



# “Gluon nature of multiparticle production”

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Конференция  
«Физика частиц при средних и высоких  
энергиях»  
(*Particle physics at intermediate and high  
energies*)

Протвино 2–5 июня 2026 г

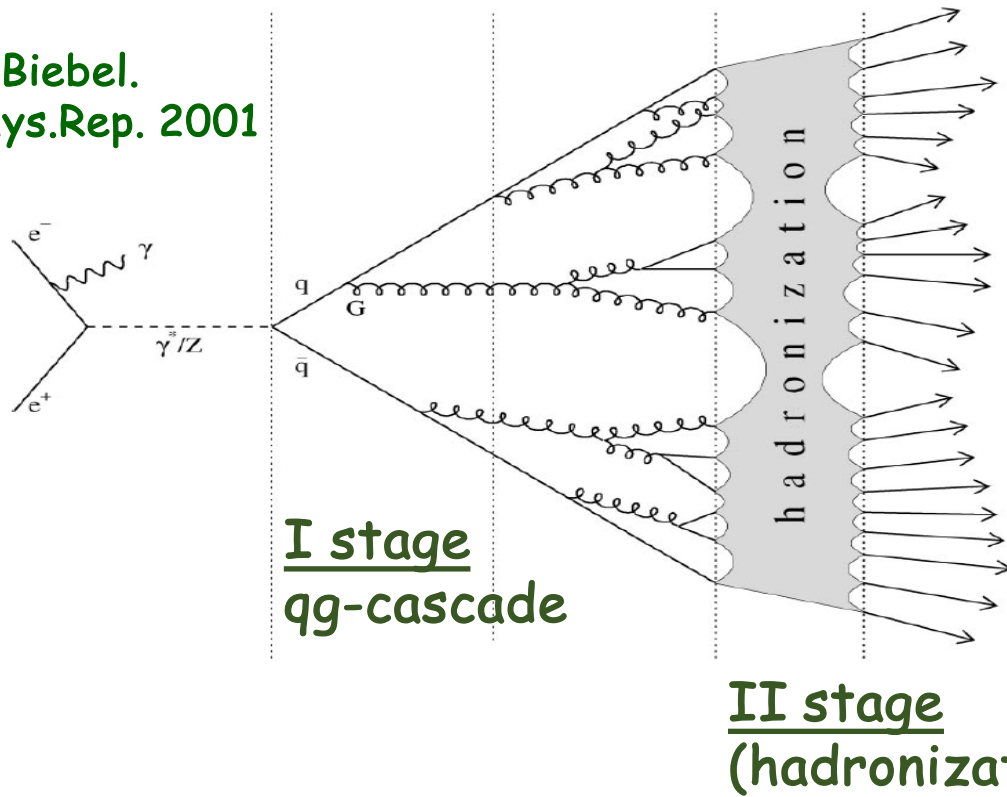
# Multiparticle processes in HEP

- Electron-positron annihilation ( $e^+e^-$ ).
- Proton-antiproton annihilation ( $p\bar{p}$ ).
- Hadron-hadron ( $pp$ ) interactions.
- Relativistic heavy ion collisions ( $AA$ )
- Three-gluon decay of quarkonium ( $\Upsilon$ ).

# $e^+e^-$ - annihilation

$$e^+e^- \rightarrow \gamma(Z^0) \rightarrow q\bar{q} \rightarrow (q, g) \rightarrow ? \rightarrow \text{hadrons}$$

O.Biebel.  
Phys.Rep. 2001



**Multiplicity  
Distribution (MD)**

$$P_n(s) = \frac{\sigma_n}{\sum_m \sigma_m}$$

**Generation  
function (GF):**

$$Q(s, z) = \sum_n P_n(s) z^n$$

**GF  $\leftrightarrow$  MD**

$$P_n(s) = \frac{1}{n!} \frac{\partial^n}{\partial z^n} Q(s, z) \Big|_{z=0}$$

**Correlative moments,  $F_k$ :**

$$F_k(s) = \overline{n(n-1)\dots(n-k+1)} = \frac{\partial^k}{\partial z^k} Q(s, z) \Big|_{z=1}$$

# $e^+e^-$ - annihilation

Konishi et al. & Giovannini [NP, 1979] described the  $qg$ -cascade in pQCD as Markovian branching processes of elementary events:

- 1) quark bremsstrahlung -  $q \rightarrow q + g, (\tilde{A})$
- 2) gluon fission -  $g \rightarrow g + g, (A)$
- 3) quark-antiquark pair creation from gluon - (B)  $g \rightarrow q + \bar{q}$  .

$$\frac{\partial G}{\partial Y} = -AG + AG^2,$$

$$\frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG.$$

System of diff. eq. describing branching processes, leads to **Pólya (NBD)** for  $q$ -jet and **Yule-Furry MD** for  $g$ -jet:

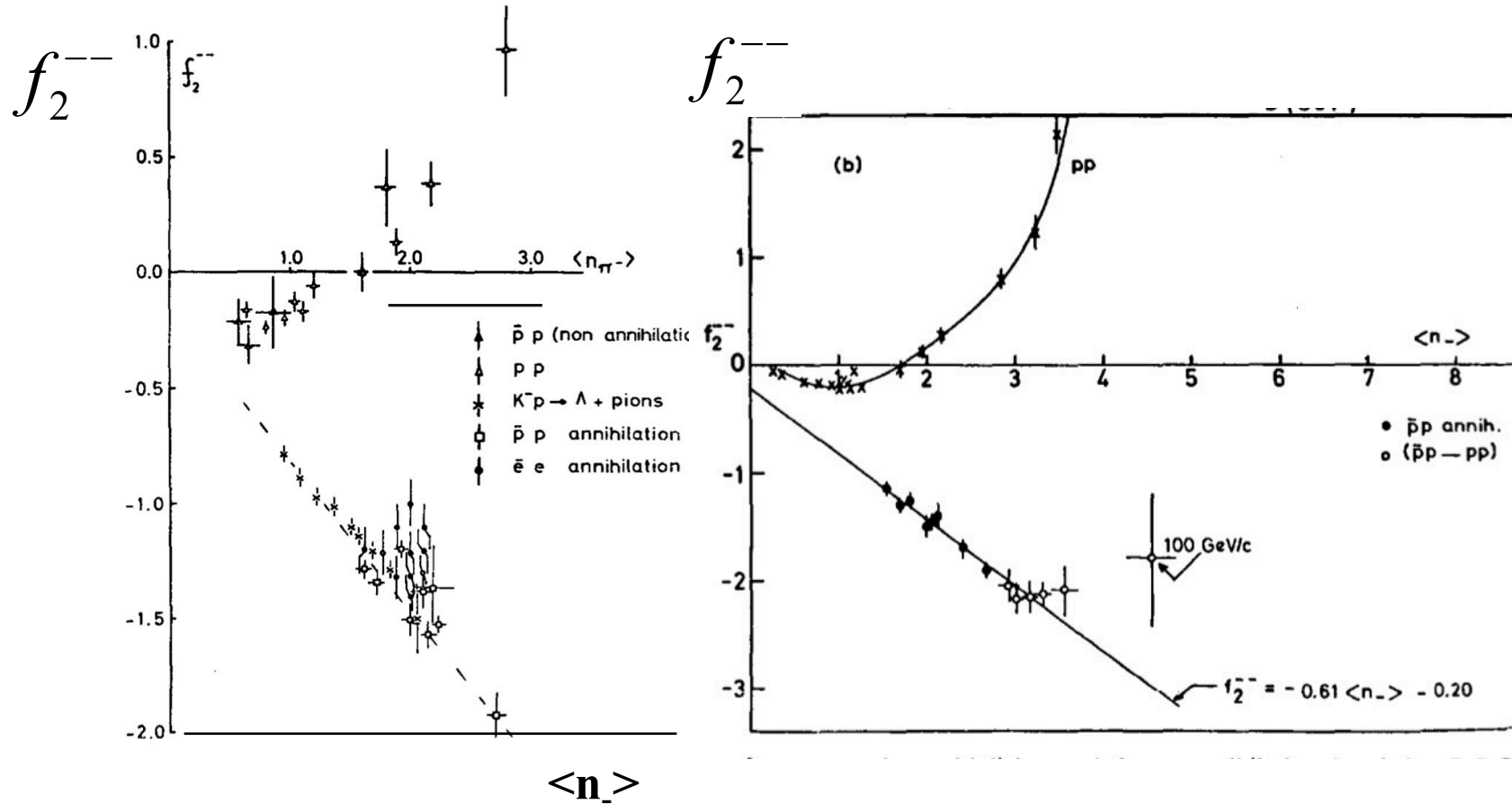
$$P_m^g = \frac{1}{\bar{m}} \left(1 - \frac{1}{\bar{m}}\right)^{m-1},$$

$$P_m^q = \frac{k_p(k_p+1)\dots(k_p+m-1)}{m!} \left(\frac{\bar{m}}{\bar{m}+k_p}\right)^m \left(\frac{k_p}{\bar{m}+k_p}\right)^{k_p}.$$

