Elastic proton-proton and pion-proton scattering in holographic QCD

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Based on:
Zhibo Liu, Wei Xie, and AW, PRD 107, 014018 (2023)
Zhibo Liu and AW, PRD 108, 034010 (2023)
Yu-Peng Zhang, Xun Chen, Xiao-Hua Li, and AW, PRD 108, 066001 (2023)
Yu-Peng Zhang, Xun Chen, Xiao-Hua Li, and AW, PRD 109, 074010 (2024)

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Outline

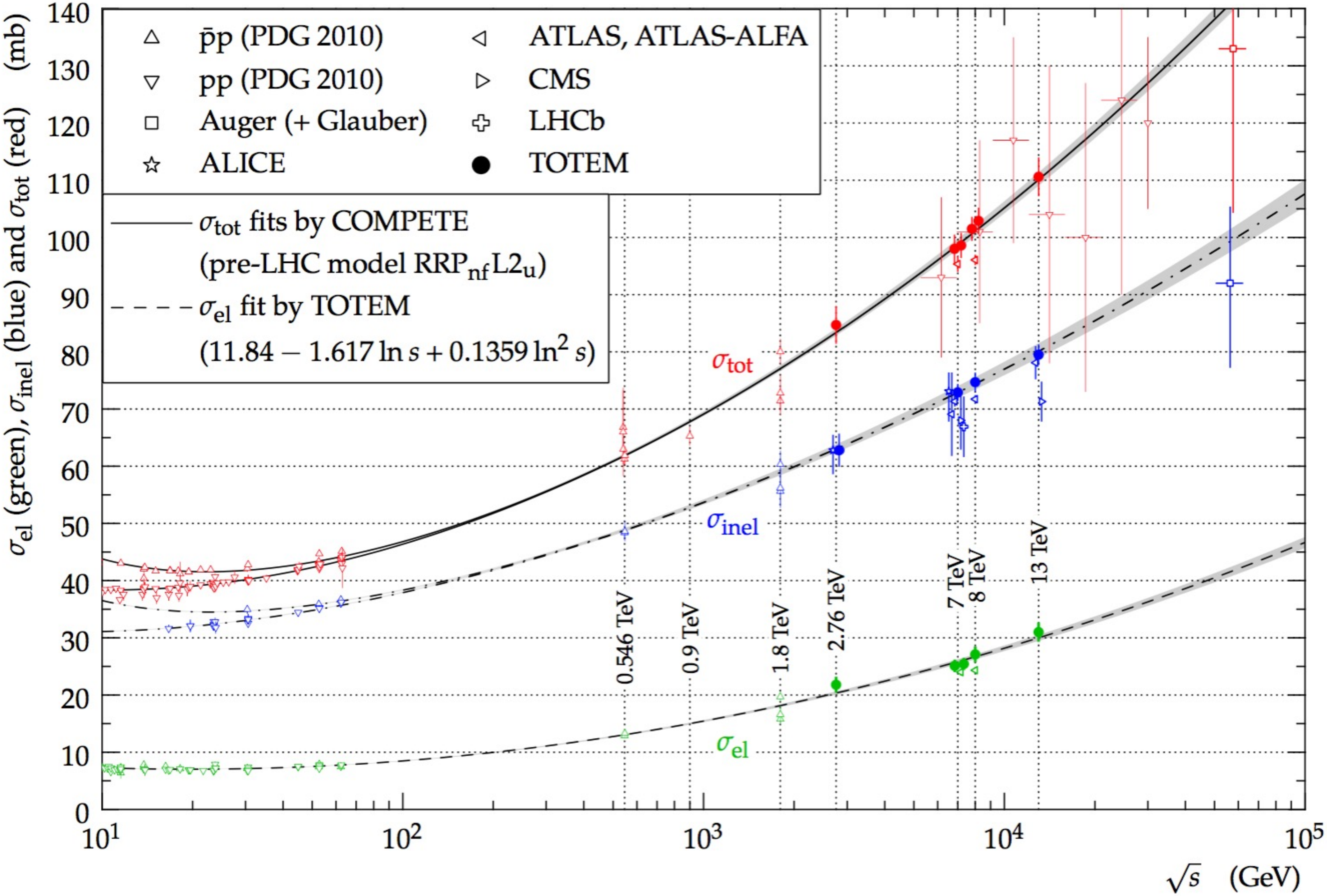
1. Introduction
2. Three attempts for elastic proton-(anti)proton scattering in the Regge regime via holography
    1. Pomeron exchange
       (for high energy region)
    2. Adding Reggeon exchange contribution
       (for intermediate energy region)
    3. Adding Coulomb interaction contribution
       (for very small momentum transfer region)
3. Analysis on elastic pion-proton scattering
4. Summary

Motivation

- Understanding the quark-gluon structure of hadrons is one of the most important problems in high energy physics.
- High energy hadron scattering cross sections include information on the internal structure of the involved hadrons. Elastic hadron-hadron scattering is one of the simplest processes.
- Since the cross sections are basically nonperturbative physical quantities, first principle calculations are almost impossible in practice.
- Improving effective methods is essential.
- As an effective approach, a holographic QCD model is applied for the analysis on elastic hadron-hadron scattering processes.

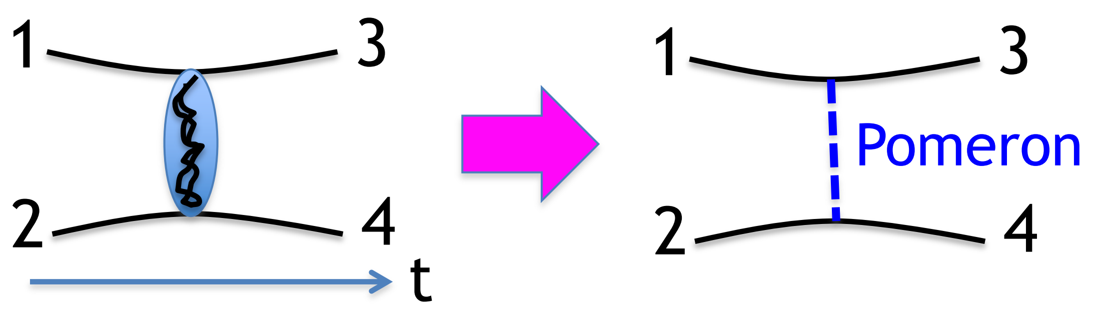
13TeV data from TOTEM

TOTEM Collaboration (2017)



Pomeron exchange picture

- A description of high energy scattering before QCD
- Pomeron: a color singlet gluonic object
- It is known that total cross sections of high energy two-body scattering can be well described with this picture



$$\sigma_{tot}(s) \sim s^{\alpha_0 - 1}$$

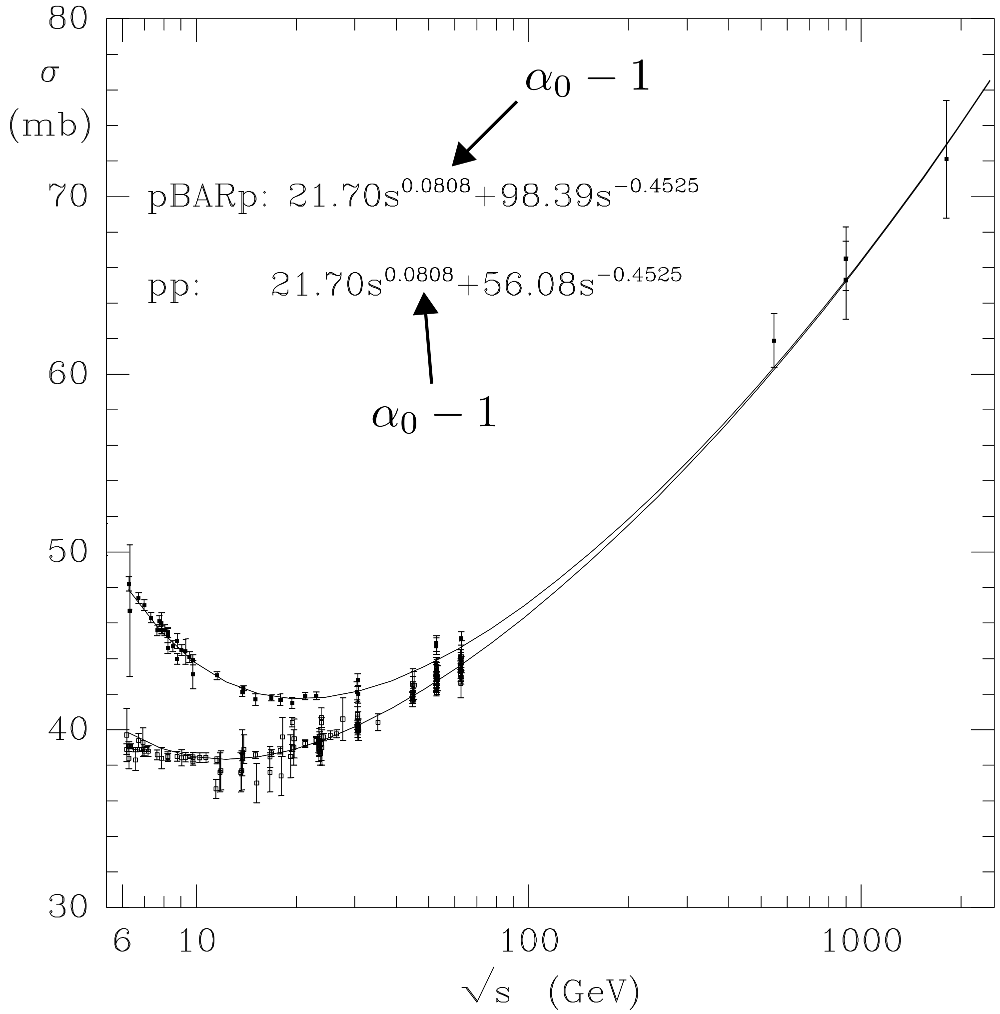
where

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

Total cross sections can be expressed with a single parameter (Pomeron intercept)

Total cross sections via soft Pomeron exchange

proton-(anti)proton total cross section



Donnachie-Landshoff (1992)

