Regge trajectories in QCD

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Outline:

- **Introduction & Motivation**
- **Regge-Gribov approach prior QCD**
- **Reggeons (Pomeron, etc.) in perturbative QCD**
- **Reggeons (Pomeron, etc.) in nonperturbative QCD**
- **Reggeons beyond QCD**
- **Highlights: selected references**

Regge trajectories before QCD: foundation highlights

High-energy scattering asymptotics

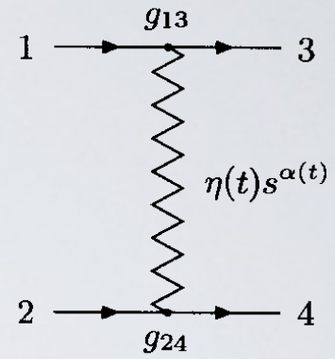
I.Ya. Pomeranchuk, JETP 34 (1958) 725

Non-relativistic scattering

Orbital momentum generalization to complex plane

Regge poles and trajectories

T. Regge, Nuov. Cim. 14 (1959) 951, Nuov. Cim. 28 (1960) 947



$$A(s,t) = \beta(t)\,\eta(t)\,s^{\alpha(t)}$$

Relativistic scattering

Mandelstam representation

S. Mandelstam, (1959)

Gribov-Froissart transformation

V.N. Gribov, Nucl. Phys. (1961)

M. Froissart, Phys. Rev. 123 (1961) 1053

$$\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}t} = F(t)\,s^{2\alpha(0)-2}\,\mathrm{e}^{-2\alpha'|t|\ln s}$$

$$\sigma_{\mathrm{tot}} \underset{s\to\infty}{\simeq} \frac{1}{s}\,\mathrm{Im}\,A(s,t=0) \underset{s\to\infty}{\sim} s^{\alpha(0)-1}$$

Froissart-Martin x-section asymptotic bound ≤ $\log^2(s)$

M. Froissart, Phys. Rev. 123 (1961) 1053

Vacuum singularity Pomeron: $a_P(t) = a_P(0) + a_P'(t)|t| + \ldots = 1 + a_P'(t)|t| + .$

V.N. Gribov JETP (1961) 1962

G.F. Chew, S.C. Frautschi, Phys. Rev. Lett. 7 (1961) 394

Pomeron before QCD: developments

Reggeon field theory

V.N. Gribov (1967)

multi-Pomeron exchanges

V.N. Gribov, A.A. Migdal (1968-1970)
K.A. Ter-Martirosyan, A.A. Migdal, A.M. Polyakov 1972-1975
A.B. Kaidalov K.A. Ter-Martirosyan 1973-1979

supercritical Pomeron $\alpha_{IP}(0) > 1$

V.N. Gribov, A.A. Migdal, A.M. Polyakov 1970-1975

strongly-interacting supercritical Pomeron

V.N. Gribov, A.A. Migdal, A.M. Polyakov 1969

LL & NLL Pomeron in perturbative QCD

Born approximation: two-gluon Pomeron

F.E. Low, Phys. Rev. D12 (1975) 163

S. Nussinov, Phys. Rev. Lett. 34 (1975) 1286

Leading logarithmic approximation: LL BFKL Pomeron

V.S. Fadin, E.A. Kuraev, L.N. Lipatov, Phys. Lett. B 60 (1975) 50

L.N. Lipatov, Yad. Fiz. 28 (1976) 1597 [Sov. J. Nucl. Phys. 23 (1976) 338]

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, ZhETF 71 (1976) 840 [JETP 45 (1977) 79]

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, ZhETF 72 (1977) 377 [JETP 45 (1977) 79]

I.I. Balitsky, L.N. Lipatov, Yad. Fiz. 28 (1978) 1597 [Sov. J. Nucl. Phys. 28 (1978) 822]

reggeized gluon

$$\Sigma C_n(\alpha_S \log s)^n$$

Next-to-leading logarithmic approximation: NLL BFKL Pomeron

V.S. Fadin, L.N. Lipatov, Phys. Lett. B 429 (1998) 127

E.A. Camici, L.N. Ciafaloni, Phys. Lett. (1998)

