sigma(600)$ production is much less than the intensity of the $f_0(980)$ production ($(f_+^\sigma(0)/f_+^{f_0}(0))^2 < 0.003$).

Discussion and conclusion

That is we find the direct evidence of decoupling of $\sigma(600)$ with the $s\bar{s}$ pair. As far as we know, this is truly a new result, which agrees well with the decoupling of $\sigma(600)$ with the $K\bar{K}$ states, obtained in PRD 85, 094016 (2012)

$g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-}$ =0.04, 0.001, 0.01, 0.01, 0.003, 0.025 for six fits.

The decoupling of $\sigma(600)$ with the $K\bar{K}$ states means also the decoupling of $\sigma(600)$ with $\sigma_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ because σ_q results in $g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-} = 1/4$.

Fit 1 describes the $\pi^+\pi^-$ spectrum on better than others, $(f_+^{\sigma}(0)/f_+^{f_0}(0))^2 = (0.039)^2, g^2_{\sigma K^+K^-}/g^2_{\sigma\pi^+\pi^-} = 0.04$.

So,the CLEO experiment gives new support in favour of the four-quark, $ud\bar{u}\bar{d}$, structure of the $\sigma(600)$ meson.

Discussion and conclusion

In the chirally symmetric model of the NJL type the coupling constants of the pseudoscalar and scalar partners with quarks are equal to each other, i.e., $g_{s\bar{s}\eta_s} = g_{s\bar{s}f_{0s}}$, where $f_{0s} = s\bar{s}$. If to neglect the strange quark mass as compared with the charmed quark mass ($m_s/m_c \ll 1$) in the numerators of the integrands for the decay diagrams, then $F_{f_0} = F_{\eta'}$ and we find that $g^2_{s\bar{s}f_0}/g^2_{s\bar{s}\eta_s} \approx 0.3$. So, the $f_{0s} = s\bar{s}$ part in the $f_0(980)$ wave function is near thirty percent.

Taking into account the suppression of the $f_0(980)$ meson coupling with the $\pi\pi$ system, $g^2_{f_0\pi^+\pi^-}/g^2_{f_0K^+K^-} = 0.154$, one can conclude that the $f_{0q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ part in the $f_0(980)$ wave function is suppressed also.

So, the CLEO experiment gives new support in favour of the four-quark, $(sd\bar{s}\bar{d} + sd\bar{s}\bar{d})/\sqrt{2}$, structure of the $f_0(980)$ meson, too.

Outlook

Certainly, there is an extreme need in experiment on the $D_s^+ \to s\bar{s}\, e^+\nu \to \pi^+\pi^-\, e^+\nu$ decay with high statistics.

Of great interest is the experimental search for the decays $D^0 \to d\bar{u}\, e^+\nu \to a_0^-(980)\, e^+\nu \to \pi^-\eta\, e^+\nu$ and $D^+ \to d\bar{d}\, e^+\nu \to a_0^0(980)\, e^+\nu \to \pi^0\eta\, e^+\nu$ (or the charge conjugate ones), which will give the information about the $a_q^- = d\bar{u}$ (or $a_q^+ = u\bar{d}$) component in the $a_0^-(980)$ (or $a_0^+(980)$) wave function and $a_q^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ component in the a_0^0 wave function.

Now it is known that

$$BR(D^0 \to d\bar{u}\, e^+\nu \to \pi^-\, e^+\nu) = (2.89 \pm 0.08) \times 10^{-3}$$ and

$$BR(D^+ \to d\bar{d}\, e^+\nu \to \pi^0\, e^+\nu) = (4.05 \pm 0.18) \times 10^{-3}.$$

Outlook

No less interesting is also search for the decays
$D^+ \to d\bar{d}\, e^+\nu \to [\sigma(600) + f_0(980)]\, e^+\nu \to \pi^+\pi^-\, e^+\nu$
(or the charge conjugate ones), which will give the information about the $\sigma_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $f_{0q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ components in the $\sigma(600)$ and $f_0(980)$ wave functions respectively.

Now it is known that
$BR(D^+ \to d\bar{d}\, e^+\nu \to \eta\, e^+\nu) = (1.14 \pm 0.10) \times 10^{-3}$ and
$BR(D^+ \to d\bar{d}\, e^+\nu \to \eta'\, e^+\nu) = (2.2 \pm 0.5) \times 10^{-4}$.

Comparative research of light scalar and pseudoscalar mesons in semileptonic decays of B quarkonia at super B-factories is very tempting. Now it is known that

Outlook

$$BR(B^0 \to d\bar{u}\, e^+\nu \to \pi^-\, e^+\nu) = (1.44 \pm 0.05) \times 10^{-4},$$

