«Physics of Fundamental Interactions»

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Forward observables in hadronic elastic collisions $\sigma = Im[A]$ and $\rho = \frac{Re[A]}{Im[A]}$

(experiment vs HPRIR2 model)

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Protvino



Experimental data & fit

All σ data & fit Common energy interval



All σ data & fit Individual collision energy intervals



HPRIR2 model

$$\sigma^{a^{\mp}b} = \begin{cases} H_{M}Log^{2}(\frac{s}{s^{ab}}) \iff \text{Heisenberg} \\ + P^{ab} \iff \text{Pomeranchuk} \\ + R_{1}^{ab}(\frac{s}{s^{ab}})^{-\eta_{1}} \iff \text{Regge C}^{+} \\ \pm R_{2}^{ab}(\frac{s}{s^{ab}})^{-\eta_{2}} \iff \text{Regge C}^{-} \end{cases}$$

$$\rho^{a^{\mp}b} = \frac{1}{\sigma^{a^{\mp}b}} \begin{bmatrix} \pi H_{M}Log(\frac{s}{s^{ab}_{M}}) & \Leftrightarrow & \text{Heisenberg} \\ - R_{1}^{ab}(\frac{s}{s^{ab}_{M}})^{-\eta_{1}}tan(\frac{\eta_{1}\pi}{2}) & \Leftrightarrow & \text{Regge C}^{+} \\ \pm R_{2}^{ab}(\frac{s}{s^{ab}_{M}})^{-\eta_{2}}cot(\frac{\eta_{2}\pi}{2}) & \Leftrightarrow & \text{Regge C}^{-} \end{bmatrix}$$

$$H_{M} = \pi \frac{(\hbar c)^{2}}{M^{2}}, \quad s_{M}^{ab} = (m_{a} + m_{b} + M)^{2}$$
$$H_{M}^{d} = \lambda H_{M} \text{ for deuterium target}$$

$$\begin{split} H_{M}^{\gamma(p,d)} &= \delta(1, \lambda) H_{M}, \quad P^{\gamma(p,d)} = \delta P^{p(p,d)} \text{ for } \gamma \text{ beam} \\ H_{M}^{\gamma\gamma} &= \delta^{2} H_{M}, \quad P^{\gamma\gamma} = \delta^{2} P^{p(p,d)} \text{ for } \gamma \gamma \text{ collisions} \end{split}$$