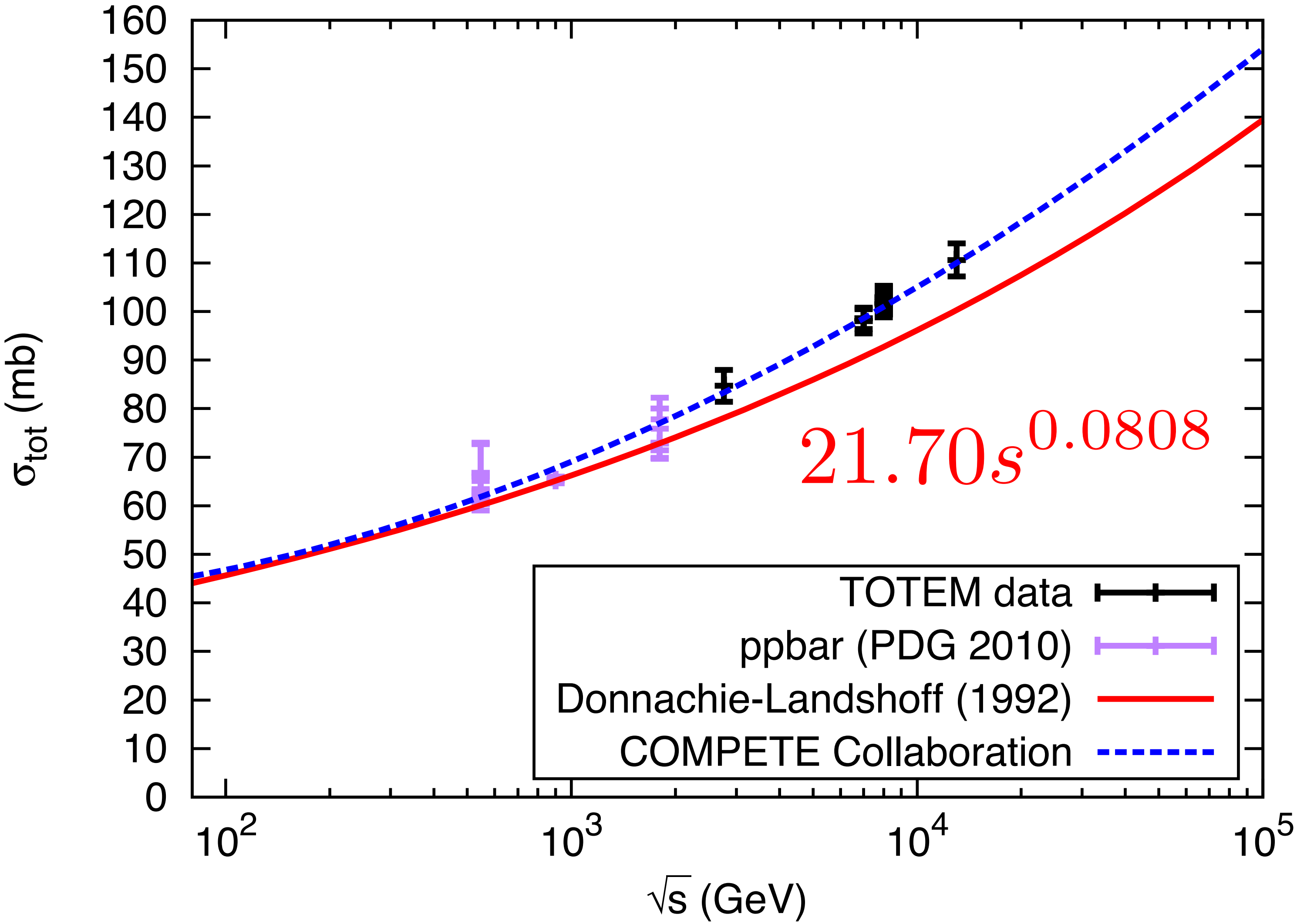
$$\sigma_{tot}(s) \sim s^{\alpha_0 - 1}$$

$$\alpha_0 = 1.0808$$

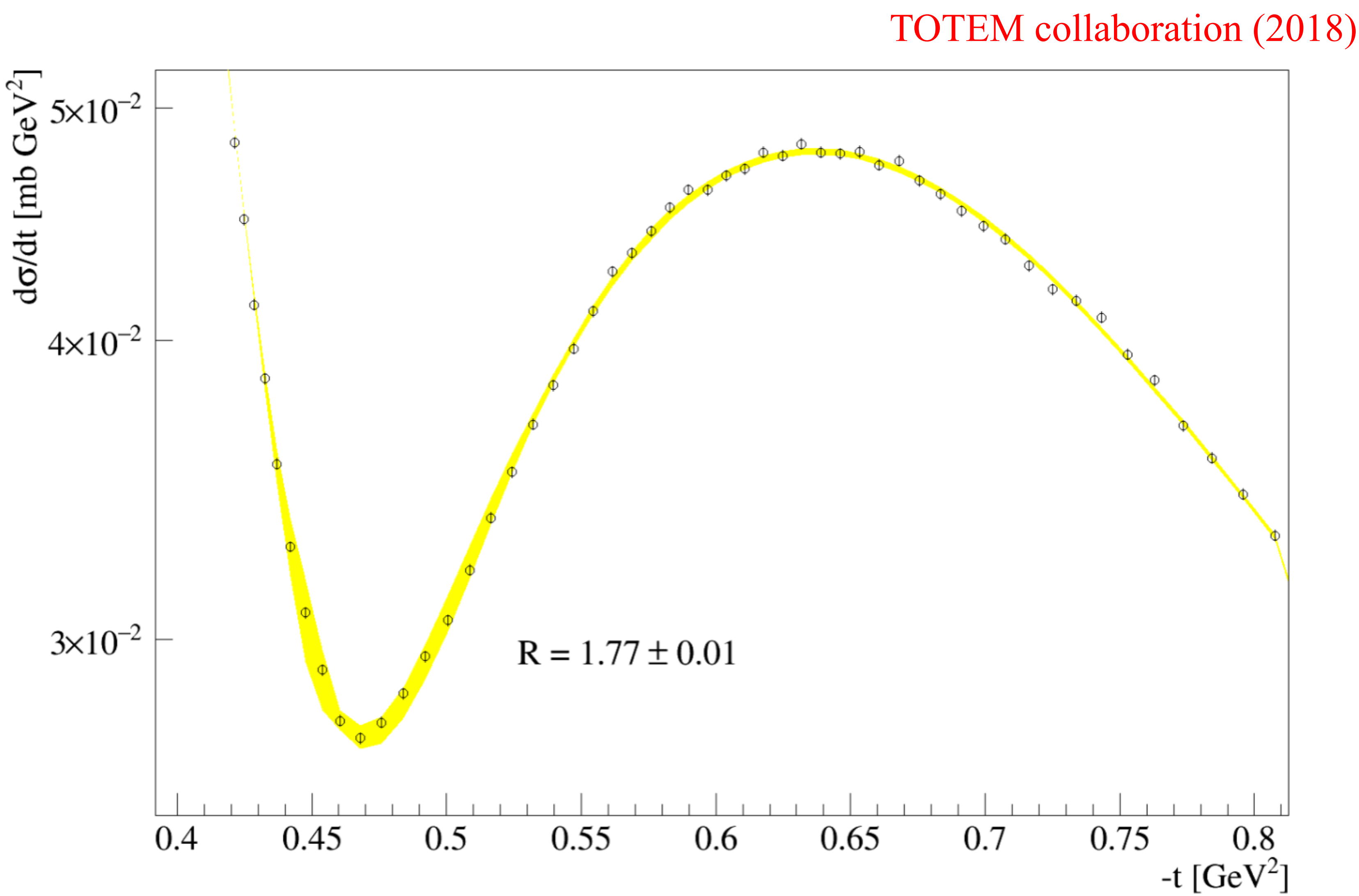
(soft Pomeron intercept)

well describing the experimental data

Can the soft Pomeron reproduce the data?



Differential cross section data from TOTEM



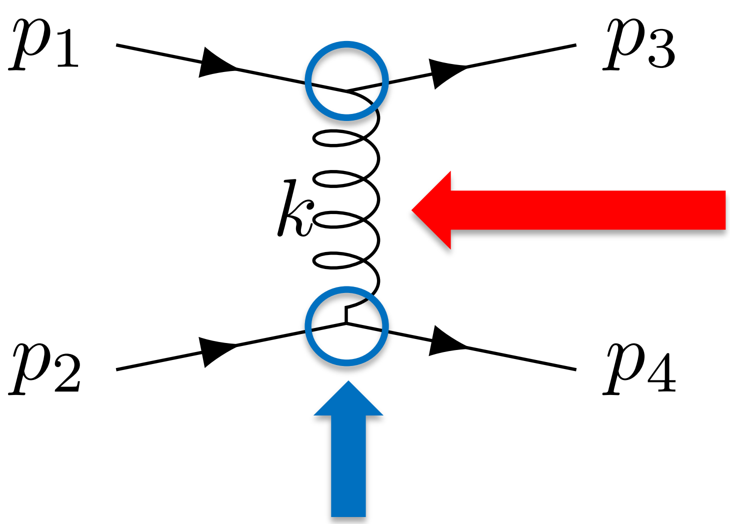
Holographic QCD

- Holographic QCD, which is constructed based on the AdS/CFT correspondence, has a potential to be a powerful tool for analysis in hadron physics.

type IIB supergravity theory on $S^5 \times AdS_5$ ⟺ strong coupling 4D N=4 supersymmetric Yang-Mills (SYM) theory

supergravity theory (classical theory) on AdS_5 ⟺ usual 4D QCD at strong coupling

Model setup



Domokos-Harvey-Mann (2009)

Reggeized spin-2 particle propagator

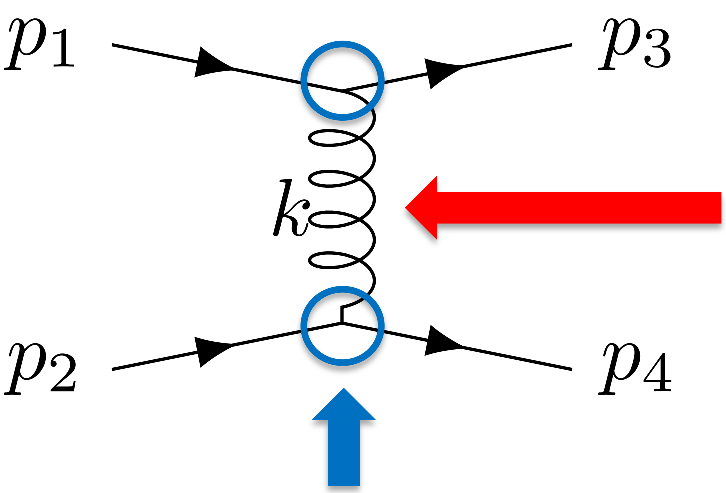
- 3 adjustable parameters are determined with data

Gravitational form factor

- Calculable with the bottom-up AdS/QCD model
- Model parameters are fixed with hadron properties

Applicable for other hadron-hadron scattering processes by replacing the form factors

Spin-2 glueball exchange



$$\frac{d_{\alpha\beta\gamma\delta}(k)}{k^2 + m_g^2}$$

$$\langle p', s' | T_{\mu\nu}(0) | p, s \rangle = \bar{u}(p', s') \left[A(t) \frac{\gamma_\alpha P_\beta + \gamma_\beta P_\alpha}{2} + B(t) \frac{i(P_\alpha \sigma_{\beta\rho} + P_\beta \sigma_{\alpha\rho}) k^\rho}{4 m_p} + C(t) \frac{(k_\alpha k_\beta - \eta_{\alpha\beta} k^2)}{m_p} \right] u(p, s)$$

Spin-2 glueball exchange

$$\mathcal{M}_g = \frac{\lambda^2 d_{\alpha\beta\gamma\delta}}{4(t-m_g^2)}\left[A(t)(\bar{u}_1\gamma^\alpha u_3)(p_1+p_3)^\beta + \frac{iB(t)}{2m_p}(p_1+p_3)^\beta k_\rho(\bar{u}_1\sigma^{\alpha\rho}u_3) + \frac{C(t)}{m_p}(\bar{u}_1 u_3)(k^\alpha k^\beta - \eta^{\alpha\beta}t)\right]$$
$$\times\left[A(t)(\bar{u}_2\gamma^\gamma u_4)(p_2+p_4)^\delta + \frac{iB(t)}{2m_p}(p_2+p_4)^\delta k_\lambda(\bar{u}_2\sigma^{\gamma\lambda}u_4) + \frac{C(t)}{m_p}(\bar{u}_2 u_4)(k^\gamma k^\delta - \eta^{\gamma\delta}t)\right]$$

where

$$d_{\alpha\beta\gamma\delta} = \frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}) - \frac{1}{2m_g^2}(k_\alpha k_\delta \eta_{\beta\gamma} + k_\alpha k_\gamma \eta_{\beta\delta} + k_\beta k_\delta \eta_{\alpha\gamma} + k_\beta k_\gamma \eta_{\alpha\delta})$$
$$+\frac{1}{24}\left[\left(\frac{k^2}{m_g^2}\right)^2 - 3\left(\frac{k^2}{m_g^2}\right) - 6\right]\eta_{\alpha\beta}\eta_{\gamma\delta} - \frac{k^2 - 3m_g^2}{6m_g^4}(k_\alpha k_\beta \eta_{\gamma\delta} + k_\gamma k_\delta \eta_{\alpha\beta}) + \frac{2k_\alpha k_\beta k_\gamma k_\delta}{3m_g^4}$$