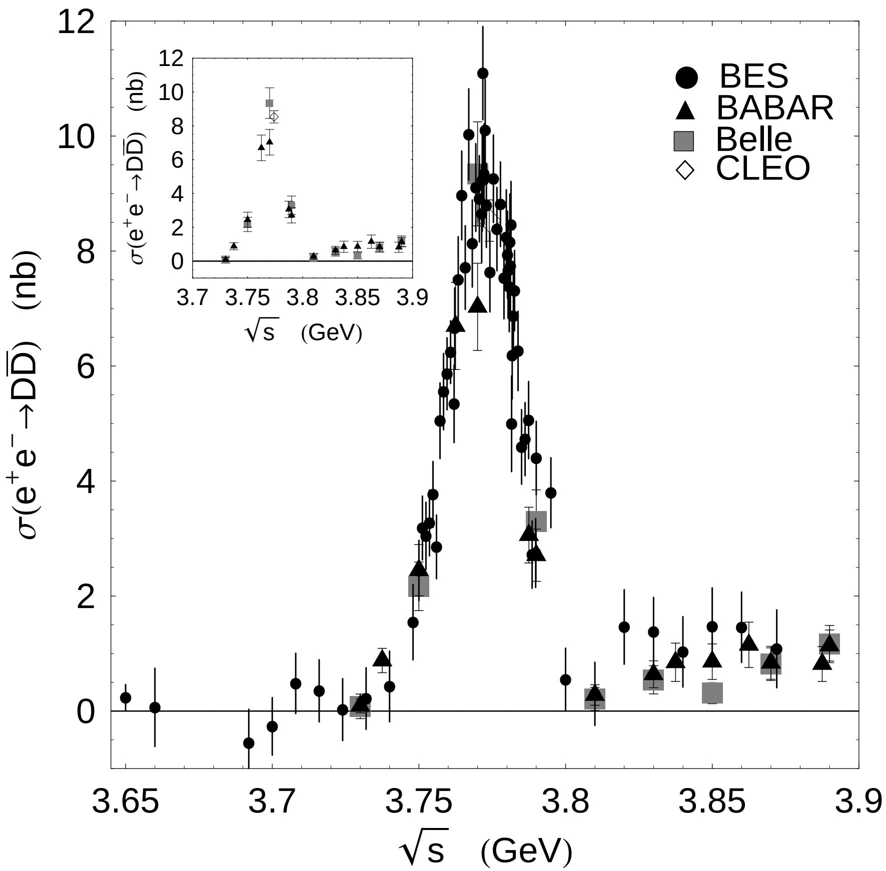
$$BR(B^+ \to u\bar{u}\, e^+\nu \to \pi^0\, e^+\nu) = (7.79 \pm 0.26) \times 10^{-5},$$

$$BR(B^+ \to u\bar{u}\, e^+\nu \to \eta\, e^+\nu) = (3.8 \pm 0.6) \times 10^{-5}$$ **and**

$$BR(B^+ \to u\bar{u}\, e^+\nu \to \eta'\, e^+\nu) = (2.3 \pm 0.8) \times 10^{-5}.$$

Interference phenomena in the $\psi(3770)$ resonance region

Based on N.N. Achasov and G.N. Shestakov, Physical Review D 86, 114013 (2012) and Physical Review D 87, 057502 (2013)



The D meson electromagnetic form factor F_D^0

A similar representation of the $e^+e^- \to D\bar{D}$ reaction amplitude used for the data description guarantees the unitarity requirement on the model level.

The sum of the $e^+e^- \to D^0\bar{D}^0$ and $e^+e^- \to D^+D^-$ reaction cross sections is expressed in terms of F_D^0 in the following way

$$\sigma(\gamma\gamma \to D\bar{D}) = \frac{8\pi\alpha^2}{3s^2}\left|F_D^0(s)\right|^2 \nu(s),$$

where $\nu(s) = [p_0^3(s) + p_+^3(s)]/\sqrt{s}\,, \quad p_{0,+}(s) = \sqrt{s/4 - m^2_{D^0,+}}\,.$

The model for F_D^0 with the mixing ψ'' and $\psi(2S)$ resonances

It is clear that the main sources of the background in the ψ'' region are the tails from the J/ψ, $\psi(2S)$, $\psi(4040)$, $\psi(4160)$ and other resonances. It is easy to incorporate the right number of resonances in our scheme.

Here we present the simplest variant of the model taking into account the background contribution from the nearest neighbor resonance $\psi(2S)$ and also discuss how it can be checked.

In the considered model the ψ'' and $\psi(2S)$ resonances mix via transitions $\psi'' \to D\bar{D} \to \psi(2S)$.

The model for F_D^0 with the mixing ψ'' and $\psi(2S)$ resonances

$$F_D^0(s) = \frac{\mathcal{R}_{D\bar{D}}(s)}{D_{\psi''}(s)D_{\psi(2S)}(s) - \Pi^2_{\psi''\psi(2S)}(s)}\,,$$

$$where \;\; D_{\psi''}(s) = m^2_{\psi''} - s - i\sqrt{s}\Gamma_{\psi'' D\bar{D}}(s),$$

$$D_{\psi(2S)}(s) = m^2_{\psi(2S)} - s - i\sqrt{s}\Gamma_{\psi(2S) D\bar{D}}(s),$$

$$\Gamma_{\psi'' D\bar{D}}(s) = \frac{g^2_{\psi'' D\bar{D}}}{6\pi}\,\frac{\nu(s)}{\sqrt{s}},\;\; \Gamma_{\psi(2S) D\bar{D}}(s) = \frac{g^2_{\psi(2S) D\bar{D}}}{6\pi}\,\frac{\nu(s)}{\sqrt{s}},$$

the $\psi'' - \psi(2S)$ mixing amplitude caused by $\psi'' \to D\bar{D} \to \psi(2S)$ transitions via the real $D\bar{D}$ intermediate states has the form

$$\Pi_{\psi''\psi(2S)}(s) = i\, g_{\psi'' D\bar{D}} g_{\psi(2S) D\bar{D}}\, \nu(s)/(6\pi)\,,$$