HPR1R2 at √s ≥5 GeV	M = 2.122 ± 0.014 [GeV] H = 0.2717 ± 0.0037 [mb] $\eta_1 = 0.448 \pm 0.013$ $\eta_2 = 0.5487 \pm 0.007$				$FQ_{INT} = 0.96$ $FQ_{EXT} = 0.96$	
P [mb]	$\delta = (3.062 \pm 0)$	$.021) \times 10^{-5}$	$l = 1.625 \pm 0.048$ Ream/Target Npt = 1047		χ^2/npt	
r [mb]	KI [III0]	K ₂ [mb]	Dealing Tanget	npt	by groups	
34.43 ± 0.21	13.06 ± 0.25	7.39 ± 0.12	$\overline{p}(p) / p$	257	1.14	
34.73 ± 0.25	12.5 ± 0.47	6.66 ± 0.22	$\overline{p}(p) / n$	67	0.48	
$34.7 \pm 2.$	$-47. \pm 28.$	$-49. \pm 29.$	Σ^- / p	9	0.37	
18.77 ± 0.18	9.55 ± 0.21	1.767 ± 0.043	π^{\mp}/p	183	1.02	
16.37 ± 0.13	4.28 ± 0.18	3.407 ± 0.06	K [∓] / p	121	0.82	
16.32 ± 0.14	3.69 ± 0.26	1.825 ± 0.1	K [∓] / n	64	0.58	
	0.0139 ± 0.0017		γ / p	41	0.62	
	$(-4. \pm 25.) \times 10^{-6}$		γ/γ	37	0.75	
	0.0369 ± 0.0028		γ / d	13	0.9	
64.47 ± 0.56	29.63 ± 0.64	14.94 ± 0.24	$\overline{p}(p) / d$	85	1.52	
36.67 ± 0.44	18.73 ± 0.58	0.34 ± 0.12	π^{\mp}/d	92	0.72	
32.07 ± 0.32	7.68 ± 0.49	5.61 ± 0.12	K [∓] / d	78	0.79	
HPR1R2	$M = 2.122 \pm 0.0$	014 [GeV] H	$I = 0.2717 \pm 0.0$	0037 [mb]	$FO_{\rm DW} = 0.96$	
HPR1R2 at $\sqrt{s} > 5$ CeV	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$	014 [GeV] H 013 η ₂	$I = 0.2717 \pm 0.0$ $_{2} = 0.5487 \pm 0.0$	0037 [mb] 007	$FQ_{INT} = 0.96$ FQ_{INT} = 0.96	
HPR1R2 at √s ≥5 GeV	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0.0)$	014 [GeV] H 013 η_2 .021)×10 ⁻³ 2	$I = 0.2717 \pm 0.0$ $_{2} = 0.5487 \pm 0.0$ $_{3} = 1.625 \pm 0.0$	0037 [mb] 007 048	$FQ_{INT} = 0.96$ $FQ_{EXT} = 0.96$	
HPR1R2 at √s ≥5 GeV P [mb]	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ R: [mb]	014 [GeV] H 013 η_2 .021)×10 ⁻³ 7 R ₂ [mb]	$I = 0.2717 \pm 0.0$ $_{2} = 0.5487 \pm 0.0$ $A = 1.625 \pm 0.0$ Beam/Target	0037 [mb] 007 048 Npt = 1047	$FQ_{INT} = 0.96$ $FQ_{EXT} = 0.96$ χ^{2}/npt	
HPR1R2 at √s ≥5 GeV P [mb]	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ R ₁ [mb]	014 [GeV] H 013 η_2 .021)×10 ⁻³ 7 R ₂ [mb]	$I = 0.2717 \pm 0.0$ $_{2} = 0.5487 \pm 0.0$ $X = 1.625 \pm 0.0$ Beam/Target	0037 [mb] 007 048 Npt = 1047 npt	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26	014 [GeV] H 013 η_2 .021)×10 ⁻³ λ R ₂ [mb] 7.39 ± 0.11	$I = 0.2717 \pm 0.0$ $a_2 = 0.5487 \pm 0.0$ $a_1 = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$	0037 [mb] 007 048 Npt = 1047 npt 257	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21 34.73 ± 0.25	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26 12.5 ± 0.48	014 [GeV] H 013 η_2 .021)×10 ⁻³ η_2 R ₂ [mb] 7.39 ± 0.11 6.66 ± 0.22	$I = 0.2717 \pm 0.0$ $_{2} = 0.5487 \pm 0.0$ $I = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$ $\overline{p}(p) / n$	0037 [mb] 007 048 Npt = 1047 npt 257 67	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14 0.48	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21 34.73 ± 0.25 34.7 ± 2.	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26 12.5 ± 0.48 $-47. \pm 30.$	014 [GeV] H 013 η_2 .021)×10 ⁻³ λ R ₂ [mb] 7.39 ± 0.11 6.66 ± 0.22 -49. ± 30.	$I = 0.2717 \pm 0.0$ $a_{2} = 0.5487 \pm 0.0$ $A = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$ $\overline{p}(p) / n$ Σ^{-} / p	0037 [mb] 007 048 Npt = 1047 npt 257 67 9	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14 0.48 0.37	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21 34.73 ± 0.25 34.7 ± 2. 