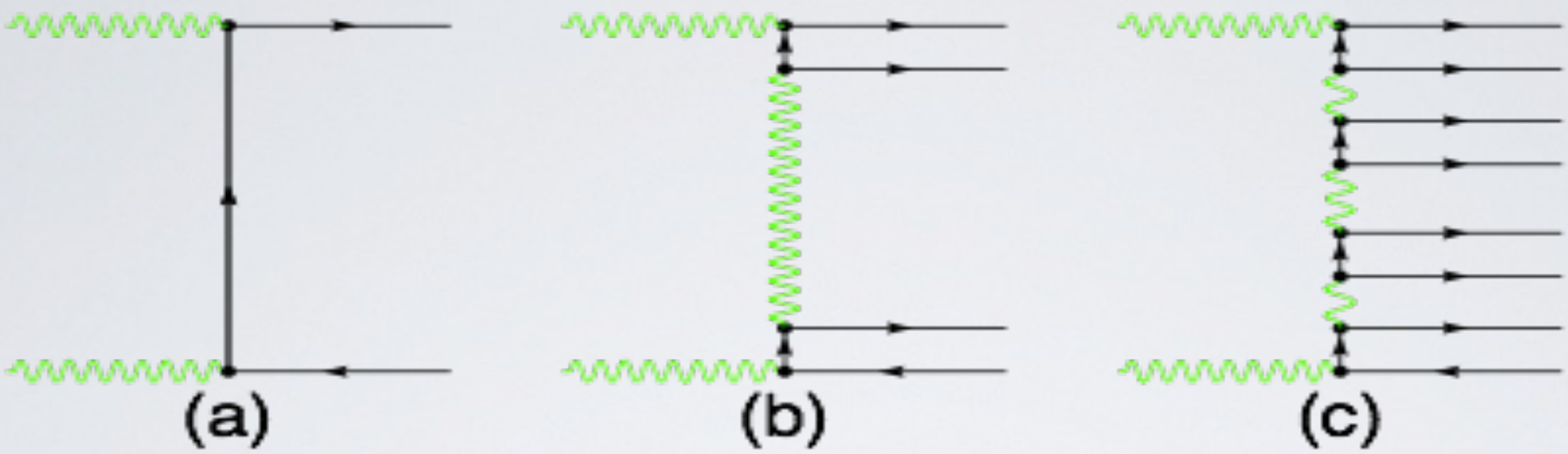
S.J. Brodsky V.S. Fadin, V.T.K., L.N. Lipatov, G.B. Pivovarov (BFKLP),

Pisma ZhETF 70 (1999) 161 [JETP Lett.]

reggeized gluon

$$+ \Sigma C_{1n} (\alpha_S^2 \log s)^n + \Sigma C_{2n}(\alpha_S^2 \log\log s)^n + \Sigma C_{3n}(\alpha_S^2 \log s \log Q^2)^n$$

Asymptotics of QED cross sections



$\sigma \sim (\alpha_{QED})^2 \log(s)/s$ $\sigma \sim (\alpha_{QED})^4 \text{ const}(s)$