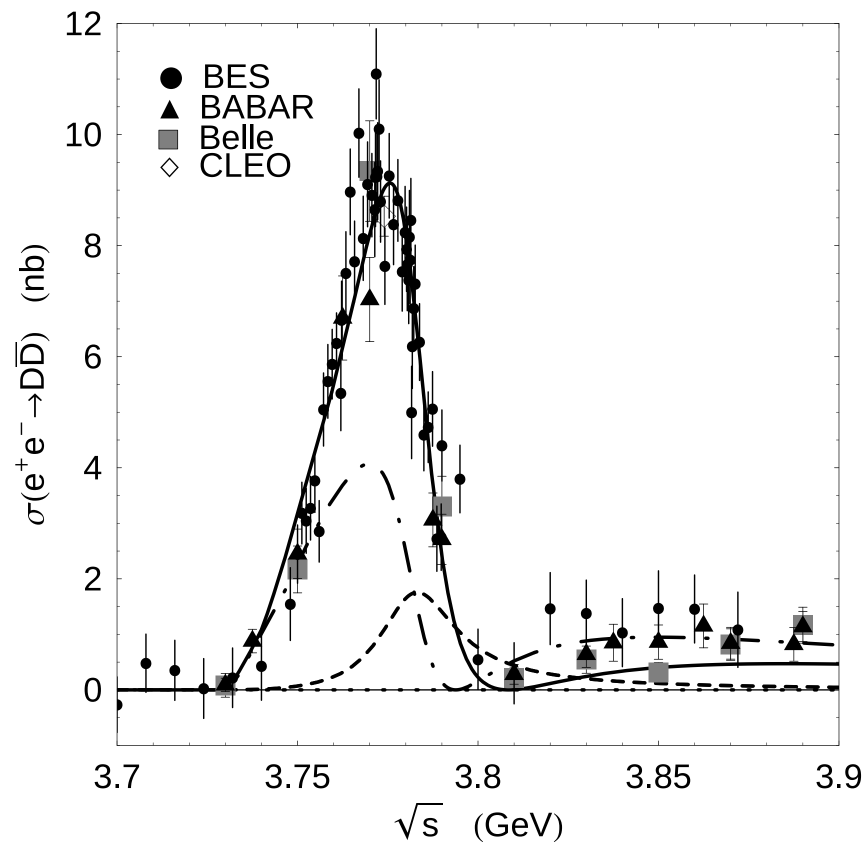
$$\mathcal{R}_{D\bar{D}}(s) = (m^2_{\psi''} - s) g_{\psi(2S)\gamma} g_{\psi(2S) D\bar{D}} + (m^2_{\psi(2S)} - s) g_{\psi''\gamma} g_{\psi'' D\bar{D}}\,.$$

The model for F_D^0 with the mixing ψ'' and $\psi(2S)$ resonances

$m_{\psi''}$, $g_{\psi'' D\bar{D}}$, $g_{\psi''\gamma}$, and $g_{\psi(2S)D\bar{D}}$ are determined by fitting; $m_{\psi(2S)}$ and $g_{\psi''\gamma}$ are fixed by the PDG data.

Note that F_D^0 in the considered model is proportional to the first-degree polynomial in s with real coefficients (see $\mathcal{R}_{D\bar{D}}(s)$ above). Hence the dip observed in $\sigma(e^+e^- \to D\bar{D})$ near 3.81 GeV can be explained by the $F_D^0(s)$ zero, caused by compensation between the ψ'' and $\psi(2S)$ contributions.

The simplest variant of the $\psi'' - \psi(2S)$ mixing model for F_D^0



The solid curve is the fit to the data. The dashed and dot-dashed curves show the ψ'' and $\psi(2S)$ contributions, respectively. **Bare parameters:** $m_{\psi''}$ = **3.794 GeV,** $\Gamma_{\psi'' D\bar{D}}$ = **56.8 MeV,** $\Gamma_{\psi'' e^+e^-}$ = **0.062 keV,** $g^2_{\psi(2S)D\bar{D}}/4\pi$ = **32.2.**

