Dispersive estimation of the LO hadronic contribution to the muon $g - 2$: what can we get from the e^+e^- data?

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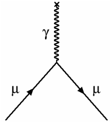
Introduction

$$\vec{\mu}_\mu = -g_\mu \frac{e}{2m_\mu} \vec{S}$$

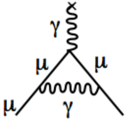
- $a_\mu = (g_\mu - 2)/2$ measured by FNAL Muon g-2 experiment to 124 ppb
- Long standing $\sim$ 4-5σ theory/experiment tension *(resolved by lattice QCD?)*
- $O(100)$ ppb precision SM test, sensitive to multi-TeV scale New Physics
  - Theory uncertainty mostly due to QCD

Tree level



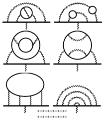
$a_\mu = 0$

Schwinger



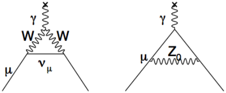
$+\frac{\alpha}{2\pi} = 11614097.3 \times 10^{-10}$

QED 2-5 loops



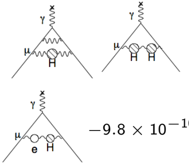
$+44374.6 \times 10^{-10}$

Electroweak



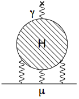
$+15.4 \times 10^{-10}$

Hadron VP NLO



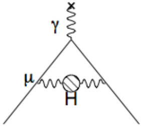
-9.8×10^{-10}

Hadron light-by-light



$+9.2 \times 10^{-10}$

Hadron VP LO

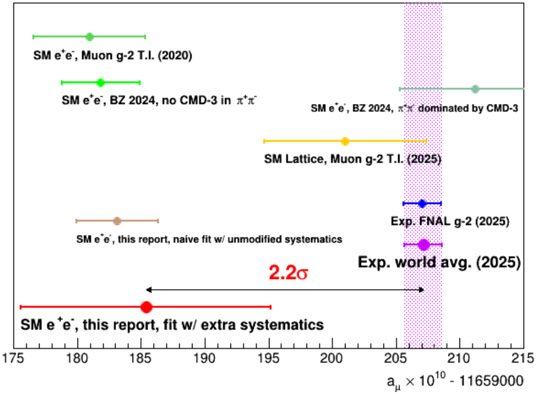


$+(\simeq 700\pm?) \times 10^{-10}$

$\longleftarrow$ **This talk**

- Uncertainty of the lattice QCD calculation became good enough by 2025.
- Dispersive calculation was the only sufficiently precise method before 2020s: long standing $\sim 4-5\sigma$ tension with the experiment.
  - Dramatic controversy between e^+e^- inputs since 2023.

$g_\mu - 2$: experiment vs theory



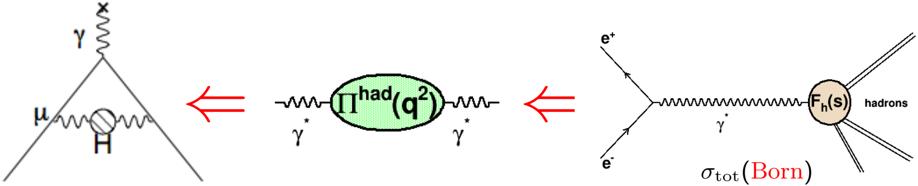
▶ Has the $g_\mu - 2$ anomaly gone with the lattice HVP estimate?

▶ The dispersive e^+e^- HVP is the only cross-check ⇒ try to treat incompatible $\sigma(e^+e^- \to hadrons)$ inputs consistently.

- World average including FNAL g-2 Run-1-6 [*Muon g-2 Collaboration, Phys. Rev. Lett. 135 (2025) 10, 101802*]: $a_\mu = 11659207.15(1.45) \times 10^{-10}$ **(124 ppb)**.
- SM prediction uncertainty mostly comes from the hadron LO VP term.
- Before CMD-3 $\pi^+\pi^-$ measurement: dispersive e^+e^- HVP was too low ⇒ $\sim 4-5\sigma$ tension with the experiment [*The White Paper '20 (Muon g-2 T.I.), Phys. Rept. 887 (2020) 1*].
- After CMD-3 [*Phys.Rev.D 109 (2024) 11, 112002*]: precise $\sigma(\pi^+\pi^-)$ measurements by CMD-3, BaBar, KLOE are incompatible at $\sim 3-5\sigma$ level [*see, e.g., Phys. Part. Nucl. 55 (2024) 6, 1432 and refs. therein*].

  ▶ HVP calculation in lattice QCD gives the SM a_μ with ~ 500 ppb uncertainty, consistent with the experiment [*The White Paper '25 (Muon g-2 T.I.), Phys. Rept. 1143 (2025) 1*].

$a_\mu(\mathrm{had, LO})$ via $\sigma(e^+e^- \to \mathrm{hadrons})$



The dispersion relation [*A. Petermann, Phys. Rev. 105 (1957) 1931*]:

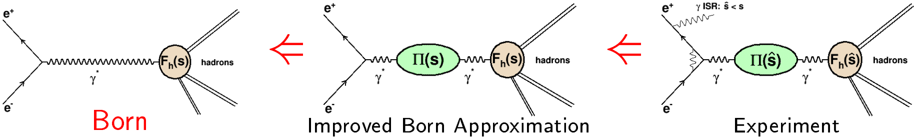
$$a_\mu(\mathrm{had, LO}) = 4\alpha_0^2 \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) \frac{1}{\pi} \mathrm{Im}\, \Pi^{\mathrm{had}}(s) = \frac{\alpha_0^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) R^{\mathrm{had}}(s)$$

$$R^{\mathrm{had}}(s) = \sigma_{\mathrm{tot}}(e^+e^- \to hadrons, \mathrm{Born}) \Big/ \frac{4\pi\alpha_0^2}{3s}$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)} \, .$$

$a_\mu(\mathrm{had,LO})$ via $\sigma(e^+e^- \to \mathrm{hadrons})$

[*Details of the procedure:* *Phys. Part. Nucl. 55 (2024) 6, 1432* *& our previous talk*]