$s \gg |t|$

$$\frac{d\sigma}{dt} = \frac{\lambda^4 s^2 A^4(t)}{16\pi(t-m_g^2)^2}$$

Needs to be Reggeized to include the higher spin states

Reggeized Spin-2 particle exchange

$$\frac{d\sigma}{dt} = \frac{\lambda^4 A^4(t)\Gamma^2[-\chi]\Gamma^2\left[1-\frac{\alpha_c(t)}{2}\right]}{16\pi\Gamma^2\left[\frac{\alpha_c(t)}{2}-1-\chi\right]}\left(\frac{\alpha_c' s}{2}\right)^{2\alpha_c(t)-2}$$

$$\sigma_{tot} = \frac{\pi\lambda^2\Gamma[-\chi]}{\Gamma\left[\frac{\alpha_c(0)}{2}\right]\Gamma\left[\frac{\alpha_c(0)}{2}-1-\chi\right]}\left(\frac{\alpha_{c'} s}{2}\right)^{\alpha_c(0)-1}$$

where

$$\alpha_c(x) = \alpha_c(0) + \alpha_c' x$$

$$\chi = \alpha_c(s) + \alpha_c(t) + \alpha_c(u) = 4\alpha_c' m^2 + 3\alpha_c(0)$$

3 parameters: $\lambda,\ \alpha_c(0),\ \alpha_c'$

to be determined with experimental data

Nucleon gravitational form factors

Abidin-Carlson (2009)

Matrix element of the energy momentum tensor in respect to spin 1/2 particle:

$$\langle p_2, s_2 | T^{\mu\nu}(0) | p_1, s_1 \rangle = u(p_2, s_2)\left(A(t)\gamma^{(\mu} p^{\nu)} + B(t)\frac{ip^{(\mu}\sigma^{\nu)\alpha} q_\alpha}{2m} + C(t)\frac{q^\mu q^\nu - q^2 \eta^{\mu\nu}}{m} \right) u(p_1, s_1)$$

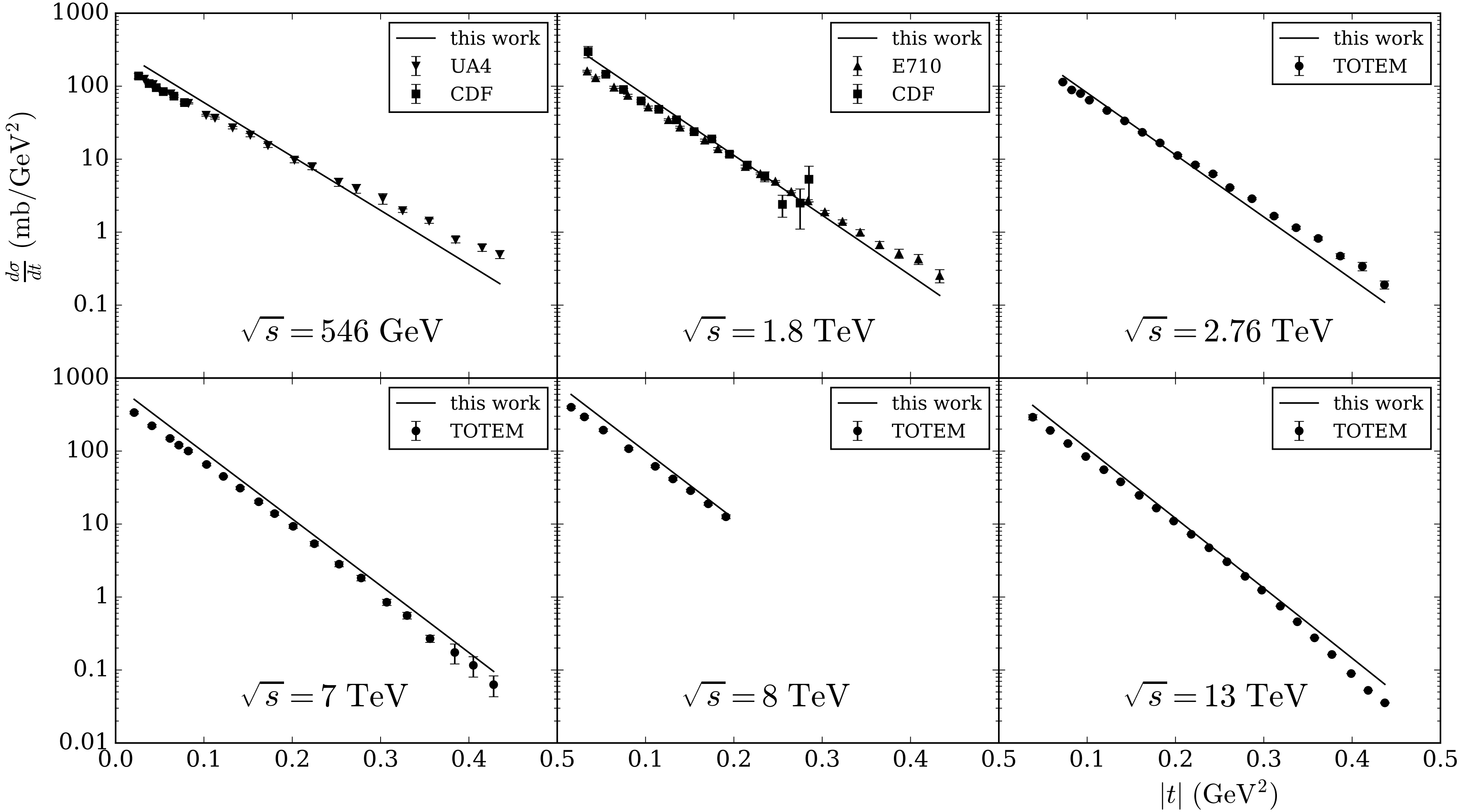
A bottom-up AdS/QCD model of the nucleon:

$$S_F = \int d^5x \sqrt{g} e^{-\kappa^2 z^2} \left(\frac{i}{2} \bar{\Psi} e_A^N \Gamma^A D_N \Psi - \frac{i}{2} (D_N \Psi)^\dagger \Gamma^0 e_A^N \Gamma^A \Psi - (M + \kappa^2 z^2) \bar{\Psi}\Psi \right)$$

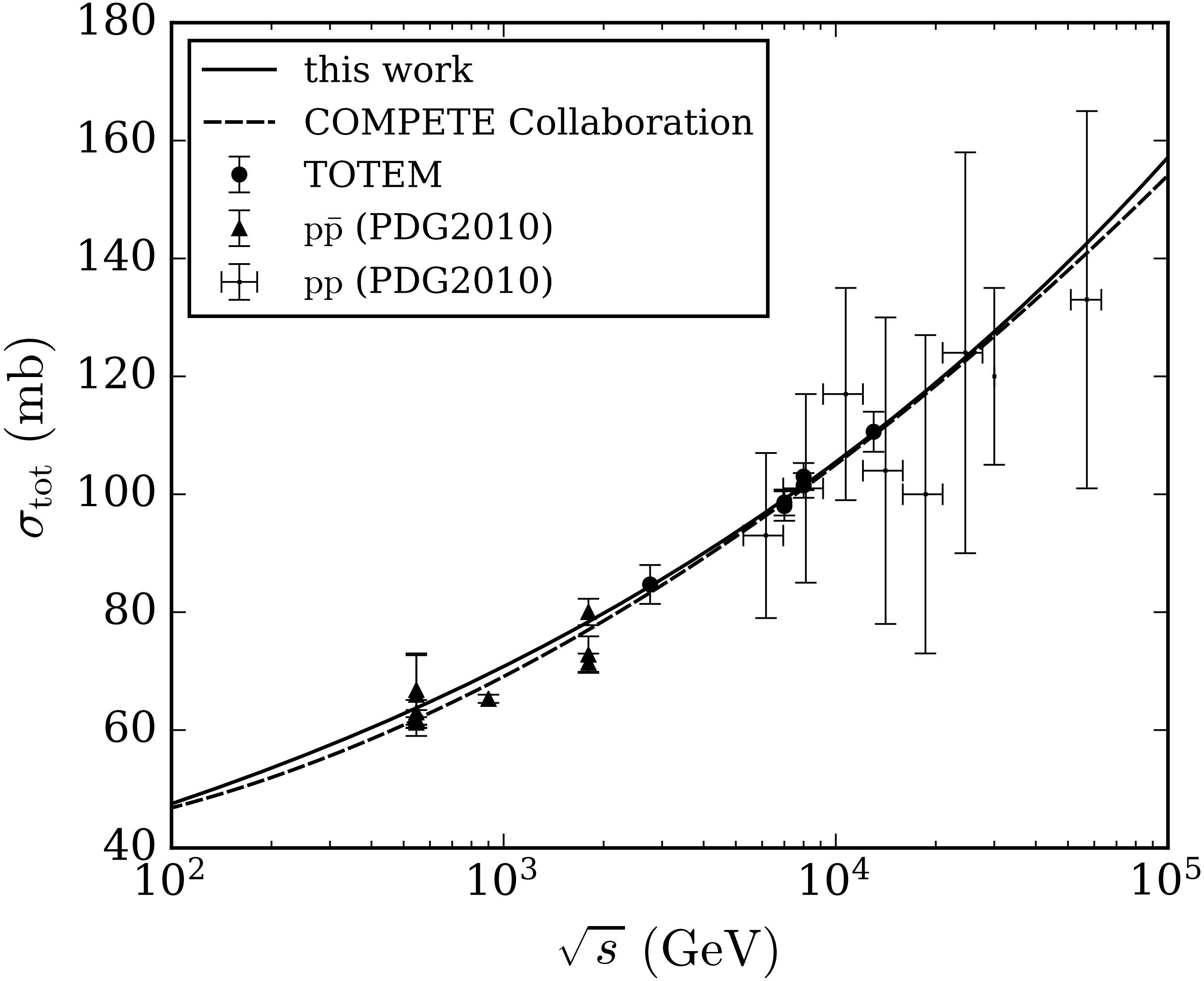
5D AdS space: $ds^2 = g_{MN} dx^M dx^N = \frac{1}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right)$

Introduce the metric perturbation, $\eta_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$, in the 5D classical action, and pick up the hψψ terms.
By comparing the Lorentz structure of them, one can obtain the form factors.

