Multiplicity Distributions and Modified Combinants in the Multipomeron Model of pp Interaction at High Energies

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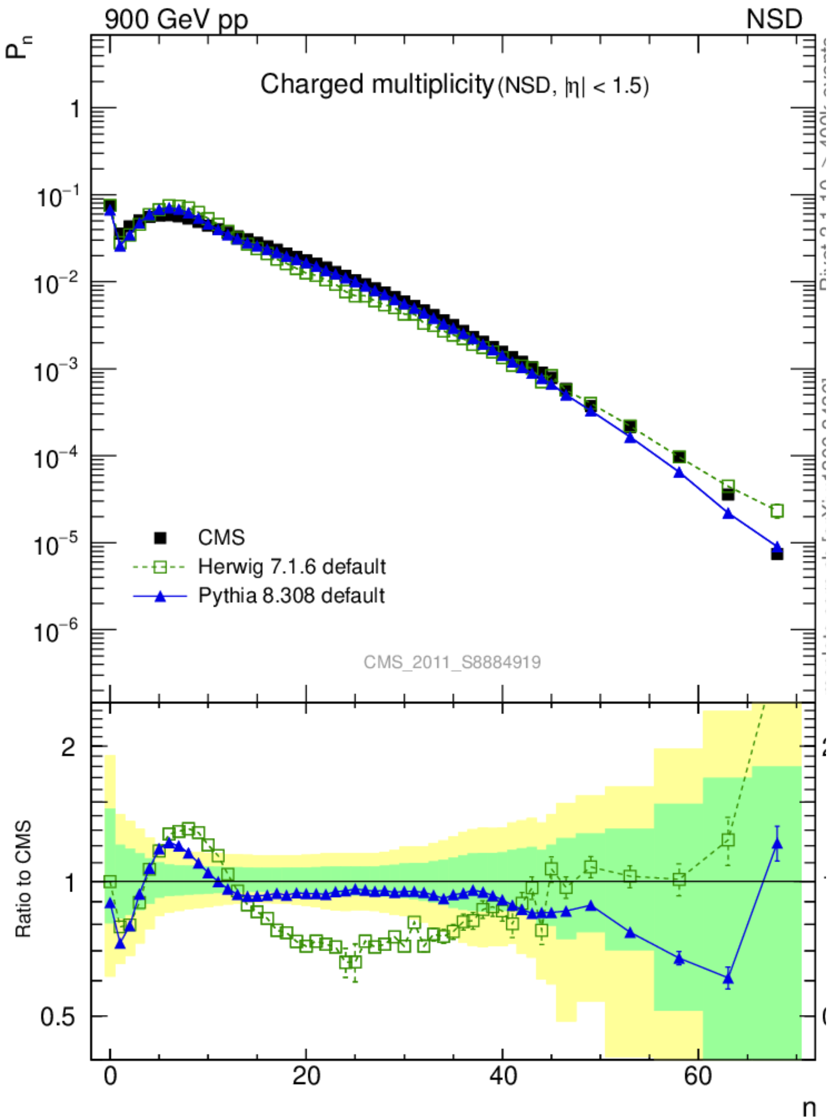
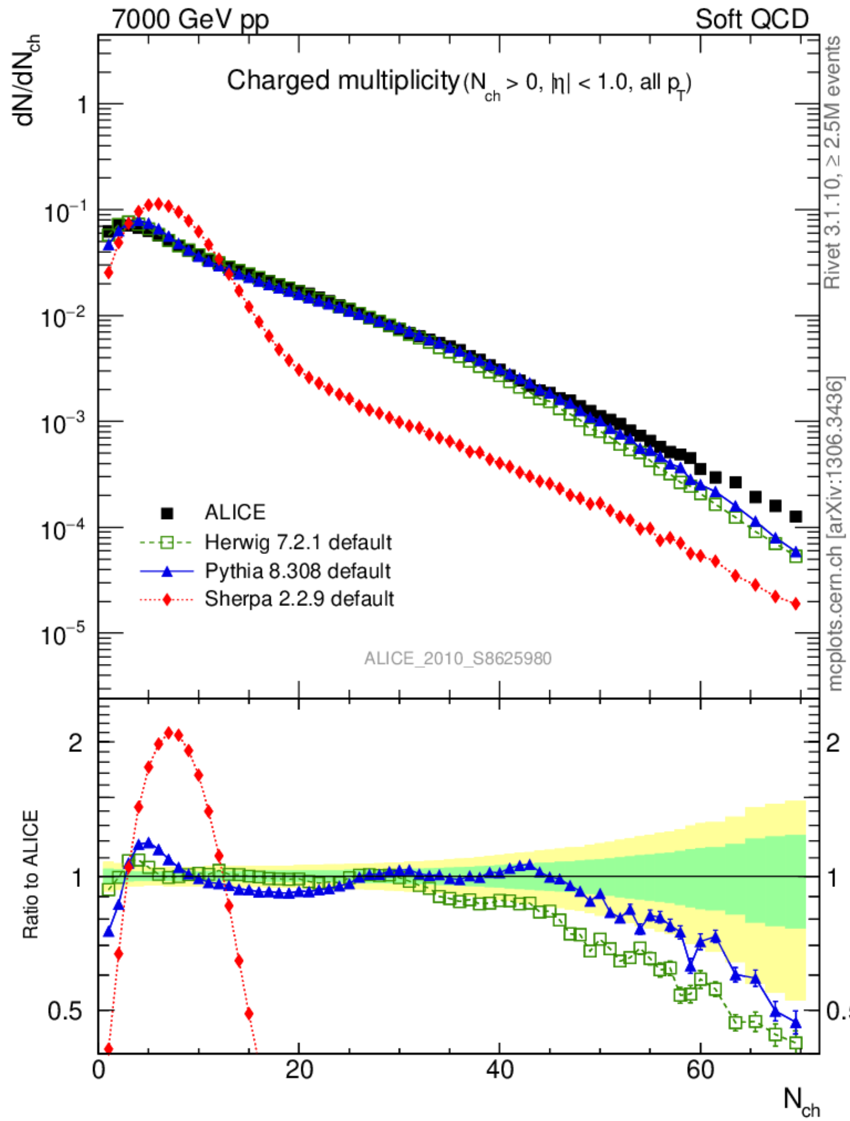
Multiplicity distributions





https://mcplots.cern.ch

Multiplicity distribution and combinants

- Generating function for multiplicity distribution:

$$G(t) = \sum_{N=0}^{\infty} P(N)\, t^N$$

- Generating function for combinants:

$$F(t) = \sum_{j=0}^{\infty} C^*(j)\, t^j \,, \qquad \text{where} \qquad F(t) = \ln G(t)$$

- Recurrence relation for combinants:

$$N\, P(N) = \sum_{j=1}^{N} j\, C^*(j)\, P(N-j)$$

Modified combinants

$$C(j) \equiv \frac{j+1}{\langle N \rangle} C^*(j+1)\,, \quad \text{where} \quad \langle N \rangle = \sum_{N=1}^{\infty} N\,P(N)$$

- Recurrence relation for modified combinants:

$$(N+1)\,P(N+1) = \langle N \rangle \sum_{j=0}^{N} C(j)\,P(N-j)$$

Relationships of this type or even simpler ones:

$(N+1)\,P(N+1) = g(N)\,P(N)$, where *g(N)* is a linear function,

are found in many models (clans, cascade models).

See e.g. formula (32) in

M.A. Braun, C. Pajares, V.V. Vechernin, Physics Letters B 493 (2000) 54–64,
“On the forward–backward correlations in a two-stage scenario”.

$$X(j) \equiv \langle N \rangle C(j) = (j+1)\,C^*(j+1)$$

Using the recurrence relation to calculate combinants

$$X(N) = (N+1)\frac{P(N+1)}{P(0)} - \sum_{j=0}^{N-1} X(j)\frac{P(N-j)}{P(0)}$$

Explicit solution for the first few $X(j) \equiv \langle N\rangle C(j) =$

$$= (j+1)\,C^{*}(j+1)$$

$$\overline{P}(N) \equiv \frac{P(N)}{P(0)}$$

$$X(0) = \overline{P}(1) = \frac{P(1)}{P(0)}$$

$$X(1) = 2\overline{P}(2) - \overline{P}^{2}(1) = 2\frac{P(2)}{P(0)} - \left(\frac{P(1)}{P(0)}\right)^{2}$$

$$X(2) = 3\overline{P}(3) - 3\overline{P}(1)\overline{P}(2) + \overline{P}^{3}(1)$$

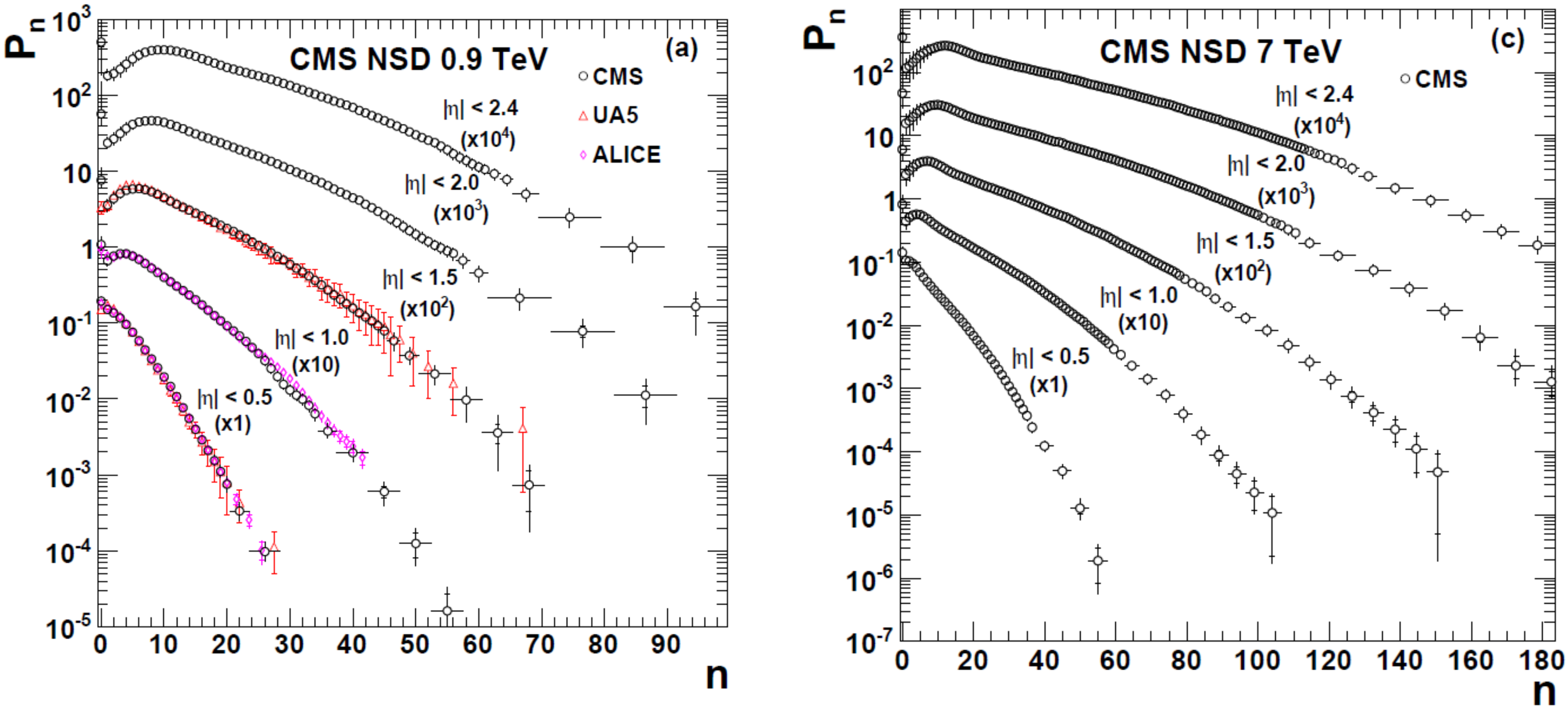
$$X(3) = 4\overline{P}(4) - 4\overline{P}(1)\overline{P}(3) - 2\overline{P}^{2}(2) + 4\overline{P}^{2}(1)\overline{P}(2) - \overline{P}^{4}(1)$$

Why combinants?