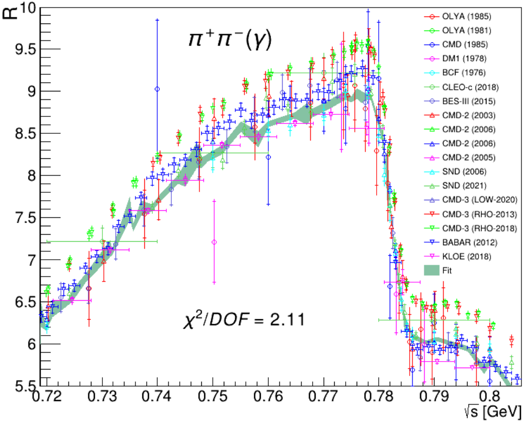


- We need first to uniformly rescale all published measurements to Born cross section.
- All experiments publish cross sections corrected for ISR + e^+e^- vertex loops. See the compilation of $\sigma_{\mathrm{tot}}(e^+e^- \to \mathit{hadrons})$ in the *PPDS CrossSection* database.
- Some experiments correct for photon VP, others leave the VP correction to readers.
- FSR correction. Additional hard γ's are rejected in the event selection to suppress backgrounds from other final states. Experimentalists then 'undress' the cross section, i.e. correct it for soft FSR using certain FSR model. Need to add the FSR back: $\sigma(\mathrm{hadrons}(+\gamma'\mathrm{s})) = \sigma(\mathrm{hadrons})C_{\mathrm{FSR}} = \sigma(\mathrm{hadrons})\left[1+\eta(s)\frac{\alpha}{\pi}\right]$, with $\eta(s)$ from scalar QED for $\pi^\pm$, $K^\pm$. The approximation: $C_{\mathrm{FSR}} = (1+0.004\pm0.004)^{N_{\mathrm{charged}}}$.
- $\sigma_{\mathrm{tot}}(e^+e^- \to \mathit{hadrons})$ is measured mostly inclusively at $\sqrt{s} > 2$ GeV and for (semi)exclusive final states at $\sqrt{s} < 2$ GeV. Most final states are measured by multiple experiments.
- Parameterize Born cross section in each final state by continuous piecewise linear function w/ positions of the nodes in $\sqrt{s}$ given by clusterization of experimental points.
- Perform a standard χ^2 fit of the parameterized cross section taking into account correlated uncertainties within each experiment and between experiments *[details in backup]*. The fitted $\sigma_{\mathrm{tot}}(\mathrm{Born})$ can be plugged into the dispersion integral to find the contribution to photon VP and thus $a_\mu(\mathrm{had,LO})$ at $0.3 < \sqrt{s} < 11.2$ GeV. Outside this range use analytic parameterizations of $\sigma_{\mathrm{tot}}(\mathrm{Born})$.

The problem: incompatible $\sigma(e^+e^- \to \text{hadrons})$ data

/ . . . Parameterize and fit Born cross section in each final state . . . /

- Significant tensions are present between cross sections of the same final state as measured by different experiments. The most dramatic effect is seen in the $\pi^+\pi^-$ channel comprising $\simeq 70\%$ of $a_\mu(\text{had}, \text{LO})$. KLOE, BaBar and CMD-3 claim $\sim 1\%$ systematic uncertainties but deviate at $\sim 3-5\sigma$ level $\longrightarrow$
/ new SND data support CMD-3! /
- At glance, this results in a poor fit $\longrightarrow$ of the cross section and makes the dispersive estimate of $a_\mu(\text{had}, \text{LO})$ senseless until the origin of the discrepancies is pinned down.
(E.g., Muon $g-2$ T.I. refused to quote dispersive $a_\mu(\text{had}, \text{LO})$ in 2025; possible origins of BaBar/KLOE/CMD-3 tensions have been widely discussed in the literature, still inconclusively).



▶ Incompatible measurements are presumably affected by some *missing systematics* which could not be evaluated in the experiments *in situ* but can be estimated *post factum* by comparing independent measurements. $\Longrightarrow$ **Infer missing systematics from the data and thus obtain a realistic uncertainty for the fitted cross section and hence for $a_\mu(\text{had}, \text{LO})$.**

The problem: incompatible $\sigma(e^+e^- \to \text{hadrons})$ data

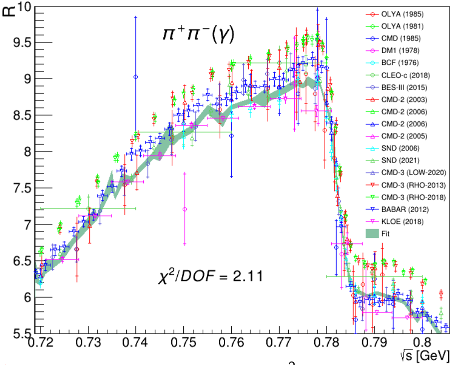
Is χ^2 distributed uniformly over degrees of freedom in the standard fit?

The error matrices are parameterized as

$$C = \mathrm{diag}(\sigma_{stat}{}_1^2, \ldots, \sigma_{stat}{}_n^2) + v_{sys} \otimes v_{sys}^T,$$
$$v_{sys} = (\sigma_{sys}{}_1, \ldots, \sigma_{sys}{}_n).$$

▶ For each experiment, find eigenvectors of the error matrix C, project the discrepancy $\Delta(s_i) = R_{exp}(s_i) - R_{fit}(s_i)$ onto the eigenvectors and find the contribution to χ^2 from each projection. ▶ Simplification: take $n_{sys} = v_{sys}/|v_{sys}|$ as the approximation for the first eigenvector, find the projection of $\Delta_i = R_{exp\,i} - R_{fit\,i}$ onto it: $\Delta_{sys} = n_{sys}(n_{sys}^T\Delta)$; compute $\Delta\chi^2_{sys} = \Delta_{sys}^T C^{-1}\Delta_{sys}$. Then find the residual: $\Delta_{res} = \Delta - \Delta_{sys}$, and compute $\Delta\chi^2_{res} = \Delta_{res}^T C^{-1}\Delta_{res}$.