18.77 ± 0.18	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26 12.5 ± 0.48 $-47. \pm 30.$ 9.55 ± 0.22	$\begin{array}{ccc} 014 \ [GeV] & H\\ 013 & \eta_2\\ .021) \times 10^{-3} & \lambda\\ \hline R_2 \ [mb] \\ \hline 7.39 \pm 0.11 \\ \hline 6.66 \pm 0.22 \\ -49. \pm 30. \\ \hline 1.767 \pm 0.042 \end{array}$	$I = 0.2717 \pm 0.0$ $a = 0.5487 \pm 0.0$ $a = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$ $\overline{p}(p) / n$ Σ^{-} / p π^{\mp} / p	0037 [mb] 007 048 Npt = 1047 npt 257 67 9 183	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14 0.48 0.37 1.02	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21 34.73 ± 0.25 34.7 ± 2. 18.77 ± 0.18 16.37 ± 0.13	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26 12.5 ± 0.48 $-47. \pm 30.$ 9.55 ± 0.22 4.28 ± 0.19	$\begin{array}{cccc} 014 [GeV] & H \\ 013 & \eta_{2} \\ .021) \times 10^{-3} & 7 \\ \hline R_{2} [mb] \\ \hline 7.39 \pm 0.11 \\ \hline 6.66 \pm 0.22 \\ -49. \pm 30. \\ \hline 1.767 \pm 0.042 \\ \hline 3.407 \pm 0.06 \\ \end{array}$	$I = 0.2717 \pm 0.0$ $I_{2} = 0.5487 \pm 0.0$ $I = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$ $\overline{p}(p) / n$ Σ^{-} / p $\pi^{-\mp} / p$ $K^{-\mp} / p$	0037 [mb] 007 048 Npt = 1047 npt 257 67 9 183 121	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14 0.48 0.37 1.02 0.82	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21 34.73 ± 0.25 34.7 ± 2. 18.77 ± 0.18 16.37 ± 0.13 16.32 ± 0.14	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26 12.5 ± 0.48 $-47. \pm 30.$ 9.55 ± 0.22 4.28 ± 0.19 3.69 ± 0.27	$\begin{array}{cccc} 014 \ [GeV] & H\\ 013 & \eta_{2}\\ .021) \times 10^{-3} & 2\\ \hline R_{2} \ [mb] \\ \hline 7.39 \pm 0.11 \\ 6.66 \pm 0.22 \\ -49. \pm 30. \\ \hline 1.767 \pm 0.042 \\ \hline 3.407 \pm 0.06 \\ \hline 1.83 \pm 0.1 \\ \end{array}$	$I = 0.2717 \pm 0.0$ $I = 0.5487 \pm 0.0$ $I = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$ $\overline{p}(p) / n$ Σ^{-} / p π^{\mp} / p K^{\mp} / p K^{\mp} / n	0037 [mb] 007 048 Npt = 1047 npt 257 67 9 183 121 64	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14 0.48 0.37 1.02 0.82 0.58	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21 34.73 ± 0.25 34.7 ± 2. 18.77 ± 0.18 16.37 ± 0.13 16.32 ± 0.14	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26 12.5 ± 0.48 $-47. \pm 30.$ 9.55 ± 0.22 4.28 ± 0.19 3.69 ± 0.27 0.0139 ± 0.0017	$\begin{array}{cccc} 014 [\text{GeV}] & \text{H} \\ 013 & \eta_2 \\ .021) \times 10^{-3} & 7 \\ \hline R_2 [\text{mb}] \\ \hline 7.39 \pm 0.11 \\ \hline 6.66 \pm 0.22 \\ -49. \pm 30. \\ \hline 1.767 \pm 0.042 \\ \hline 3.407 \pm 0.06 \\ \hline 1.83 \pm 0.1 \\ \hline \end{array}$	$I = 0.2717 \pm 0.0$ $I = 0.5487 \pm 0.0$ $I = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$ $\overline{p}(p) / n$ Σ^{-} / p π^{\mp} / p K^{\mp} / p K^{\mp} / n γ / p	0037 [mb] 007 048 Npt = 1047 npt 257 67 9 183 121 64 41	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14 0.48 0.37 1.02 0.82 0.58 0.62	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21 34.73 ± 0.25 34.7 ± 2. 18.77 ± 0.18 16.37 ± 0.13 16.32 ± 0.14	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26 12.5 ± 0.48 $-47. \pm 30.