Covariant reggeization framework and cross-sections of diffractive processes

R. A. Ryutin[1]

[1] *NRC Kurchatov Institute– IHEP, Protvino, Moscow region, 142281, Russia*

A version of the Regge approach is considered, in which the Reggeon is initially treated as a quantum field with arbitrary spin in Minkowski space, followed by an analytical continuation of the diffractive amplitudes into the complex angular momentum domain. Results are presented for irreducible tensors and their contractions, from which cross sections of all basic diffraction processes are calculated. The three-Reggeon approximation and the limit at small momentum transfer is also considered.

INTRODUCTION

In this paper, we consider Reggeons as objects arising from irreducible tensors of the Poincar group in Minkowski space $\mathbb{M}_D = \mathbb{R}^{1,D-1}$ for arbitrary dimension D and spin J after contracting them over all indices and then analytically continuing the resulting cross-sections to the domain of complex angular momentum. This sentence contains the entire description of the approach.

This work was started quite a long time ago [1]-[4] (see also old papers [5]-[9]) and was motivated by several facts:

- First, study of the spin-parity analysis in the exclusive central diffraction [1],[2], where the covariant approach yields strictly defined cross-section behavior for fixed spins. However, since calculations for arbitrary spins, as they should be in principle, are quite cumbersome, many authors (see [10] for example) mix integer spins and trajectories in their approach, this is incorrect. For high energies, this is less critical, and may give similar results, but if the model is used for low (like in U-70, for example) and intermediate energies, significant discrepancies are possible.

- Secondly, the direct application of this approach leads to zeros in the cross sections in the limit of small momentum transfer [4], which was demonstrated quite a long time ago for conserved currents [11]. This is particularly visible in single dissociation and central production. However, experimental data do not show such a behaviour. Of course, reggeon is a rather complex object, and the requirement for current conservation is likely unnecessarily strong. However, the idea arose to try to eliminate this behavior near zero momentum transfer squared in various ways, so that the model could be used for data analysis. It was shown that unitarization can smooth out the zeros effect[4]. Another solution is to assume that the current is conserved in more dimensions; then, in four dimensions, it will be effectively non-conserved.

- Third, we would like to have a clearer understanding of the reggeon from a quantum field theory perspective, and what the obtained cross sections for its scattering by hadrons mean. We can conditionally consider it analogous to an atom with its energy levels and scattering, or a string, or simply a mathematical object corresponding to a mixture of various processes. Or we can consider it, among other things, as a reggeized quantum field of arbitrary spin. In the last case it leads to rigorous, rather beautiful mathematical consequences, which can be tested experimentally to determine the extent to which they are fulfilled or violated.

- The next question to be solved is whether we can extract reggeonhadron and reggeon-reggeon cross sections from data, which was made first by Kaidalov [12], and what is the physical meaning of these cross sections, which can be of the order of hadronic cross sections, as was shown in the previous paper [4].

I. BASICAL SCHEME AND TENSORS FOR DIFFRACTIVE PROCESSES

We start with the field $\Phi^{\mu_1 \cdots \mu_J}(x)$ and its current tensor $\mathcal{I}^{\mu_1 \cdots \mu_J}$ corresponding to the spin J, which are an irreducible tensors of the Poincar group in Minkowski space $\mathbb{M}_D = \mathbb{R}^{1,D-1}$ of arbitrary dimension D, satisfying, together with its current, the standard Rarita-Schwinger conditions: transverseness, symmetry, tracelessness (TST).