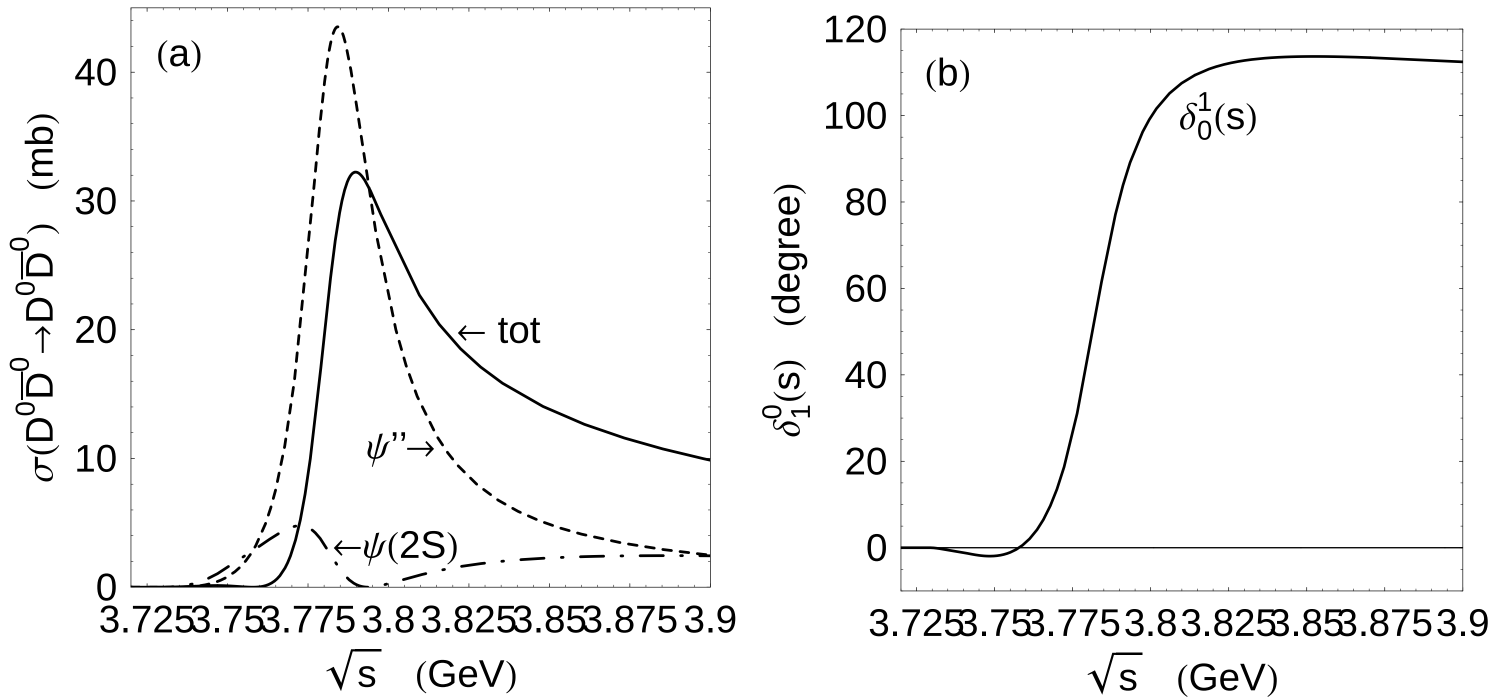
The simplest variant of the $\psi'' - \psi(2S)$ mixing model for T_1^0

From the fitting of the $e^+e^- \to D\bar{D}$ data we all know, at the model level, about the $I = 0$ P wave $D\bar{D}$ elastic scattering amplitude T_1^0:

$$T_1^0(s) = e^{i\delta_1^0(s)} \sin \delta_1^0(s) =$$

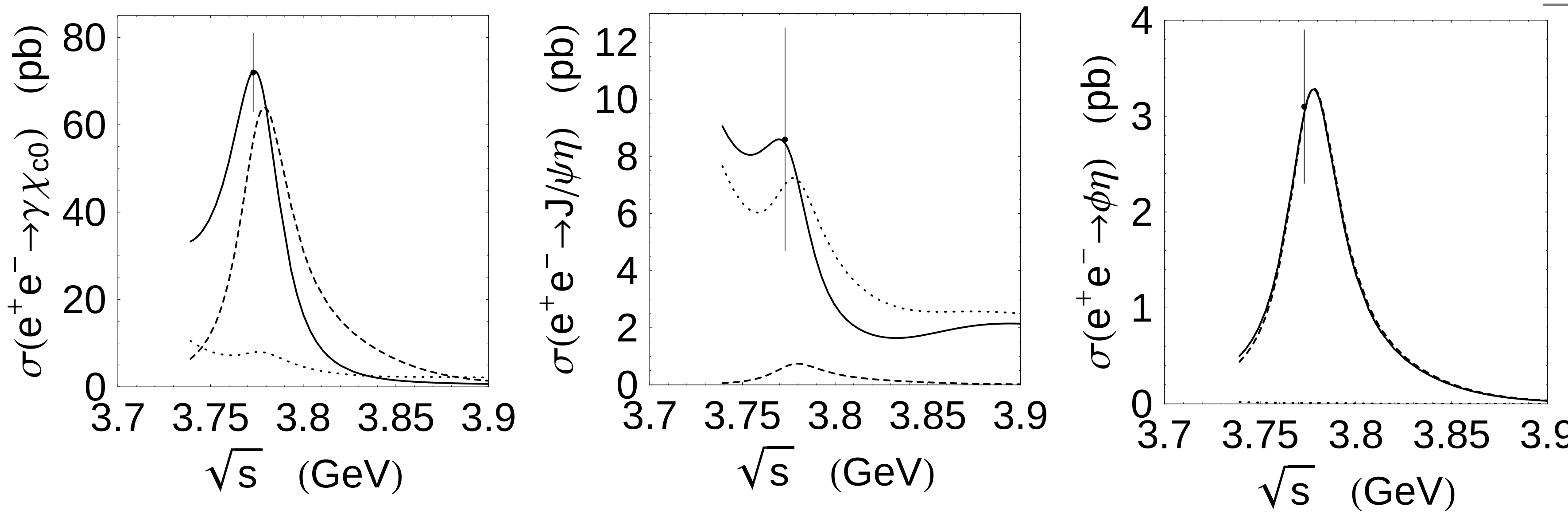
$$= \frac{\nu(s)}{6\pi} \left[\frac{(m^2_{\psi''} - s)g^2_{\psi(2S)D\bar{D}} + (m^2_{\psi(2S)} - s)g^2_{\psi''D\bar{D}}}{D_{\psi''}(s)D_{\psi(2S)}(s) - \Pi^2_{\psi''\psi(2S)}(s)} \right].$$

Cross section and phase for $D\bar{D}$ elastic scattering in the P wave



(a) The cross section $\sigma(D^0\bar{D}^0 \to D^0\bar{D}^0) = 3\pi|\sin\delta_1^0(s)|^2/p_0^2(s)$ and (b) the phase $\delta_1^0(s)$ for the simplest variant of the $\psi'' - \psi(2S)$ mixing model. Unfortunately, these predictions are not possible to verify. However, there are many other reactions which can be measured experimentally.

