z-Scaling:
Self-similarity from hundreds of MeV to TeV energies

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ABSTRACT– A brief review of z-scaling concept for analysis of hadron production over a range from MeV to TeV energies is given. The concept is based on fundamental principles of self-similarity, locality, and fractality of hadron interactions. This is verified over a wide range of collision energy and transverse momentum for different particle species, jets and top quarks produced at U70, ISR, RHIC, Tevatron and LHC. A hadron spectrum is described by scaling function $\psi(z)$ as a function of self-similarity variable z. Properties of $\psi(z)$ such as energy, angular, and flavor independence are described. The shape of the scaling function reveals a power law at low and high z. Verification of the z-scaling and search for new physics in high-p_T and cumulative regions are discussed.

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1. INTRODUCTION

Symmetry principles have played a central role in much of the physics of the past century, from the Lorentz invariance of classical electrodynamics to the gauge symmetries underlying the standard model of strong and electroweak interactions [1]. Symmetry is generally accepted to be an important tool for study of such phenomena in particle and nuclear physics as structure and interactions of hadron constituents and phase transitions in nuclear medium. Fundamental symmetry principles dictate the basic laws of physics, control the structure of matter and define the fundamental forces in nature [2, 3]. The local gauge symmetry is used to formulate classical and quantum electrodynamics, quantum chromodynamics, electroweak theory and gravity [1]. The concepts of "scaling" and "universality" have been widely developed to explain the critical phenomena [4, 5]. In high energy collisions, the scaling regularities were subject of intense investigations. Most famous among them are Bjorken, Feynman and Polyakov-Koba-Nielsen-Olesen scaling, Matveev-Muradyan-Tavkhelidze and Brodsky-Farrar quark counting rules and others (see [6] and references therein).

The z-scaling [7, 8] is one of the methods allowing systematic analysis of data on inclusive cross sections over a wide range of the collision energies, multiplicity densities, transverse momenta, and angles of various particles found in high energy proton-(anti)proton collisions. The approach was also applied for description of hadron production in heavy ion collisions [6, 9]. The concept of the z-scaling [10] is based on the principles of self-similarity, locality and fractality reflecting general features of particle interactions at high energy and transverse momentum. The approach can be used to search for and study of new symmetries that reflect fractal properties of hadron interactions at constituent level [11, 12]. We believe that fractal dimensions, like other particle characteristics such as mass, spin, charge, and flavor, have their origins in the structure of spacetime and in the properties of gravity at small scales.

2. z-SCALING

Here, we briefly remind the physics concept of z-scaling [10]. We assume that the collision of protons or nuclei at high energies is considered as an ensemble of individual interactions of their constituents. Colliding objects and produced particles are characterized by thier masses M_1, M_2, momenta P_1, P_2, and model parameters – fractal dimensions δ_1, δ_2 and ϵ_a, ϵ_b describing structure and fragmentation process, respectively. The constituents of the incoming objects carry their fractions x_1, x_2. The registered particle with mass m_a and the

4-momentum p carries the fraction y_a of the 4-momentum of the scattered constituent. Its hadron counterpart (antiparticle) with mass m_b carries the y_b fraction of the produced recoil. The sub-processes are considered as binary collisions of the constituents with masses (x_1M_1) and (x_2M_2) which result in the scattered and recoil objects in the final state with masses (m_a/y_a) and M_X, respectively. We assume that the momentum conservation law written in the form

$$(x_1P_1 + x_2P_2 - p/y_a)^2 = M_X^2 \tag{1}$$

is fullfiled in every sub-process, where $M_X = (x_1M_1 + x_2M_2 + m_b/y_b)$ is named as a recoil mass. The associate production of the particle with mass m_b ensures the conservation of additive quantum numbers. The structure of the colliding nuclei and fragmentation of the objects produced in constituent collisions are characterized by the function

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\epsilon_a}(1-y_b)^{\epsilon_b}. \tag{2}$$

The function $\Omega(x_1, x_2, y_a, y_b)$ has physical meaning of the relative number of all such configurations in the initial and final state, which contain the configuration defined by the fractions x_1, x_2, y_a and y_b. The self-similarity variable z is defined as follows $z = z_0\Omega^{-1}$, where z_0 is proportional to the transverse kinetic energy of the selected sub-process consumed on the production of the inclusive particle with 4-momentum p and mass m_a, and its counterpart with mass m_b. The quantity z has the property of a fractal measure, whereas Ω^{-1} has meaning of a resolution with respect to all underlying binary sub-processes. The minimality requirement of resolution Ω^{-1} allows us to determine the momentum fractions and fractal dimensions simultaneously fulfilling the condition (1) and scaling behavior of $\psi(z)$.