$ 9.55 ± 0.22 4.28 ± 0.19 3.69 ± 0.27 0.0139 ± 0.0017 $(-4. \pm 25.) \times 10^{-6}$	$\begin{array}{cccc} 014 \ [GeV] & H\\ 013 & \eta_{2}\\ .021) \times 10^{-3} & \lambda\\ R_{2} \ [mb] \\ \hline 7.39 \pm 0.11 \\ 6.66 \pm 0.22 \\ -49. \pm 30. \\ 1.767 \pm 0.042 \\ 3.407 \pm 0.06 \\ \hline 1.83 \pm 0.1 \end{array}$	$I = 0.2717 \pm 0.0$ $I = 0.5487 \pm 0.0$ $I = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$ $\overline{p}(p) / n$ Σ^{-} / p π^{+} / p K^{+} / p K^{+} / n γ / p γ / γ	0037 [mb] 007 048 Npt = 1047 npt 257 67 9 183 121 64 41 37	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14 0.48 0.37 1.02 0.82 0.58 0.62 0.75	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21 34.73 ± 0.25 34.7 ± 2. 18.77 ± 0.18 16.37 ± 0.13 16.32 ± 0.14	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26 12.5 ± 0.48 $-47. \pm 30.$ 9.55 ± 0.22 4.28 ± 0.19 3.69 ± 0.27 0.0139 ± 0.0017 $(-4. \pm 25.) \times 10^{-6}$ 0.0369 ± 0.0028	$\begin{array}{cccc} 014 \ [GeV] & H\\ 013 & \eta_{2}\\ .021) \times 10^{-3} & 7\\ \hline R_{2} \ [mb] \\ \hline 7.39 \pm 0.11 \\ \hline 6.66 \pm 0.22 \\ -49. \pm 30. \\ \hline 1.767 \pm 0.042 \\ \hline 3.407 \pm 0.06 \\ \hline 1.83 \pm 0.1 \end{array}$	$I = 0.2717 \pm 0.0$ $I = 0.5487 \pm 0.0$ $I = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$ $\overline{p}(p) / n$ Σ^{-} / p π^{\mp} / p K^{\mp} / p K^{\mp} / n γ / p γ / γ γ / d	0037 [mb] 007 048 Npt = 1047 npt 257 67 9 183 121 64 41 37 13	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14 0.48 0.37 1.02 0.82 0.58 0.62 0.75 0.9	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21 34.73 ± 0.25 34.7 ± 2. 18.77 ± 0.18 16.37 ± 0.13 16.32 ± 0.14	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26 12.5 ± 0.48 $-47. \pm 30.$ 9.55 ± 0.22 4.28 ± 0.19 3.69 ± 0.27 0.0139 ± 0.0017 $(-4. \pm 25.) \times 10^{-6}$ 0.0369 ± 0.0028 29.63 ± 0.61	$\begin{array}{cccc} 014 \ [GeV] & H\\ 013 & \eta_{2}\\ .021) \times 10^{-3} & 7\\ \hline R_{2} \ [mb] \\ \hline 7.39 \pm 0.11 \\ 6.66 \pm 0.22 \\ -49. \pm 30. \\ \hline 1.767 \pm 0.042 \\ \hline 3.407 \pm 0.06 \\ \hline 1.83 \pm 0.1 \\ \hline \\ 14.94 \pm 0.24 \\ \end{array}$	$I = 0.2717 \pm 0.0$ $I = 0.5487 \pm 0.0$ $I = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$ $\overline{p}(p) / n$ Σ^{-} / p π^{\mp} / p K^{\mp} / p K^{\mp} / n γ / p γ / γ γ / d $\overline{p}(p) / d$	0037 [mb] 007 048 Npt = 1047 npt 257 67 9 183 121 64 41 37 13 85	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14 0.48 0.37 1.02 0.82 0.58 0.62 0.75 0.9 1.52	
HPR1R2 at √s ≥5 GeV P [mb] 34.43 ± 0.21 34.73 ± 0.25 34.7 ± 2. 18.77 ± 0.18 16.37 ± 0.13 16.32 ± 0.14 64.47 ± 0.53 36.67 ± 0.42	$M = 2.122 \pm 0.0$ $\eta_1 = 0.448 \pm 0.0$ $\delta = (3.062 \pm 0)$ $R_1 \text{ [mb]}$ 13.06 ± 0.26 12.5 ± 0.48 $-47. \pm 30.$ 9.55 ± 0.22 4.28 ± 0.19 3.69 ± 0.27 0.0139 ± 0.0017 $(-4. \pm 25.) \times 10^{-6}$ 0.0369 ± 0.0028 29.63 ± 0.61 18.73 ± 0.58	$\begin{array}{c c} 014 [GeV] & H\\ 013 & \eta_{2}\\ 021) \times 10^{-3} & 7\\ \hline R_{2} [mb] \\ \hline 7.39 \pm 0.11 \\ \hline 6.66 \pm 0.22 \\ -49. \pm 30. \\ \hline 1.767 \pm 0.042 \\ \hline 3.407 \pm 0.06 \\ \hline 1.83 \pm 0.1 \\ \hline \\ 14.94 \pm 0.24 \\ \hline 0.34 \pm 0.12 \\ \end{array}$	$I = 0.2717 \pm 0.0$ $I = 0.5487 \pm 0.0$ $I = 1.625 \pm 0.0$ Beam/Target $\overline{p}(p) / p$ $\overline{p}(p) / n$ Σ^{-} / p π^{+} / p K^{+} / p K^{+} / n γ / p γ / q $\overline{p}(p) / d$ $\overline{p}(p) / d$	0037 [mb] 007 048 Npt = 1047 npt 257 67 9 183 121 64 41 37 13 85 92	FQ _{INT} = 0.96 FQ _{EXT} = 0.96 χ^2/npt by groups 1.14 0.48 0.37 1.02 0.82 0.58 0.62 0.75 0.9 1.52 0.72	