Multiplicity distributions in pp collisions at LHC energies

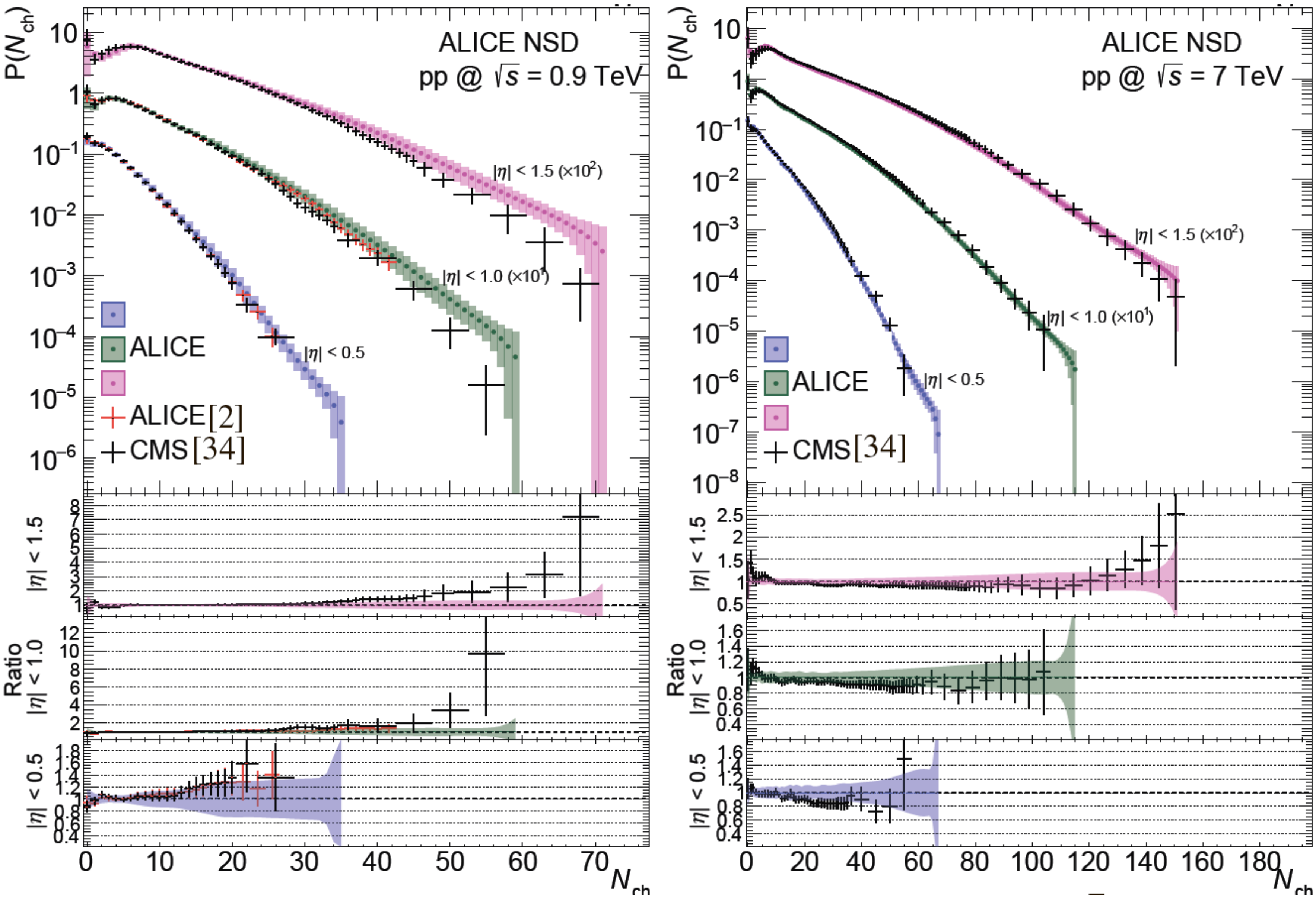
ALICE Collaboration, Eur.Phys.J.C68, 89 (2010); ibid C68, 345(2010),

CMS Collaboration, JHEP 01, 079 (2011).



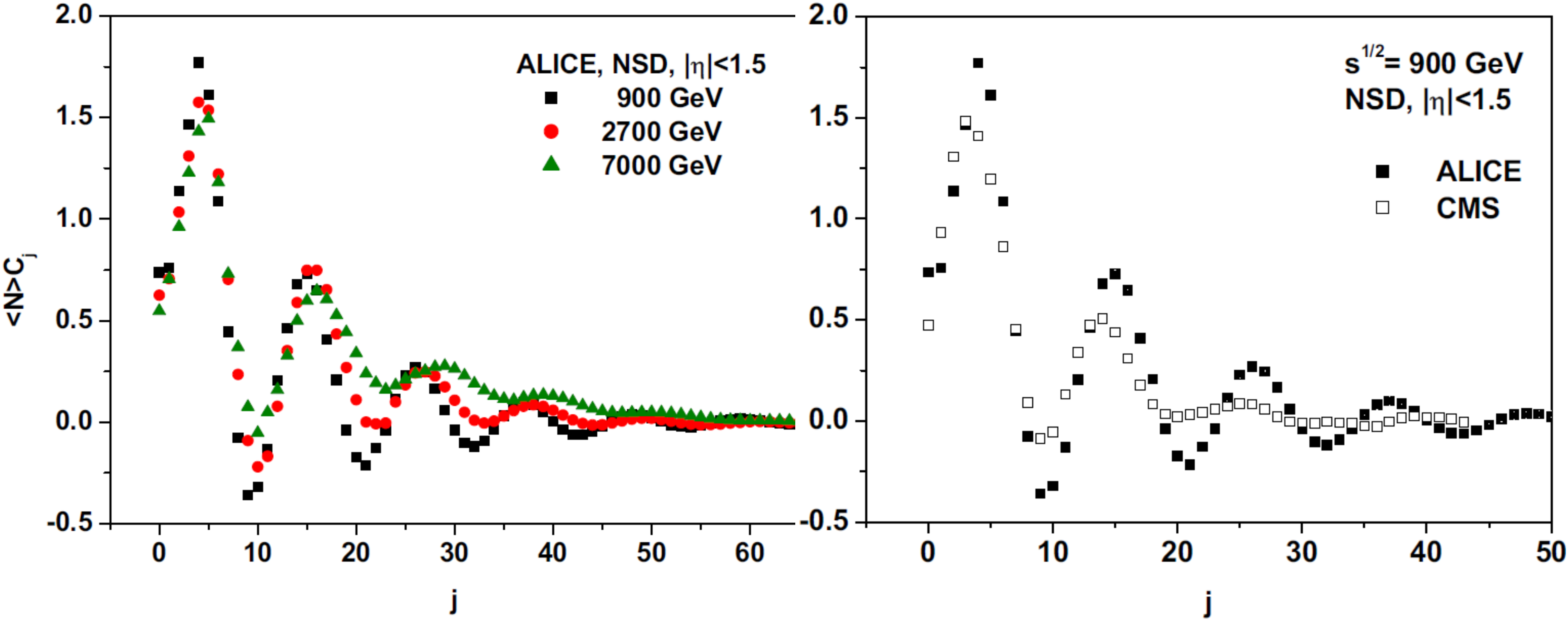
Multiplicity distributions in pp collisions at LHC energies

ALICE Collaboration, Eur.Phys.J.C77, 33 (2017).



Combinants extracted from the experimental data

G. Wilk, Z. Włodarczyk, J. Phys. G 44, 015002 (2017).



ALICE Collaboration, Eur.Phys.J.C77, 33 (2017),

CMS Collaboration, JHEP 01, 079 (2011).

Multi-Pomeron Exchange Model

N. Armesto, D.A. Derkach, G.A. Feofilov, Phys. Atom. Nucl. 71, 2087 (2008),
E.O. Bodnya, V.N. Kovalenko, A.M. Puchkov, G.A. Feofilov, AIP Conf.Proc. 1606, 273 (2015),
V.N. Kovalenko, A.M. Puchkov, G.A. Feofilov, Bull. Russ. Acad. Sci. Phys. 80, 966 (2016),
G. Feofilov, V. Kovalenko, A. Puchkov, Eur. Phys. J.: Web of Conf. 171, 18003 (2018),
E.V. Andronov, V.N. Kovalenko, Theor.Math.Phys. 200, 1282 (2019),
V. Kovalenko, G. Feofilov, A. Puchkov, F. Valiev, Universe 8, 246 (2022).
V. Vechernin, E. Andronov, V. Kovalenko, A. Puchkov. Universe 10, 56 (2024).

$$P(N) = C(z) \sum_{N_{pom}} \frac{1}{z \cdot N_{pom}} \left(1 - e^{-z} \sum_{l=0}^{N_{pom}-1} \frac{z^l}{l!} \right) \cdot P_{N_{pom}}(N)$$

$$z = \frac{2C\gamma s^{\Delta}}{R^2 + \alpha' \log(s)}, \ \Delta = 0.139, \ \alpha' = 0.21 \text{ GeV}^{-2}, \ \gamma = 1.77 \text{ GeV}^{-2}, \ R_0^2 = 3.18 \text{ GeV}^{-2}, \ C = 1.5$$