n_p	$\Delta\chi^2/n_p$	$\Delta\chi^2_{sys}$	$\Delta\chi^2_{res}/n_p$	Pull	Exp.	Run
13	4.09	44.31	0.95	0.048	CMD-3	Low-2020
82	1.79	54.14	1.01	0.072	CMD-3	Rho-2013
114	1.99	93.79	1.11	0.071	CMD-3	Rho-2018
323	1.32	13.44	1.23	0.020	BaBar	2012
85	2.97	1.94	3.00	-0.009	KLOE	Comb-2018
60	0.86	0.29	0.85	0.002	BES-III	2015
35	0.67	1.36	0.63	0.016	CLEO-c	2018
43	0.90	8.37	0.81	0.018	CMD-2	2003
36	0.75	0.46	0.66	0.071	CMD-2	2005
10	1.71	8.10	1.35	-0.052	CMD-2	Low-2006
29	1.14	1.83	1.05	0.011	CMD-2	Rho-2006
45	1.53	0.01	1.54	-0.001	SND	2006
36	2.15	2.73	2.05	0.013	SND	2021
24	1.56	0.14	1.58	-0.007	CMD	1985
79	0.97	0.00	0.97	0.004	OLYA	1985



⟵ For a good fit, one expects $\Delta\chi^2 \sim 1$ for all degrees of freedom. The anomaly: $\Delta\chi^2_{sys} \gg 1$ for projections onto "systematic" eigenvectors while $\Delta\chi^2 \sim 1$ for the residual d.o.f. *(Except KLOE: bad parameterization of the error matrix? The original one seems to be lost.)*.

Birge scaling of the entire error matrices by χ^2/dof is inadequate: statistical errors are not severely underestimated as the residuals give $\Delta\chi^2_{res} \sim 1$ per d.o.f; on the other hand, $\Delta\chi^2_{sys} \gg 1$ for the "systematic" d.o.f. indicates that only systematic errors are underestimated.

⇒ Estimate missing systematics from the fit itself.

Missing systematic uncertainties

- Some heuristics:

Take the experiments with $\Delta\chi^2_{sys} > 6$, $\rightarrow$ compute their integral pulls relative to the fit

$$\mathrm{Pull} = \int (R_{exp}(s) - R_{fit}(s))\, ds \Big/ \int R_{fit}(s) ds\,,$$

and estimate the missing systematic uncertainty as $\epsilon = \sqrt{\langle \mathrm{Pull}^2 \rangle}$. This extra uncertainty has to be assigned to *all* experiments (neglect the correlations):

$$C_{mod} = \mathrm{diag}(\sigma_{stat}{}_1^2, \ldots, \sigma_{stat}{}_n^2) + v_{sys} v_{sys}^T + \epsilon^2 R_{fit} R_{fit}^T$$

n_p	$\Delta\chi^2/n_p$	$\Delta\chi^2_{sys}$	$\Delta\chi^2_{res}/n_p$	Pull	Exp.	Run
13	4.09	44.31	0.95	0.048	CMD-3	Low-2020
82	1.79	54.14	1.01	0.072	CMD-3	Rho-2013
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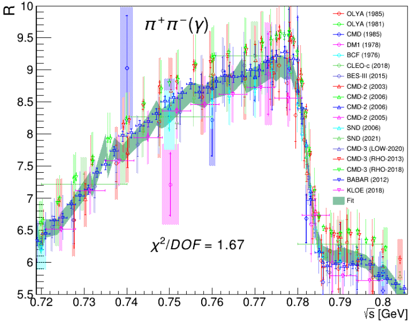
323	1.32	13.44	1.23	0.020	BaBar	2012
85	2.97	1.94	3.00	-0.009	KLOE	Comb-2018
60	0.86	0.29	0.85	0.002	BES-III	2015
35	0.67	1.36	0.63	0.016	CLEO-c	2018
43	0.90	8.37	0.81	0.018	CMD-2	2003
36	0.75	0.46	0.66	0.071	CMD-2	2005
10	1.71	8.10	1.35	-0.052	CMD-2	Low-2006
29	1.14	1.83	1.05	0.011	CMD-2	Rho-2006
45	1.53	0.01	1.54	-0.001	SND	2006
36	2.15	2.73	2.05	0.013	SND	2021
24	1.56	0.14	1.58	-0.007	CMD	1985
79	0.97	0.00	0.97	0.004	OLYA	1985
16	0.82	1.49	0.75	-0.042	DM1	1978

$\epsilon = \sqrt{\langle \mathrm{Pull}^2 \rangle} = 0.05$

- Now repeat the fit with the modified error matrices. $\Rightarrow$ $\Delta\chi^2_{sys}$ "peaks" are reduced as expected:



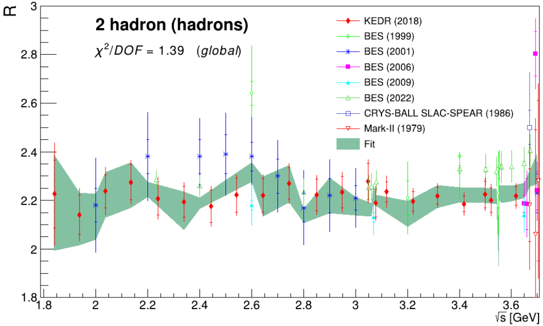
n_p	$\Delta\chi^2/n_p$	$\Delta\chi^2_{sys}$	$\Delta\chi^2_{res}/n_p$	Pull	Exp.	Run
13	1.10	1.01	0.95	0.053	CMD-3	Low-2020
82	1.15	3.19	0.99	0.059	CMD-3	Rho-2013
114	1.11	5.36	1.01	0.058	CMD-3	Rho-2018
323	1.20	0.05	1.18	0.007	BaBar	2012
85	2.14	1.39	2.27	-0.022	KLOE	Comb-2018
60	0.79	0.02	0.80	-0.011	BES-III	2015
35	0.66	0.02	0.65	0.003	CLEO-c	2018
43	0.87	0.06	0.85	0.007	CMD-2	2003
36	0.74	0.12	0.68	0.037	CMD-2	2005
10	1.21	1.10	1.28	-0.056	CMD-2	Low-2006
29	1.02	0.00	1.02	-0.001	CMD-2	Rho-2006
45	1.36	0.08	1.41	-0.012	SND	2006
36	2.07	0.00	2.06	0.002	SND	2021
24	1.56	0.00	1.56	-0.017	CMD	1985
79	0.87	0.07	0.90	-0.009	OLYA	1985
16	0.75	0.65	0.77	-0.053	DM1	1978