Evolutinary parameter:

$$Y = \frac{1}{2\pi b} \ln[1 + ab \ln(Q^2 / \mu^2)], \quad \tilde{A} \text{ и } A - \text{probabilities of 1) и 2) events, } k_p = \tilde{A}/A.$$

# Variation of $f_2$ with $\langle n_- \rangle$ for annihilation & non-annihilation data

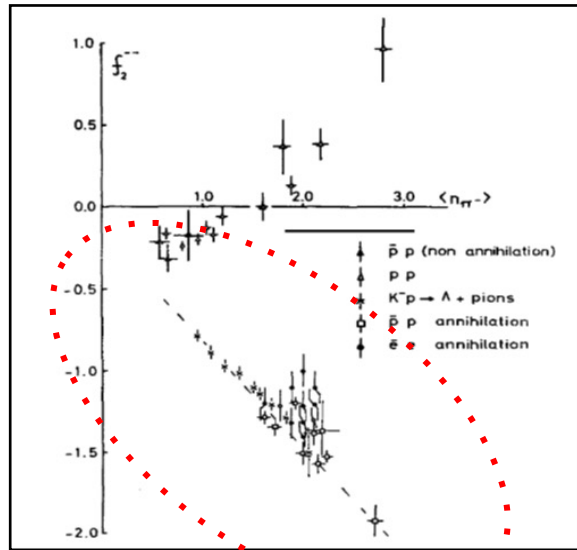


$$f_2 = \langle n(n-1) \rangle - \langle n \rangle^2 = D_2 - \langle n \rangle < 0 \text{ at low energy;}$$

J.G. Rushbrooke,  
B.R. Webber. Phys. Rep.  
44 (1978) 1

# $e^+e^-$ annihilation - II stage

pQCD is unable to describe hadronization. The choice of MD at this stage is based on experimental behavior of the second correlative moment  $f_2$ . It is always **positive** for **Pólya (NBD)** (and **Furry** also):



J.G. Rushbrooke,  
B.R. Webber. Phys. Rep.  
44 (1978) 1

$$\text{NBD} \rightarrow f_2 = \overline{n(n-1)} - \bar{n}^2 \rightarrow \frac{\overline{m^2}}{k_p} > 0$$

We chose **binomial MD (Bernoulli)** for II-stage:

$$P_P^H(n) = C_{N_p}^n \left( \frac{\bar{n}_p^h}{N_p} \right)^n \left( 1 - \frac{\bar{n}_p^h}{N_p} \right)^{N_p - n}, P = q, g.$$

$$f_2 = - \frac{(\bar{n}_p^h)^2}{N_p} < 0.$$

## Convolution of two stages. Two-stage model

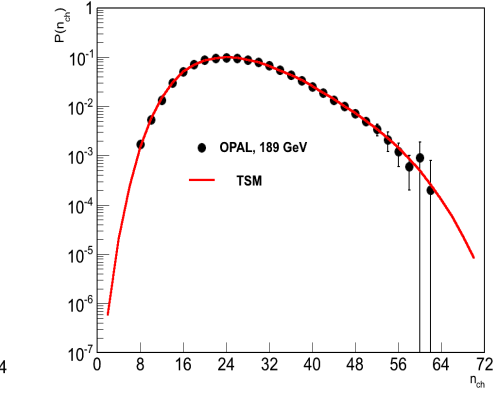
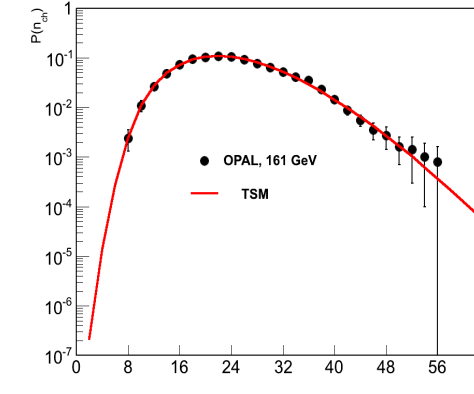
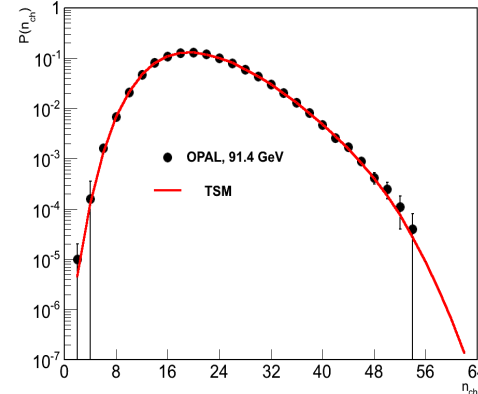
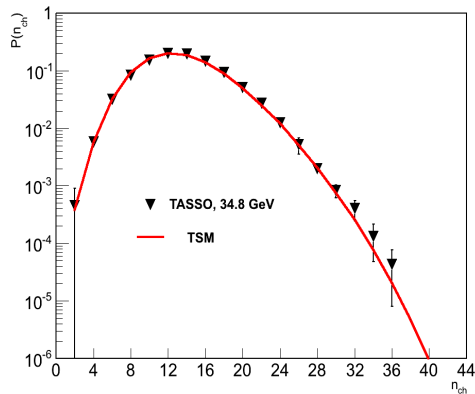
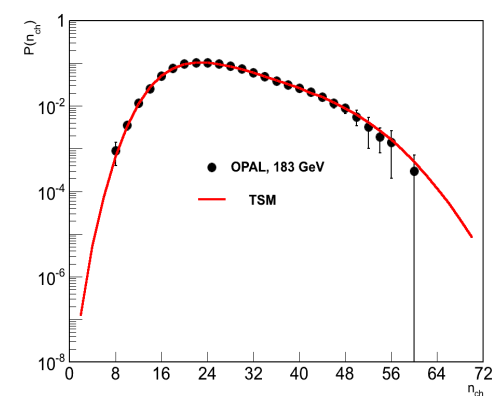
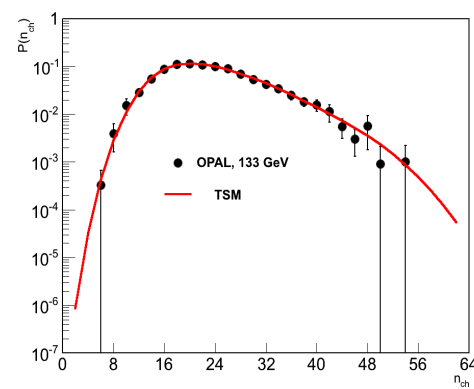
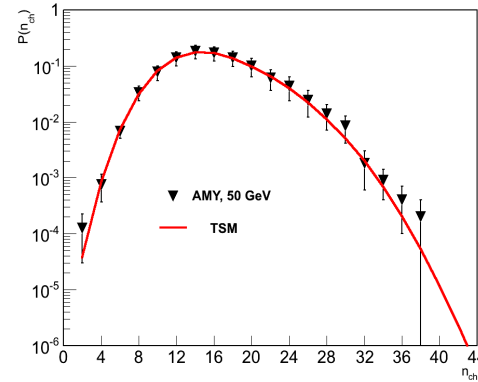
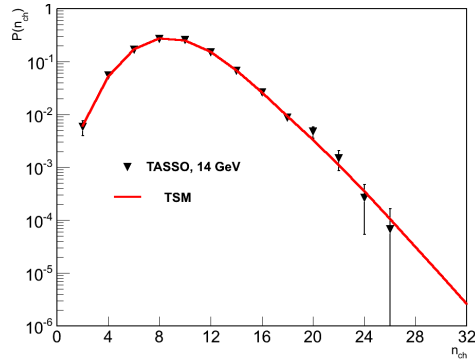
$$Q(s, z) = \sum_m P_m^P Q^H(m, s, z) \quad (\text{soft discoloration}).$$

$$P_n(s) = \Omega \sum_{m=0}^{M_g} P_m^P C_{(2+\alpha m)N}^n \left( \frac{\bar{n}^h}{N} \right)^n \left( 1 - \frac{\bar{n}^h}{N} \right)^{(2+\alpha m)N-n}$$

$$Q_p^H = \left[ 1 + \frac{\bar{n}_p^h}{N_p} (z-1) \right]^{N_p}, \quad \mathbf{p} = \mathbf{q}, \mathbf{g},$$

**Model parameters:**  $k_p$ ,  $\bar{m}$ ,  $N_q = N$ ,  $\bar{n}_q^h$ ,  $N_g = \alpha N$ ,  $\bar{n}_g^h = \alpha \bar{n}_q^h$ .