Then we obtain the scalarscalartensor vertex $\mathcal{V}$ and the propagator $\mathcal{P}$

$$\mathcal{V}_{\mu_1 \dots \mu_J}(p,q) = < p-q|\mathcal{I}_{(\mu_1 \dots \mu_J)}|p>, \tag{1}$$

$$\begin{aligned}\mathcal{P}_{(\mu_1 \dots \mu_J),(\nu_1 \dots \nu_J)}(q) &= \int d^4x \; e^{iq(x-y)} \, \langle 0| \; \Phi_{\mu_1 \dots \mu_J}(x) \Phi_{\nu_1 \dots \nu_J}(y) \; |0\rangle = \\ &= \Pi_{(\mu_1 \dots \mu_J),(\nu_1 \dots \nu_J)}(q)/(m^2(J) - q^2),\end{aligned} \tag{2}$$

which have the poles at $m^2(J) - q^2 = 0$, i.e. $J = \alpha_{\mathbb{R}}(q^2) \equiv \alpha_{\mathbb{R}}$, after an appropriate analytic continuation of the signatured amplitudes in J, where $\alpha_{\mathbb{R}}$ is the reggeon trajectory (reggeization prescription):

$$\sum_J F^J/(q^2 - m^2) \to \alpha'_{\mathbb{R}} \eta_{\mathbb{R}}(q^2) \Gamma(-\alpha_{\mathbb{R}}) F^{\alpha_{\mathbb{R}}}/2. \tag{3}$$

TST conditions for them are

$$q_\lambda \mathcal{T}_{\mu_1 \ldots \lambda \ldots \mu_J} = 0, \quad \mathcal{T}_{\ldots \mu_i \ldots \mu_j \ldots} = \mathcal{T}_{\ldots \mu_j \ldots \mu_i \ldots}, \quad g_{\mu_i \mu_j} \mathcal{T}_{\mu_1 \ldots \mu_i \ldots \mu_j \ldots \mu_J} = 0. \tag{4}$$

for any irreducible $\mathcal{T}_{\ldots} = \mathcal{T}_{\ldots}(\{q_i\}, \{p_i\})$,

$$\begin{array}{rcccccc} \mathcal{T} & = & \mathcal{V} & \mathcal{F} & \mathcal{W}(\text{or } \mathcal{P}) & \mathcal{Y} & \mathcal{H} \\ \Omega_T & = & \{i\} & \{i, i^\star\} & \{i, i'\} & \{1,2,3\} & \{1,1',2,2'\} \end{array} \tag{5}$$

$$1^\star = 2, 2^\star = 1; \qquad (i^\star)' = (i')^\star; \qquad i \in \{1, 1', 2, 2'\},$$

$$(r) \equiv \alpha_s^{(r)},\ s \in [1, J_r], \qquad r \in \{1, 1', 2, 2', 3\} \leftrightarrow \alpha^{(r)} \in \{\mu, \mu', \nu, \nu', \rho\} \tag{6}$$

Then we can construct vertexes with multiple tensor legs with different spins and also different symmetric groups of Lorentz indexes at each leg. And we have the same conditions, but for each symmetric group of indexes and corresponding momentum transfers (see Fig. 1).

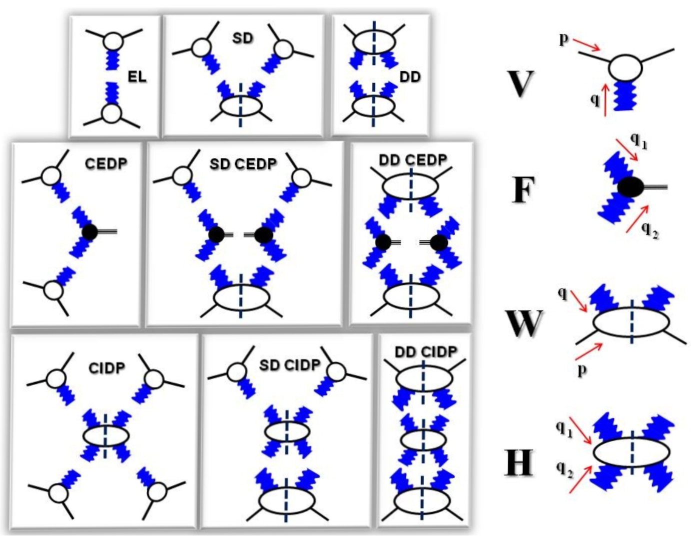

FIG. 1. Amplitudes of diffractive processes represented as contractions of basic Lorentz tensors. Abbreviations: EL (elastic), SD (single dissociation), DD (double dissociation), CEDP (central exclusive diffractive production), CIDP (central inclusive diffractive production).