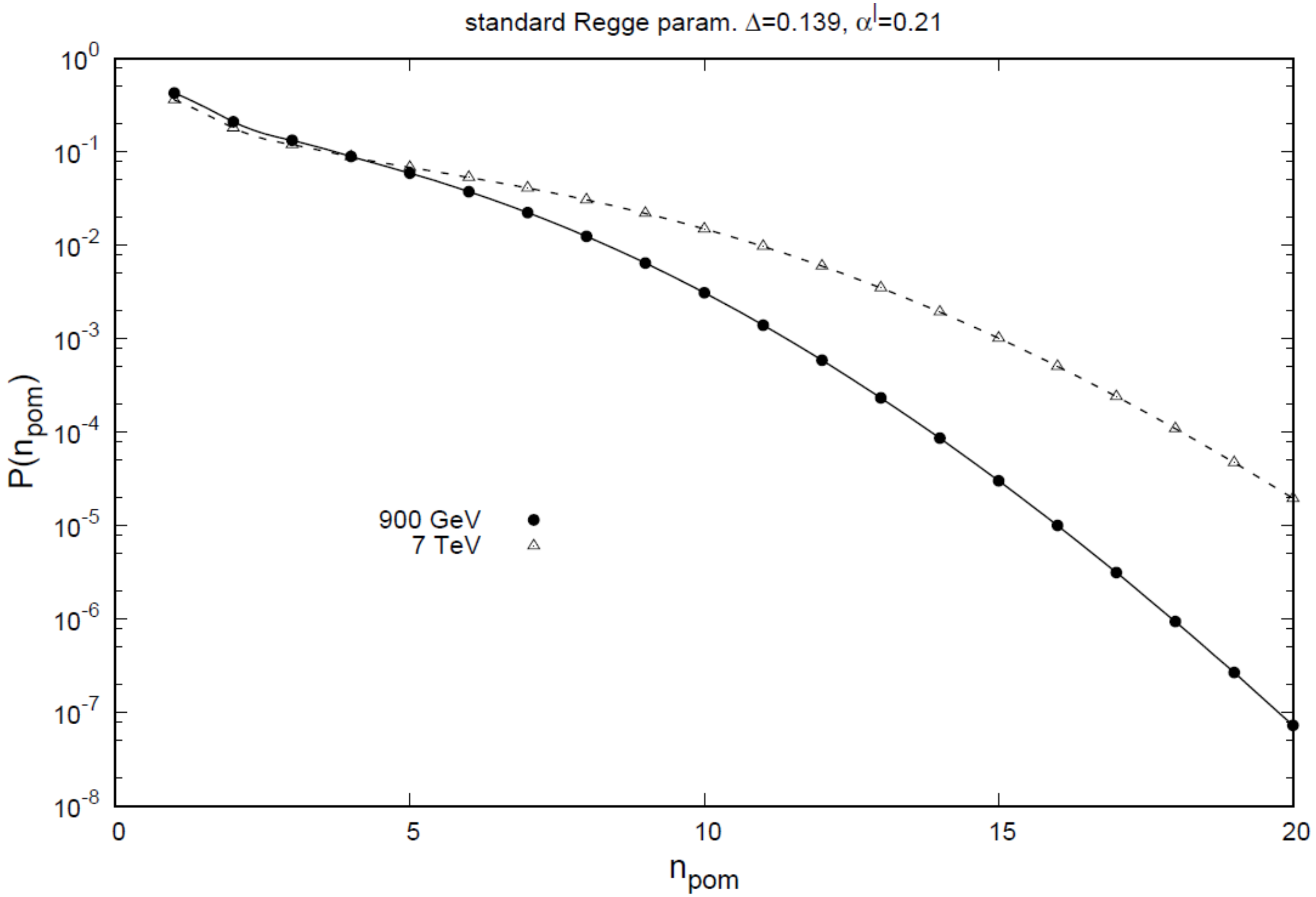
$P_{N_{pom}}(N)$ was taken to be Poisson distribution with mean $2 \cdot N_{pom} \cdot \delta\eta \cdot k(\sqrt{s})$ where $k = 0.255 + 0.0653 \cdot ln\sqrt{s}$

Regge parameters are taken from:
G. H. Arakelyan, A. Capella, A. B. Kaidalov, Yu. M. Shabelski, EPJC 26, 81 (2002).

Each cut pomeron corresponds to a pair of quark-gluon strings.

An increase in the multiplicity density with energy is explained by an increase in the mean number of cut pomerons and a growth of the average multiplicity from a single string .

Distribution in number of cut pomerons



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$$z = \frac{2C\gamma s^{\Delta}}{R^2 + \alpha' \log(s)}, \ \Delta = 0.139, \ \alpha' = 0.21 \ \mathrm{GeV}^{-2}, \ \gamma = 1.77 \ \mathrm{GeV}^{-2}, \ R_0^2 = 3.18 \ \mathrm{GeV}^{-2}, \ C = 1.5$$

Modeling a particle distribution from *n* cut pomerons

$$P(N) = \sum_{N_{pom}=1}^{\infty} P(N_{pom})\, P_{N_{pom}}(N) = \sum_{n=1}^{\infty} P(n)\, P_n(N)\,, \qquad n \equiv N_{pom}$$

$$\langle N \rangle_n = 2n\, \delta\eta\, k(s) = 2n\, \langle N \rangle_{str} \equiv 2n\, \mu_{str}$$

Poisson Distribution

$$P_n(N) = e^{-\langle N \rangle_n} \frac{\langle N \rangle_n^{\,N}}{N!}\,, \qquad P_{str}(N) = e^{-\mu_{str}} \frac{\mu_{str}^N}{N!}$$

Negative Binomial Distribution

$$P_n(N) = \frac{p^N}{q^{N+\kappa}} \frac{\Gamma(N+\kappa)}{\Gamma(\kappa)\, N!}\,, \qquad g_{NBD}(t) = (q - p\, t)^{-\kappa}$$

$$q = \omega \equiv \frac{\langle N^2 \rangle_n - \langle N \rangle_n^2}{\langle N \rangle_n} = \frac{\langle N^2 \rangle_{str} - \langle N \rangle_{str}^2}{\langle N \rangle_{str}} \equiv \omega_{str}$$

=> **true also for merged, but identical strings**

$$p = q - 1 = \omega_{str} - 1\,, \qquad \kappa = \frac{\langle N \rangle_n}{p} = 2n \frac{\mu_{str}}{\omega_{str} - 1}$$

Fluctuation in the number of particles from one string

V. Vechernin, Nucl.Phys.A 939, 21 (2015)

The pair correlation function of a single string:

$$\Lambda(\eta_1,\eta_2) \equiv \frac{\lambda_2(\eta_1,\eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 = \frac{\lambda_2(\eta_1-\eta_2)}{\mu_0^2} - 1 = \Lambda(\eta_1-\eta_2)$$

The one- and two-particle rapidity distributions of particles from one string decay:

$$\lambda(\eta) \equiv \frac{dN}{d\eta} = \frac{\mu_{str}}{\delta\eta} = \mu_0 = k(s)\ , \qquad \lambda_2(\eta_1,\eta_2) \equiv \frac{d^2N}{d\eta_1\, d\eta_2} = \lambda_2(\eta_1-\eta_2)$$

Resulting two-particle correlation function (for all strings):

$$C_2(\eta_1,\eta_2;\phi_1-\phi_2) = \frac{\rho_2(\eta_1,\eta_2;\phi_1-\phi_2)}{\rho_1(\eta_1)\rho_1(\eta_2)} - 1 \qquad C_2(\eta_1,\eta_2;\phi) = \frac{\omega_N + \Lambda(\eta_1,\eta_2;\phi)}{\langle N\rangle}$$

Forward-backward correlation coefficient can be calculated using:

$$\langle n_F n_B\rangle - \langle n_F\rangle\langle n_B\rangle = \langle n_F\rangle\langle n_B\rangle I_{FB}$$

$$D_{n_F} = \langle n_F\rangle + \langle n_F\rangle^2 I_{FF}\ ,$$

$$b_{rel} = \frac{\langle n_F\rangle}{\langle n_B\rangle} b = \frac{\langle n_F\rangle I_{FB}}{1+\langle n_F\rangle I_{FF}}$$

$$\langle n_F\rangle = \int_{\delta\eta_F} d\eta\, \rho_1(\eta)\ ,$$

$$I_{FB} = \frac{1}{\langle n_F\rangle\langle n_B\rangle}\int_{\delta\eta_B} d\eta_1 \int_{\delta\eta_F} d\eta_2\, \rho_1(\eta_1)\rho_1(\eta_2)C_2(\eta_1,\eta_2)$$

$$I_{FF} = \frac{1}{\langle n_F\rangle^2}\int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_F} d\eta_2\, \rho_1(\eta_1)\rho_1(\eta_2)C_2(\eta_1,\eta_2)$$

Fluctuation in the number of particles from one string

V. Vechernin, Nucl.Phys.A 939, 21 (2015)

$$b_{rel} = \frac{\langle n_F \rangle}{\langle n_B \rangle} b = \frac{\langle n_F \rangle I_{FB}}{1 + \langle n_F \rangle I_{FF}}$$

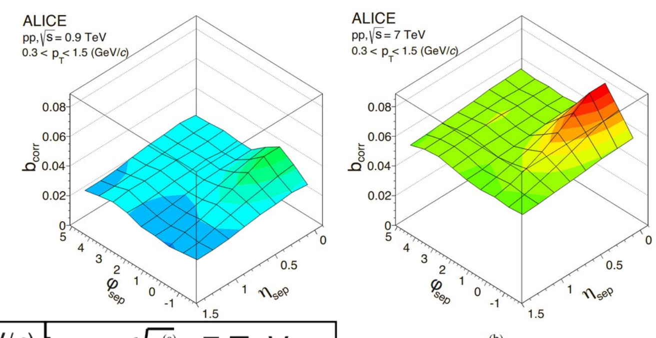
$$I_{FF} = \frac{1}{\langle n_F \rangle^2} \int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_F} d\eta_2 \, \rho_1(\eta_1)\rho_1(\eta_2) C_2(\eta_1, \eta_2)$$

$$I_{FB} = \frac{1}{\langle n_F \rangle \langle n_B \rangle} \int_{\delta\eta_B} d\eta_1 \int_{\delta\eta_F} d\eta_2 \, \rho_1(\eta_1)\rho_1(\eta_2) C_2(\eta_1, \eta_2)$$

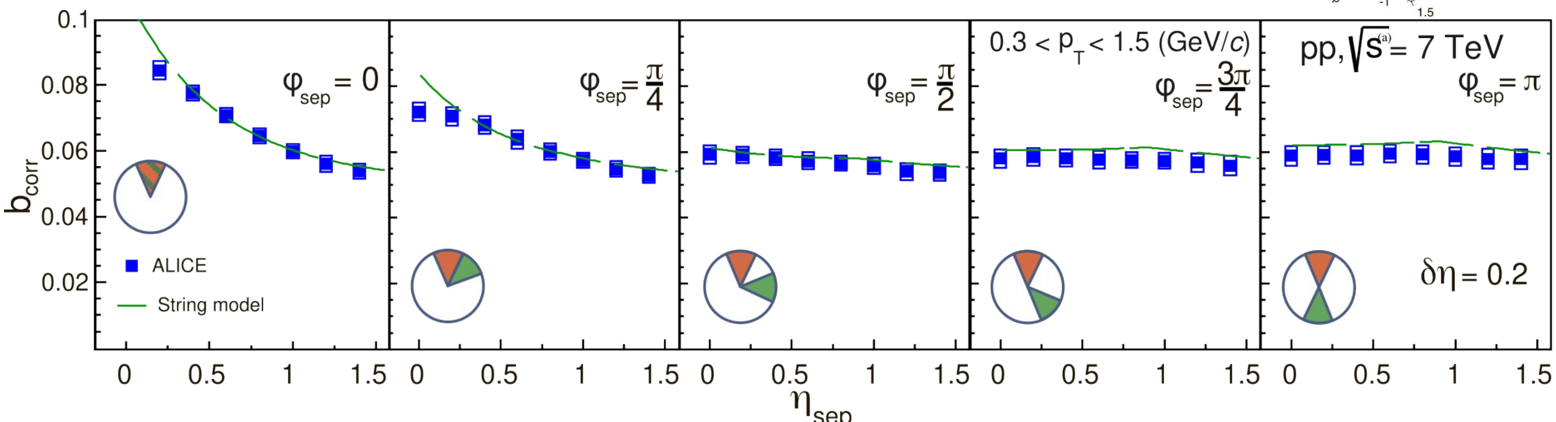
For sufficiently small windows in both rapidity and azimuth

$$b_{rel} = b_{rel}^{LR} + b_{rel}^{SR}; \quad b_{rel}^{LR} = \frac{\delta_F \mu_0 \omega_N}{1 + \delta_F \mu_0 [\omega_N + \Lambda(0,0)]}, \quad b_{rel}^{SR} = \frac{\delta_F \mu_0}{1 + \delta_F \mu_0 [\omega_N + \Lambda(0,0)]} \Lambda(\eta_{FB}, \phi_{FB}) .$$

that allows extracting Λ from the experimental data



ALICE Collaboration, JHEP 05, 097 (2015)