# MD in $e^+e^-$ annihilation (14 -189 GeV)



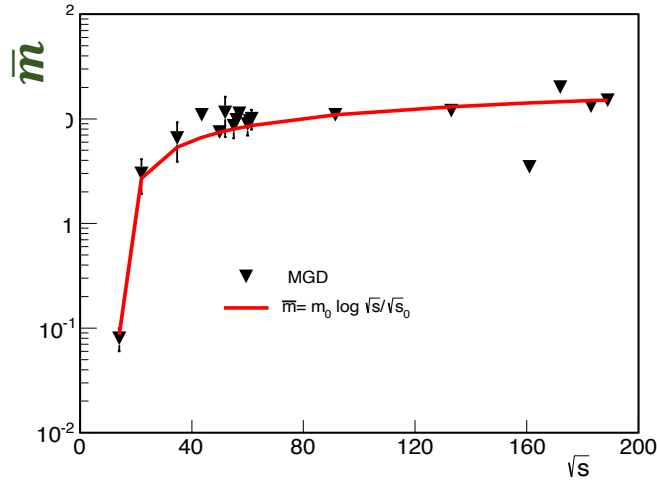
## Second correlative moment, $f_2$

$$\bar{n}(s) = \frac{\partial Q(s, z)}{\partial z} \Big|_{z=1} = (2 + \alpha \bar{m}) \bar{n}^h$$

$$f_2 = F_2 - F_1^2 = \left[ \alpha^2 \frac{\bar{m}^2}{k_p} + \alpha^2 \bar{m} - \frac{2 + \alpha \bar{m}}{N} \right] (\bar{n}^h)^2$$

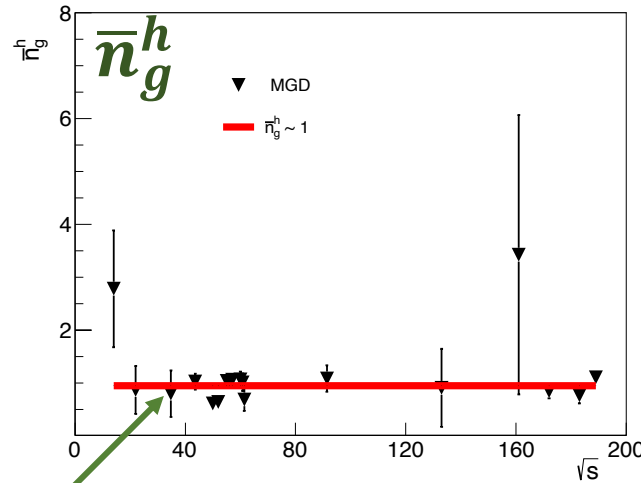
$$f_2 < 0 \quad \text{at} \quad \bar{m} \ll 1 \quad (\alpha < 1, k_p > 1)$$

# TSM parameters (2002)

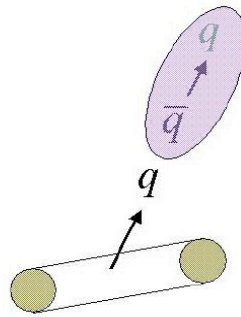


$\bar{m} \sim \log s .$

Hypothesis of Local Parton-Hadronic Duality (LoPHD)  
 $\langle m \rangle = \rho \langle n \rangle, \rho \sim 1.$

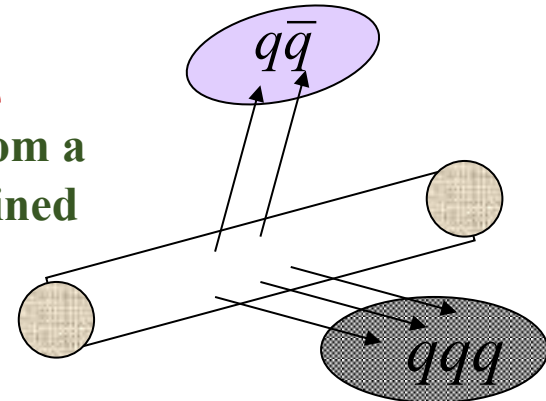


Average number of hadrons,  $\bar{n}_g^h$ , formed from gluon, is close to 1, which testifies the **fragmentation** mechanism of hadronization.



Fragmentation mechanism (in vacuum)

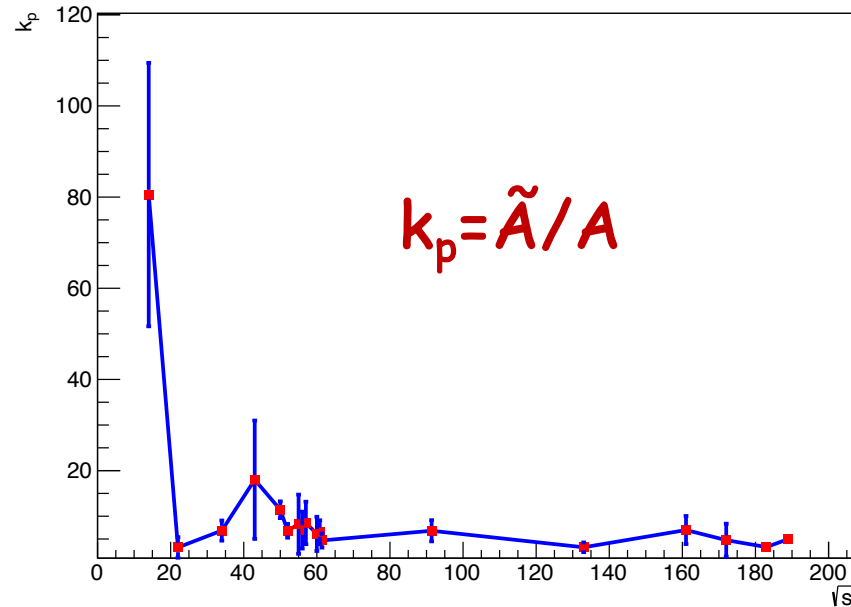
Recombination mechanism (from a thermal, deconfined medium)



[ B. Muller. 2004 ]

# TSM parameters: $k_p, \langle m_g \rangle$

$k_p$ , parameter of quark-gluon cascade



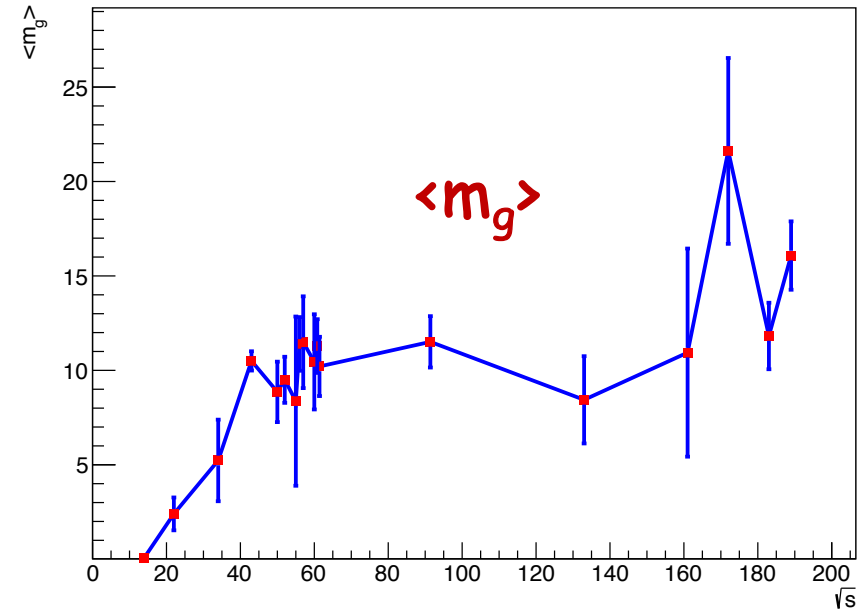
Low energies:

q-bremsstrahlung, g-fission is suppressed

High energies:

q-bremsstrahlung/ g-fission  $\sim 6 \div 8$

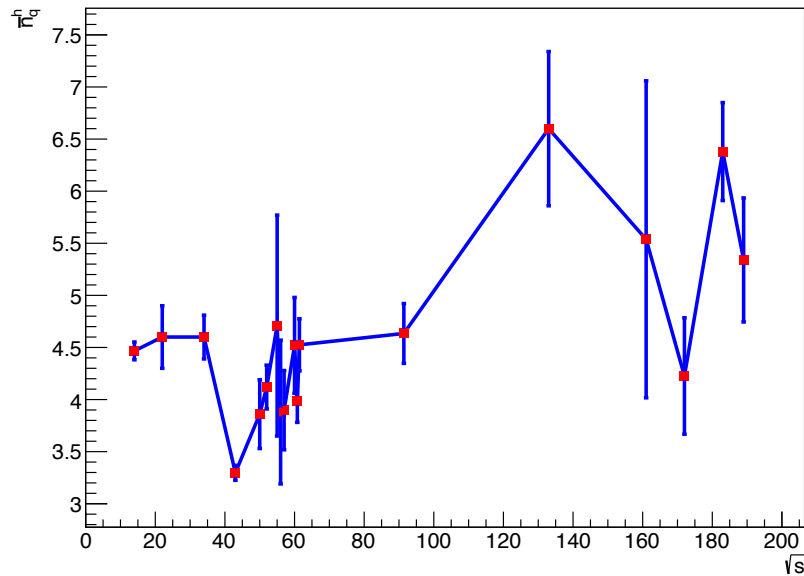
$\langle m_g \rangle$ , average multiplicity of gluons in the quark-gluon cascade