In our approach we consider only totally symmetric tensors (in each symmetric group of Lorentz indices) without mixed symmetry for $D > 4$. This is an effective, fairly simple model that requires further refinement and generalization, at least to curved spaces of the anti-de Sitter type, but the fundamental influence of additional dimensions can be traced even in this way.

Let us consider the "harmonical" basis, where we have basic transverse symmetric tensors $S^{T\,\vec{J}}_{\vec{k}',\vec{n}}$ which are made of all possible combinations (marked by vector indexes) of simple transverse tensors

$$\mathcal{G}_{\alpha\beta}(v_1, v_2) = g_{\alpha\beta} + \frac{v_2^2 v_{1\alpha} v_{1\beta} + v_1^2 v_{2\alpha} v_{2\beta} - (v_1 v_2)(v_{1\alpha} v_{2\beta} + v_{2\alpha} v_{1\beta})}{(v_1 v_2)^2 - v_1^2 v_2^2} \tag{7}$$

$$P_\alpha(v_1, v_2) = (v_{1\alpha} - \frac{v_1 v_2}{v_2^2} v_{2\alpha}) / \sqrt{|v_1^2 - \frac{(v_1 v_2)^2}{v_2^2}|} \tag{8}$$

for different $v_2 = q, q_i$ (momentum transfer) and $v_1 = p, p_i$ (incoming/outcoming momenta of a vertex) (see [13] for details). Powers of tensors $\mathcal{G}$, P denote symmetrized products (sum of unique terms without any multiplier) of these tensors in each group of Lorentz indices:

$$S^{T\,\vec{J}}_{\vec{k}\,\vec{n}} \equiv \left(\prod_{i \in \Omega_T} P^{\otimes J_i^*}_{i(i)} \mathcal{G}^{\otimes n_i}_{(ii)} \prod_{\substack{\forall r \neq s \\ r,s \in \Omega_T}} \mathcal{G}^{\otimes k_{rs}}_{(rs)} \right), \quad J_i^* = J_i - 2n_i - \kappa_i, \quad \kappa_i = \sum_{r \neq i} k_{ir}. \tag{9}$$

Each irreducible tensor is decomposed over a basis of irreducible representations, the choice of which is quite arbitrary, and contains a certain number of independent components corresponding to the degrees of freedom for the chosen spin values. In the work [13] two different basises were considered. In every basis tracelessness is ensured by the tensor $G_{\alpha\beta} = g_{\alpha\beta} - q_{i\alpha} q_{i\beta} / q_i^2$, which is essentially a metric orthogonal only to the momentum transfer and "lives" in Minkowski space $\mathbb{M}_{D-1}$.

The formulas and equations for the coefficients in the "harmonical" basis are much simpler, and the basic tensors are finite in the limit of small momenta transfers, which is very convenient for calculations. Any irreducible tensor can be represented as follows

$$\boxed{\mathcal{T}^{\vec{J}} = \sum_{\bar{\Omega}^T_{\vec{k}}} \hat{\tau}^{\vec{J}}_{\vec{k}} \mathcal{T}^{*\,\vec{J}}_{\vec{k}}, \qquad \mathcal{T}^{*\,\vec{J}}_{\vec{k}} = \sum_{\tilde{\Omega}^T_{\vec{k}'\,\vec{n}}} \tau^{\vec{k}'(\vec{k};\vec{J})}_{\vec{n}} S^{T\,\vec{J}}_{\vec{k}'\,\vec{n}},} \tag{10}$$