Fluctuation in the number of particles from one string

V. Vechernin, Nucl.Phys.A 939, 21 (2015)

In the presence of *correlations* between particles, there can be *no Poisson distribution* from the source.

$$\frac{\langle N^2\rangle_{str} - \langle N\rangle_{str}^2}{\langle N\rangle_{str}} \equiv \omega_{str} = 1 + \mu_{str} J \,, \qquad J \equiv \frac{1}{\delta\eta^2} \int_{\delta\eta} d\eta_1 \int_{\delta\eta} d\eta_2 \, \Lambda(\eta_1 - \eta_2)$$

The pair correlation function of a single string:

$$\Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 = \frac{\lambda_2(\eta_1 - \eta_2)}{\mu_0^2} - 1 = \Lambda(\eta_1 - \eta_2)$$

The one- and two-particle rapidity distributions of particles from one string decay:

$$\lambda(\eta) \equiv \frac{dN}{d\eta} = \frac{\mu_{str}}{\delta\eta} = \mu_0 = k(s) \,, \qquad \lambda_2(\eta_1, \eta_2) \equiv \frac{d^2 N}{d\eta_1 \, d\eta_2} = \lambda_2(\eta_1 - \eta_2)$$

The parametrization for the pair correlation function of a single string $\Lambda(\Delta\eta, \Delta\phi)$
using the data from *ALICE Collaboration, JHEP 05, 097 (2015),*
Forward-backward multiplicity correlations in pp collisions at 0.9, 2.76 and 7 TeV

Then integrating over azimuth we find :

$$\Lambda(\Delta\eta) = \frac{1}{\pi} \int_0^{\pi} \Lambda(\Delta\eta, \Delta\phi) \, d\Delta\phi$$ **well approximated by** $$\Lambda(\Delta\eta) = \Lambda_0 \, e^{-\frac{|\Delta\eta|}{\eta_{corr}}}$$

V. Vechernin, EPJ Web Conf. 191, 04011 (2018),
E. Andronov, V. Vechernin, Eur. Phys. J. A 55, 14 (2019).

Fluctuation in the number of particles from one string

For the exponential pair correlation function, the integral can be calculated explicitly:

S. Belokurova, Phys. Part. Nucl. 53, 154 (2022)

$$\omega_{str} = 1 + \mu_{str} J \,, \qquad J = \frac{2\Lambda_0\, \eta_{corr}}{(\delta\eta)^2} \left[\delta\eta - \eta_{corr}\left(1 - e^{-\delta\eta/\eta_{corr}}\right)\right]$$

Then we find:

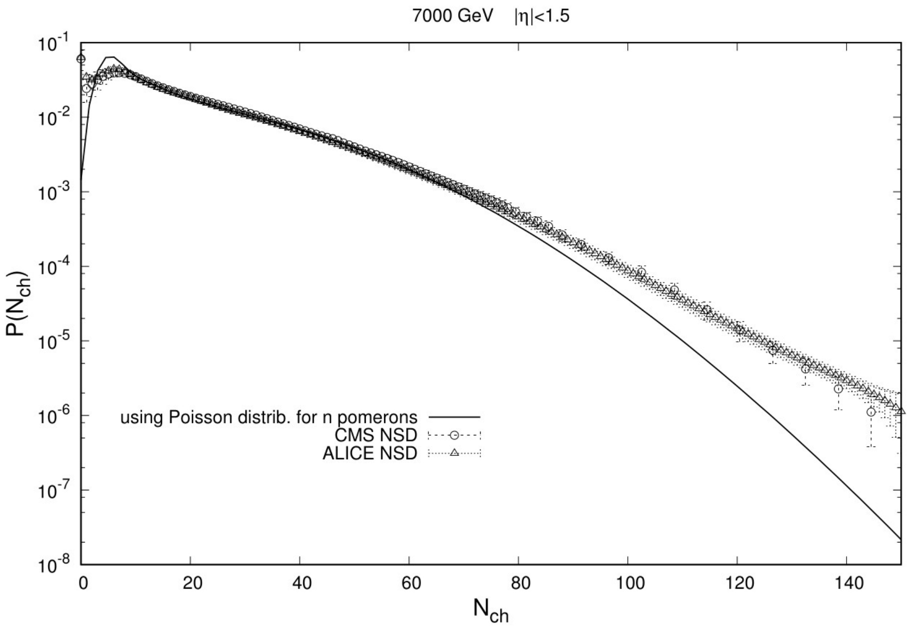
$$\frac{\langle N^2\rangle_{str} - \langle N\rangle^2_{str}}{\langle N\rangle_{str}} \equiv \omega_{str}$$

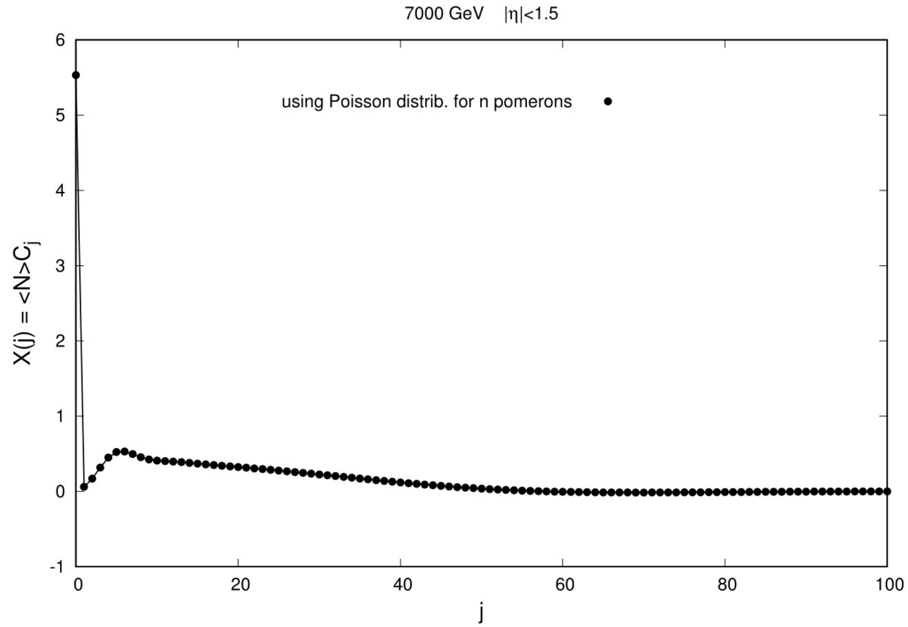
$\sqrt{s}$, TeV	0.9	7.0
$\delta\eta = 3$	3.1	3.5
$\delta\eta = 4.8$	3.6	4.0

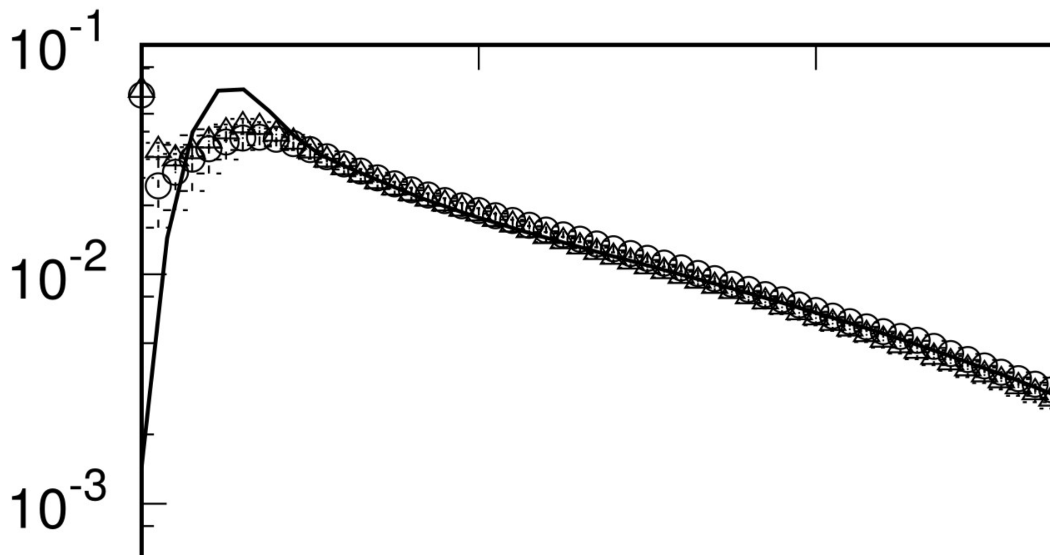
Recall that in the multipomeron exchange model:

$$\mu_{str} = k(s)\,\delta\eta$$

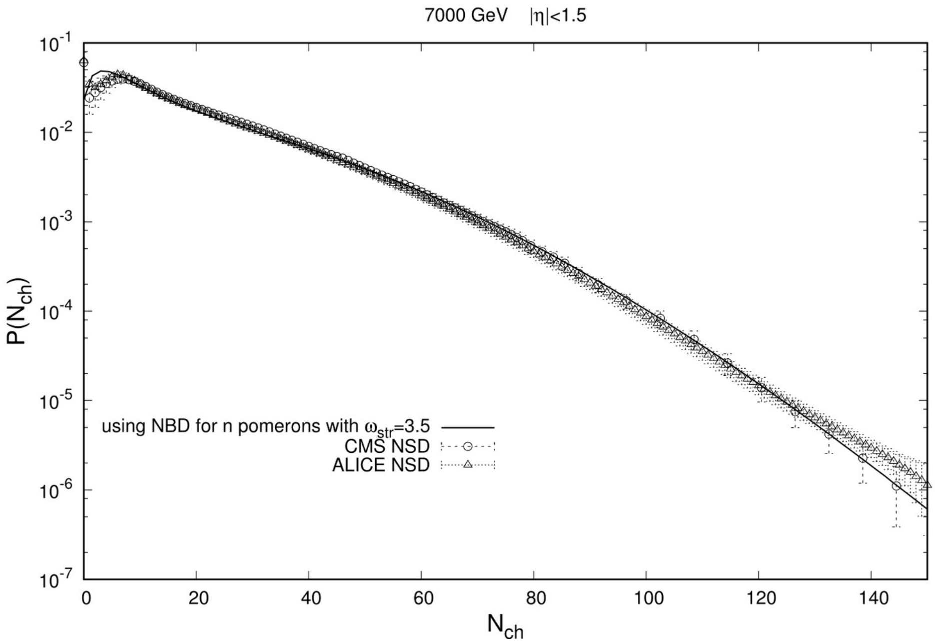
Assuming a Poisson distribution of particles from one string