Steady growth trend

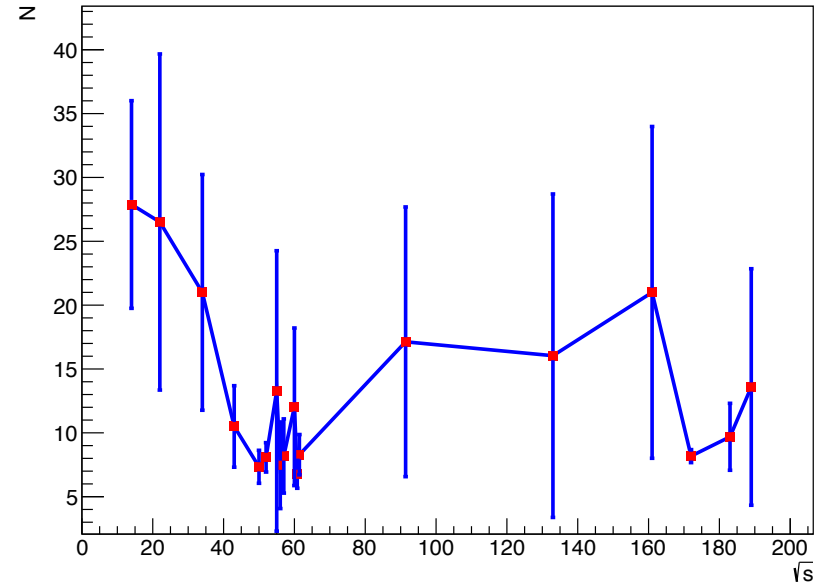
# TSM parameters: $\bar{n}_q^h, N_q$

$\bar{n}_q^h$ , parameter of hadronization for quark



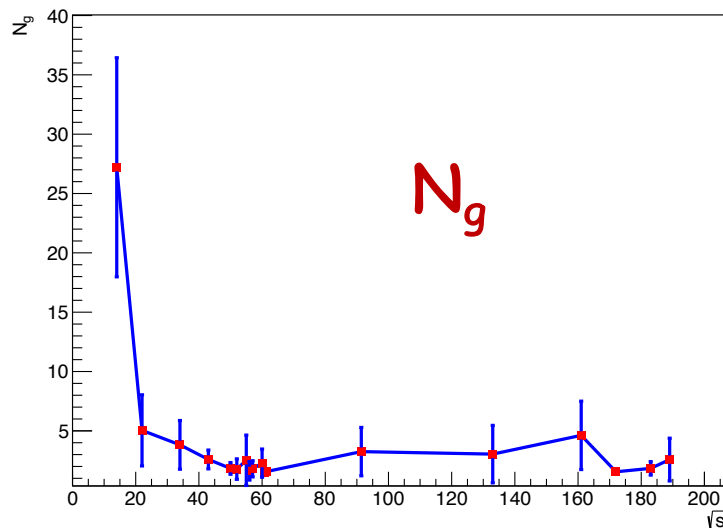
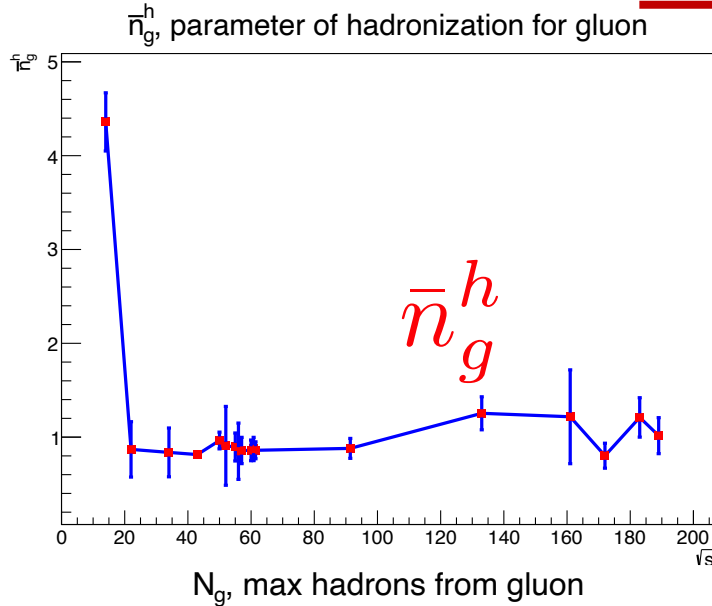
$\bar{n}_q^h$  Almost constant  $\rightarrow$   
growth with oscillations

$N_q$ , max hadrons from quark



$N_q$  steady decline  $\rightarrow$   
almost constant

# TSM parameters: $\bar{n}_g^h, N_g$



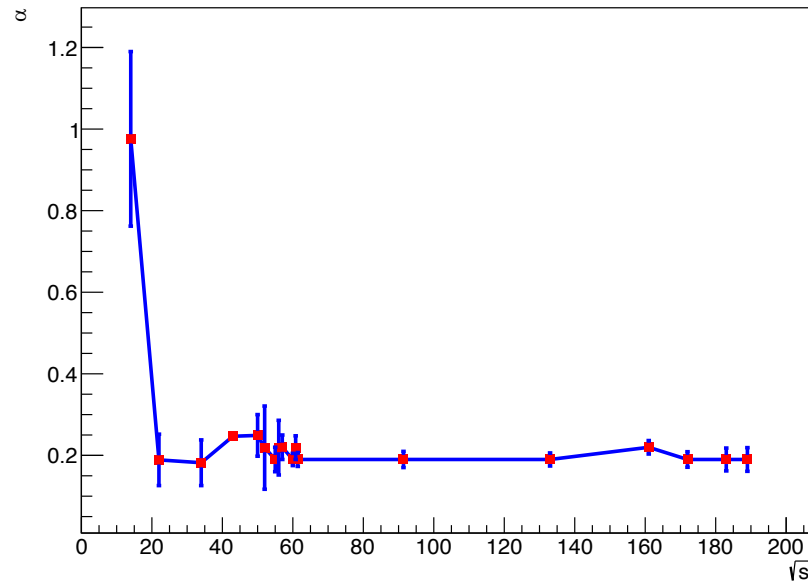
$\sqrt{s} < 10$  GeV.  $\bar{m} \ll 1$ , g-fission are virtually absent.

$\sqrt{s} > 10$  GeV.  $\bar{m} \geq 1$ , multiplicity of g-fission increases,  $\bar{n}_g^h$  remains constant and smaller than 1 up to  $\sim 100$  GeV

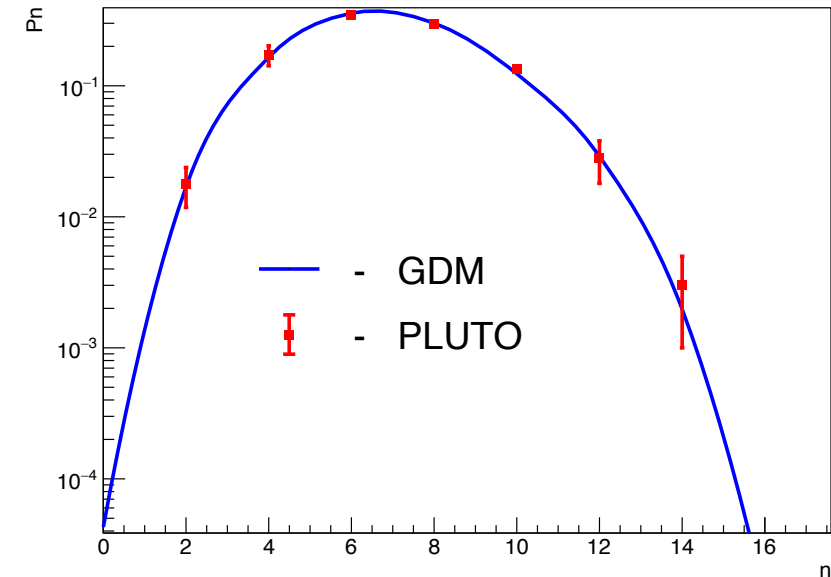
(fragmentation mechanism of hadronization), then it starts to grow and gently exceeds 1 (recombination mechanism of hadronization) at  $\gtrsim 130$  GeV. Data from the 90s have weak indication of such behavior (HERA).