The ψ'' shapes in non-$D\bar{D}$ decay channels



The solid curves show predictions of the model with the mixing ψ'' and $\psi(2S)$ resonances for the ψ'' peak shapes in the $e^+e^- \to \gamma\chi_{c0}$, $e^+e^- \to J/\psi\eta$, and $e^+e^- \to \phi\eta$ cross sections; the dashed and dotted curves show the contributions from ψ'' and $\psi(2S)$ production amplitudes proportional to $g_{\psi''ab}$ and $g_{\psi(2S)ab}$, respectively ($ab = \gamma\chi_{c0},\ J/\psi\eta,\ \phi\eta$). The points with errors are the CLEO data.

Conclusion

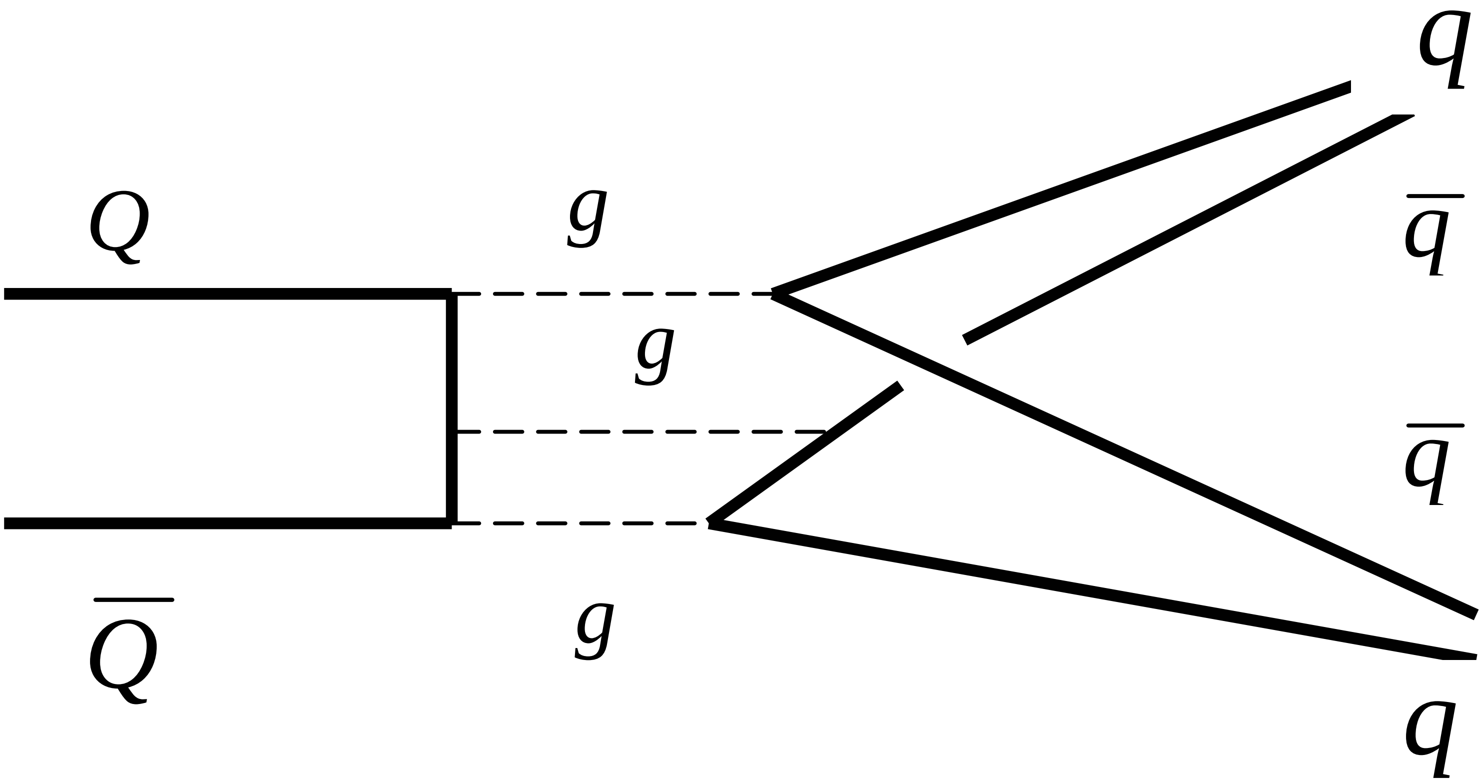
1. The ψ'' resonance shape keep important information about the production mechanism and interference with background. Its description requires taking into account the unitarity.
2. The simplest model mixing the ψ'' and $\psi(2S)$ resonances satisfies the unitarity requirement and describes the current data on the $e^+e^- \to D\bar{D}$ reaction cross section very well. We also extracted from experiment $g^2_{\psi(2S)D\bar{D}}/4\pi \approx 13 - 30$.
3. New high-statistics data on the reactions $e^+e^- \to D\bar{D}$ should help reveal the complex mechanism of the ψ'' production.
4. The measurements of mass spectra in the ψ'' region in the non-$D\bar{D}$ channels, such as $e^+e^- \to \gamma\chi_{c0}$, $J/\psi\eta$, $\phi\eta$, etc., will promote comprehensive study of the ψ'' resonance physics and effective selection of theoretical models.