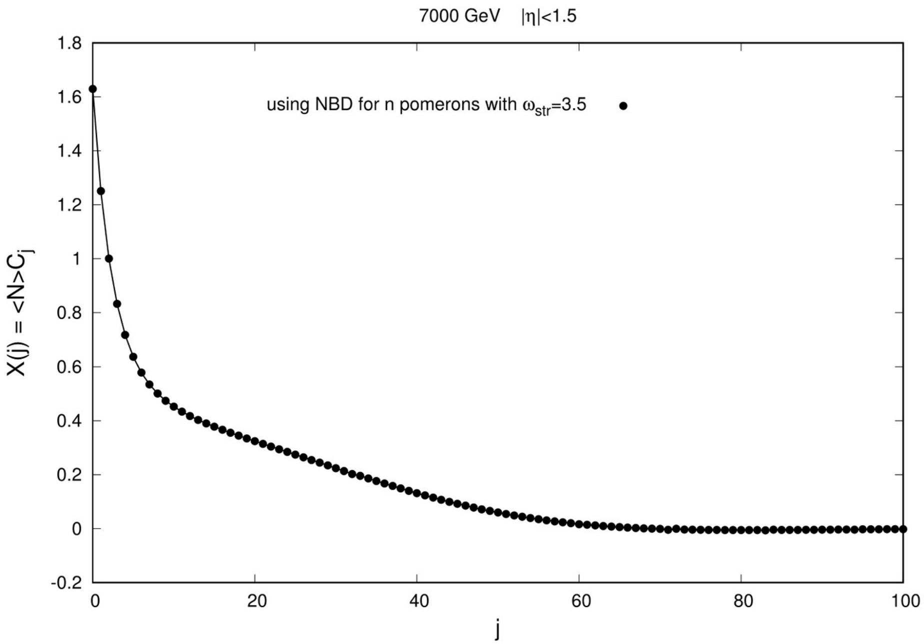


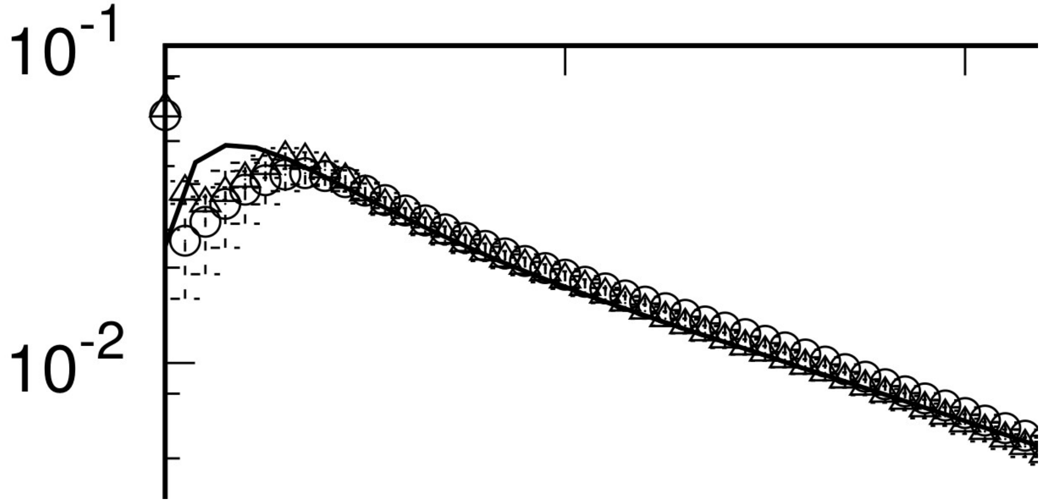




Assuming a Negative Binomial distribution from one string







Assuming a Gaussian distribution for $P_n(N)$

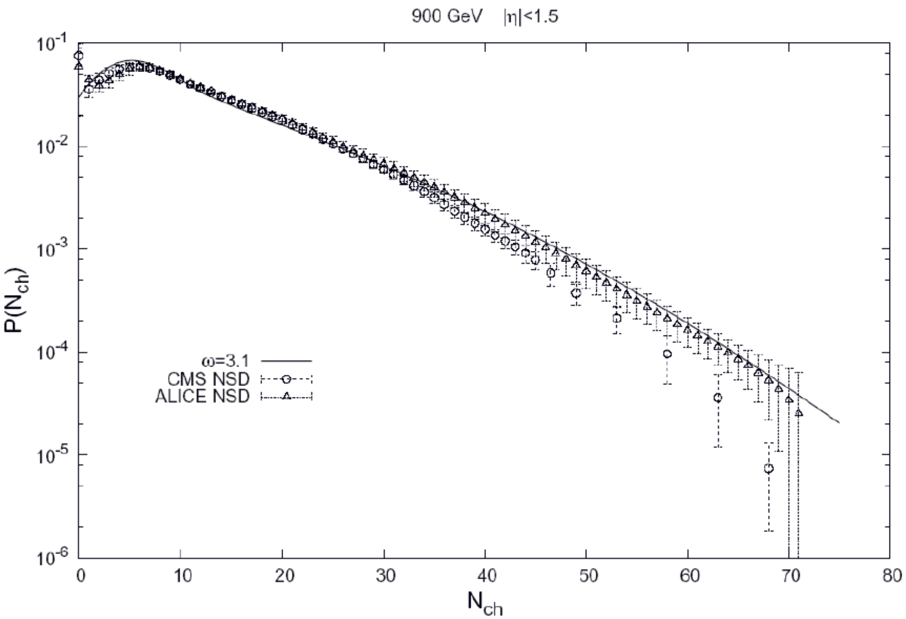
$$P_n(N) = C \exp\left[-\frac{(N - 2n\mu_{str})^2}{2\omega_{str}\, 2n\mu_{str}}\right], \qquad \sum_{N=0}^{\infty} P_n(N) = 1$$

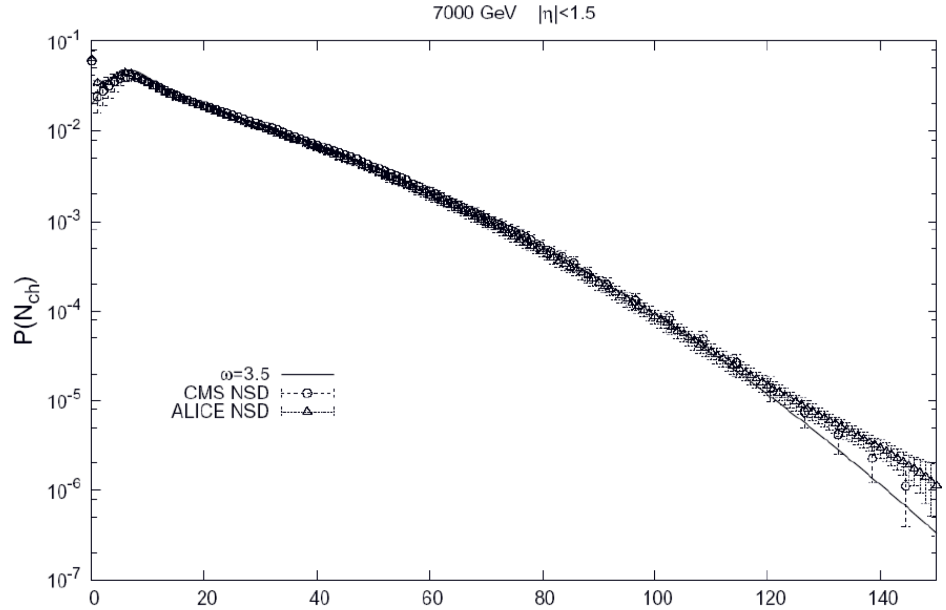
$$C^{-1} = \sum_{N=0}^{\infty} \exp\left[-\frac{(N - 2n\mu_{str})^2}{2\omega_{str}\, 2n\mu_{str}}\right]$$

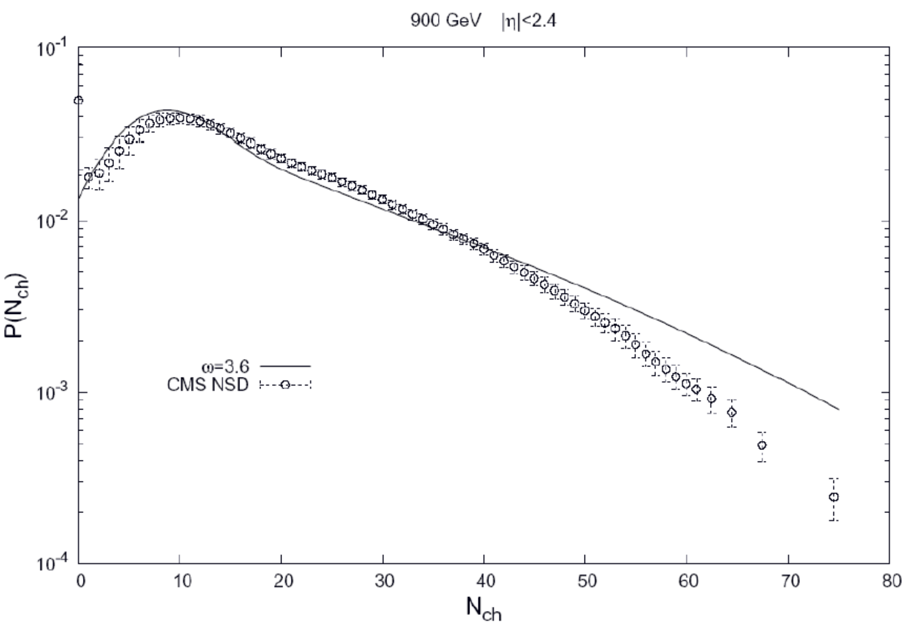
For $2n\mu_{str} \gg 1$ we have

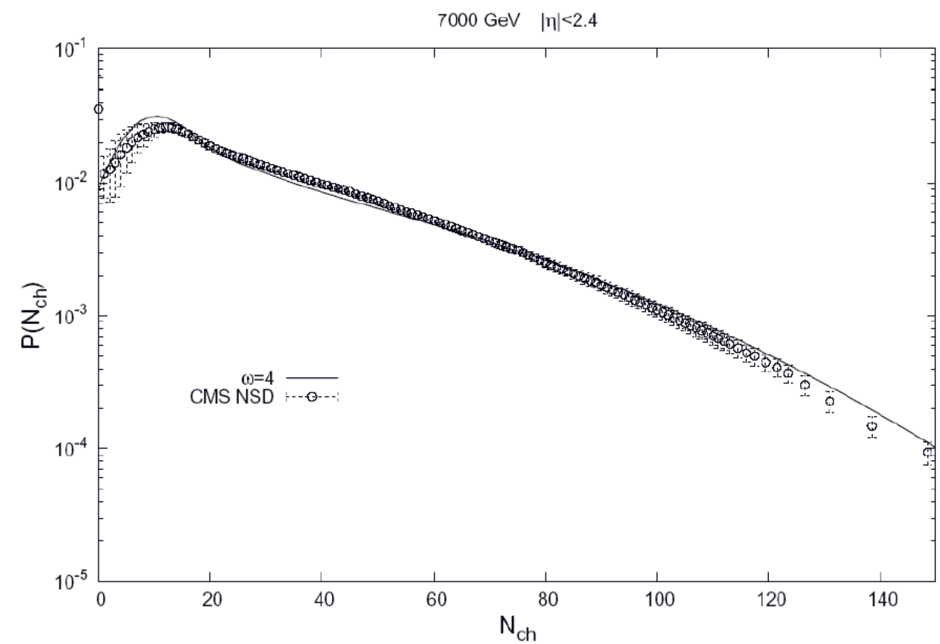
$$\langle N \rangle_n \equiv \sum_{N=1}^{\infty} N\, P_n(N) \to 2n\mu_{str}$$