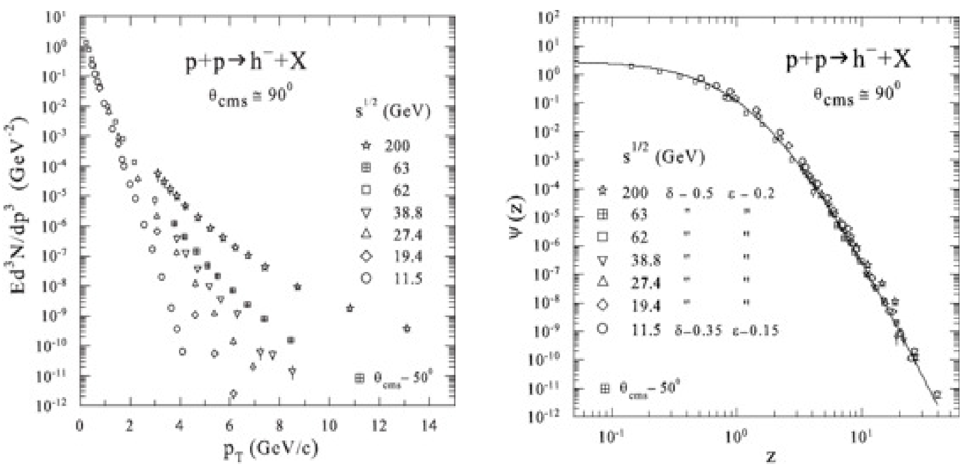
The scaling function $\psi(z)$ is expressed in terms of the experimentally measured inclusive invariant cross section $Ed^3\sigma/dp^3$, the multiplicity density $dN/d\eta$ and the total inelastic cross section σ_{in} as follows

$$\psi(z) = \frac{\pi}{(dN/d\eta)\ \sigma_{in}} J^{-1} E\frac{d^3\sigma}{dp^3} \tag{3}$$

Here J is the Jacobian of the transformation from $\{p_T^2, y\}$ to $\{z, \eta\}$. The multiplicity density concerns particular hadrons species. The integral of $\psi(z)$ over the interval $(0, \infty)$ is normalized to unity. This allows us to interpret the scaling function as a probability density for producing a hadron in an inclusive process with a corresponding value of z.

3. HADRON PRODUCTION IN $P-P$ COLLISIONS

The experimental data on negative hadron spectra measured in proton-proton collisions at U70, FNAL, ISR and RHIC over an energy range $\sqrt{s} = 11.5 - 200$ GeV and $\theta_{cms} \simeq 90^0$ in p_T- and z-presentation are shown in Fig.1 (see [6] and referencies therein).



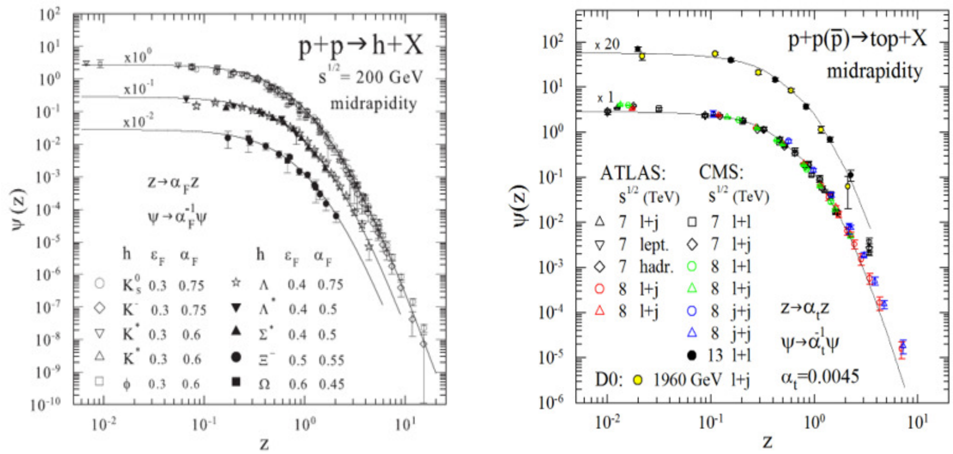
a) b)