Differential cross section

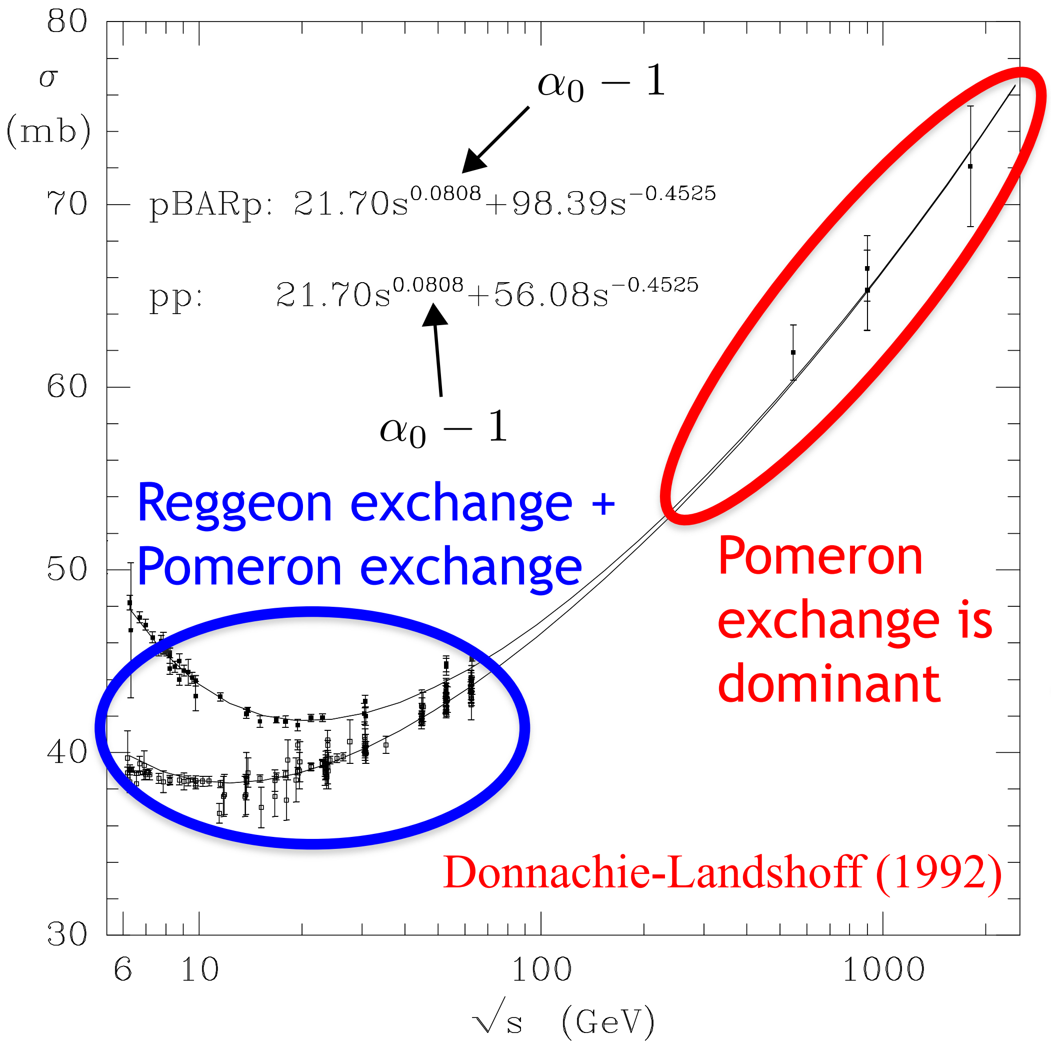


Total cross section



Description with Reggeon + Pomeron

proton-(anti)proton total cross section



$$\sigma_{tot}(s) \sim s^{\alpha_0 - 1}$$

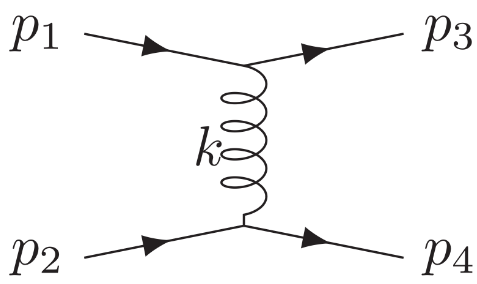
$$\alpha_0 = 1.0808$$

(soft Pomeron intercept)

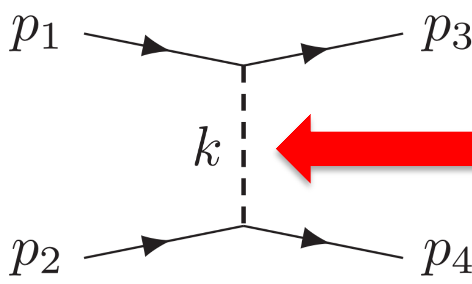
well describing the experimental data

Description with Pomeron + Reggeon

Pomeron exchange



Reggeon exchange



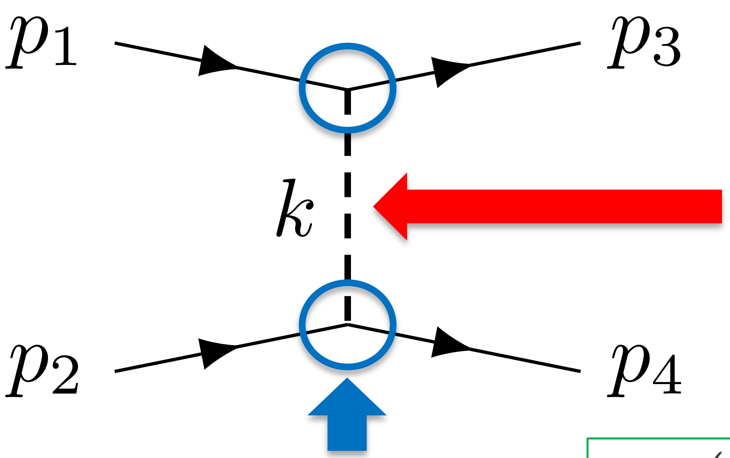
Anderson-Domokos-Mann (2017)

Reggeized vector meson propagator

- Dominant in high energy region
- Same treatment as the previous

- This contribution raises the total cross section and decreases with the energy
- Required to describe the intermediate energy region (Sqrt(s) < 100GeV)

Vector meson exchange



$$D^{v}_{\mu\nu}(k) = \frac{i}{k^2 + m_v}\eta_{\mu\nu}$$

$$\Gamma^{\mu}_{v} = -i\lambda_v \gamma^{\mu}$$

$$\mathcal{A}^{pp(p\bar{p})}_{v} = (\bar{u}_1 \Gamma^{\mu}_{v} u_3) D^{v}_{\mu\nu}(k) (\bar{u}_2 \Gamma^{\nu}_{v} u_4)$$
$$= -\frac{i\lambda_v^2}{k^2 + m_v^2}\eta_{\mu\nu}(\bar{u}_1 \gamma^{\mu} u_3)(\bar{u}_2 \gamma^{\nu} u_4)$$

- For the Pomeron related parameters, we use the previously determined ones.
- The newly introduced parameter λ_v is determined with the total cross section data, and differential cross sections can be predicted without any adjustable parameters.

Total scattering amplitude

$$\mathcal{A}_{\mathrm{tot}}^{pp(p\bar{p})} = \frac{-i\lambda_g^2}{8(t-m_g^2)}\left[2sA^2(t)(\bar{u}_1\gamma^\alpha u_3)(\bar{u}_2\gamma_\alpha u_4) + 4A^2(t)p_2^\alpha p_1^\beta(\bar{u}_1\gamma_\alpha u_3)(\bar{u}_2\gamma_\beta u_4)\right] + \frac{i\lambda_v^2}{t-m_v^2}\eta_{\mu\nu}(\bar{u}_1\gamma^\mu u_3)(\bar{u}_2\gamma^\nu u_4)$$

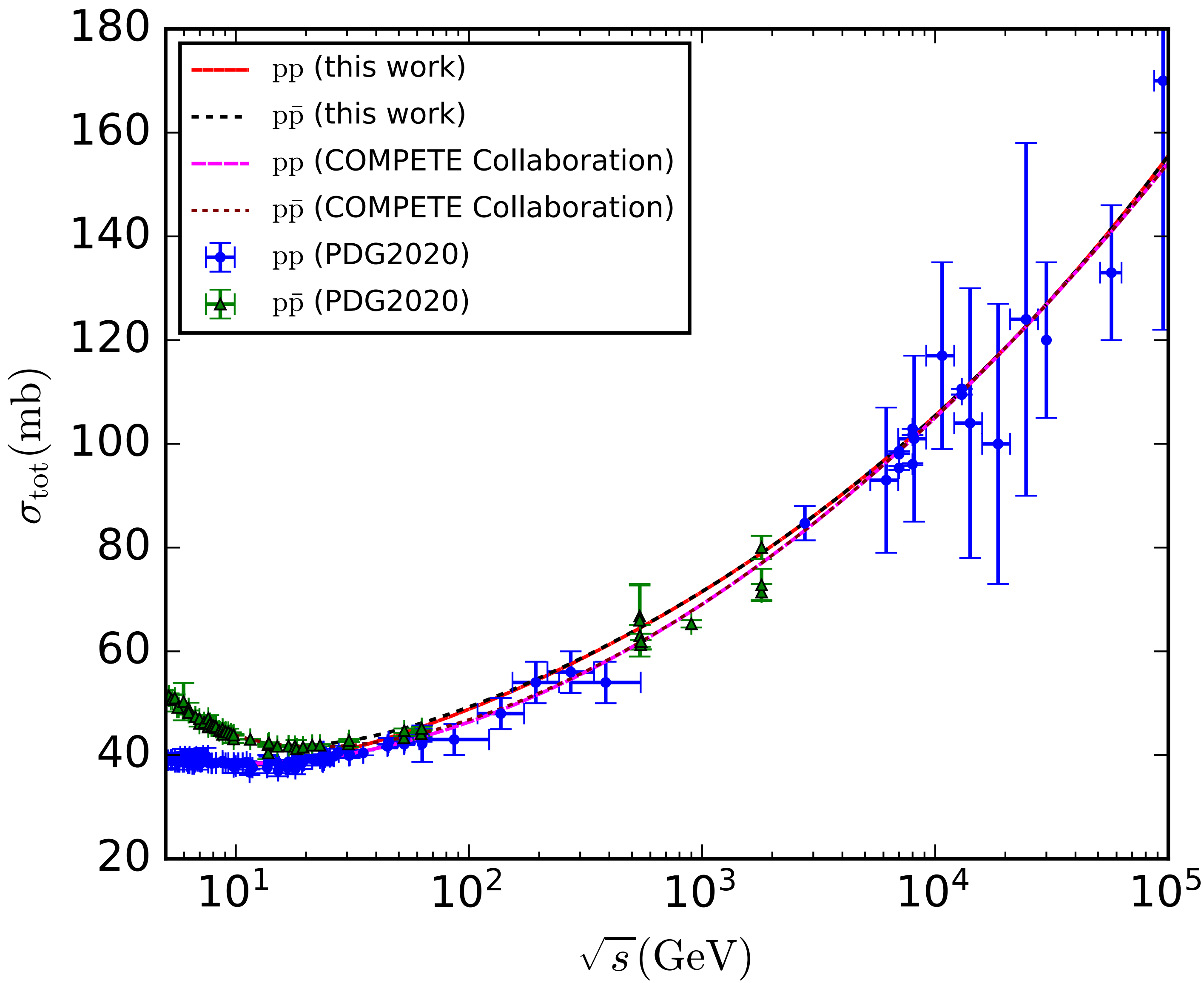
$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}|\mathcal{A}_{\mathrm{tot}}|^2 = \frac{\lambda_g^4 s^2 A^2(t)}{16\pi|t-m_g^2|^2} + \frac{\lambda_g^2\lambda_v^2 A^2(t)s}{8\pi}\left[\frac{1}{(t-m_g^2)^*}\times\frac{1}{t-m_v^2} + \frac{1}{(t-m_v^2)^*}\times\frac{1}{t-m_g^2}\right] + \frac{\lambda_v^4}{4\pi|t-m_v^2|^2}$$