$\uparrow$ Stop upon this iteration as all $\Delta\chi^2_{sys} < 6$

Note: incompatible inclusive $R(s)$ measurements

- Inclusive $R(s)$ is used by us for the dispersion integral at $1.937 < \sqrt{s} < 11.2$ GeV.
- Incompatibility issues are less severe here than in the $\pi^+\pi^-(\gamma)$ channel, still worth to mention.
- Inclusive $R(s)$ in continuum can be confronted to pQCD with $\alpha_S(\mu)$ / $\Lambda_{\overline{\rm MS}}^{f=3,4,5}$ parameters extracted from fits to "orthogonal" data, particularly, at scales $\mu \gg m_{c,b}$.

▶ The *uds* continuum region: KEDR vs BES-III vs pQCD(!) issues
[see A.L. Kataev, K.Yu. Todyshev, arXiv:2603.29803 and refs. therein]



n_p	$\Delta\chi^2/n_p$	$\Delta\chi^2_{sys}$	$\Delta\chi^2_{res}/n_p$	Pull	Exp.	Run
22	0.51	0.06	0.50	0.006	KEDR	2018
6	1.70	0.99	1.17	0.074	BES	1999
85	1.03	0.06	1.01	0.012	BES	2001
68	1.60	0.13	1.56	0.013	BES	2006
3	0.97	2.20	0.31	-0.049	BES	2009
14	0.27	3.68	0.04	0.044	BES-III	2022
98	0.71	0.43	0.64	0.038	CRYS-BALL	1986
24	2.12	0.50	1.88	0.042	Mark-II	1979

Only our model-independent fit is shown.

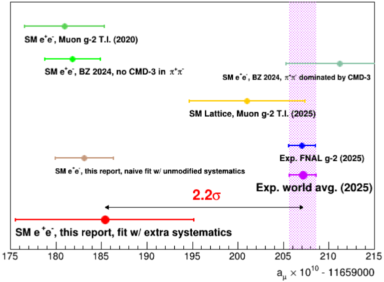
$\mathcal{O}(\alpha_S^4)$ pQCD (plus correction for quark-hadron duality violation) predicts R_{uds} lower than both KEDR and BES-III data *[D. Boito, M. Caram, Phys. Rev. D 112 (2025) 9, 9]*

Missing $\sigma_{\rm tot}(e^+e^- \to hadrons)$ systematics and $a_\mu(\rm had, LO)$

- The technique outlined on previous slides is used to estimate missing systematics in all channels affected by incompatible measurements of the $\sigma_{\rm tot}$: $\pi^+\pi^-(\gamma)$, K^+K^-, $\pi^+\pi^-\pi^0$, $\eta\gamma$, $\omega\pi$. The extra systematic uncertainty is then propagated to $a_\mu(\rm had, LO)$. [*Open questions, todo's ...*]
- In the literature, uncertainties related to incompatible $\sigma_{\rm tot}(s)$ measurements are usually described in terms of the $a_\mu(\rm had, LO)$ dispersive integral in the given s range using $\sigma_{\rm tot}(s)$ from individual (mutually incompatible) experiments: e.g., let's integrate over CMD-3 $\pi^+\pi^-(\gamma)$ data at $\sqrt{s} = 0.6$–0.9 GeV, then compare this to the integral over CMD-2, SND, BaBar, KLOE, *etc.*, and point out (in)compatibility of the integrals.
- Our technique allows for a more detailed estimate of the extra uncertainty at a level of $\sigma_{\rm tot}(s)$ fits rather than dispersive integrals over individual $\sigma_{\rm tot}(s)$ measurements. The uncertainty of the extra uncertainty (see σ`xsys` on the next slide) is estimated by varying the $\Delta\chi^2_{sys}$ threshold in 2.2–10 range (the nominal indicator of missing systematic is $\Delta\chi^2_{sys} > 6$).

In fact, our technique involves simultaneous extremization of both χ^2 and its "entropy": to make χ^2 distribution over d.o.f. uniform, the fit is "warmed up" by extra systematics accounting for incompatibility between experiments.

Result: $a_\mu(\text{had,LO}) = (697.6 \pm 9.0_{e^+e^-} \pm 0.8_{\sigma xsys} \pm 2.4_{par} \pm 2.3_{rad}) \times 10^{-10}$



- Assigning extra systematic uncertainties to $\sigma_{tot}(e^+e^- \to hadrons)$ measurements we account for incompatibilities between them and obtain the dispersive estimate of $a_\mu(\text{had,LO})$ with a realistic uncertainty.
- $a_\mu^{SM} - a_\mu^{exp}$ tension is reduced to 2.2σ. The dispersion based $a_\mu(\text{had,LO})$ is compatible with the lattice estimate.
- $g_\mu - 2$ experimental accuracy is ahead of theory. No $g_\mu - 2$ anomaly? Refined $\sigma_{tot}(e^+e^- \to \pi^+\pi^-, \dots)$ needed.