Simultaneous fit results

HPR1R2 at √s ≥7 GeV	$M = 2.074 \pm 0.0$ $\eta_1 = 0.411 \pm 0.0$ $\delta = (3.113 \pm 0.0)$	$FQ_{INT} = 0.86$ $FQ_{EXT} = 0.87$						
D [mh]	D [mb]	D [mb]	Room/Torgot	Npt = 837	χ^2/npt			
r [mo]	\mathbf{K}_1 [mb]	\mathbf{K}_2 [mb]	Deally Larger	npt	by groups			
33.7 ± 0.3	13.7 ± 0.3	7.77 ± 0.17	$\overline{p}(p) / p$	220	1.08			
33.75 ± 0.35	14.07 ± 0.58	6.94 ± 0.27	$\overline{p}(p) / n$	48	0.39			
33.2 ± 3.7	$-14. \pm 43.$	$-14. \pm 48.$	Σ^-/p	8	0.41			
18.06 ± 0.26	10.46 ± 0.29	1.977 ± 0.073	$\pi \mp / p$	137	0.91			
15.82 ± 0.18	5.14 ± 0.25	3.539 ± 0.089	$\overline{K^{\mp}/p}$	85	0.76			
15.71 ± 0.2	4.83 ± 0.37	1.87 ± 0.13	K^{\mp}/n	48	0.56			
	0.0133 ± 0.0021		γ / p	34	0.56			
	$(-60. \pm 31.) \times 10^{-6}$		γ/γ	31	0.68			
	0.0257 ± 0.0041		γ / d	3	0.31			
64.75 ± 0.68	27.09 ± 0.78	15.47 ± 0.34	$\overline{p}(p) / d$	75	0.97			
36.63 ± 0.57	17.91 ± 0.76	0.38 ± 0.13	π^{\mp}/d	81	0.71			
32.26 ± 0.43	7.05 ± 0.66	5.74 ± 0.15	K^{\mp}/d	67	0.67			
	$M = 2.074 \pm 0.0^{\circ}$	= 0.86						
$\frac{11}{\sqrt{2}} = \frac{1}{\sqrt{2}} = $	$\eta_1 = 0.411 \pm 0.0$	$FQ_{INT} = 0.00$						
at √s ≥7 GeV	$\delta = (3.113 \pm 0.0$	$\mathbf{r}\mathbf{Q}_{\mathrm{EXT}} = 0.07$						
D [].]	D [mh]	D [mh]	Boom/Torgot	Npt = 837	χ^2/npt			
τ [πιν]	$\mathbf{K}_1 [\mathbf{m}\mathbf{D}] = \mathbf{K}_2 [\mathbf{m}\mathbf{D}]$		Deally I alger	npt	by groups			
33.7 ± 0.33	13.7 \pm 0.33 7.77 \pm 0.18 $\overline{p}(p) / p$ 220				1.08			
33.75 ± 0.38	14.07 ± 0.63	6.94 ± 0.29	$\overline{p}(p) / n$	48	0.39			