Cross sections after Reggeization

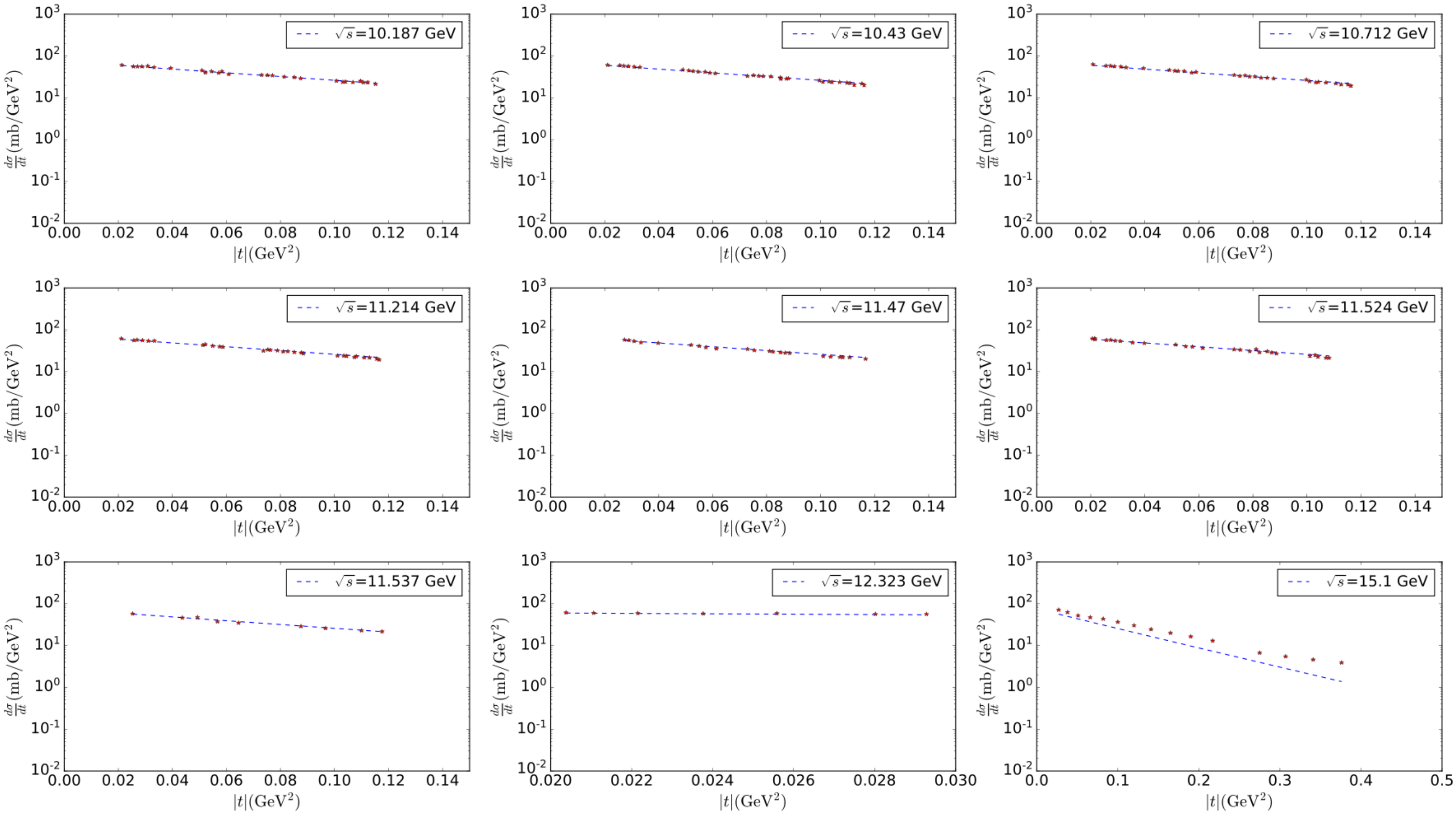
$$
\begin{aligned}
\frac{d\sigma}{dt} =& \frac{\lambda_g^4 s^2 A^4(t)}{16\pi} \left[\frac{\alpha_g'}{2} \frac{\Gamma\left[3-\frac{\chi_g}{2}\right]\Gamma\left[1-\frac{\alpha_g(t)}{2}\right]}{\Gamma\left[2-\frac{\chi_g}{2}+\frac{\alpha_g(t)}{2}\right]} \left(\frac{\alpha_g' s}{2}\right)^{\alpha_g(t)-2} \right]^2 \\
&- \frac{\lambda_g^2 \lambda_v^2 s A^2(t)}{4\pi} \left[\frac{\alpha_g'}{2} \frac{\Gamma\left[3-\frac{\chi_g}{2}\right]\Gamma\left[1-\frac{\alpha_g(t)}{2}\right]}{\Gamma\left[2-\frac{\chi_g}{2}+\frac{\alpha_g(t)}{2}\right]} \left(\frac{\alpha_g' s}{2}\right)^{\alpha_g(t)-2} \right] \\
&\quad \times \left[\alpha_v' \sin\left(\frac{\pi\alpha_v(t)}{2}\right) (\alpha_v' s)^{\alpha_v(t)-1} \Gamma[-\alpha_v(t)] \right] \cos\left[\frac{\pi}{2}(\alpha_g(t)-\alpha_v(t))\right] \\
&+ \frac{\lambda_v^4}{4\pi} \left[\alpha_v' \sin\left(\frac{\pi\alpha_v(t)}{2}\right) (\alpha_v' s)^{\alpha_v(t)-1} \Gamma[-\alpha_v(t)] \right]^2
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{tot} =& \frac{1}{s} \mathrm{Im}\mathcal{A}(s, t=0) \\
=& \lambda_g^2 \sin\left(\frac{\pi\alpha_g(0)}{2}\right) \frac{\Gamma\left[3-\frac{\chi_g}{2}\right]\Gamma\left[1-\frac{\alpha_g(t)}{2}\right]}{\Gamma\left[2-\frac{\chi_g}{2}+\frac{\alpha_g(t)}{2}\right]} \left(\frac{\alpha_g' s}{2}\right)^{\alpha_g(0)-1} \\
&- 2\lambda_v^2 \alpha_v' \sin^2\left(\frac{\pi\alpha_v(0)}{2}\right) (\alpha_v' s)^{\alpha_v(0)-1} \Gamma[-\alpha_v(0)]
\end{aligned}
$$