Final state	$a_\mu(\text{had,LO}) \times 10^{10}$ (e^+e^-)	$(\sigma$xsys.)	(par.)	(rad.)	$\sqrt{s}$ [GeV]	Extra sys. %	$\frac{\chi^2}{\text{dof}}$
$\pi^+\pi^-(\gamma)$	508.792 (8.892)	(0.187)	(2.018)	(2.150)	0.3 ÷ 1.937	5.2	1.67
$\pi^+\pi^-\pi^0$	48.986 (0.933)	(0.251)	(0.447)	(0.035)	0.56024 ÷ 1.937	5.6	1.31
$\pi^+\pi^-2\pi^0$	19.294 (0.436)	(0.312)	(0.063)	(0.064)	0.85 ÷ 1.937		1.17
$2\pi^+2\pi^-$	14.411 (0.174)	(0.000)	(0.183)	(0.012)	0.6125 ÷ 1.93		1.41
K^+K^-	22.566 (0.987)	(0.447)	(1.787)	(0.032)	0.985 ÷ 1.937	6.6	2.26
K_SK_L	13.106 (0.106)	(0.059)	(0.000)	(0.000)	1.00028 ÷ 1.937		0.95
$\pi^0\gamma$	4.359 (0.093)	(0.048)	(0.049)	(0.000)	0.59986 ÷ 1.38		1.70
$K_SK^+\pi^- + K_SK^-\pi^+$	1.814 (0.100)	(0.000)	(0.000)	(0.000)	1.24 ÷ 1.937		0.99
$2\pi^+2\pi^-\pi^0$	1.219 (0.076)	(0.000)	(0.017)	(0.001)	1.0125 ÷ 1.937		0.58
$2\pi^+2\pi^02\pi^-$	1.381 (0.172)	(0.000)	(0.011)	(0.000)	1.3125 ÷ 1.937		1.49
$2\pi^+2\pi^-3\pi^0$	0.099 (0.013)	(0.000)	(0.002)	(0.001)	1.575 ÷ 1.937		0.57
$3\pi^+3\pi^-$	0.254 (0.014)	(0.000)	(0.002)	(0.012)	1.3125 ÷ 1.937		1.52
$3\pi^+3\pi^-\pi^0$	0.020 (0.004)	(0.000)	(0.001)	(0.000)	1.6 ÷ 1.937		0.65
$\eta\gamma$	0.648 (0.063)	(0.000)	(0.036)	(0.000)	0.59986 ÷ 1.354	5.7	2.56
$\eta\pi^+\pi^-$	0.575 (0.019)	(0.000)	(0.000)	(0.000)	1.15 ÷ 1.937		1.18
$K^+K^-\pi^0$	0.202 (0.050)	(0.000)	(0.000)	(0.001)	1.44 ÷ 1.937		0.54
$K^+K^-\pi^0\pi^0$	0.100 (0.011)	(0.000)	(0.000)	(0.000)	1.5 ÷ 1.937		1.32
$K^+K^-\pi^+\pi^-$	0.810 (0.038)	(0.000)	(0.009)	(0.000)	1.4125 ÷ 1.937		1.91
$K^+K^-\pi^+\pi^-\pi^0$	0.129 (0.024)	(0.000)	(0.000)	(0.000)	1.6125 ÷ 1.937		1.63
$K_SK_L\eta$	0.238 (0.059)	(0.000)	(0.000)	(0.000)	1.575 ÷ 1.937		1.31
$K_SK_L\pi^0$	0.839 (0.114)	(0.000)	(0.000)	(0.000)	1.425 ÷ 1.937		1.50
$K_SK_L\pi^0\pi^0$	0.137 (0.043)	(0.000)	(0.000)	(0.000)	1.35 ÷ 1.937		0.00
$K_SK_L\pi^+\pi^-$	0.166 (0.028)	(0.000)	(0.000)	(0.000)	1.425 ÷ 1.937		0.00
$K_SK^+\pi^-\pi^0 + K_SK^-\pi^+\pi^0$	0.640 (0.044)	(0.000)	(0.000)	(0.000)	1.51 ÷ 1.937		1.08
$K_SK_S\pi^+\pi^-$	0.066 (0.007)	(0.000)	(0.000)	(0.000)	1.63 ÷ 1.937		1.37
$\phi(1020) < X - K_SK_L - K^+K^- > \eta$	0.068 (0.003)	(0.001)	(0.002)	(0.000)	1.56 ÷ 1.937		0.97
$\omega(783)\eta$	0.035 (0.002)	(0.000)	(0.000)	(0.000)	1.34 ÷ 1.937		0.85
$\omega(783) < \pi^0\gamma > \pi^0$	0.878 (0.038)	(0.020)	(0.077)	(0.000)	0.75 ÷ 1.937	9.2	1.13
$\omega(783) < \pi^+\pi^-\pi^0 > \pi^+\pi^-$	0.092 (0.005)	(0.000)	(0.000)	(0.000)	1.15 ÷ 1.937		0.90
$\omega\eta\pi^0$	0.189 (0.045)	(0.000)	(0.091)	(0.000)	1.5 ÷ 1.937		0.30
$\pi^+\pi^-2\pi^0\eta$	0.117 (0.019)	(0.000)	(0.000)	(0.000)	1.625 ÷ 1.937		0.85
$\pi^+\pi^-3\pi^0$	1.067 (0.112)	(0.000)	(0.000)	(0.000)	1.125 ÷ 1.937		0.68
$\pi^+\pi^-\pi^0\eta$	0.663 (0.075)	(0.000)	(0.000)	(0.000)	1.394 ÷ 1.937		0.82
$p\bar{p}$	0.033 (0.003)	(0.000)	(0.001)	(0.000)	1.889 ÷ 1.937		0.70
$n\bar{n}$	0.026 (0.005)	(0.000)	(0.000)	(0.000)	1.89 ÷ 1.937		1.48
$2hadron(hadrons)$	42.704 (0.552)	(0.119)	(0.248)	(0.000)	1.937 ÷ 11.199		1.39
pQCD	2.065		(0.002)		> 11.1990		
ChPT $\pi\pi, \pi^0\gamma$	0.538		(0.013)		0.2792 ÷ 0.3000		
$\Psi(1S)$	6.495		(0.124)		3.0969		
$\Psi(2S)$	1.631		(0.057)		3.6861		
$\Upsilon(1S)$	0.054		(0.002)		9.4604		
$\Upsilon(2S)$	0.021		(0.003)		10.0234		
$\Upsilon(3S)$	0.014		(0.002)		10.3551		
$\Upsilon(4S)$	0.010		(0.001)		10.5794		
Total	**697.552** (9.030)	(0.760)	(2.427)	(2.309)			