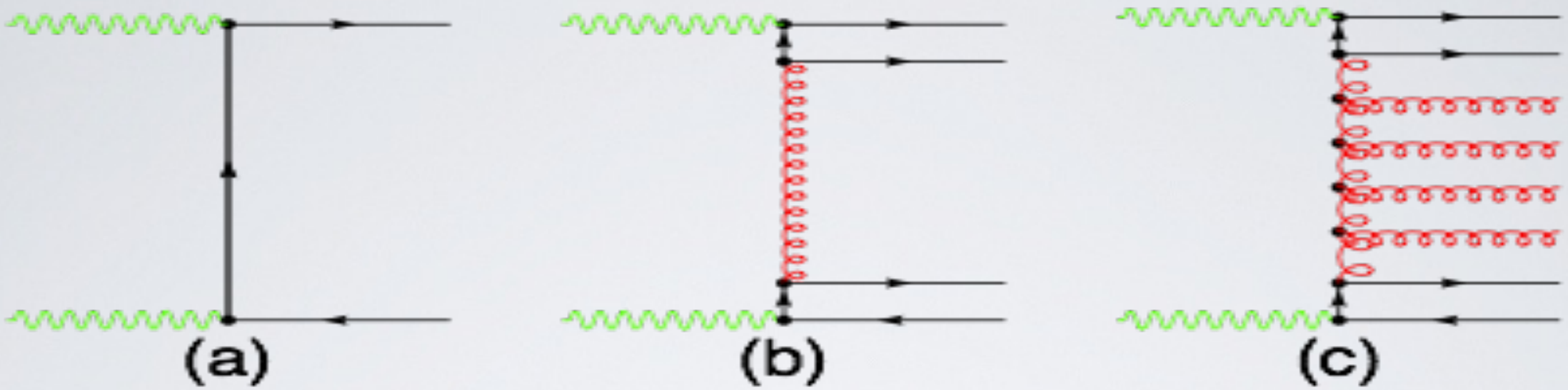
All orders: V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)
H. Cheng & T.T. Wu (69-70)

Cross section at s -> ∞: $\sim (\alpha_{QED})^4 (S/S_0)^{(aP-1)}$

$a_P = 1 + C (\alpha_{QED})^2 \approx 1.002$

photon: no reggeization!

High-energy limit pQCD as LL BFKL: γγ



$\sigma \sim (\alpha_{QED})^2 \log(s)/s$ $\sigma \sim (\alpha_{QED})^2 (\alpha_S)^2 \text{const}(s)$

Resummation of all leading logarithms: LL BFKL

non-Abelian interaction → gluon reggeization!

Cross section at s -> ∞: $\sim (\alpha_{QED})^2 (\alpha_S)^2 (S/S_0)^{(aP-1)}$

$a_P = 1 + C\,\alpha_S \approx 1.5$ **LL BFKL** S. Brodsky & F. Hautmann (96)

$a_P = 1 + C\,\alpha_S \approx 1.2$ **NLL BFKL**

S.Brodsky, V Fadin, VK,L. Lipatov, G. Pivovarov (2001-02)

LL BFKL remarkable properties: effective theories

2D-conformal properties
BFKL(Schredinger eq) as ”quantization”of RG-DGLAP (Euler-Lagrange eq)
L.N. Lipatov (1986)

Effective action for reggeized gluons
L.N. Lipatov (1995)

LL BFKL 2D-conformal block symmetry:
Feynman-like rules for inclusive x-sections
V.T.K., G.B. Pivovarov (1997)

LL BFKL 2D-conformal block symmetry
H. Navelet, R. Peschanski (1998-1999)

Effective Regge QCD: gluon trajectory as RG constant
V.T.K., G.B. Pivovarov (1997)

Feynman rules for Reggeized gluons
E.N. Antonov, E.A. Kuraev, L.N. Lipatov, I. Cherednikov (2005)

BFKL observables

Heavy quark production
I.I. Balitsky, L.N. Lipatov (1978)

Inclusive jet
M.G. Ryskin (1980)

Lepton pair production
M.G. Ryskin, E.M. Levin (1981)

Deep inelastic processeses -> small-x physics
unitarization -> small-x shadowing
L.V. Gribov, M.G. Ryskin, E.M. Levin (1981-83)

Most forward/backward (Mueller-Navelet) dijets:
x-section ~ exp(|Δ|y)
A. Mueller & H. Navelet, Nucl. Phys. B (1987)

LL BFKL motivated approaches

LL BFKL Pomeron
2D conformal symmetry and 1/N expansion
➥ factorization into integrable theory
high-energy QCD -> integrable system!
L.N. Lipatov (1994)
L.D. Faddeev, G.P. Korchemsky (1994)

LL BFKL Pomeron with 1/N expansion
Dipole Pomeron
A.H. Mueller (1994)
N.N. Nikolaev, B.G. Zakharov (1994)

Reggeon field theory with BFKL Pomeron
E.M. Levin, A. Kovner, M. Lublinsky (2024)

Pomeron and Reggeon applications

LL BFKL for DIS, inclusive processes

E.M. Levin, M.G. Ryskin, (1971 - 1990)