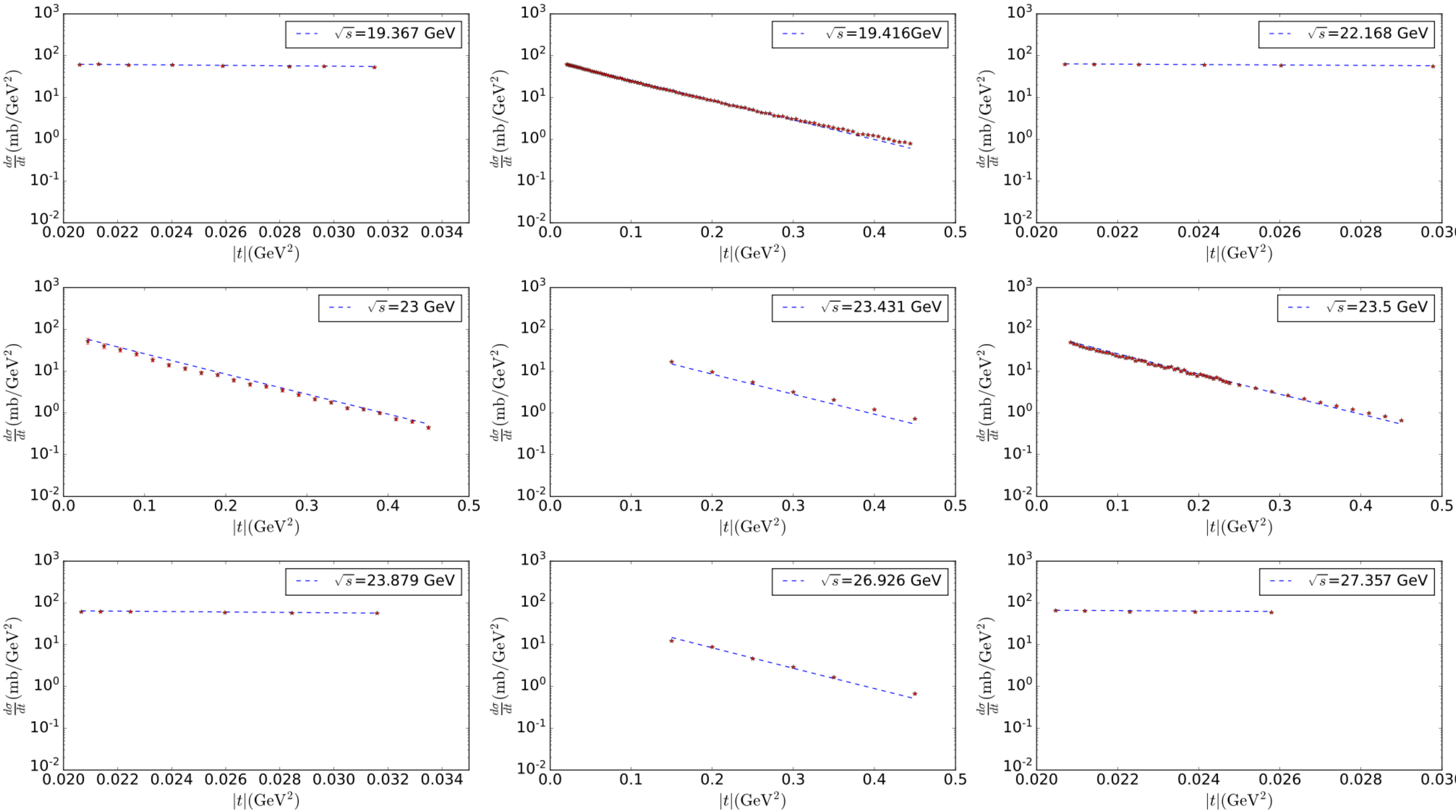
Total cross sections



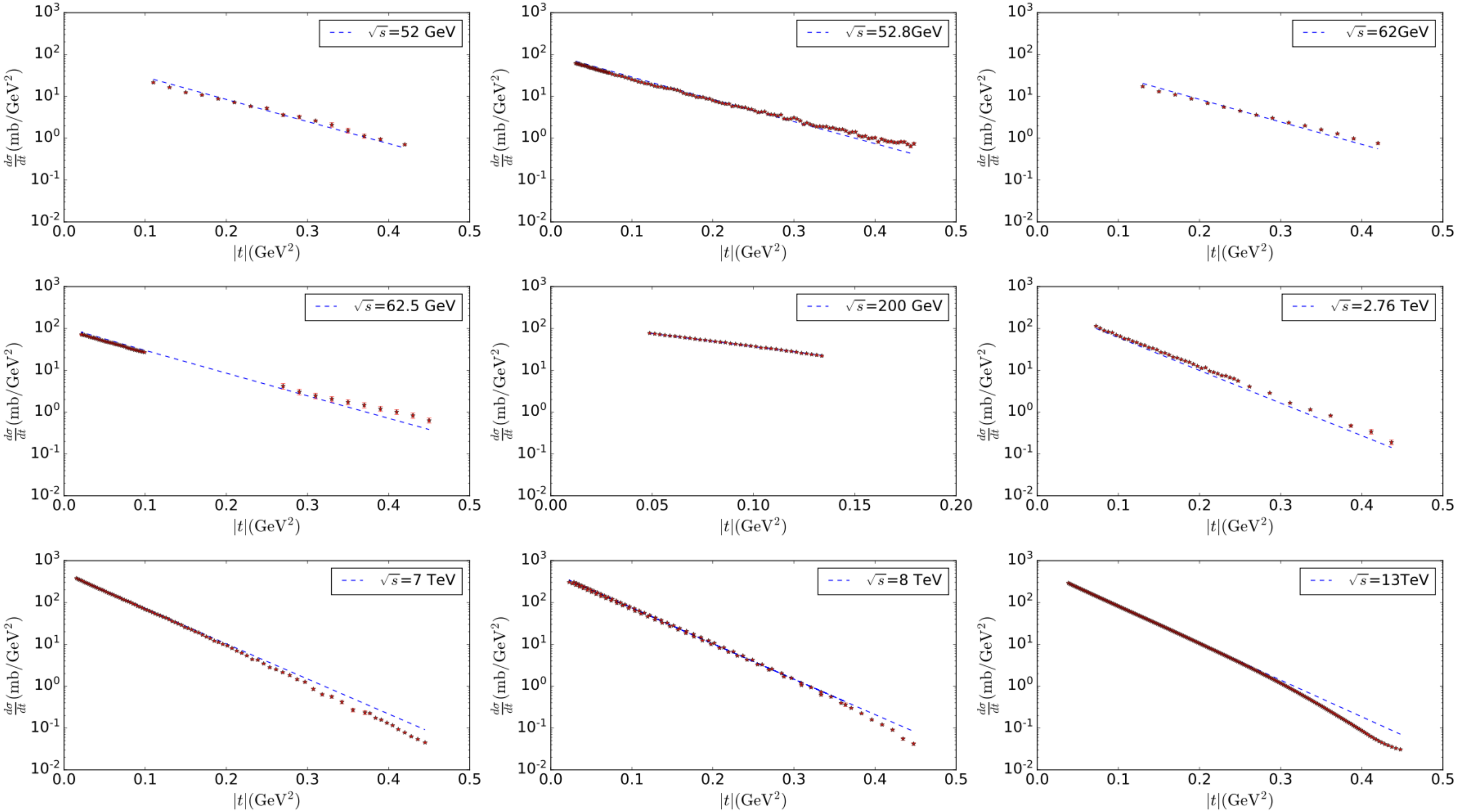
Differential pp cross section (1/3)



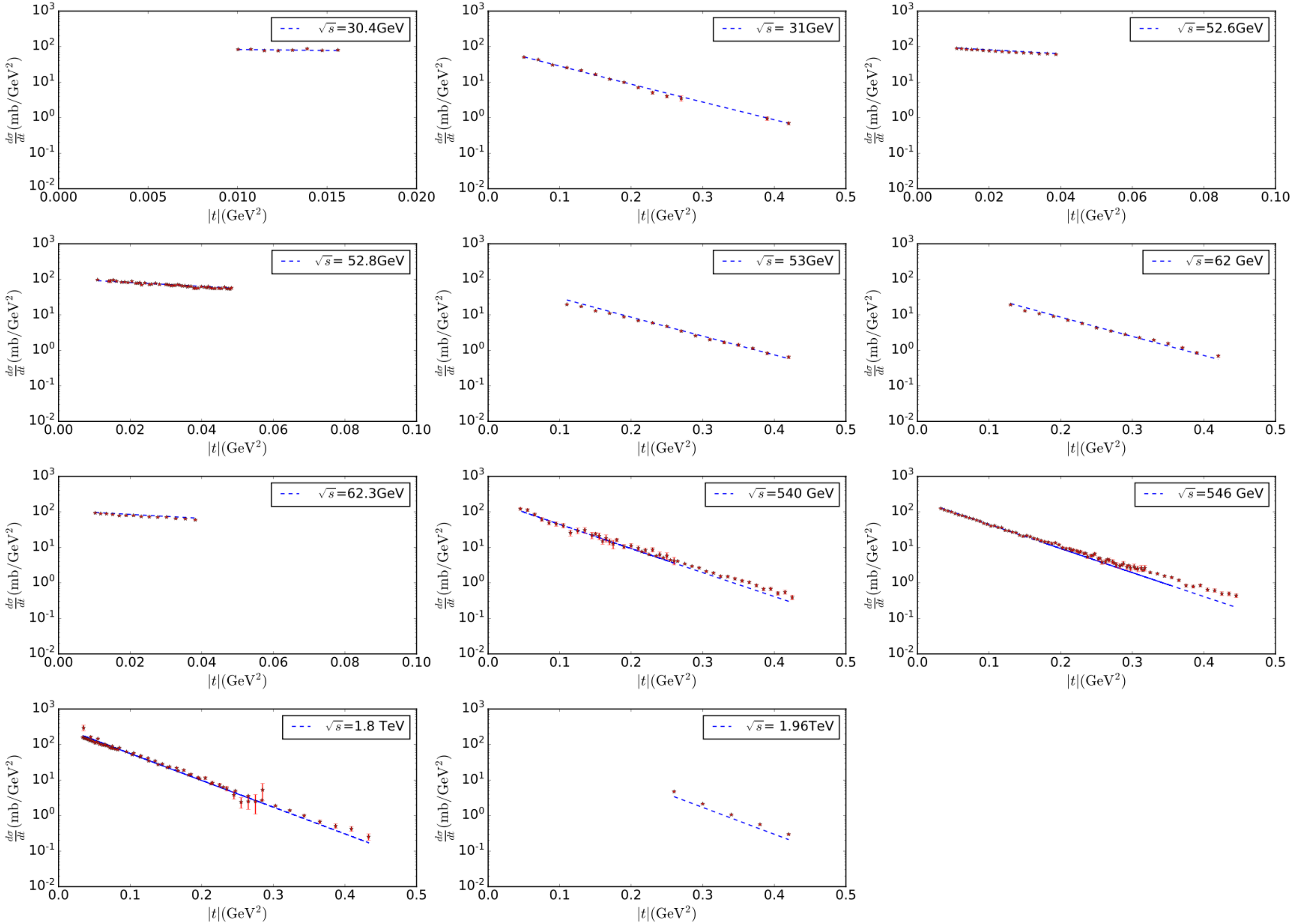
Differential pp cross section (2/3)



Differential pp cross section (3/3)



Differential ppbar cross section



Contribution of Coulomb interaction

Taking into account the Coulomb interaction (one photon exchange), the total scattering amplitude can be expressed as

$$\mathcal{A}_{\text{tot}} = \mathcal{A}_N + e^{i\alpha\phi}\mathcal{A}_C$$

$$\mathcal{A}_C(s,t) = \mp\frac{8\pi\alpha s}{|t|}G_{\text{eff}}^2(t)$$

Negative (positive) sign corresponds to identical (opposite) charges case

For the Coulomb phase, the expression proposed by Cahn is employed:

$$\phi_{\text{Cahn}} = -\left[\ln\left(\frac{B|t|}{2}\right) + \gamma + C\right]$$

Cahn (1982)

where

$$C = \ln\left(1 + \frac{8}{B\Lambda^2}\right) + \frac{4|t|}{\Lambda^2}\ln\left(\frac{4|t|}{\Lambda^2}\right) + \frac{2|t|}{\Lambda^2}$$

The parameter Λ^2 can be determined with the electromagnetic form factor obtained in the bottom-up AdS/QCD model

Total scattering amplitude

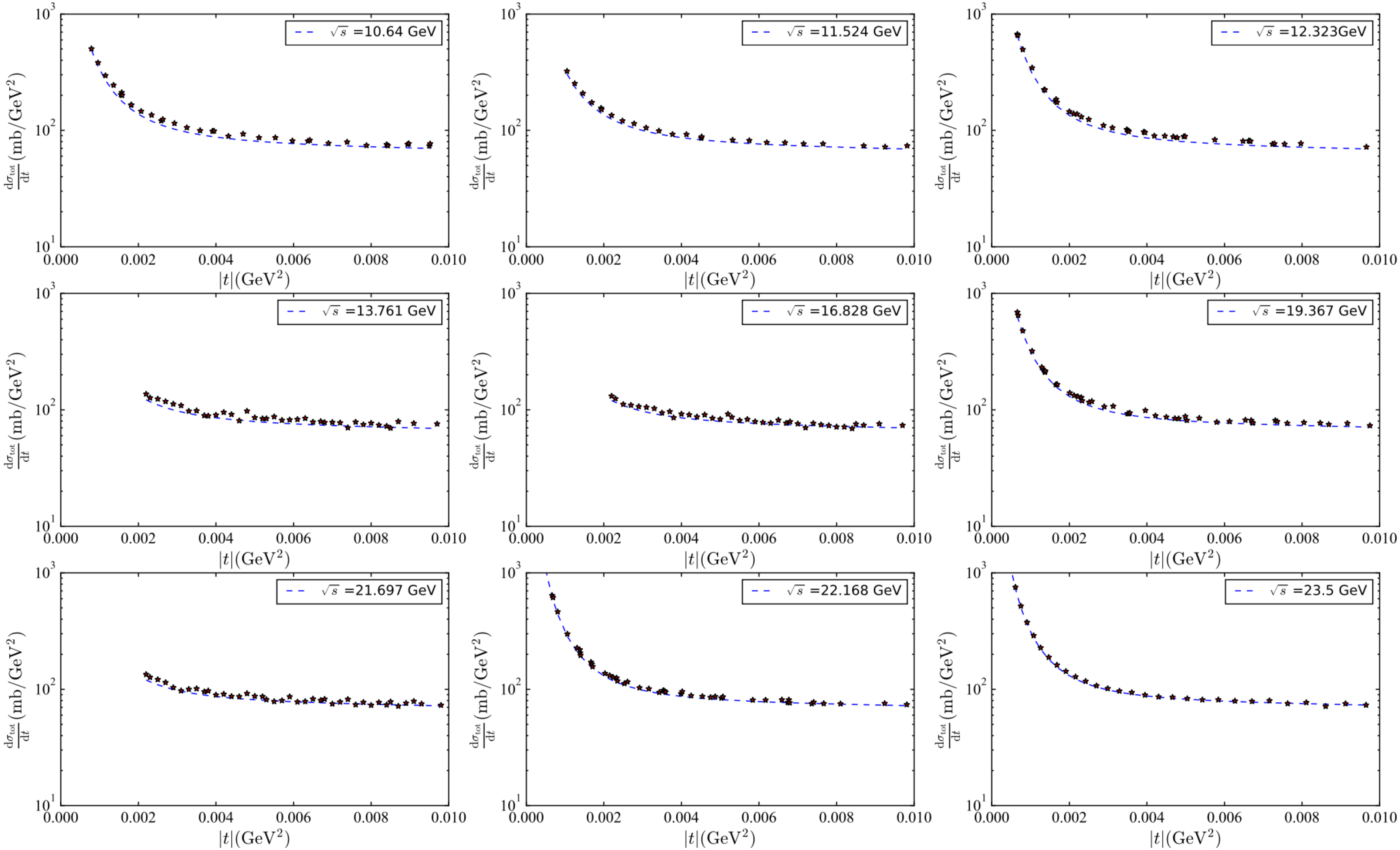
Involving the one photon exchange contribution, the total scattering amplitude is obtained as

$$
\begin{aligned}
\mathcal{A}_{\mathrm{tot}} = &\mp e^{i\alpha\phi}\frac{8\pi\alpha s}{|t|}G_{\mathrm{eff}}^{2}(t) \\
&- s\lambda_g^2 A^2(t) e^{-\frac{i\pi\alpha_g(t)}{2}} \frac{\Gamma\left[-\chi_g\right]\Gamma\left[1-\frac{\alpha_g(t)}{2}\right]}{\Gamma\left[\frac{\alpha_g(t)}{2}-1-\chi_g\right]}\left(\frac{\alpha_g' s}{2}\right)^{\alpha_g(t)-1} \\
&+ s\lambda_v^2 \alpha_v' e^{-\frac{i\pi\alpha_v(t)}{2}} \sin\left[\frac{\pi\alpha_v(t)}{2}\right]\left(\alpha_v' s\right)^{\alpha_v(t)-1}\Gamma\left[-\alpha_v(t)\right]
\end{aligned}
$$