Summary

- Using an up-to-date compilation of $\sigma_{\rm tot}(e^+e^- \to hadrons)$ data we estimated the LO hadronic contribution to the muon $g-2$.
- Incompatibilities between independent $\sigma_{\rm tot}(e^+e^- \to hadrons)$ measurements are mitigated by extra systematic uncertainties estimated using the requirement of uniformity of χ^2 distribution over degrees of freedom.
- Tensions in the e^+e^- input data translate into an expanded uncertainty of the $a_\mu(\rm had, LO)$:

$$a_\mu(\mathrm{had, LO}) = (697.6 \pm 9.0_{e^+e^-} \pm 0.8_{xsys} \pm 2.4_{par} \pm 2.3_{rad}) \times 10^{-10}$$

Given this, we obtain the SM prediction for the muon $g-2$

$$a_\mu^{\rm SM} = 11659185.4(9.8)\,,$$

lower than the experimental world average $a_\mu^{\rm exp}$ at 2.2σ level.

Accuracy of both dispersive and lattice $a_\mu(\rm had, LO)$ is behind the experiment.

New $e^+e^- \to \pi^+\pi^-(\gamma)^{[SND!]}, 3\pi, 4\pi, \ldots 2hadron(hadrons)$ data?

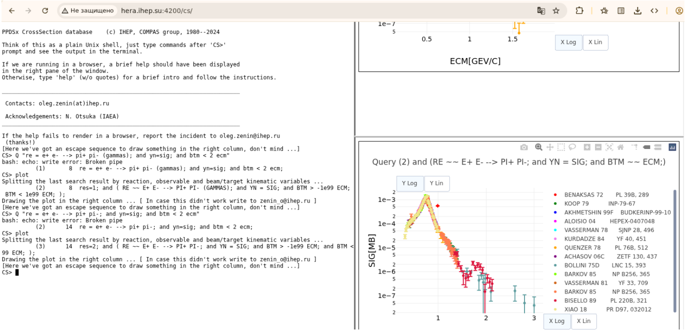
Backup

PPDS CrossSection database

- Originates from the PPDS CrossSection database maintained at IHEP (Protvino) since 1980s.
- Implemented from scratch for Unix in 2017-2020 (no legacy code).
- Covers total cross section measurements published since 1947: 22165 data records (one record $\sim$ one reaction from one paper).
- The data are encoded in a formal language (protection against meaningless content and some input mistakes). Special query language (not SQL).

Command line web interface with interactive plots:

http://hera.ihep.su:4200/cs/

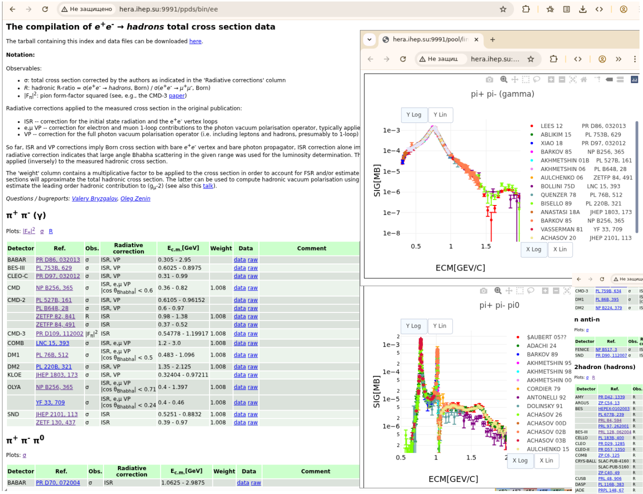


- Fragmentary coverage of world data published since $\sim$ 2000 (lack of manpower!). However, $\sigma_{tot}(e^+e^- \to hadrons)$ and $\sigma_{tot,inel}(hadron\ hadron \to X)$ compilations are kept up-to-date by former authors of the PDG σ_{tot} mini-reviews.
- No problem to create a more traditional web interface like INSPIRE (except lack of manpower!).
- Custom web pages are generated from the database "on the fly" for specific topics, e.g., for the $\sigma_{tot}(e^+e^- \to hadrons)$ compilation (next slide). $\longrightarrow$

back

PPDS CrossSection database: $e^+e^- \to$ *hadrons*

http://hera.ihep.su:9991/ppds/bin/ee



hera.ihep.su:9991/ppds/bin/ee

The compilation of $e^+e^- \to$ *hadrons* total cross section data

The tarball containing this index and data files can be downloaded here.

Notation:

Observables:

- σ: total cross section corrected by the authors as indicated in the 'Radiative corrections' column
- *R*: hadronic R-ratio = σ($e^+e^- \to$ *hadrons*, Born) / σ($e^+e^- \to \mu^+\mu^-$, Born)
- $|F_\pi|^2$: pion form-factor squared (see, e.g., the CMD-3 paper)

Radiative corrections applied to the measured cross section in the original publication:

- ISR -- correction for the initial state radiation and the e^+e^- vertex loops
- e,μ VP -- correction for electron and muon 1-loop contributions to the photon vacuum polarisation operator, typically applie
- VP -- correction for the full photon vacuum polarisation operator (i.e. including leptons and hadrons, presumably to 1-loop)

So far, ISR and VP corrections imply Born cross section with bare e^+e^- vertex and bare photon propagator, ISR correction alone im radiative correction indicates that large angle Bhabha scattering in the given range was used for the luminosity determination. Th applied (inversely) to the measured hadronic cross section.

The 'weight' column contains a multiplicative factor to be applied to the cross section in order to account for FSR and/or estimate sections will approximate the total hadronic cross section. The latter can be used to compute hadronic vacuum polarisation using estimate the leading order hadronic contribution to (g_μ-2) (see also this talk).