FIG. 1. (a) The inclusive spectra and (b) the scaling function $\psi(z)$ for negative hadrons produced in $p-p$ collisions at $\sqrt{s_{NN}} = 11.5 - 200$ GeV and $\theta_{cms} = 90^0$ in p_T- and z-presentation [6]. The symbols correspond to the data obtained at U70, FNAL, ISR and RHIC. The solid line is a fitted line for these data.

One can see a strong dependence of the spectra on the transverse momentum p_T, as the collision energy $\sqrt{s}$ increases. The collapse of data points onto a single curve, $\psi(z)$, is clearly observed. Evidence for the power behavior of $\psi(z)$ at low and high values of z has been found. The scaling behavior agrees with the constant values of fractal dimensions, δ, and "specific heat", c. Decreases of δ are found for $\sqrt{s} < 20$ GeV. These results support the assumption about self-similarity of the particle structure, interactions and fragmentation process of their constituents over a wide range of scales.

3. FLAVOR UNIVERSALITY OF z-SCALING

The type of hadron is determined by the quark flavor (u, d, c, s, t, b) content. The flavor origin is one of the mysteries of nature. Figure 2 shows the flavor universality of the scaling function $\psi(z)$ for strange hadrons (a) produced at RHIC and top quark (b) produced at Tevatron, and LHC energies, respectively (see [8, 9, 13] and referencies therein).



a) b)

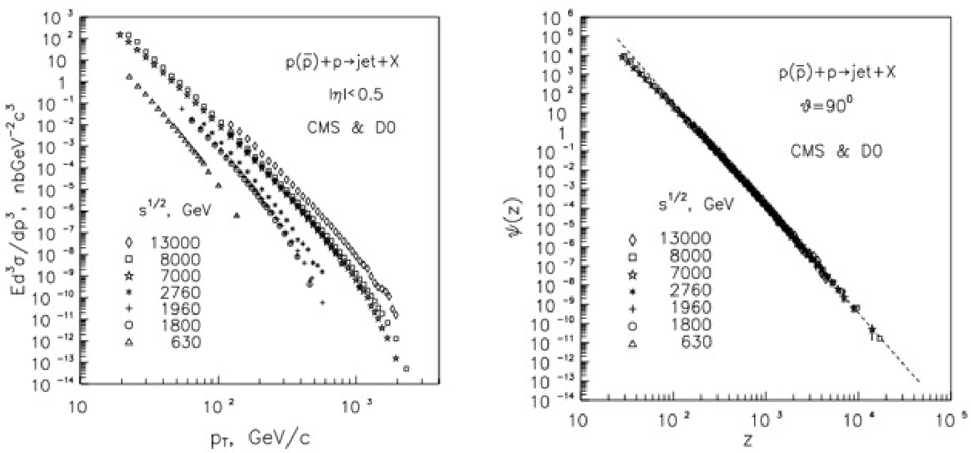
FIG. 2. (a) The scaling function $\psi(z)$ for strange hadron production in $p{-}p$ collisions at RHIC [8, 9]. (b) The scaling function $\psi(z)$ for top-quark production in $\bar{p}-p$ and $p-p$ collisions at Tevatron and LHC [13].

One can see from Fig. 2 the collapse of experimental data points onto a single curve for particles with various flavors. The mass of the produced particles varies over a range of 0.14–172 GeV/c^2. A power law of $\psi(z)$ is observed up to $z \simeq 10^{-2}$ for low z and 10^2 for high z. We note that the structural fractal dimension is the same for protons and anti-protons, while the value of the fragmentation fractal dimension depends on the type of particle. We expect that the fractal dimensions of both colliding and produced particles will be different for mesons and baryons. It should be noted that for point-like particles, both values should be zero. The results of our analysis support the assumption of the universality of $\psi(z)$. Nevertheless, further experimental verification with higher accuracy is needed.