Branching ratios of decays $\psi(3770)$, $\psi(4040)$, and $\Upsilon(10580)$ into light hadrons

Based on N.N. Achasov and A.A. Kozhevnikov, Phys. Rev. D 49, 275 (1994) and Yad. Fiz. 69, 1017 (2006) [Phys. At. Nucl. 69, 988 (2006)].

Exclusive decays of the ground-state $c\bar{c}$ and $b\bar{b}$ quarkonia $J/\psi(1S)$ and $\Upsilon(1S)$ into light hadrons are qualitatively similar in that their branching ratios are very small, $\sim 10^{-3} - 10^{-4}$. Since in the framework of the quark-gluon picture such decays are originated from the 3-gluon annihilation, a rough estimate gives $\Gamma((Q\bar{Q} \to (q\bar{q}) + (q\bar{q})) \sim \alpha_s^3 \Gamma((Q\bar{Q} \to 3gluons)$, where Q (q) denotes heavy (light) quark, means that each of above listed decays has the branching ratio which is much lower than the branching ratio of the decay into 3 gluons.

The three-gluon annihilation

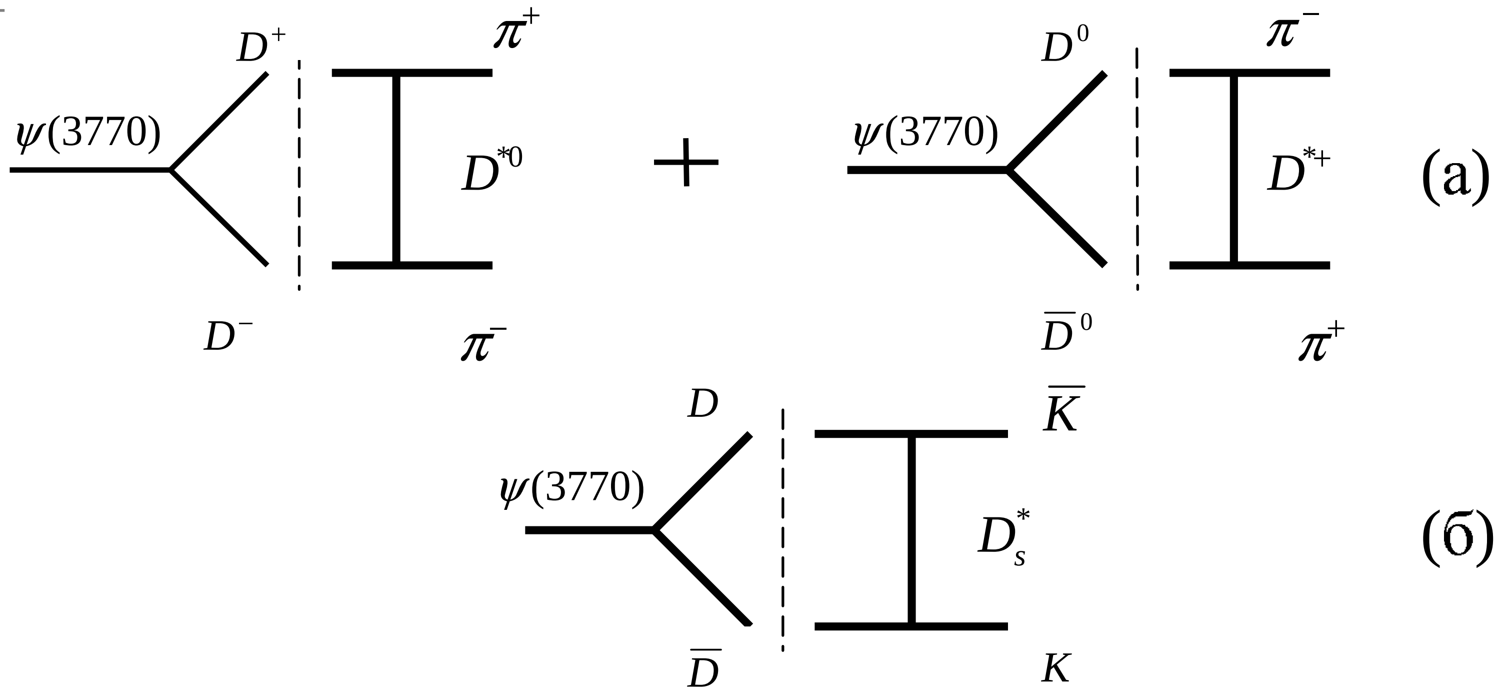


Let us try to understand this suppression in the language of intermediate hadronic states, i.e. in the framework of dispersion approach.