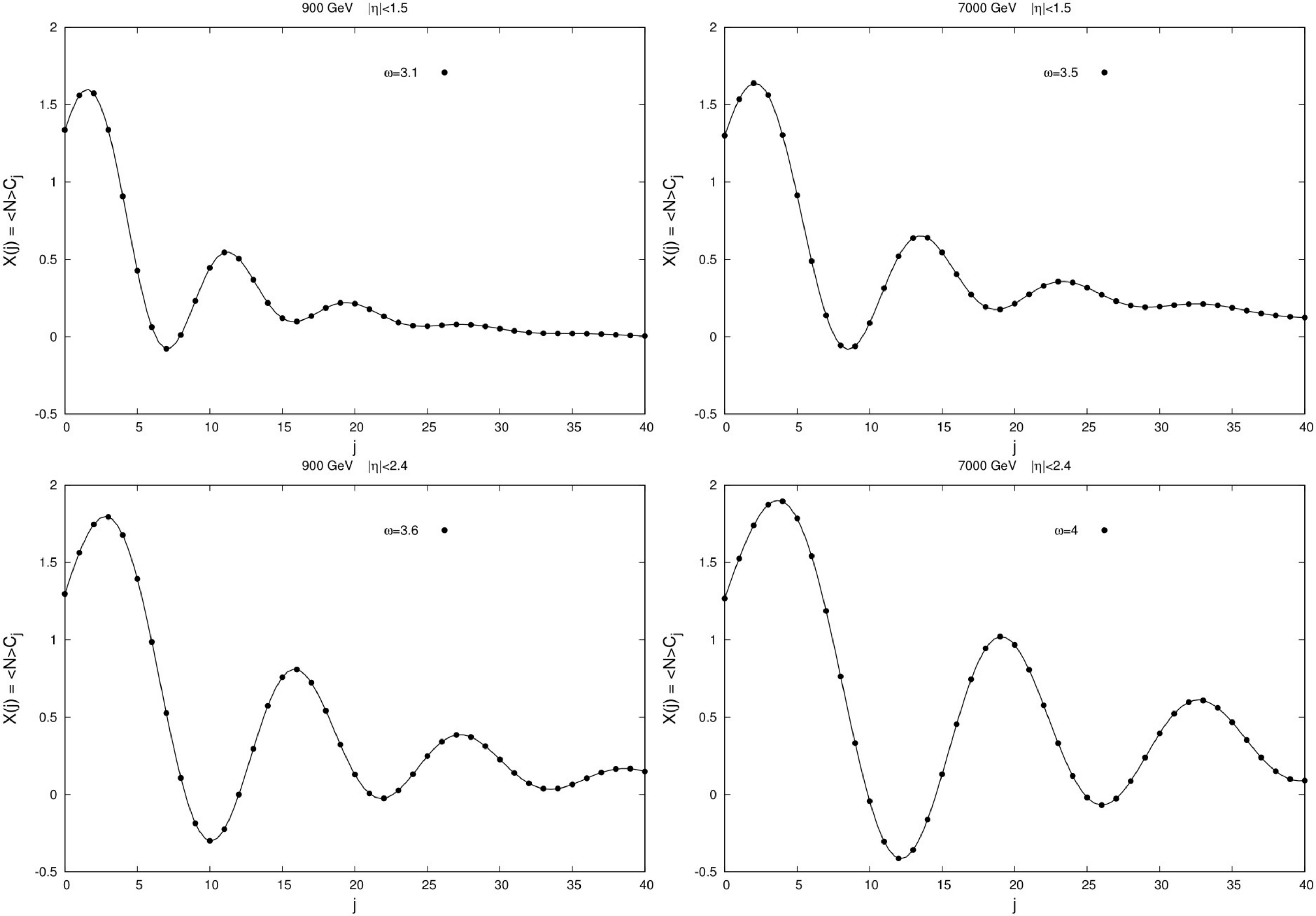
$$\omega_n[N] \equiv \frac{\langle N^2 \rangle_n - \langle N \rangle_n^2}{\langle N \rangle_n} \to \omega_{str}$$