# TSM parameter $\alpha$ & MD at 9.4 GeV

$\alpha$ , the ration of q bremsstrahlung to gluon fission

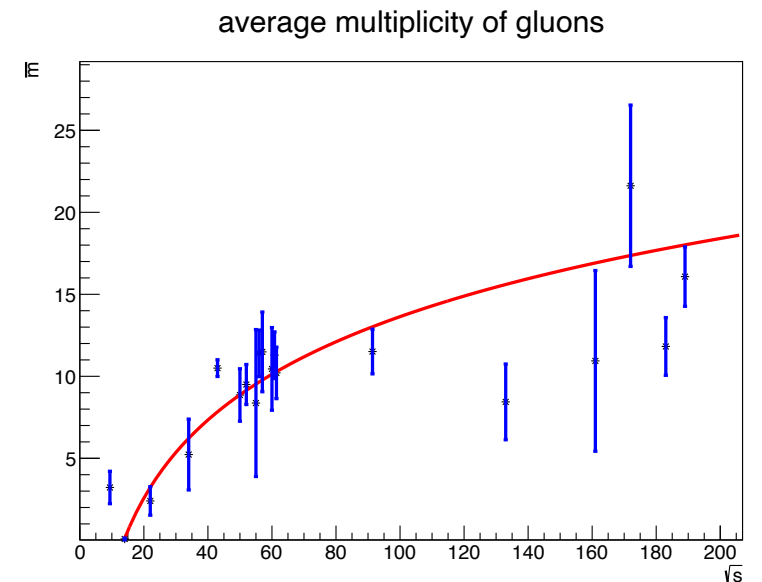
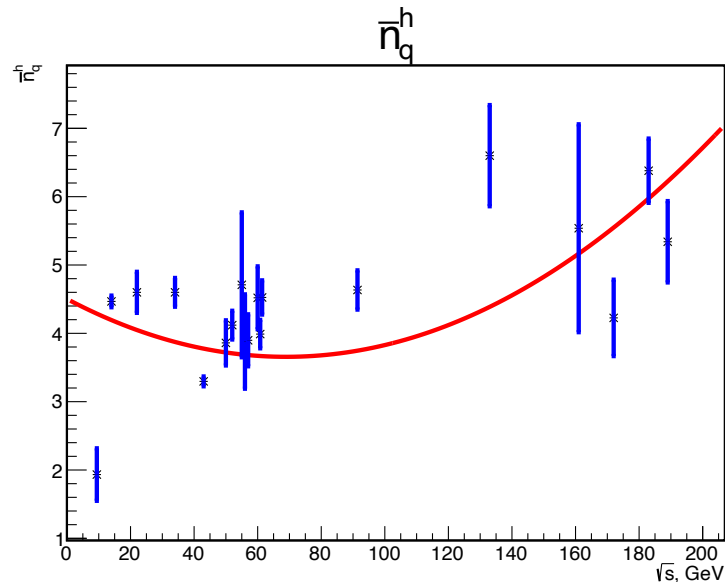
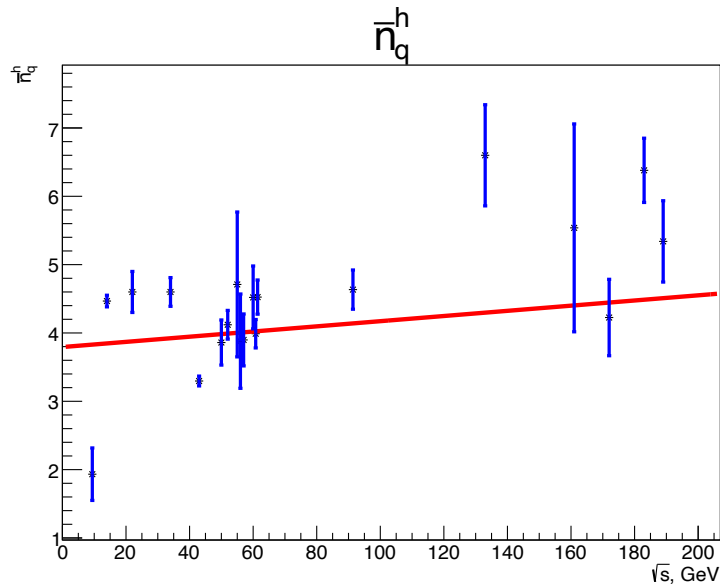


Multiplicity Distribution in  $e^+e^-$  at 9.4 GeV/c



(On the left) Parameter  $\alpha$  as the ration  $\bar{n}_g^h / \bar{n}_q^h$ . It almost stays constant and close to 0.2 (one fifth) as a steady process of qg-cascade up to  $\sim 200$  GeV  
(On the right) MD at low energy where  $f_2$  is negative. Only q-bresstrahlung conrtributes.

# TSM parameter's fits for average multiplicity:



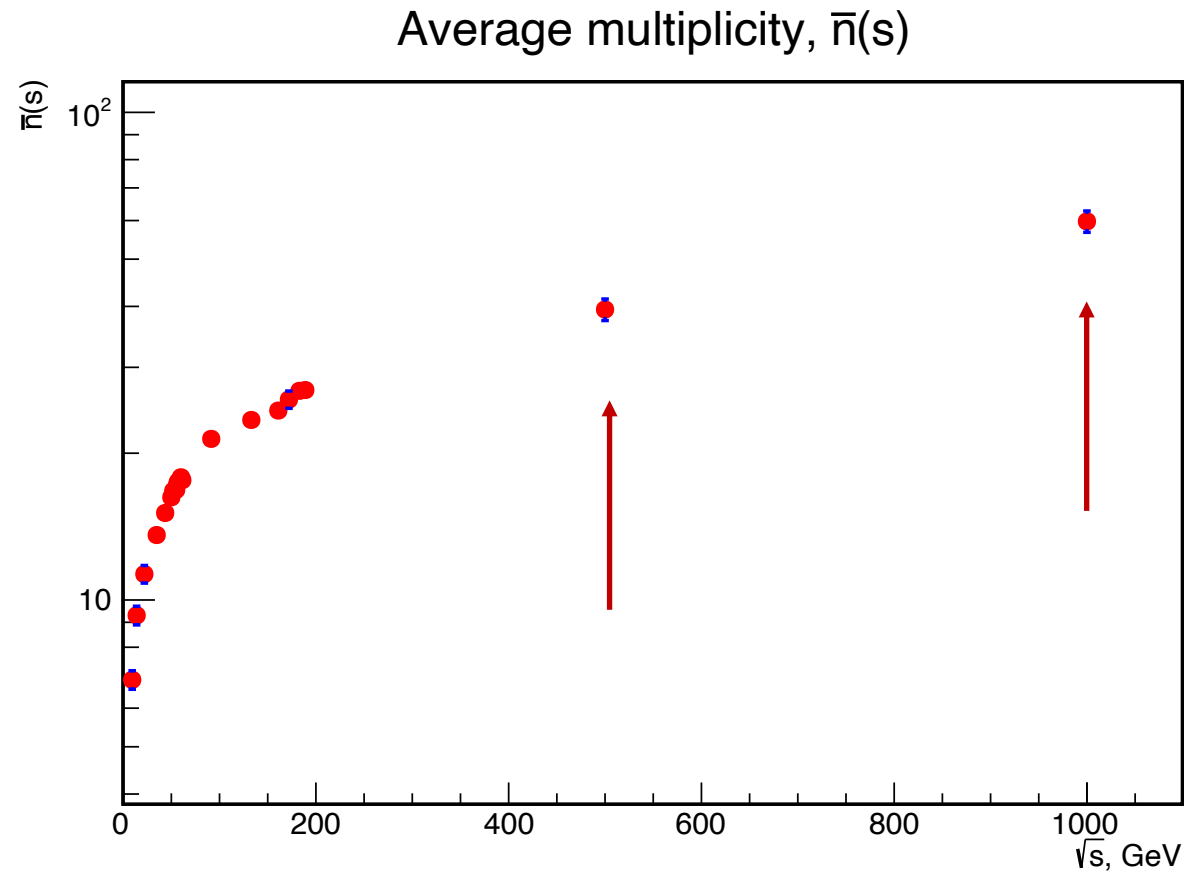
$$\bar{n}(s) = (2 + \alpha \bar{m}) \bar{n}^h$$

(left) Linear growth for  $\bar{n}_q^h$

(middle) quadratic growth for  $\bar{n}_q^h$

(right) logarithmic growth for  $\bar{m}$

# Prediction for mean multiplicity at 500 GeV and 1 TeV:



Prediction:  $\bar{n}(500 \text{ GeV}) = 32 \div 39$  &  $\bar{n}(1 \text{ TeV}) = 37 \div 60$ .

# Summary from $e^+e^-$ -annihilation study:

1. The process of  $e^+e^-$ -annihilation is the **simple one** for the QCD description for the first (parton) stage because the creation of gluons and following its fission occurs in a vacuum at  $\sqrt{s} < \sim 100 \text{ GeV}$ .
2. pQCD is not applied at the 2nd stage (hadronization). Our choice of MD at the hadronization is based on the data:  $f_2$  for quark and gluon jets is always positive. Experimental  $f_2$  is negative at low energies and becomes positive at higher energies.
3. At low energy, hadronization dominates over the  $qg$ -cascade. We use **Binomial distribution** for hadronization. **Convolution** of those two stages **describes well** experimental MD up to 200 GeV including low energies.