Questions / bugreports: Valery Bryzgalov, Oleg Zenin

$\pi^+ \pi^- (\gamma)$

Plots: $|F_\pi|^2$ σ R

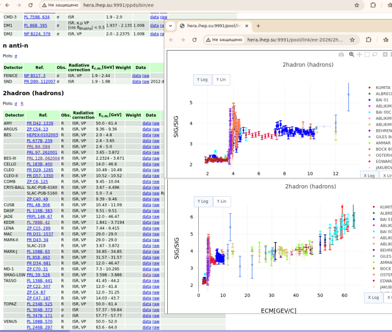
Detector	Ref.	Obs.	Radiative correction	$E_{c.m.}$[GeV]	Weight	Data	Comment
BABAR	PR D86, 032013	σ	ISR, VP	0.305 - 2.95		data raw	
BES-III	PL 753B, 629	σ	ISR, VP	0.6025 - 0.8975		data raw	
CLEO-C	PR D97, 032012	σ	ISR, VP	0.31 - 0.99		data raw	
CMD	NP B256, 365	σ	ISR, e,μ VP $\lvert\cos\theta_{Bhabha}\rvert < 0.6$	0.36 - 0.82	1.008	data raw	
CMD-2	PL 527B, 161	σ	ISR, VP	0.6105 - 0.96152		data raw	
	PL B648, 28	σ	ISR, VP	0.6 - 0.97		data raw	
	ZETFP 82, 841	R	ISR	0.98 - 1.38	1.008	data raw	
	ZETFP 84, 491	σ	ISR	0.37 - 0.52		data raw	
CMD-3	PR D109, 112002	$\lvert F_\pi\rvert^2$	ISR	0.54778 - 1.19917	1.008	data raw	
COMB	LNC 15, 393	σ	ISR, e,μ VP	1.2 - 3.0	1.008	data raw	
DM1	PL 76B, 512	σ	ISR, e,μ VP $\lvert\cos\theta_{Bhabha}\rvert < 0.5$	0.483 - 1.096	1.008	data raw	
DM2	PL 220B, 321	σ	ISR, VP	1.35 - 2.125	1.008	data raw	
KLOE	JHEP 1803, 173	σ	ISR, VP	0.32404 - 0.97211		data raw	
OLYA	NP B256, 365	σ	ISR, e,μ VP $\lvert\cos\theta_{Bhabha}\rvert < 0.71$	0.4 - 1.397	1.008	data raw	
	YF 33, 709	σ	ISR, e,μ VP $\lvert\cos\theta_{Bhabha}\rvert < 0.24$	0.4 - 0.46	1.008	data raw	
SND	JHEP 2101, 113	σ	ISR	0.5251 - 0.8832	1.008	data raw	
	ZETF 130, 437	σ	ISR	0.39 - 0.97	1.008	data raw	

$\pi^+ \pi^- \pi^0$

Plots: σ

Detector	Ref.	Obs.	Radiative correction	$E_{c.m.}$[GeV]	Weight	Data	Comment
BABAR	PR D70, 072004	σ	ISR	1.0625 - 2.9875		data raw	

- The web page is generated "on the fly" from the database.
- $\sigma_{\rm tot}$, $R^{\rm had}$ and $|F_{had}|^2$ data for 36 final states from $\pi^0\gamma$, $\pi^+\pi^-(\gamma)$ to inclusive $2hadron(hadrons)$.
- Metadata include radiative corrections, weights accounting for FSR, missing channels, *etc.*, just enough to feed the data into our program for the dispersive computation of $a_\mu(\rm had, LO)$.
- References to raw records, data tables, original papers.
- Interactive plots.



To our knowledge, the only similar public compilation is PrecisionSM [L. Cotrozzi, A. Driutti, F. Ignatov, A. Lusiani, and G. Venanzoni, PoS EPS-HEP2025, 287 (2026)]: only $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$ and $\pi^0\gamma$ channels so far, more detailed specifications of radiative corrections, . . .

Fitting the $R^{\rm had}$ data

A standard χ^2 minimization:

$$\chi^2 = \sum_{i,j} \left[\frac{1}{\Delta\sqrt{s_i}} \int_{\Delta\sqrt{s_i}} R^{\rm had}_{fit}(s) d\sqrt{s} - R^{\rm had}_i \right] \times {\rm COV}^{-1}_{ij} \times \left[\frac{1}{\Delta\sqrt{s_j}} \int_{\Delta\sqrt{s_j}} R^{\rm had}_{fit}(s) d\sqrt{s} - R^{\rm had}_j \right],$$

where $R^{\rm had}_{fit}(s)$ is the fitted parameterisation, $R^{\rm had}_i$ are the measurements in $\Delta\sqrt{s_i}$ bins, and ${\rm COV}_{ij}$ is the full covariance matrix between measurements:

$$\begin{aligned}{\rm COV}_{ij} = \delta_{ij}\sigma^2_{{\rm stat},i} \quad + \quad & \frac{1}{\Delta\sqrt{s_i}} \int_{\Delta\sqrt{s_i}} R^{\rm had}_{fit}(s) d\sqrt{s} \times \frac{1}{\Delta\sqrt{s_j}} \int_{\Delta\sqrt{s_j}} R^{\rm had}_{fit}(s) d\sqrt{s} \times \\ & \times \left\{ \begin{array}{l} \Delta_{{\rm sys},i}\Delta_{{\rm sys},j}, \text{ if } i,j \text{ are from the same experiment} \\ \Delta_{{\rm sys},i}\Delta_{{\rm sys},j} \times ({\rm cross-experiment\ covariation}), \\ \qquad\qquad \text{if } i,j \text{ are from different experiments} \end{array} \right\},\end{aligned}$$

where $\Delta_{{\rm sys},i}$ are relative systematic uncertainties as quoted in the original papers.

Why $R^{\rm had}_{fit}(s)$ in the systematic term of ${\rm COV}_{ij}$? Naively taking individual measurements $R^{\rm had}_{i,j}$ for the systematic uncertainty leads to a biased ${\rm COV}_{ij}$ and to a biased fit, as far as $R^{\rm had}_{i,j}$ *are already biased themselves* – a manifestation of the well known *Peele's Pertinent Puzzle (PPP)*: "... a phenomenon exhibiting unexpected mean values for experimental data affected by statistical and systematic errors" [*R. Frühwirth et al, EPJ Web of Conf., Vol. 27 (2012), 00008*]

The problem: $\delta\chi^2/\delta R^{\rm had}_{fit}(s)$ is non-linear w.r.t. $R^{\rm had}_{fit}(s)$ ⇒ run the fit iteratively $\longrightarrow$ ...