4. SELF-SIMILARITY OF JET PRODUCTION

The production of jets with high transverse momentum in high energy particle collisions is usually considered as a direct manifestation of the hadron structure at small scales. The search for substructure of colliding particles, interactions of their constituents, and fragmentation process at small scales, is associated with existence of new phenomena such as quark compositeness, phase transitions, extra dimensions, and spacetime fractality.

Figure 3 demonstrates the spectra of jet production at Tevatron and LHC energies in p_T- and z-presentation (see [10, 14] and referencies therein). The collision energy $\sqrt{s}$ and transverse momentum p_T cover the ranges 630-13000 GeV and 20-2400 GeV/c, respectively. The dependence of the spectra on transverse momentum enhances with the collision energy. The same spectra in z-presentation demonstrate the straite line coreponding to the power law in double-log scale.



e) f)

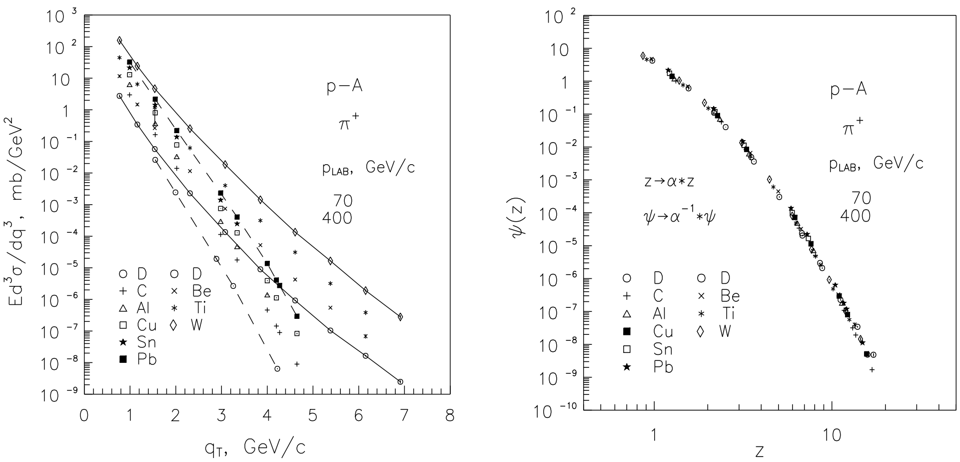
FIG. 3. (a) The inclusive spectra and (b) the scaling function $\psi(z)$ for jet production in $\bar{p}-p$ and $p-p$ collisions at Tevatron and LHC [14, 15].

The verification of the power law found in jet production at higher p_T is of greate interest for searching for and study fractality of momentum space, verification of Lorentz transformation and self-similarity of tagged jets.

4. SELF-SIMILARITY OF CUMULATIVE HADRON PRODUCTION

The effects of the nuclear matter environment on the particle production are studied in $p-A$ collisions (see [16] and referencies therein). One assume that the sensitivity of constituent interactions to nuclear effects manifest itself more evident at high p_T and especially near a kinematic boundary. The structural fractal dimension of a nucleus δ_A is a new characteristic of the composite system.

Figure 4 shows inclusive spectra in dependence on transverse momentum of π^+ mesons produced in $p-A$ collisions, measured at U70 in IHEP (Protvino) and PS in FNAL (Batavia). The momentum of incident proton was 70 GeV/c and 400 GeV/c and the angle $\theta_{cms} \simeq 90^0$, respectively. The atomic weight of the nuclear target is varied from 2 for deuteron up to 207 for lead. One can see the strong dependence of the inclusive spectra on p_T, and the