L.V. Gribov, E.M. Levin, M.G. Ryskin, (1981 - 1984)

LL BFKL Pomeron for diffractive physics, various processes DIS, pp-, pA-, AA- collisions …

V.A. Khoze, A.D. Martin, M.G. Ryskin, (1997 -)

Jet production

A.H. Mueller, H. Navelet Nucl. Physc. B (1987)

V.Del Duca, C. Schmidt (1994)

W. J. Striling , Nucl. Phys. B (1994)

V.T.K., G.B. Pivovarov (1996-1997)

kT-factorization, Pomeron for inclusive processes

A.V. Lipatov, S.P. Baranov, H. Jung, M.A. Malyshev, Eur. Phys. J. C78 (2018) 1

S.P. Baranov, A.V. Lipatov, M.A. Malyshev, A.A. Prokhorov, Phys. Rev. D 110 (2024) 054001

A.V. Karpishkov, M.A. Nefedov, V.A. Saleev, A.V. Shipilova, Phys.Rev. D 91 (2015) 054009

A.V. Karpishkov, V.A. Saleev, A.V. Shipilova, Phys.Rev. D 94 (2016) 114012 (Er)

Pomeron and Reggeons: elastic & diffractive processes, total x-sections

V.A. Petrov, R.A. Ryutin, Int. J. Mod. Phys. A 31 (2016) 1650049

V.A. Petrov, V. Prokudin, Phys. Rev. D 87 (2013) 036003

A.A. Godizov, V.A. Petrov, Phys. Part. Nucl. 39 (2008) 89

Effective field theory in high-energy QCD limit

Effective gauge-invariant action, based on BFKL reggeons

L.N. Lipatov, Nucl. Phys. B 721 (1995) 111
L.N. Lipatov, Phys. Rep. 286 (1997) 131
E.N. Antonov L.N., I. Cherednikov, Lipatov, E.A. Kuraev, Nucl. Phys. (2005)

Effective gluon fields:
fast right- and Left- movers

V.T.K., G.B. Pivovarov, Phys. Rev. Lett. 89 (1997) 809

Renormalization ”constant” ->
Regge trajectory of gluon in BFKL

LL BFKL: problems

LL BFKL: designed for infinite collision energies multi-Regge-kinematics

LL BFKL problems (at finite energies):
- fixed (non-running) coupling a_s
- energy-momentum conservation
- transverse momentum conservation

Cross section in LL BFKL:

$\sigma = \sigma_0 (S/S_0)^{(aP-1)}$ $\qquad a_P = 1 + C\, a_s \approx 1.5\text{-}1.6$