Simultaneous fit results

HPR1R2 at √s ≥7 GeV	$M = 2.074 \pm 0.0$ $\eta_1 = 0.411 \pm 0.0$ $\delta = (3.113 \pm 0.0)$	$FQ_{INT} = 0.86$ $FQ_{EXT} = 0.87$			
P [mb]	R ₁ [mb]	R ₂ [mb]	Beam/Target	Npt = 837 npt	χ^2/npt by groups
33.7 ± 0.33	13.7 ± 0.33	7.77 ± 0.18	$\overline{p}(p) / p$	220	1.08
33.75 ± 0.38	14.07 ± 0.63	6.94 ± 0.29	$\overline{p}(p) / n$	48	0.39
33.2 ± 3.9	$-14. \pm 47.$	$-14. \pm 52.$	Σ^- / p	8	0.41
18.06 ± 0.29	10.46 ± 0.32	1.977 ± 0.078	π^{\mp}/p	137	0.91
15.82 ± 0.2	5.14 ± 0.28	3.539 ± 0.095	K [∓] / p	85	0.76
15.71 ± 0.22	4.83 ± 0.4	1.87 ± 0.13	K [∓] / n	48	0.56
	0.0133 ± 0.0023		γ / p	34	0.56
	$(-60. \pm 33.) \times 10^{-6}$		γ/γ	31	0.68
	0.0257 ± 0.0044		γ / d	3	0.31
64.75 ± 0.75	27.09 ± 0.85	15.47 ± 0.37	$\overline{p}(p) / d$	75	0.97
36.63 ± 0.62	$1\overline{7.91 \pm 0.82}$	0.38 ± 0.14	π^{\mp}/d	81	0.71
32.26 ± 0.46	7.05 ± 0.71	5.74 ± 0.16	K^{\mp}/d	67	0.67



Data errors \Rightarrow **Parameter errors**

Fit to data above $\sqrt{s} \ge 5 \text{ GeV}$, $\ge 7 \text{ GeV}$



Data errors \Rightarrow **Parameter errors**

Fit to data above $\sqrt{s} \ge 8 \text{ GeV}$, $\ge 9 \text{ GeV}$

SUMMARY

$\frac{\sqrt{s}}{\text{GeV}}$	$\frac{\chi^2}{\text{ndf}}$	H _M mb	M GeV	λ	 MC 	CN_{Hess}	CN _{MC}
≥ 5	0.96	0.272(4)	2.122(14)	1.625(48)	10 ⁶	137197	26312
≥ 6	0.90	0.283(4)	2.079(15)	1.495(53)	2×10 ⁵	69375	13950
≥ 7	0.87	0.284(4)	2.074(16)	1.456(58)	3×10 ⁵	353613	14051
≥ 8	0.77	0.285(4)	2.072(15)	1.492(64)	3.5×10 ⁴	388426	13209
≥ 9	0.75	0.288(4)	2.063(16)	1.576(74)	3.5×10 ⁴	336272	11105
≥10	0.73	0.290(5)	2.054(16)	1.639(93)	3.5×10 ⁴	273180	11955

HPR1R2 model give the reliable quantitative description of the forward observables $\sigma = \text{Im}[A]$ and $\rho = \frac{\text{Re}[A]}{\text{Im}[A]}$ measured up to date at collision energies greater than 5 GeV (ECM)