The language of intermediate hadronic states

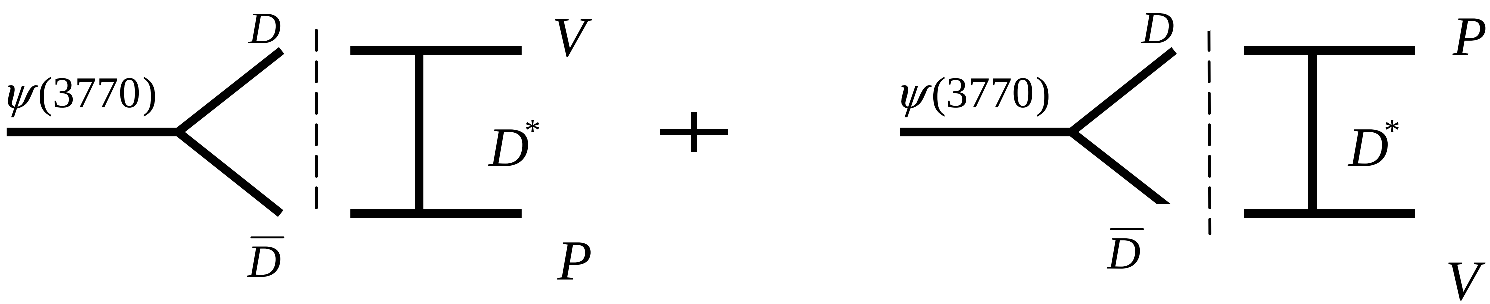
In this approach a amplitude of a decay under discussion can be represented as the sum over the contributions to the dispersion integral coming from the $D\bar{D}$, $D^*\bar{D}+c.c.$, $D^*\bar{D}^*$ etc., intermediate states in the case of the $J/\psi(1S)$ or $B\bar{B}$, $B^*\bar{B}+c.c.$, $B^*\bar{B}^*$ etc., intermediate states in the case of the $\Upsilon(1S)$. We do not see a reason for large suppression of each specific contribution. The most probable explanation of the suppression of the decays under consideration is the strong cancellation between the contributions from intermediate states listed above. However, such a cancellation could be broken when a new channel is opening. If so, the energy window may open where imaginary part of the amplitude is appreciable.

$\psi(3770) \to PP$



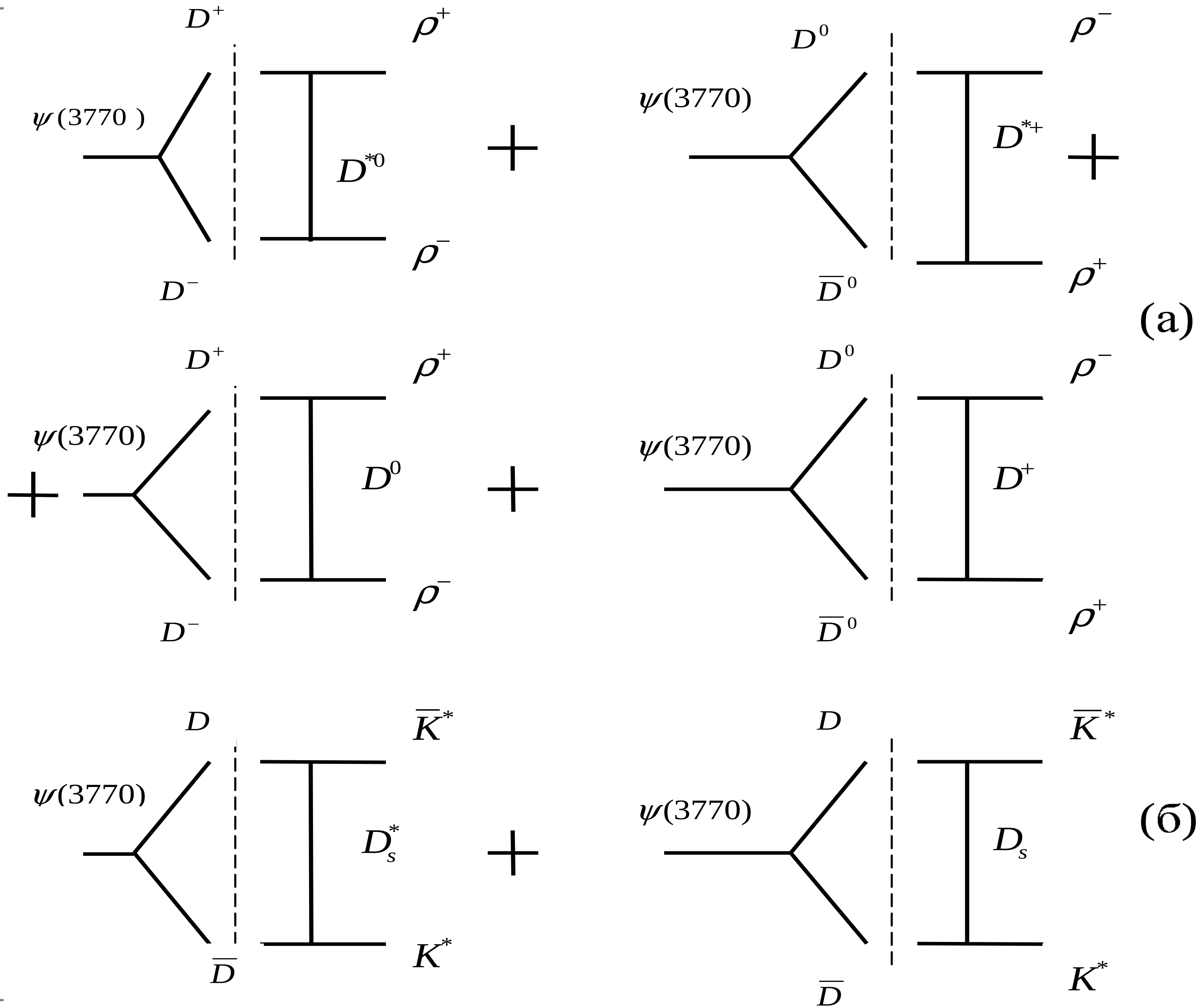
We believe that such a situation is realized for the states lying slightly above the production thresholds of open charm and beauty. Hence, the states $\psi(3770)$ and $\Upsilon(10580)$ are most promising from the point of view of the idea under consideration.