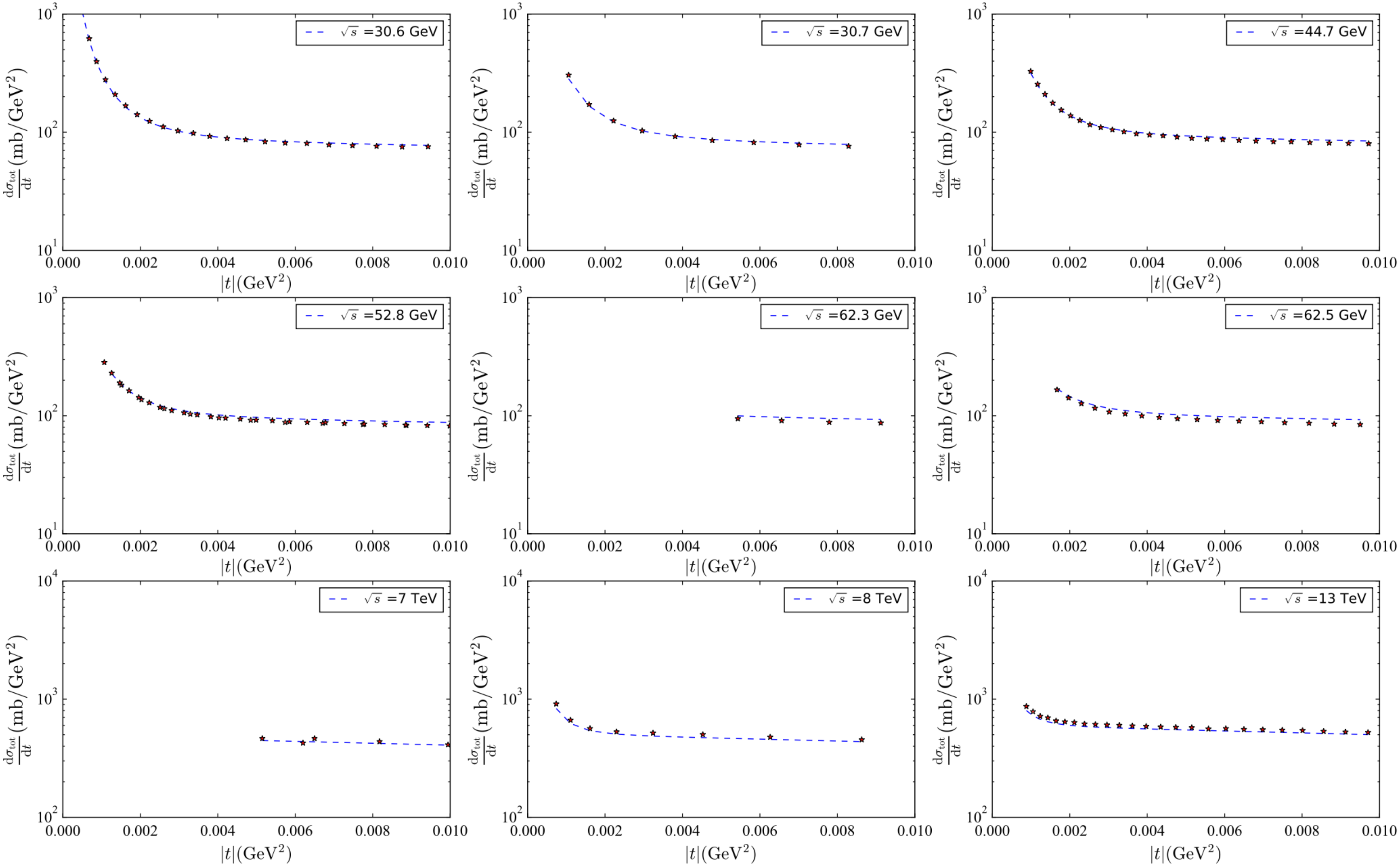
$$
\frac{d\sigma_{\mathrm{tot}}}{dt} = \frac{1}{16\pi s^2}\left|\mathcal{A}_{\mathrm{tot}}(s,t)\right|^2
$$

Cross terms --> Coulomb-nuclear interference (CNI) effect

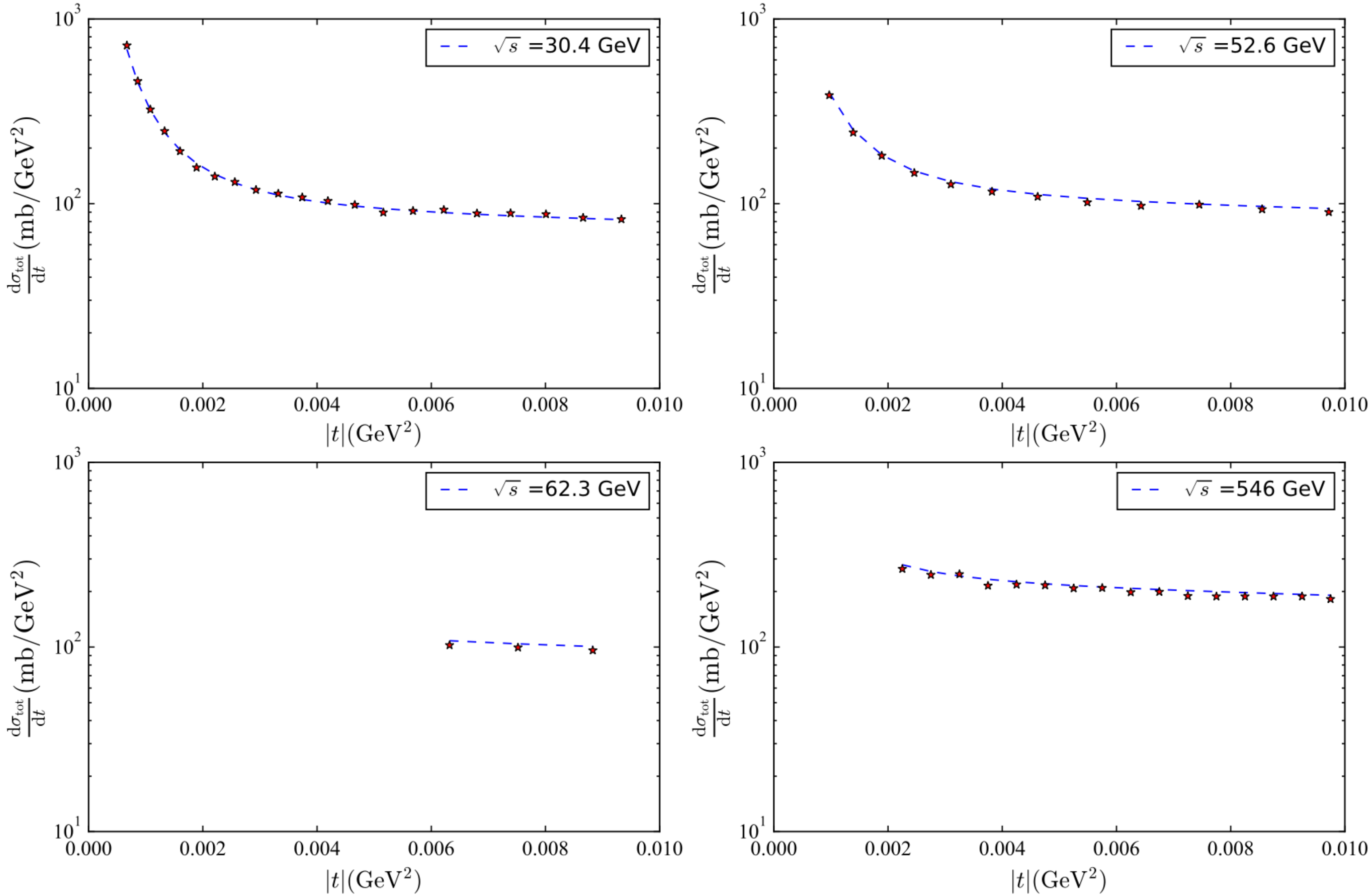
Differential pp cross section (1/2)



Differential pp cross section (2/2)



Differential ppbar cross section



Analysis on pion-proton scattering

- Procedures are the same as those for the proton-proton case.
- The gravitational and electromagnetic form factors of the pion are required to numerically evaluate the resulting cross sections, but those can also be obtained with bottom-up AdS/QCD models.

$$F_{C+N}(s,t) = -e^{i\alpha\phi}\frac{8\pi\alpha s}{|t|}G_\pi G_p - s\lambda_{g\pi\pi}\lambda_{gpp}A_\pi(t)A_p(t)e^{-\frac{i\pi\alpha_g(t)}{2}}\frac{\Gamma[-\chi_g]\Gamma\left[1-\frac{\alpha_g(t)}{2}\right]}{\Gamma\left[\frac{\alpha_g(t)}{2}-1-\chi_g\right]}\left(\frac{\alpha_g' s}{2}\right)^{\alpha_g(t)-1}$$
$$+ 2s\lambda_{v\pi\pi}\lambda_{vpp}\alpha_v' e^{-\frac{i\pi\alpha_v(t)}{2}}\sin\left[\frac{\pi\alpha_v(t)}{2}\right](\alpha_v' s)^{\alpha_v(t)-1}\Gamma[-\alpha_v(t)],$$

$$\frac{d\sigma_{C+N}}{dt} = \frac{1}{16\pi s^2}|F_{C+N}(s,t)|^2$$

A bottom-up AdS/QCD model of mesons

Erlich-Katz-Son-Stephanov (2005)

$$S_{\mathrm{AdS}} = \mathrm{Tr} \int d^4x dz \left[\frac{1}{z^3} |DX|^2 + \frac{3}{z^5} |X|^2 - \frac{1}{2g_5^2 z} \left(F_V^2 + F_A^2 \right) \right]$$