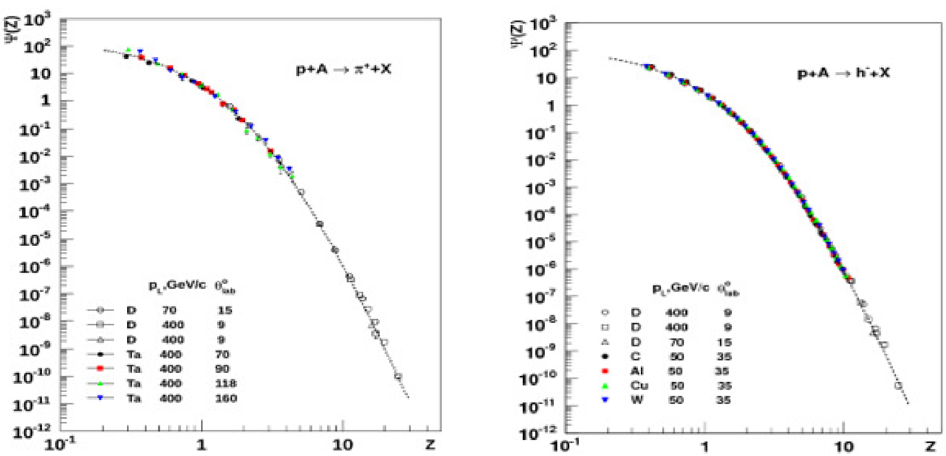


a) b)

FIG. 4. (a) The inclusive spectra and (b) the scaling function $\psi(z)$ for π^+-meson production in $p-A$ collisions at U70 and Fermilab proton synchrotron [16].

collapse of data points describing the scaling function $\psi(z)$. The scaling behavior of $\psi(z)$ is in agreement with the assumption on additivity law for the nucleus fractal dimension $\delta_A = A\delta$. Similar results were obtained for other hadrons $\pi^-, K^\pm, \bar{p}$ [16] as well.

The kinematic region for the particle production in $p-A$ collisions forbidden in free nucleon-nucleon interactions is named as a cumulative one [17–19]. The different mechanisms were proposed to describe a cumulative hadron production (rescattering and multiple scattering of hadrons and partons, fluctuation of nuclear matter – fluctons, quark clusters, and merging and splitting colored strings). Figure 5(a) shows the scaling function for high-p_T hadron production in $p-A$ collisions in comparison with the cumulative data obtained at low p_T (see [20–22] and referencies therein). The data points measured in the first region are marked by open symbols and color symbols correspond to the second region. One should be noted that both the data measured in the non-cumulative region at high p_T and the data measured in the cumulative region at low and high p_T are described by the same scaling function. A good agreement between these data in the overlaping region is observed. The transition from cumulative to non-cumulative region is found to be sufficiently smooth, and there is no discontinuity in the fractal dimension. The scaling function demonstrates transition to power behavior at high z in the overlapping region. The A-dependence of z-scaling is described by a transformation of the self-similarity variable z and the scaling function ψ as follows: $z \to \alpha(A)z$ and $\psi \to \alpha^{-1}(A)\psi$. The results indicate that the modification of



a) b)

FIG. 5. (a) The scaling function $\psi(z)$ for π^+ mesons produced in $p+A$ collisions at $p_L = 70$ and 400 GeV/c and $\theta_{lab} = 15^0$ and $9^0, 70^0, 90^0, 118^0, 160^0$ [20]. (b) The scaling function $\psi(z)$ for negative hadrons h^- produced in $p+A$ collisions at $p_L = 50, 70, 400$ GeV/c and $\theta_{lab} = 35^0, 15^0$ and 9^0 [21]. The symbols correspond to data obtained at U70 in IHEP (Protvino) and PS in FNAL (Batavia). The solid line is a reference curve for $p+D$ collisions.

constituent interaction by the nuclear environment occurs in a self-similar manner.

5. CONCLUSIONS

A brief review of the z-scaling approach to analyzing inclusive spectra of hadron and jet production in high-energy $p{-}p$, $\bar{p}{-}p$ and $p{-}A$ collisions has been given. General properties of the experimental data presented in z presentation were discussed. We have justified that the principles of self-similarity, locality and fractality reflect fundamental properties of particle structure, constituent interactions and fragmentation processes at high momentum transfer. We consider it would be interesting to verify further the universality of scaling functions, to determine the fractal dimension of hadrons and nuclei at high transverse momentum in the cumulative region and to search for signatures of phase transitions in nuclear matter.

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