$\psi(3770) \to VP$



All told about the $\psi(3770)$ is transferred to the case of $\Upsilon(10580) \equiv \Upsilon(4S)$ by means of the replacements $\psi(3770) \to \Upsilon(10580), c \to b, D \to B, D^*_s \to B^*_s$ etc.

$\psi(3770) \to VV$



Mode	$\psi(3770)$	$\Upsilon(10580)$
$\pi^+\pi^-$	$2\cdot10^{-6}(7\cdot10^{-5})$	$8\cdot10^{-8}(6\cdot10^{-6})$
$K\bar{K}$	$2\cdot10^{-5}$	$2\cdot10^{-6}$
$\omega\pi^0$	$2\cdot10^{-5}(7\cdot10^{-4})$	$5\cdot10^{-6}(4\cdot10^{-4})$
$\omega\eta$	$3\cdot10^{-4}(1\cdot10^{-5})$	$3\cdot10^{-4}(4\cdot10^{-6})$
$\omega\eta'$	$1\cdot10^{-4}(7\cdot10^{-6})$	$2\cdot10^{-4}(2\cdot10^{-6})$
$\rho\pi$	$2\cdot10^{-3}(7\cdot10^{-5})$	$1\cdot10^{-3}(2\cdot10^{-5})$
$\rho\eta$	$1\cdot10^{-5}(3\cdot10^{-4})$	$4\cdot10^{-6}(3\cdot10^{-4})$
$\rho\eta'$	$7\cdot10^{-6}(1\cdot10^{-4})$	$2\cdot10^{-6}(2\cdot10^{-4})$
$\rho^+\rho^-$	$3\cdot10^{-5}(1\cdot10^{-3})$	$1\cdot10^{-4}(8\cdot10^{-3})$
$K^*\bar{K}+c.c$	$3\cdot10^{-4}$	$4\cdot10^{-4}$
$K^*\bar{K}^*$	$7\cdot10^{-4}$	$3\cdot10^{-3}$
$J/\psi(1S)+\pi^0$	$8\cdot10^{-6}(1\cdot10^{-4})$	-
$J/\psi(1S)+\eta$	$4\cdot10^{-5}(1\cdot10^{-6})$	-
$\Upsilon(1S)+\pi^0$	-	$7\cdot10^{-9}(5\cdot10^{-7})$
$\Upsilon(1S)+\eta$	-	$2\cdot10^{-7}(3\cdot10^{-9})$
3 gluons	$2\cdot10^{-4}$	$4\cdot10^{-4}$
total	$4\cdot10^{-3}(3\cdot10^{-3})$	$5\cdot10^{-3}(13\cdot10^{-3})$

The inclusive decay of $\psi(3770)$ and $\Upsilon(4S)$ to light hadrons

$\psi(3770) \to D^+D^- + D^0\overline{D^0} \to X$ and
$\Upsilon(10580) \to B^+B^- + B^0\overline{B^0} \to X$

$$\sum_X B_{V\to P^+P^- + P^0\overline{P^0}\to X} = \left(m_V^2 v_P^2/48\pi\right) \times B_{V\to P^0\overline{P^0}} \times$$
$$\sum_X \left[1 + |c_{P^\pm}|^2 \times \left(v_{P^\pm}/v_{P^0}\right)^3\right]^2 \times \sigma_P\left(P^0\overline{P^0} \to X\right),$$

where $V = \psi(3770)$ (or $\Upsilon(10580)$); $P = D$ (or B); where the velocities $v_{D^\pm} \approx 0.128$ and $v_{D^0} \approx 0.147$, $v_{B^\pm} \approx 0.064$ and $v_{B^0} \approx 0.063$; $c_{P^\pm}$ takes into account electromagnetic interaction between P^+ and P^-; $\sum_X \sigma_P\left(P^0\overline{P^0} \to X\right)$ is the total annihilation cross-section $P^0\overline{P^0} \to X$ in the P wave with I_X=0.

The inclusive decay of $\psi(3770)$ and $\Upsilon(4S)$ to light hadrons

$$\sum_X B_{\psi(3770)\to D^+D^- + D^0\overline{D^0}\to X} = 1\% \text{ and}$$

$$\sum_X B_{\Upsilon(10580)\to B^+B^- + B^0\overline{B^0}\to X} = 1\%$$

correspond

$$\sum_X \sigma_P(D^0\overline{D^0} \to X) \approx 1.5\,\mu\text{b and}$$

$$\sum_X \sigma_P(B^0\overline{B^0} \to X) \approx 0.64\,\mu\text{b}\,.$$

$\sum_X \sigma_P(D^0\overline{D^0} \to X) \sim v_{D^0}$ and $\sum_X \sigma_P(B^0\overline{B^0} \to X) \sim v_{B^0}$

Thanks

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THANK YOU