900 GeV |η|<1.5
P(N_ch)
N_ch
ω=3.1
CMS NSD
ALICE NSD


7000 GeV |η|<1.5
P(N_ch)
ω=3.5
CMS NSD
ALICE NSD

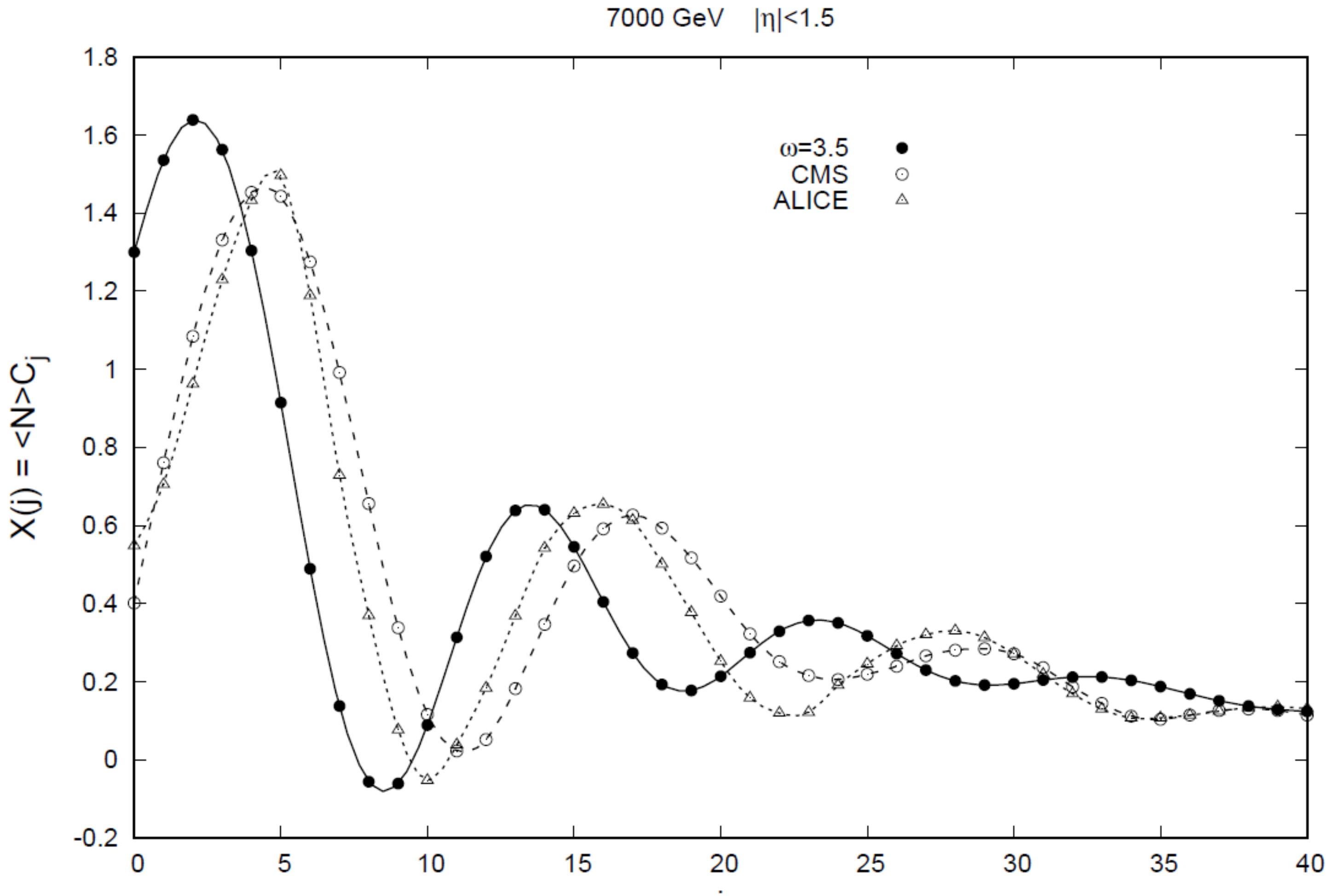

900 GeV |η|<2.4
P(N_ch)
N_ch
ω=3.6
CMS NSD


7000 GeV |η|<2.4
P(N_ch)
N_ch
ω=4
CMS NSD

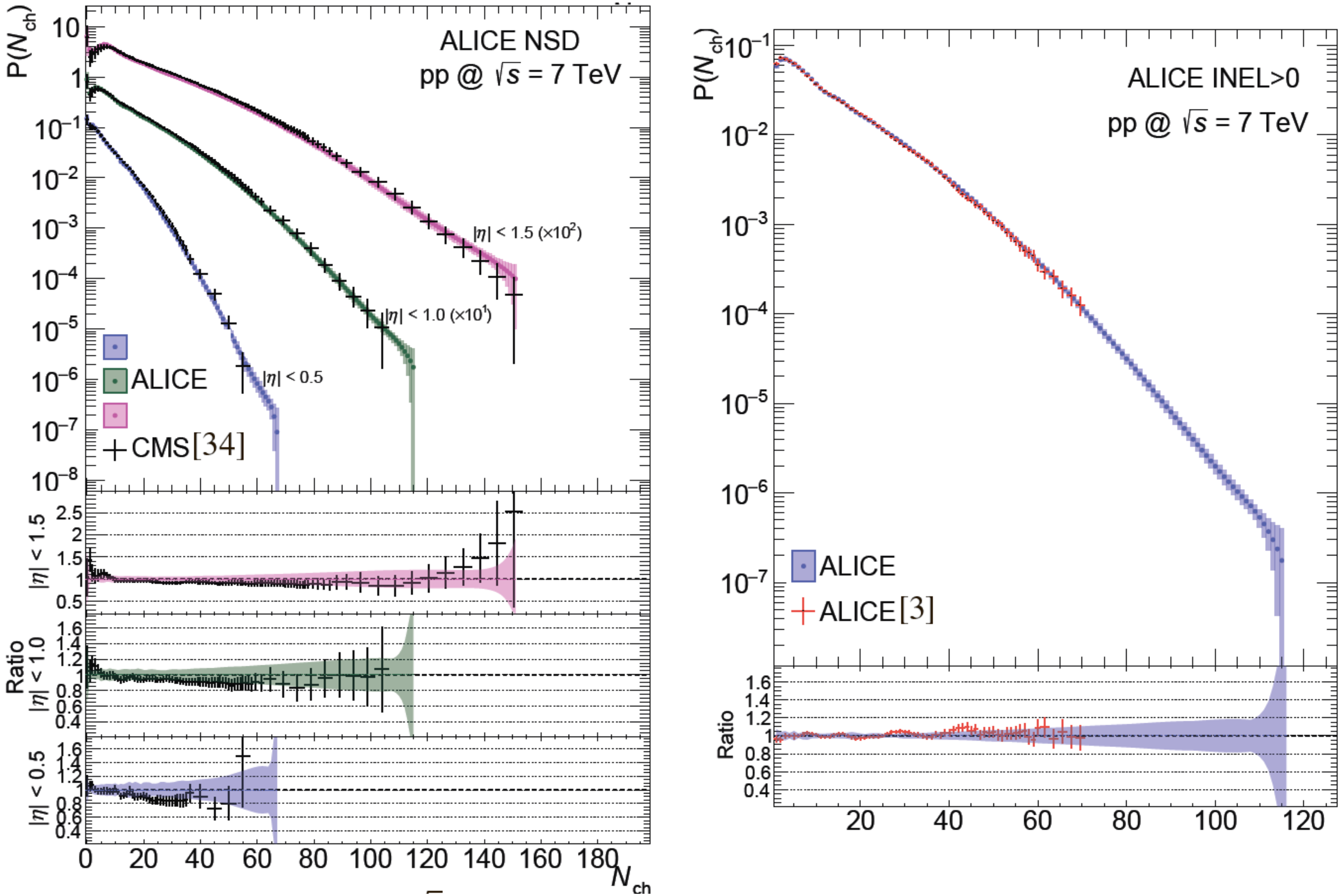


900 GeV |η|<1.5
ω=3.1
7000 GeV |η|<1.5
ω=3.5
900 GeV |η|<2.4
ω=3.6
7000 GeV |η|<2.4
ω=4
X(j) = <N>C_j
j

Comparison to the pp experimental data

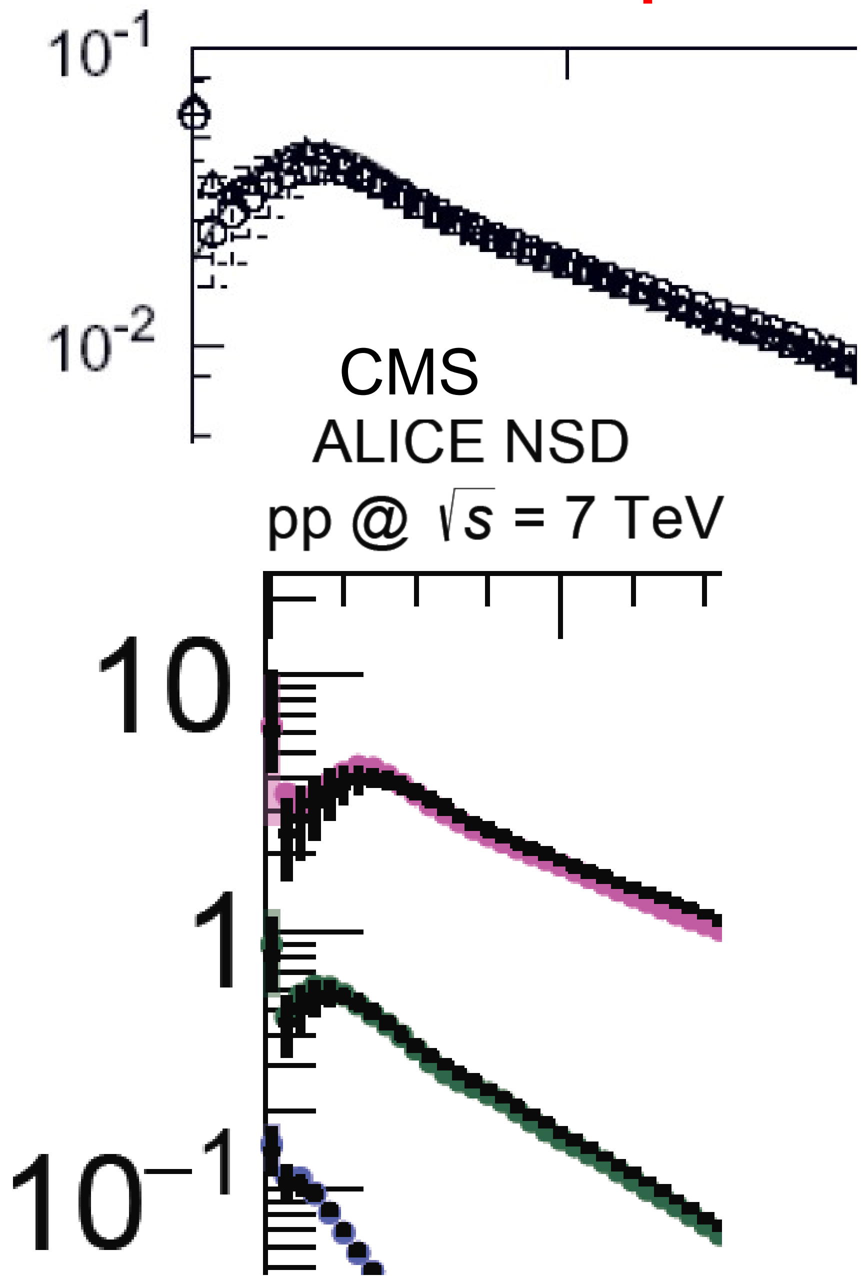


ALICE Collaboration, Eur.Phys.J.C68, 89 (2010); ibid C68, 345(2010),
CMS Collaboration, JHEP 01, 079 (2011),
ALICE Collaboration, Eur.Phys.J.C77, 33 (2017)



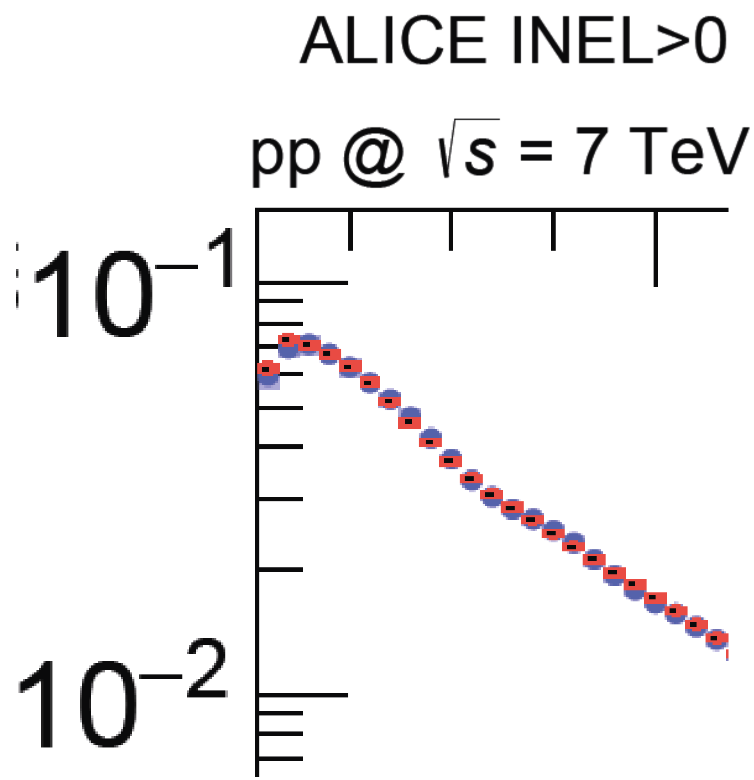
ALICE Collaboration, Eur.Phys.J.C68, 89 (2010); ibid C68, 345(2010),
CMS Collaboration, JHEP 01, 079 (2011),
ALICE Collaboration, Eur.Phys.J.C77, 33 (2017)

The problem of N=0 bin

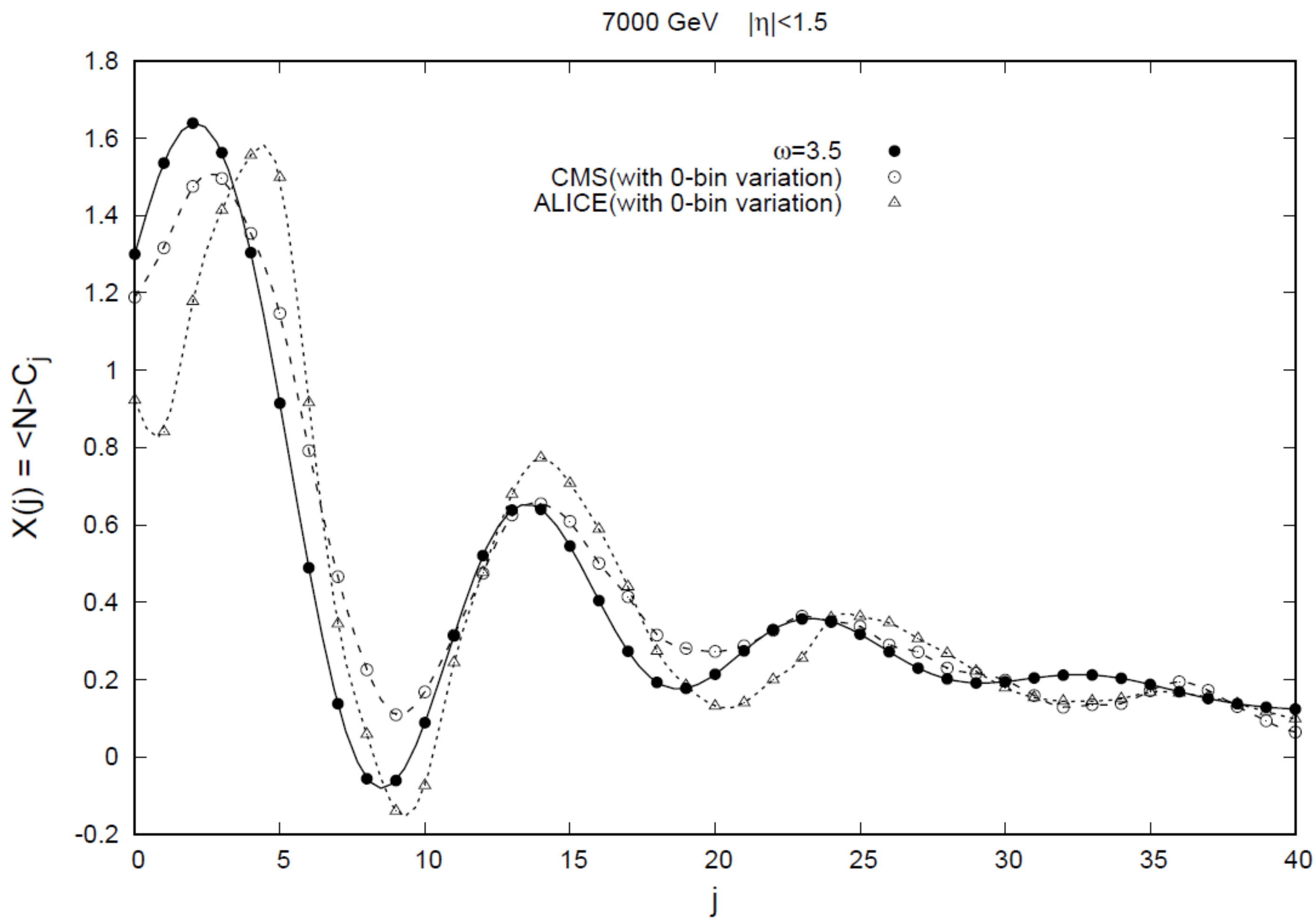


Model:
At least one cut pomeron,
n>0 =>
this excludes also the DD

Experimental data:
Non-single diffractive events



Comparison to the pp experimental data with modified 0-bin



CMS Collaboration, JHEP 01, 079 (2011),
ALICE Collaboration, Eur.Phys.J.C77, 33 (2017)

Tree ways of considering cut-Gaussian distribution

1. Fixed constants of Gaussian distribution

$$P_n(N) = C \exp\left[-\frac{(N-2n\mu_{str})^2}{2\omega_{str}\,2n\mu_{str}}\right], \qquad \sum_{N=0}^{\infty} P_n(N) = 1$$

$$C^{-1} = \sum_{N=0}^{\infty} \exp\left[-\frac{(N-2n\mu_{str})^2}{2\omega_{str}\,2n\mu_{str}}\right]$$

For $2n\mu_{str} \gg 1$ we have

$$\langle N \rangle_n \equiv \sum_{N=1}^{\infty} N\, P_n(N) \to 2n\mu_{str}$$

$$\omega_n[N] \equiv \frac{\langle N^2 \rangle_n - \langle N \rangle_n^2}{\langle N \rangle_n} \to \omega_{str}$$

V. Vechernin, E. Andronov, V. Kovalenko, A. Puchkov. Universe 10, 56 (2024).

Tree ways of considering cut-Gaussian distribution

2. Adjusted Gaussian distribution parameters for fixed values from one string

$$P_n(N) = C_n \exp\left[-\frac{(N - N_n^{max})^2}{2\alpha_n N_n^{max}}\right] \qquad \sum_{N=0}^{\infty} P_n(N) = 1$$

$$C_n^{-1} = \sum_{N=0}^{\infty} \exp\left[-\frac{(N - N_n^{max})^2}{2\alpha_n N_n^{max}}\right]$$

$$\langle N \rangle_n \equiv \sum_{N=1}^{\infty} N\, P_n(N) \qquad \omega_n[N] \equiv \frac{\langle N^2 \rangle_n - \langle N \rangle_n^2}{\langle N \rangle_n}$$

$$\langle N \rangle_n = 2 n \mu_{str} \qquad \omega_n[N] = \omega_{str} \equiv \omega.$$

So for fixed parameters from one string we can obtain distribution parameters for arbitrary number of pomeron exchanges *n*

$$\mu_{str} = \delta\eta\, k(s),\ \omega \rightarrow\ N_n^{max},\ \alpha_n$$

Parameters can be calculated using iterative algorithm of the numerical solution of the system of equations.