## Summary from $e^+e^-$ -annihilation study:

4. Average number of hadrons formed from single gluon ( $\bar{n}_g^h$ ) at the second stage is close to 1 at the wide energy region ( $\sim 10$ - $100$  GeV): **1 gluon  $\rightarrow$  1 hadron** (confirmation of the **fragmentation mechanism** of hadronization in vacuum).
5. Average gluon multiplicity ( $\bar{m}$ ) rises logarithmically.
6. Gluon fission ( $g \rightarrow g + g$ ) commences prevailing over bremsstrahlung ( $q \rightarrow q + g$ ) with increasing of energy and  $f_2$  changes sign.
7. Hadronization of gluons is softer than quark one at the hadronization.

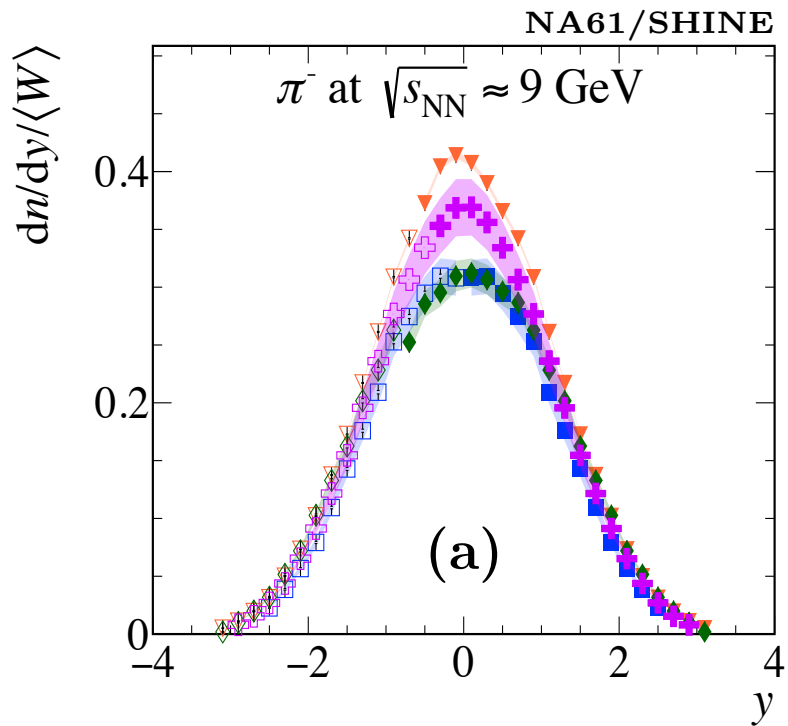
Future  $e^+e^-$  accelerators  
(CEPC and others)  
will answer questions about  
hadronization stage,  
its mechanism and  
gluon fission

Thank you for attention

# Spare slides

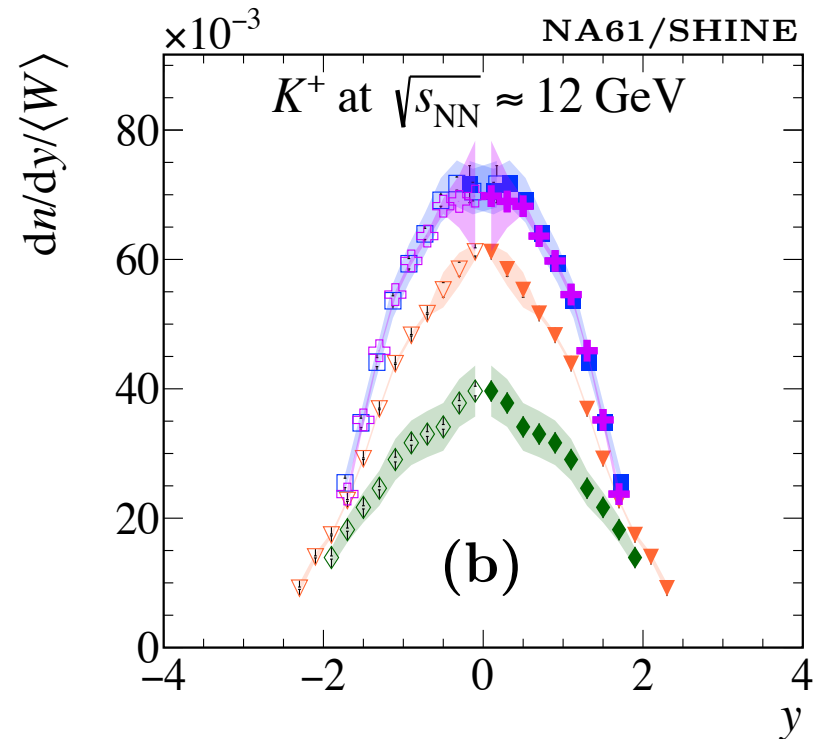
# Overview of results from NA61/SHINE.

A. Rybicki for the NA61/SHINE Coll. [arXiv:2604.20616](https://arxiv.org/abs/2604.20616) [nucl-ex]



Be + Be (4+4)  
Ar + Sc (18+21)  
Xe + La (54 +57)  
Pb +Pb (82 +82)

(a) Be+Be  $\rightarrow$  Pb+Pb  $\rightarrow$  Xe+La  $\rightarrow$  Ar+Sc



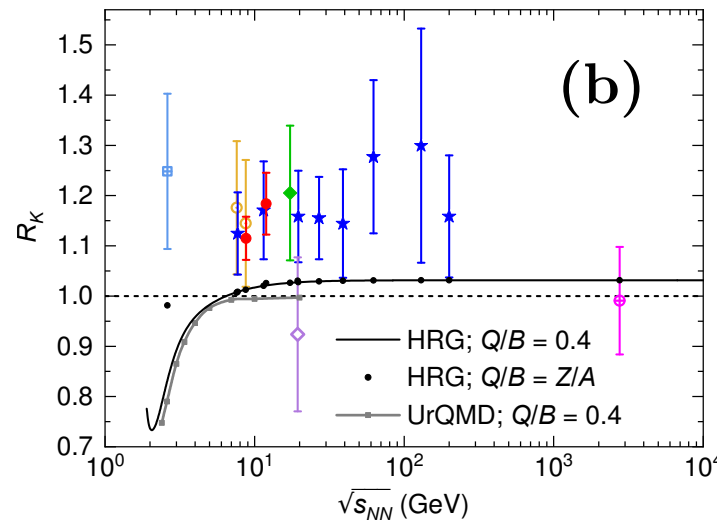
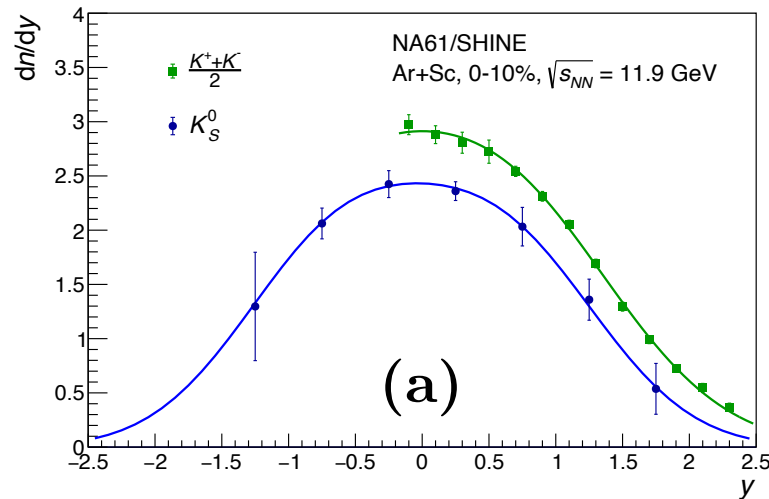
(b) Be+Be  $\rightarrow$  Ar+Sc  $\rightarrow$  Xe+La  $\rightarrow$  Pb+Pb

$K^+/\pi^+$  ratio  $\rightarrow$  (b)

# Kaon sector of MP: unexpectedly strong violation of isospin (flavor) symmetry [arXiv:2604.20616]:

$$\frac{K^+ + K^-}{2} \approx \frac{K^0 + \bar{K}^0}{2} = K_S,$$

$$R_K = \frac{K^+ + K^-}{2K_S}.$$



Flavor symmetry (QCD) in the limit of equal quark masses, the remnant approximate light u/d quark isospin symmetry implies equal production of charged and respective neutral kaons in case of colliding charge-symmetric ( $Z = N \equiv A - Z$ ) nuclei.