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Fitting the R^{had} data (continued)

. . . ⟶ run the fit iteratively:

1. Make the fit ignoring the systematic uncertainties to get zeroth approximation for $R^{\mathrm{had}}_{fit}(s)$. Though χ^2/dof is awful, there's no PPP bias in the fit using a diagonal covariance matrix.
2. Rebuild the full covariance matrix using the obtained $R^{\mathrm{had}}_{fit}(s)$.
3. Repeat the fit with the full covariance matrix.
4. In practice, the procedure converges after these 2 iterations.

Caveats:

- A simplified parameterization of error matrices $C_{ij} = \delta_{ij}\sigma^2_{\mathrm{stat},i} + R_i R_j \cdot \Delta_{\mathrm{sys},i}\Delta_{\mathrm{sys},j}$, where $\Delta_{\mathrm{sys},i}$ is the quadratic sum of relative systematic uncertainties from all sources in the i-th point, is not always adequate. Some experiments publish full covariance matrices which, generally speaking, cannot be parameterized as above. Our parameterization is actually an artifact of the simplified output format from the PPDS CrossSection database.
- Covariance between different experiments: common MC for acceptance calculation, common background models, the same GEANT4 for detector simulation, common modeling of large angle Bhabha scattering for luminosity determination, etc. Need to be an expert to estimate a degree of covariance between different experiments. We neglect cross-dataset covariations, except for CMD-2 $\pi^+\pi^-\pi^0$ measurements in three scans at different $\sqrt{s}$, and CMD-2 (2003) / SND (2006) $\pi^+\pi^-(\gamma)$ measurements (based on expert estimate by S. Eidelman).
- It is not always clear how to scale statistical errors. If $R^{\mathrm{had}}(s)$ measurement gets a normalization bias from luminosity, acceptance, efficiency, then its statistical uncertainty should be scaled in the fit just as systematics (cf. the previous slide). Additive bias from background subtraction is more problematic. So far, we conservatively keep original $\sigma_{\mathrm{stat},i}$.

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Missing systematic uncertainties: K^+K^-

Initial fit with unmodified error matrices

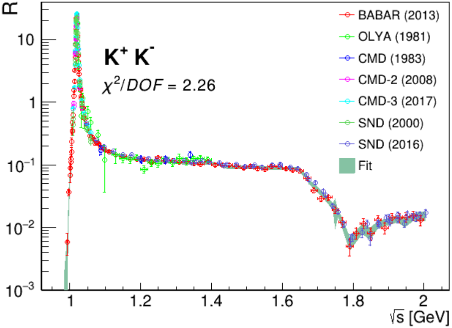
(global $\chi^2/dof = 2.35$):

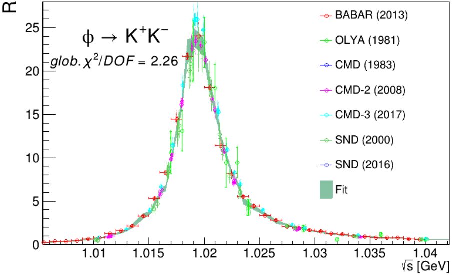
n_p	$\Delta\chi^2/n_p$	$\Delta\chi^2_{sys}$	$\Delta\chi^2_{res}/n_p$	Pull	Exp.	Run
100	1.96	0.05	1.95	-0.001	BaBar	2013
4	1.23	0.01	1.20	0.014	CMD	1983
21	1.98	1.99	1.96	-0.034	CMD-2	2008
24	2.40	6.92	1.69	0.066	CMD-3	2017
28	1.46	0.72	1.66	-0.066	SND	2000
56	0.98	0.44	0.95	0.011	SND	2016

↓

Fit with extra 6.6% systematics:

n_p	$\Delta\chi^2/n_p$	$\Delta\chi^2_{sys}$	$\Delta\chi^2_{res}/n_p$	Pull	Exp.	Run
100	1.88	0.27	1.82	0.022	BaBar	2013
4	1.41	0.13	1.23	0.056	CMD	1983
21	1.89	0.02	1.91	-0.013	CMD-2	2008
24	2.28	4.35	1.56	0.090	CMD-3	2017
28	1.43	0.26	1.56	-0.044	SND	2000
56	0.94	1.17	0.82	0.066	SND	2016





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Open questions, todo's ...

- Missing systematics:
  - Parameterization of the extra uncertainty accounting for incompatible measurements is not unique.
  - We stick to the simplest estimate: assign single additional normalization uncertainty to all experiments contributing to the fit in the given channel.
  - Expert knowledge required. Is extra systematics (partially) correlated between experiments? Is it multi-component (e.g., some components are correlated due to common MC, some other components are not)?
  - $\sqrt{s}$ dependence. We can restrict fit in the given channel to some $\sqrt{s}$ range and find missing systematics in $\sqrt{s}$ ranges.
  - Minimum χ^2 & maximum χ^2 entropy criterion:
    - ★ Formulate it using a strict definition of the entropy of the fit.
    - ★ Make missing systematics true free parameter(s) of the fit.
    - ★ This would allow to drop our heuristics and implement a well justified numerical procedure.
  - Ultimately, an estimate of the missing uncertainties should give hints to instrumental origins of incompatibilities between experiments *(currently, it does not)*.
- Covariance matrices:
  - The parameterization $C \propto \mathrm{diag}(\sigma^2_{stat}) + \sigma_{sys} \otimes \sigma^T_{sys}$ is insufficient: σ_{stat} correlations are neglected, the systematic part is better approximated as $\sum_i \sigma_{sys,i} \otimes \sigma^T_{sys,i}$, etc.
  - Recent experiments publish full covariance matrices.
  - Covariations between experiments: expert knowledge needed.
- Missing channels?