Tree ways of considering cut-Gaussian distribution

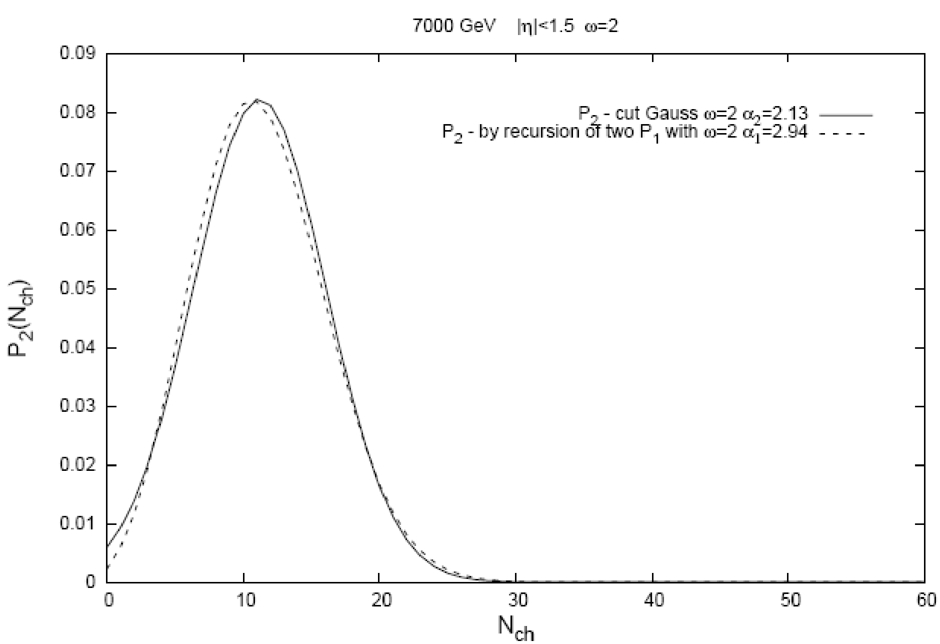
3. cut-Gaussian distribution — only for one string.

$$P_1(N) = C_1 \exp\left[-\frac{(N - N_1^{max})^2}{2\alpha_1 N_1^{max}}\right] \qquad \sum_{N=0}^{\infty} P_1(N) = 1$$

For n-pomeron exchange it will be corresponding convolution, that can be expressed using recurrent expression:

$$P_n(N) = \sum_{K=0}^{N} P_{n-1}(K)\, P_1(N-K)$$

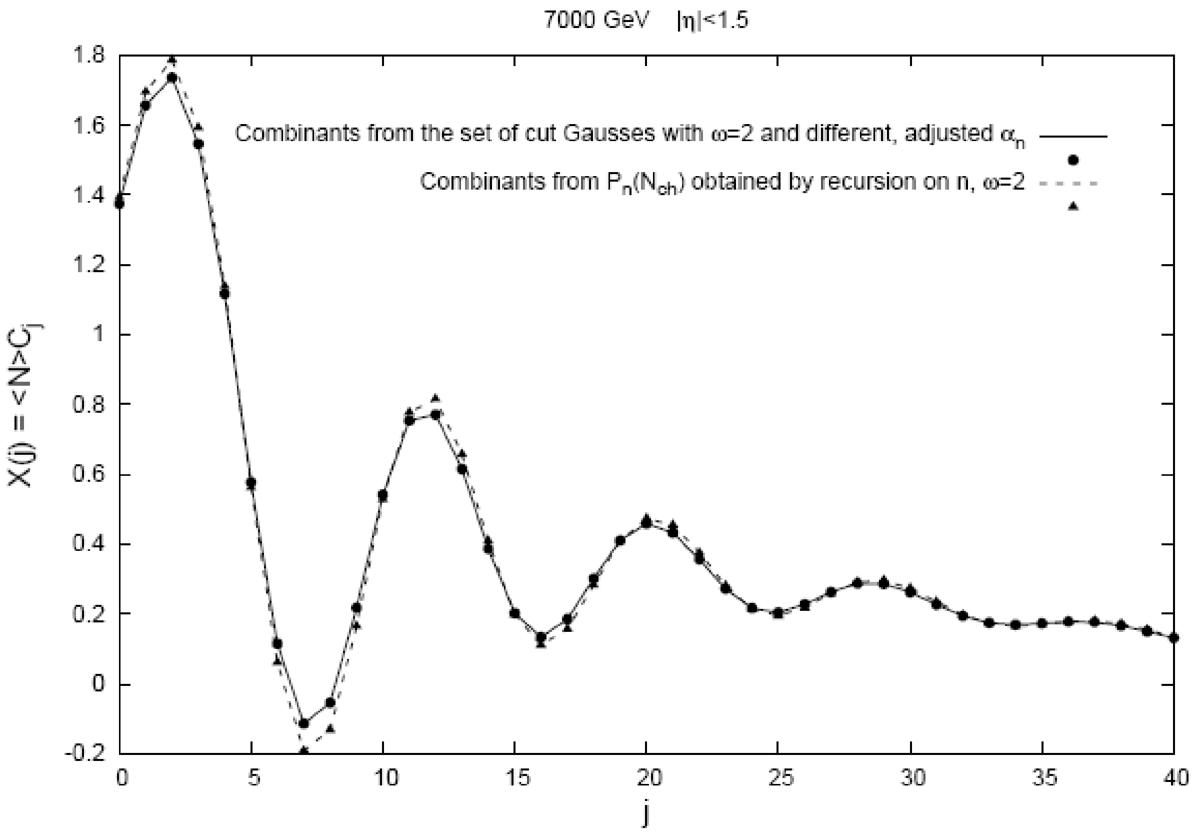
We keep $\langle N \rangle_n = 2n\mu_{str}$ $\quad \omega_n[N] = \omega_{str} \equiv \omega.$



Comparison of case 2 and case 3 distributions from one two pomerons $P_2(N_{ch})$s

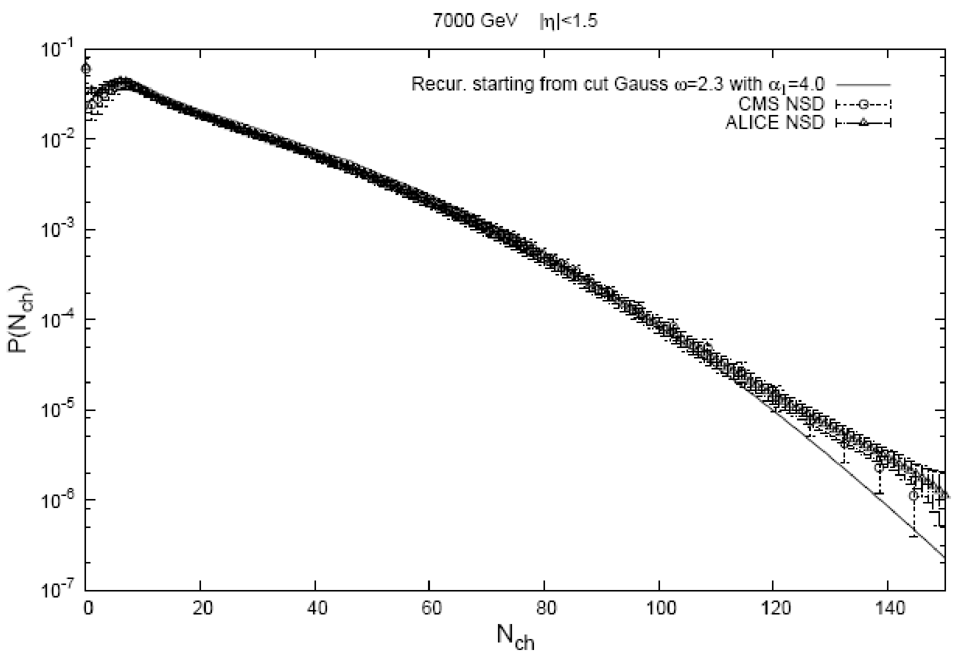
Tree ways of considering cut-Gaussian distribution

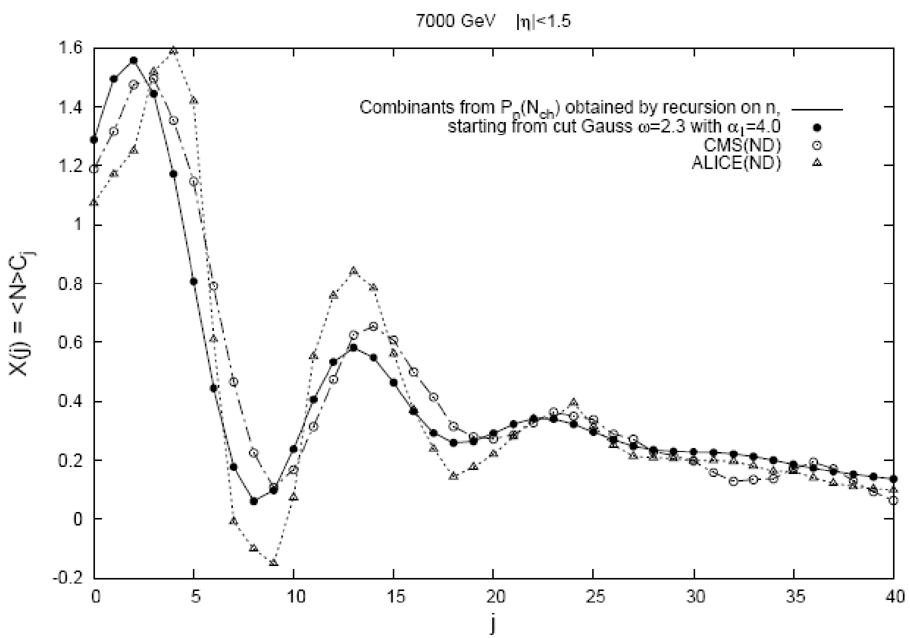
3. Combinants for the resulting multiplicity distributions



Tree ways of considering cut-Gaussian distribution

Comparison with experimental data for multiplicity distributions and combinants





Concluding remarks

- The behavior of combinants proves to be a very sensitive tool in the analysis of particle multiplicity distribution. Even minor deviations of the ALICE and CMS data, within the error, lead to considerable changes in combinants.
- It is shown that within the framework of the multipomeron exchange model with Poisson or NBD distribution of particles from one source (quark-gluon string), it is not possible to explain the experimentally observed oscillations of combinants with increase of their number.
- Oscillations of combinants with increasing number can be explained if we assume a Gaussian form of particle multiplicity distribution at a fixed number of pomerons.
- The reason for the occurrence of these oscillations is the interplay between the contributions of 1, 2, 3, etc. cut pomerons possibly influenced by single, double and central diffraction.

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Thank you!