$\hat{\tau}^{\vec{J}}_{\vec{k}}$ are form-factors, and regions of summation are

$$\tilde{\Omega}^T_{\vec{k}'\,\vec{n}}\!: J_r^* \geq 0,\ \kappa_r \geq \kappa_r',\ n_r \geq 0,\ k_{rs}' \geq 0, \quad r, s \in \Omega_T,\ r \neq s; \qquad \bar{\Omega}^T_{\vec{k}} = \tilde{\Omega}^T_{\vec{k}\,\vec{0}}. \tag{11}$$

The basis of generating functions is

$$\boxed{\mathit{\Phi}^{\star}{}^{\mathcal{T}}_{\vec{k}}(\vec{x},\vec{u},\vec{\vec{u}}) = \sum_{\tilde{\Omega}^{T}_{\vec{k}',\vec{n}}} \tau^{\vec{k}'(\vec{k};\vec{J})}_{\vec{n}} \mathcal{N}^{\vec{J}}_{\vec{k}'\vec{n}}\, \vec{x}^{\otimes \vec{J}^{*}} \vec{u}^{\otimes \vec{n}}\, \vec{\vec{u}}^{\otimes \vec{k}'} = \frac{\mathcal{N}^{\vec{J}}_{\vec{k}\vec{0}}}{\Pi_0^{\vec{J}\vec{k}}} \prod_{s\in\Omega_T} \hat{\Pi}(\Box_{\omega_s})\, \vec{x}^{\otimes \vec{J}-\vec{k}} \vec{\vec{u}}^{\otimes \vec{k}}} \tag{12}$$

$$\Pi_0^{\vec{J}\vec{k}} = \prod_{i\in\Omega_T} {}_2F_1\left(-\frac{J_i-\kappa_i}{2}, \frac{1-J_i+\kappa_i}{2}; c^{J_i}; 1\right), \tag{13}$$

$$\Box_{\omega_s} = g_{\alpha\beta}\, \partial_{\omega_s\,\alpha}\partial_{\omega_s\,\beta}, \quad \partial_{\omega_s\,\alpha} = \frac{\partial x_s}{\partial_{\omega_s\,\alpha}}\partial_{x_s} + \frac{\partial u_s}{\partial_{\omega_s\,\alpha}}\partial_{u_s} + \sum_{r\neq s}\frac{\partial u_{rs}}{\partial_{\omega_s\,\alpha}}\partial_{u_{rs}} \tag{14}$$

$$\boxed{\hat{\Pi}(\Box_{\omega_s})\vec{x}^{\otimes \vec{J}-\vec{k}}\vec{\vec{u}}^{\otimes \vec{k}} = \sum_{m_s=0}^{[J_s/2]} \frac{(u_s+x_s^2)^{m_s}}{4^{m_s} m_s! (c^{J_s})_{m_s}} (\Box_{\omega_s})^{m_s}\, \vec{x}^{\otimes\, \vec{J}-\vec{k}} \vec{\vec{u}}^{\otimes\, \vec{k}}|_{\omega_s\to 0}} \tag{15}$$

$$\boxed{\mathcal{T}^{\vec{J}} = \sum_{\tilde{\Omega}^{T}_{\vec{k}}} \hat{\tau}_{\vec{k}}\, \frac{1}{\vec{J}!} \prod_{s\in\Omega_T} \partial^{\otimes J_s}_{\omega_s}\, \mathit{\Phi}^{\star}{}^{\mathcal{T}}_{\vec{k}}(\vec{x},\vec{u},\vec{\vec{u}})|_{\{\omega_s\}\to 0}} \tag{16}$$