Data from Ar+Sc nuclei collisions at  $\sqrt{s_{NN}}=11.9$  GeV do not follow this prediction, and exhibit a charged-over-neutral kaon excess reaching  $18.4 \pm 6.1\%$  at mid-rapidity.

## Possible explanation of these findings

Gluons are active sources of secondaries, they are formed at high energy collisions (q-bremsstrahlung and g-fission). At the hadronization stage they decay into quark pairs:  $u, \bar{u}, d, \bar{d}, s, \bar{s}$ . Probabilities of their creation depend on the mass value. It is known, quark masses differ from each other. The d-quark has slightly greater mass than the u-quark (by a couple of MeV), but is already noticeably less than the s-quark. So, at hadronization, u,  $\bar{u}$  quarks will be predominant, then d,  $\bar{d}$  (a little less) and s,  $\bar{s}$  quarks will be even fewer.

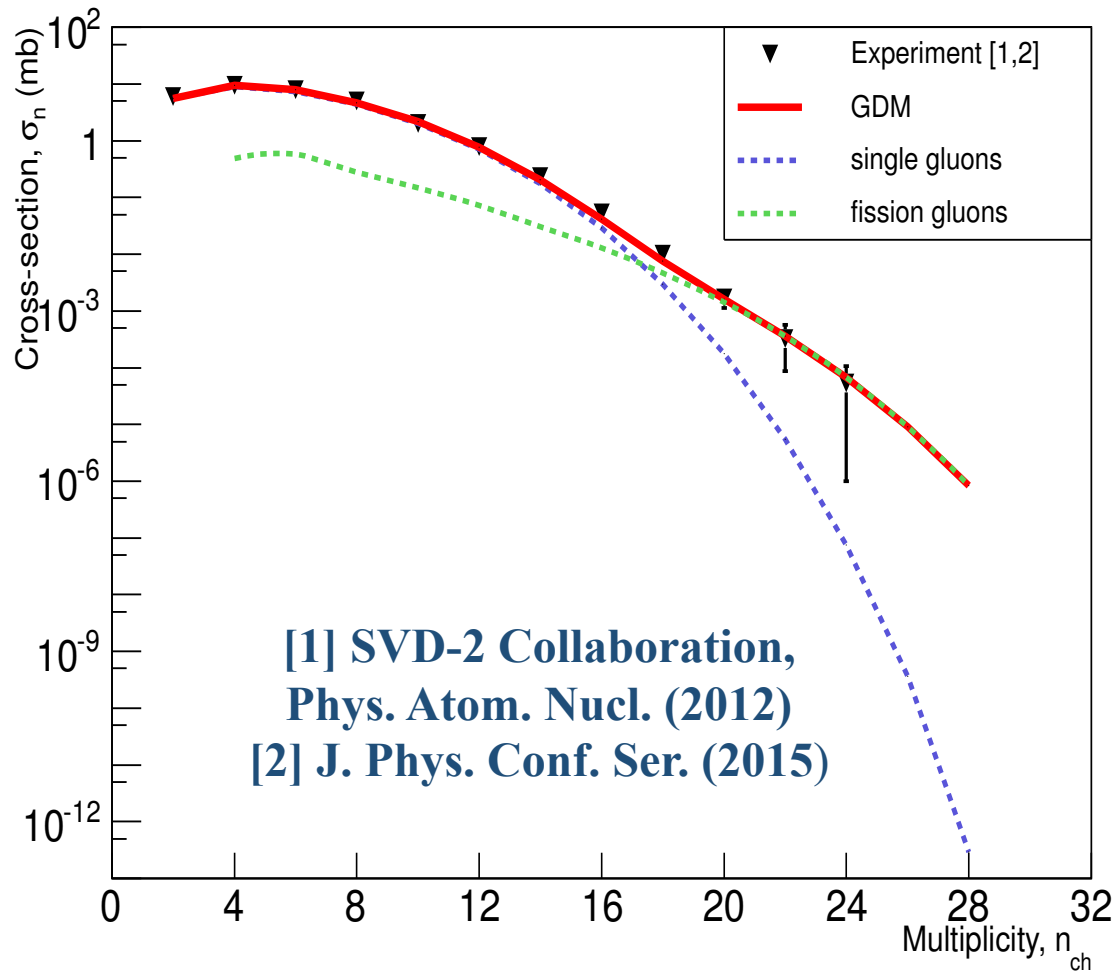
## Possible explanation of $K^\pm$ excess

Simple combinatorics shows: the number of ways in which quarks can form  $K^\pm$  must be greater than formation of  $K_S$ .  $K^+\{u\bar{s}\}$ ,  $K^-\{s\bar{u}\}$ ,  $K^0\{d\bar{s}\}$  and  $\bar{K}^0\{s\bar{d}\}$ . Let us define  $N_u$ ,  $N_d$  and  $N_s$  multiplicities of corresponding quarks and antiquarks. Experimental excess of  $K^\pm$  over  $K_S$

$$R_K \approx 18.4 \pm 6.1 \%,$$

As  $R_K = (N_u - N_d)/(N_u + N_d)$  we get the ratio between u and d quarks,  $N_d \sim 0.69 N_u$ . The fraction of d-quarks in u-quarks is close to  $2/3 N_u$ .

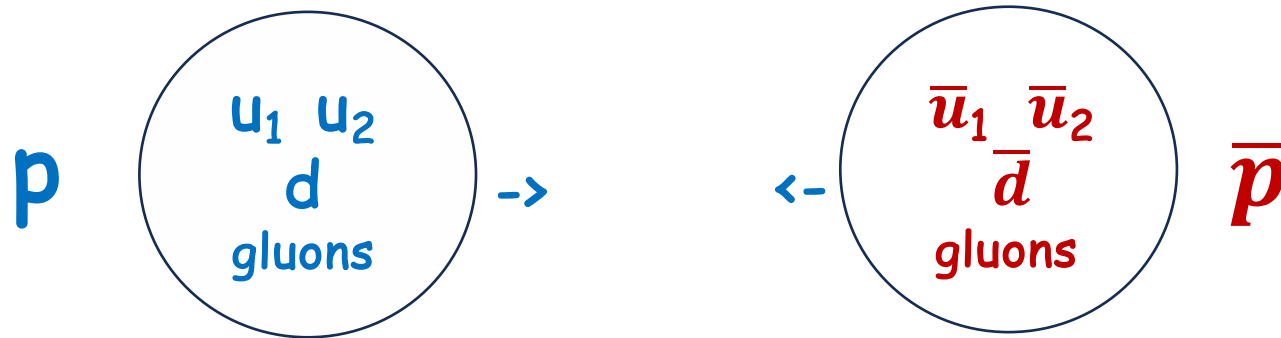
# GDM with gluon fission



SVD-2 and Mirabelle (50 GeV) have been stitched along  $\sigma_n$ . GDM in HM region takes into account 2 types of contributions: without g-fission (blue line) and with (green line). Their superposition is shown by red line. HM stipulates namely by gluon fission. Ratio of bremsstrahlung to gluon fission is equal to  $\sim 1/9$ . More than 64% of  $E$  is converted into a mass of newborn pions.

# $p\bar{p}$ annihilation

GDM offers description of  $p\bar{p}$  annihilation by the formation of 3 and more intermediate charged quark topologies with corresponding contributions  $c_0$ ,  $c_2$  and  $c_4$ , which are stipulated by all kinds of permutations of valence quarks with antiquarks and the formation of three leading pions.



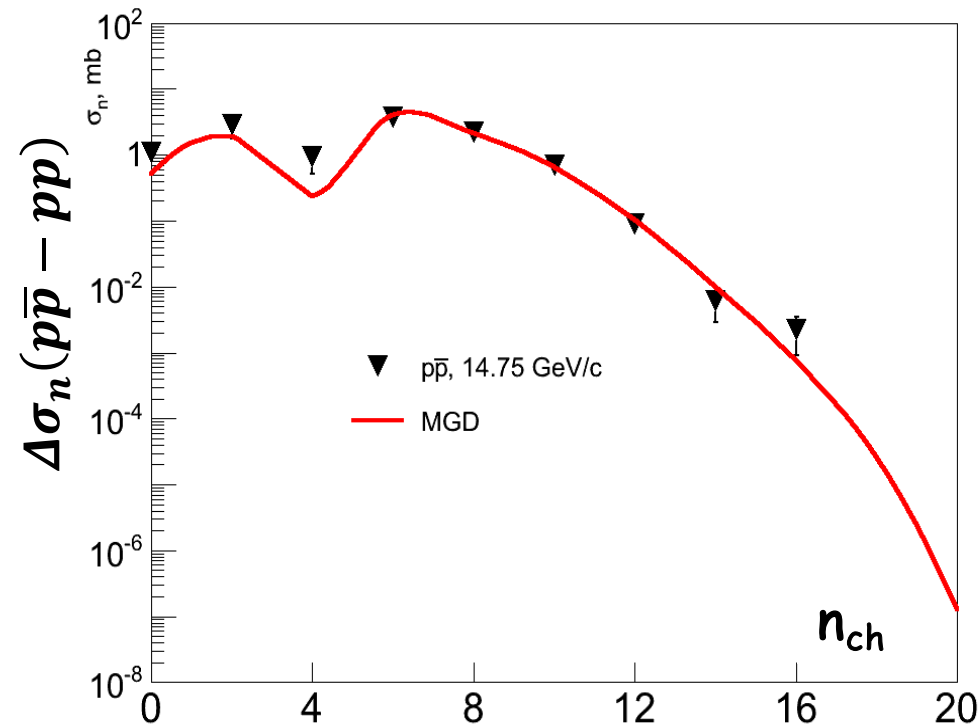
# Variants of q-topologies

2 variants of "0"- topology ( $3 \pi^0$ ):  
 $u_1 \bar{u}_1 + u_2 \bar{u}_2 + d \bar{d}$  and  $u_1 \bar{u}_2 + u_2 \bar{u}_1 + d \bar{d}$ ;