5D coupling

vector

axial vector

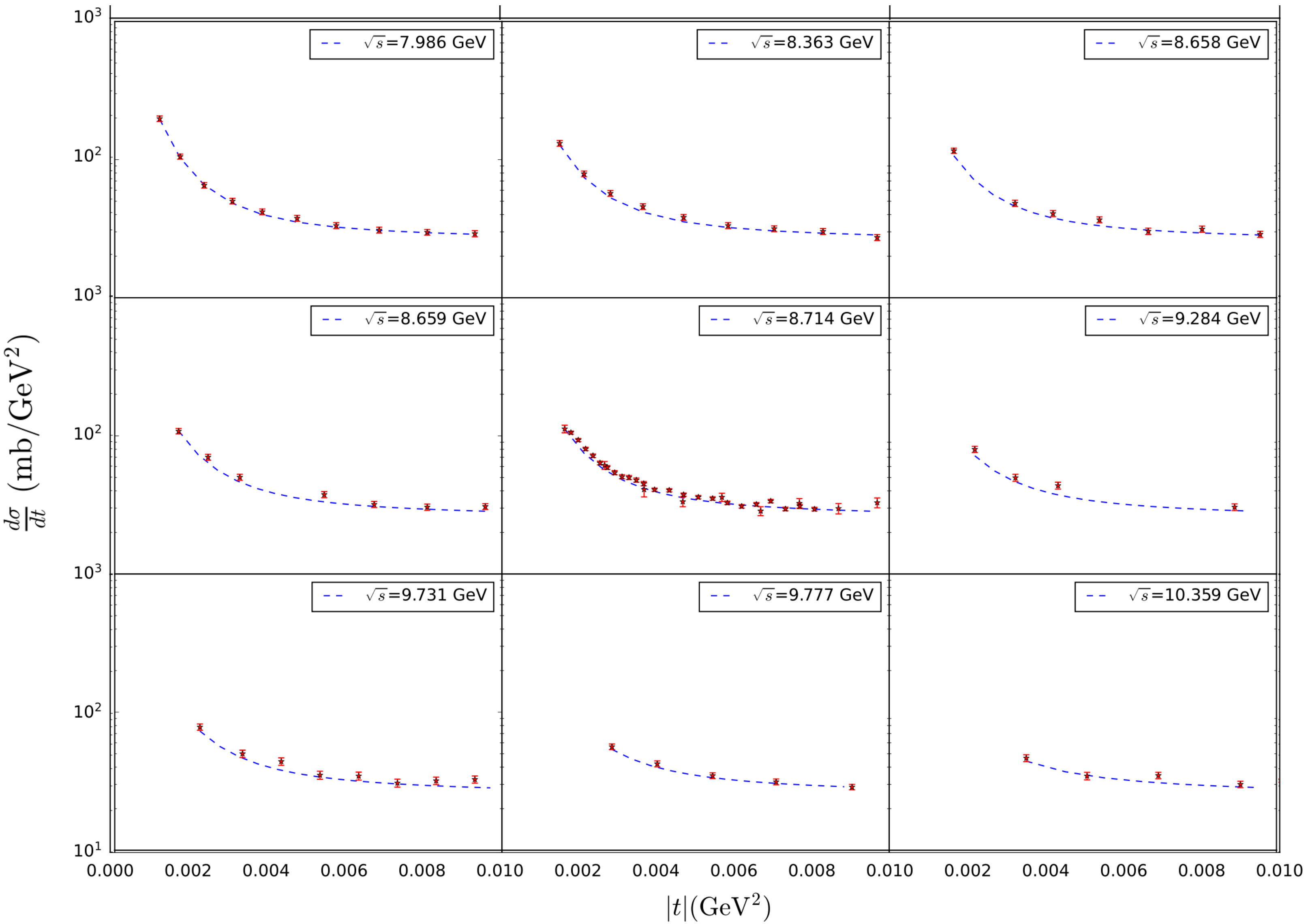
$$\begin{cases} X = X_0 \exp\left(2i\pi^a t^a\right) \\ D^M X = \partial^M X - i\left[V^M, X\right] - i\left\{A^M, X\right\} \\ F_V^{MN} \equiv \partial^M V^N - \partial^N V^M - i\left(\left[V^M, V^N\right] + \left[A^M, A^N\right]\right) \\ F_A^{MN} \equiv \partial^M A^N - \partial^N A^M - i\left(\left[V^M, A^N\right] + \left[A^M, V^N\right]\right) \\ X_0(z) = \underline{m_{\mathrm{q}} z/2 + \sigma z^3/2} \end{cases}$$

chiral symmetry breaking
(explicit and spontaneous)

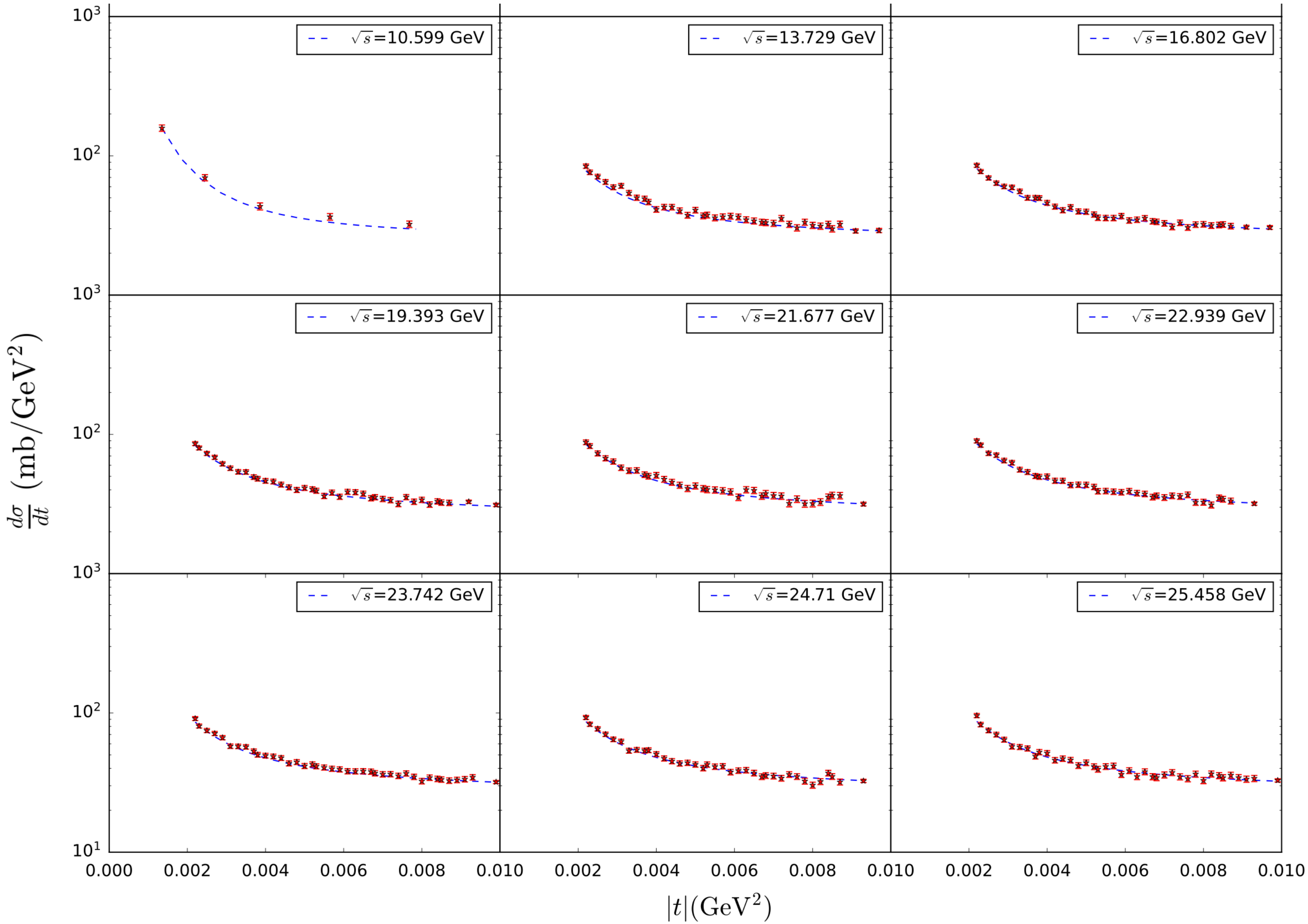
$$M, N = 0 \sim 3, z$$

$$V_z = A_z = 0$$

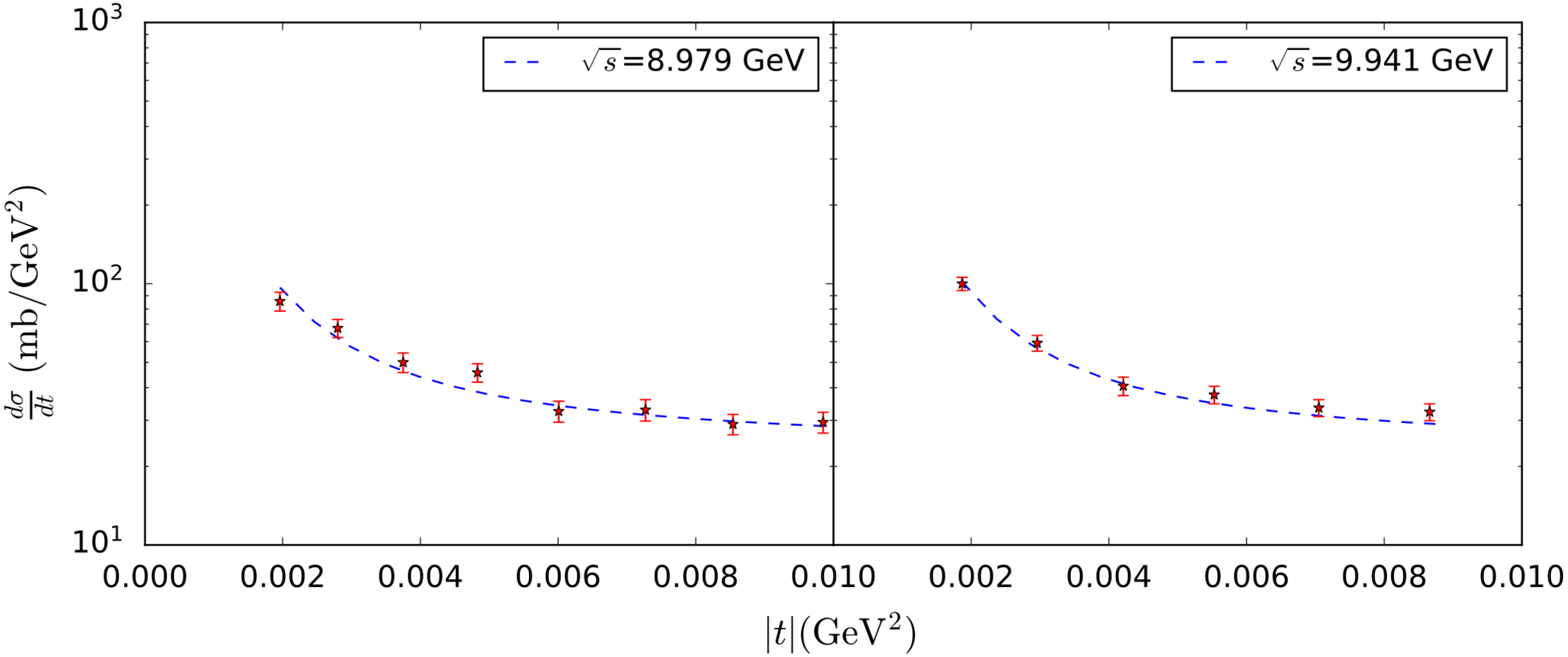
Differential π⁻p cross section (1/2)



Differential π⁻p cross section (2/2)



Differential π^+p cross section



Summary

- We have investigated elastic proton-(anti)proton and pion-proton scattering in the Regge regime in holographic QCD.
- Three contributions (Pomeron + Reggeon + Coulomb interaction) have been taken into account to calculate the cross sections.
- It is found that the experimental data of both the total and differential cross sections can be well described within the model in a wide kinematic region.
- Other hadron-hadron scattering processes can be studied in this framework.