The notations are $\vec{x} = \{x_s\}, \vec{u} = \{u_s\}, \vec{\vec{u}} = \{u_{rs}\}$,

$$x_s = P_{s(s)} \otimes \omega_s, \quad u_s = \mathcal{G}_{(ss)} \otimes \omega_s \otimes \omega_s, \quad u_{rs} = \mathcal{G}_{(rs)} \otimes \omega_r \otimes \omega_s, \tag{17}$$

$$c^{J} = -\left(J + (D-5)/2\right), \qquad \mathcal{N}^{\vec{J}}_{\vec{k}\vec{n}} = \vec{J}!/(2^{|\vec{n}|}\vec{n}!\vec{k}!\vec{J}^{*}). \tag{18}$$

$$A^{\otimes b} = A_{\alpha_1}...A_{\alpha_b}, \quad \vec{a}^{\,\vec{b}} = \prod_i a_i^{b_i}, \quad \vec{a}\,! = \prod_i a_i!, \quad |\vec{a}| = \sum_i a_i \tag{19}$$

II. GENERAL FORMULA FOR CROSS-SECTIONS

The algorithm for cross sections (convolutions of tensors) is presented below for W tensors (DD cross section). It easily generalizes to other contractions [14]. Then cross-sections can be expressed in terms of hypergeometric like functions and continued to the complex J.

$$\frac{d\sigma_{DD}}{d\Phi} = \sum_{k_1,k_2=0}^{\min(J,J')} \hat{w}_{k_1}\hat{w}_{k_2}\frac{d\sigma^{k_1k_2}_{DD}}{d\Phi}$$

$$\frac{d\sigma^{k_1k_2}_{DD}}{d\Phi} \sim \mathcal{W}^{*\vec{J}}_{k_2}(p_2,q) \otimes \mathcal{W}^{*\vec{J}}_{k_1}(p_1,q) = S^{W\,\vec{J}}_{k_2,\vec{0}}(p_2,q) \otimes \mathcal{W}^{*\vec{J}}_{k_1}(p_1,q) =$$

$$= \frac{(P_2\partial_{\omega_1})^{J-k_2}(P_2\partial_{\omega_1'})^{J'-k_2}\left[\mathcal{G}_{\alpha\beta}\otimes\partial_{\omega_{1\alpha}}\partial_{\omega_{1'\beta}}\right]^{k_2}}{k_2!(J-k_2)!(J'-k_2)!}\mathit{\Phi}^{\star}{}^{\mathcal{W}}_{k_1}(\vec{x},\vec{u},\vec{\vec{u}})|_{\Omega_{ch}}$$

$$\Omega_{ch} : x_i = P_1\omega_i \to Z = P_1P_2, \quad \xi_i = P_2\omega_i \to P_2^2, \quad u_i, u_{11'} \to 1 - \frac{Z^2}{P_1^2} \tag{20}$$

SD and DD cross-sections for $D = 4$ are presented in [4] in the forward limit.

Nevertheless, the number of form factors sometimes remains large enough, and all they need to be fitted. It makes difficult to extract any physical information from the data. Therefore, one can, for example, require the finiteness of tensors in the forward limit and get linear equations for form-factors. Then, for tensors W, F, everything is expressed in terms of a single form factor, which can be extracted from the data. For H there still remains a fairly large number of form factors, for even $|\vec{J}|$

$$N_H = \mathbb{C}^2_{J_0}, \qquad J_0 = \min\left(J_1, (J_1 + J_2 + J_3 - J_4)/2\right) \tag{21}$$

But we can use, for example, their cross-channels symmetry to reduce their number to a minimum.

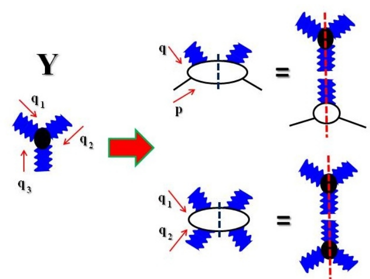

FIG. 2. Contractions of 3-Reggeon vertexes to obtain $W_{3\mathbb{R}}$ and $H_{3\mathbb{R}}$.