4 variants of "2"- topology ( $\pi^0, \pi^+, \pi^-$ ):  
 $u_1 \bar{d} + u_2 \bar{u}_1 + d \bar{u}_2$ ,  $u_1 \bar{u}_2 + u_2 \bar{d} + d \bar{u}_1$   
 $u_1 \bar{u}_1 + u_2 \bar{d} + d \bar{u}_2$ , and  $u_1 \bar{d} + u_2 \bar{u}_2 + d \bar{u}_1$ ;

Topology "4" (and higher) is formed by adding to a valence quark (an antiquark) the corresponding antiquark (quark), which are born from active gluons ( $g \rightarrow q + \bar{q}$ ):  
 $u_1 \bar{d} + \bar{u}_1 d + u_2 \bar{d} + d \bar{u}_2 + \dots (\pi^+, \pi^-, \pi^-, \pi^+)$ ;

# $p\bar{p}$ annihilation



Generation function in GDM:

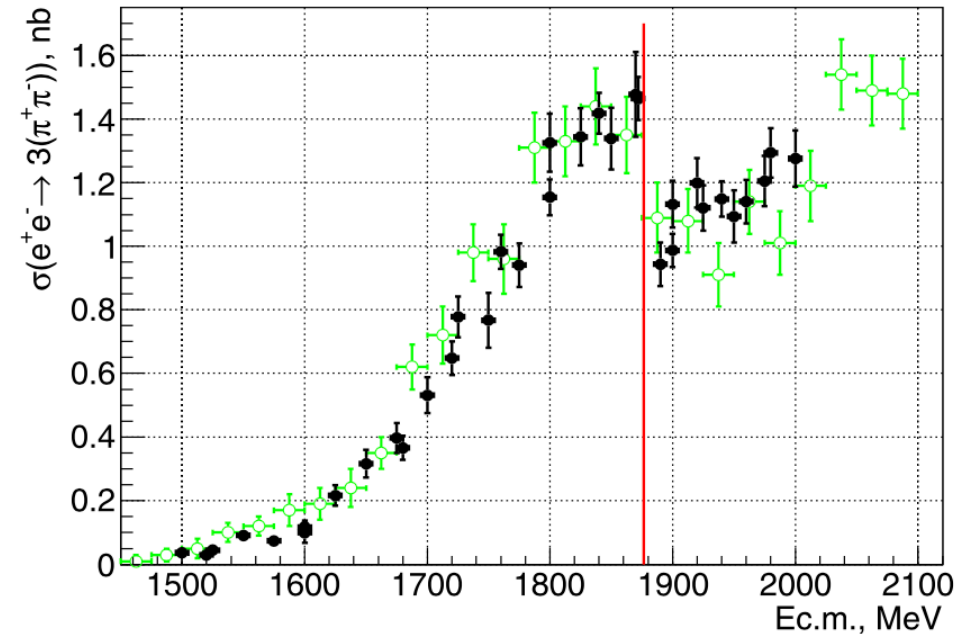
$$Q(z) =$$
$$= c_0 \sum_m P_m^G \left[1 + \frac{\bar{n}^h}{N} (z - 1)\right]^{mN} +$$
$$+ c_2 z^2 \sum_m P_m^G \left[1 + \frac{\bar{n}^h}{N} (z - 1)\right]^{mN} +$$
$$+ c_4 z^4 \sum_m P_m^G \left[1 + \frac{\bar{n}^h}{N} (z - 1)\right]^{mN} .$$

The "4" topology is responsible for the tail of HM.

## Study of the process $e^+e^- \rightarrow 3(\pi^+\pi^-)$ in the c.m. energy range 1.5-2.0 GeV with the CMD-3 detector

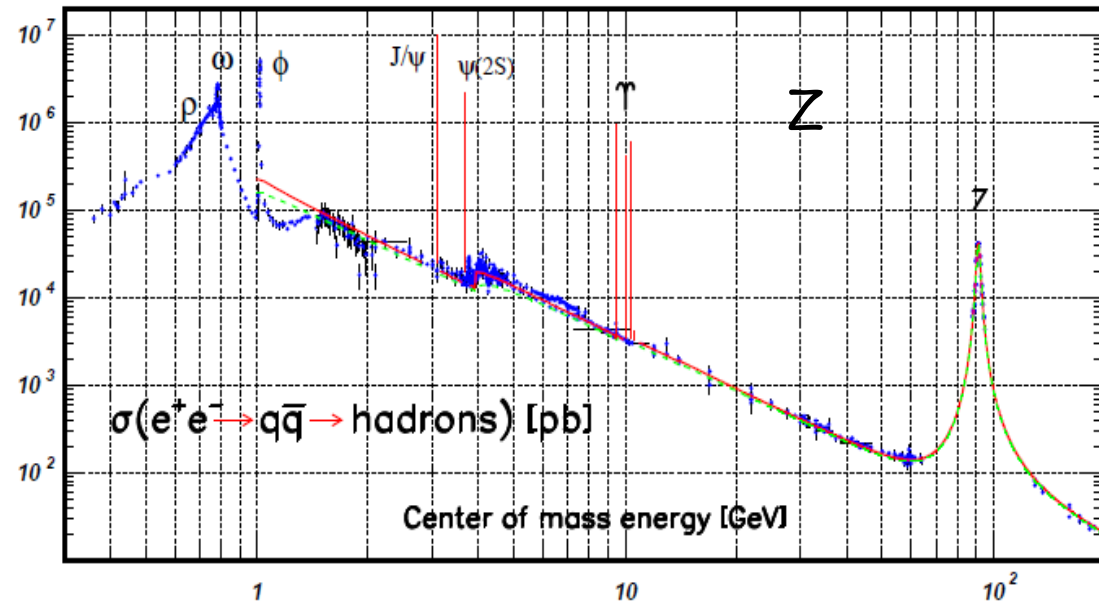
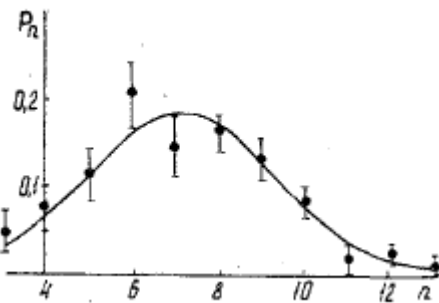
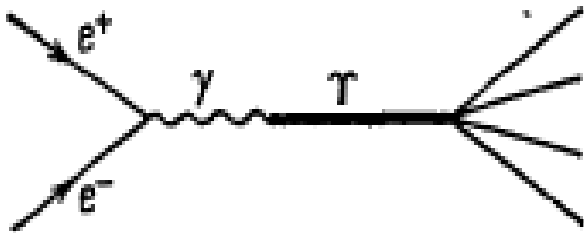
" $\sigma(e^+e^- \rightarrow 3(\pi^+\pi^-))$  is measured at the CMD-3 (VEPP-2000) in the c.m. energy range 1.5-2.0 GeV. The measured  $\sigma$  exhibits a sharp drop near the  $p\bar{p}$  threshold ( $\sim 1.88$  GeV). A first study of dynamics of 6-pion production has been performed".

R.R. Akhmetshin et al. Phys. Lett. B 723 (2013) 82–89.



$\sigma(e^+e^- \rightarrow 3(\pi^+\pi^-))$  measured with the CMD-3 at VEPP-2000 (black). BaBar data are shown by (green). The red line shows the  $p\bar{p}$  threshold.

# Three-gluon decay of quarkoniums $\Upsilon(9.46)$ , $\Upsilon(10.02)$



$$\Delta \bar{n}_{TSM}(s) = [\alpha(\bar{m}' - \bar{m}_{(q)}) - 3(\alpha - 2/3)] \bar{n}_q^h \quad \Delta \bar{n}_{exp}(s) \approx \Delta \bar{n}_{TSM}(s) \approx 0.8 \quad \bar{m} = \bar{m}' + 3.$$