A three-Reggeon approximation can be used in W and H. We compute the forward limit of Y vertices, contract them and average over an arbitrary space-like direction that appears in this limit (see Fig. 2):

$$\mathcal{W}^{\vec{J}}_{3\mathbb{R}}(p,q) = <\mathcal{Y}^{\{J,J',J_3\}}_{fwd}(q,n) \otimes \mathcal{V}^{J_3}(n)>_{p,q}, \tag{22}$$

$$\mathcal{H}^{\vec{J}}_{3\mathbb{R}}(q_{1,2}) = <\mathcal{Y}^{\{J_1,J_{1'},J_3\}}_{fwd}(q_1,n) \otimes \mathcal{Y}^{\{J_2,J_{2'},J_3\}}_{fwd}(q_2,n)>_{q_1,q_2}, \tag{23}$$

$$\mathcal{Y}^{\vec{J}}_{fwd}(q,n) = \lim_{\substack{\epsilon\to 0\\ qn=0}} \mathcal{Y}^{\vec{J}}(q_1 = q + \frac{\epsilon}{2}n, q_2 = -q + \frac{\epsilon}{2}n), \tag{24}$$

$$\begin{aligned} <n_{\alpha_1}...n_{\alpha_{2N}}>_{p,q} &= \frac{\int d^D n\ \delta(n^2+1)\ \delta(pn)\ \delta(qn)\ n_{\alpha_1}...n_{\alpha_{2N}}}{\int d^D n\ \delta(n^2+1)\ \delta(pn)\ \delta(qn)} = \\ &= \frac{(-1)^N}{2^N\left(\frac{D-2}{2}\right)_N}\left(\mathcal{G}_{(rs)}(p,q)^{\otimes N}\right)_{all\ sym.}, \qquad N = |\vec{J}|/2, \end{aligned} \tag{25}$$

where "*all sym.*" means the symmetrization of all indices (even from different groups), and r, s can be from any group of indices defined by $\Omega_{W,H}$.

III. CONCLUSIONS

In this work we shortly presented the algorithm to calculate irreducible tensors in $\mathbb{M}_D$ and express all the basic diffractive cross-sections in terms of their contractions. The general form of irreducible tensor form factors can be used not only in diffraction processes. All the details can be found in [13],[14].

[1] V. A. Petrov, R. A. Ryutin, A. E. Sobol and J.-P. Guillaud, JHEP **06**, 007 (2005).

[2] A. B. Kaidalov, V. A. Khoze, A. D. Martin et al., Eur. Phys. J. C **31**, 387 (2003).

[3] R. A. Ryutin, Eur. Phys. J. C **74**, 3162 (2014).

[4] V. A. Petrov and R. A. Ryutin, Int. J. Mod. Phys. **31**, No. 10, 1650049 (2016).

[5] H. F. Jones, M. D. Scadron and F. D. Gault, Nuovo Cimento A **66**, 424 (1970).

[6] M. D. Scadron, Phys. Rev. **165**, 1640 (1968).

[7] F. D. Gault, H. F. Jones, Nucl. Phys. B **30**, Issue 1, 68 (1971).

[8] T.M. Aye, J. Phys. A **4**, Number 6, 846 (1971).

[9] H. F. Jones, M. D. Scadron, Nuclear Physics B, **4**, Issue 2, 267 (1968).

[10] C. Ewerz, M. Maniatis, O. Nachtmann, Annals Phys. **342**, 31 (2014).

[11] F. E. Close, G. A. Schuller, Phys. Lett. B **464**, 279 (1999).

[12] A. B. Kaidalov, Phys. Rept. **50**, 157 (1979).

[13] R. Ryutin, arXiv:2507.16019 [hep-ph];

[14] R. Ryutin, Covariant reggeization framework for diffraction. Part II: Hadronic diffractive cross-sections, *in preparation.*