Data: $a_P \approx 1.2\text{-}1.3$

BFKL: next-to-leading logs (NLL) improved by running a_s

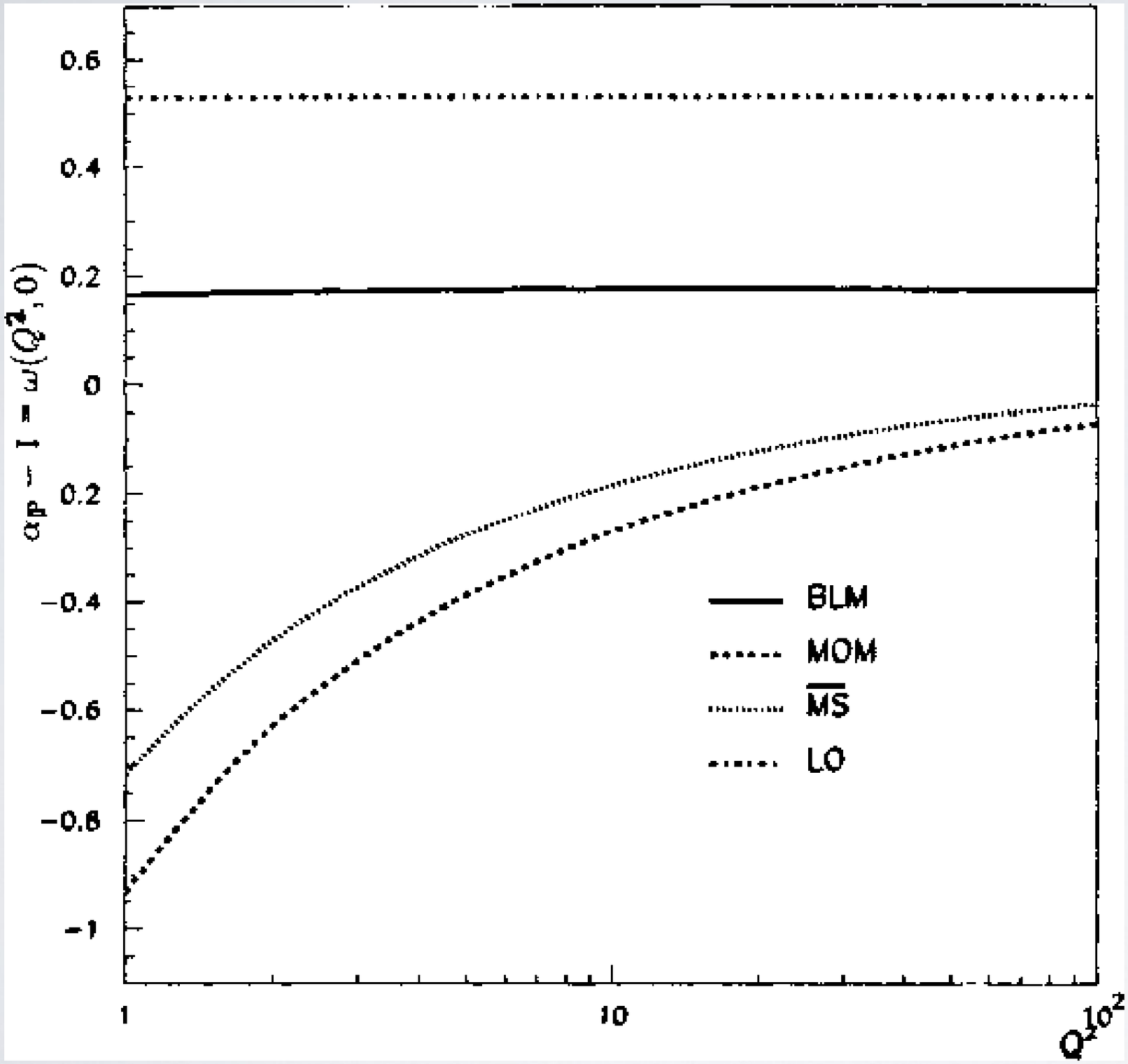
next-to-leading log approximation (NLL) BFKL
MSbar-renormalization scheme: large corrections
V.S. Fadin & L.N. Lipatov (89-98)
C.Camici & M. Ciafaloni (96-98)

BFKLP: NLL BFKL + resummation of running coupling a_S generalized for the case with non-Abelian LO
S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP

➔ BLM approach Brodsky, Lepage & Mackenzie – 1983
➔ works only (!) for the case with Abelian LO

BFKLP: NLL BFKL within generalized BLM

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP



$$\sigma \sim s^{\alpha_{IP}-1} = s^{\omega^{\max}}$$

BFKLP: NLL BFKL within generalized BLM

V.S. Fadin & L.N. Lipatov (89-98)
C.Camici & M. Ciafaloni (96-98)
next-to-leading log approximation (NLL) BFKL
MSbar-renormalization scheme: large corrections

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP
BFKLP: NLL BFKL + resummation of running coupling a_s

in physical renormalization scheme

BFKLP: Conformal BFKL kernel in NLL -> SUSY N=4
Pomeron intercept: a_P=1.2 - 1.3
Cross section: $\sigma_0 (S/S_0)^{(aP-1)}$ $a_P = 1 + C\, a_s$

First NLL BFKL observation at LHC: Mueller-Navelet dijets

BFKL with BFKLP **F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa,**
. Rev. (2015)

BFKL with BFKLP: 2.76 TeV dijet x-section **A. Egorov & VK**
Phys. Rev. (2023)

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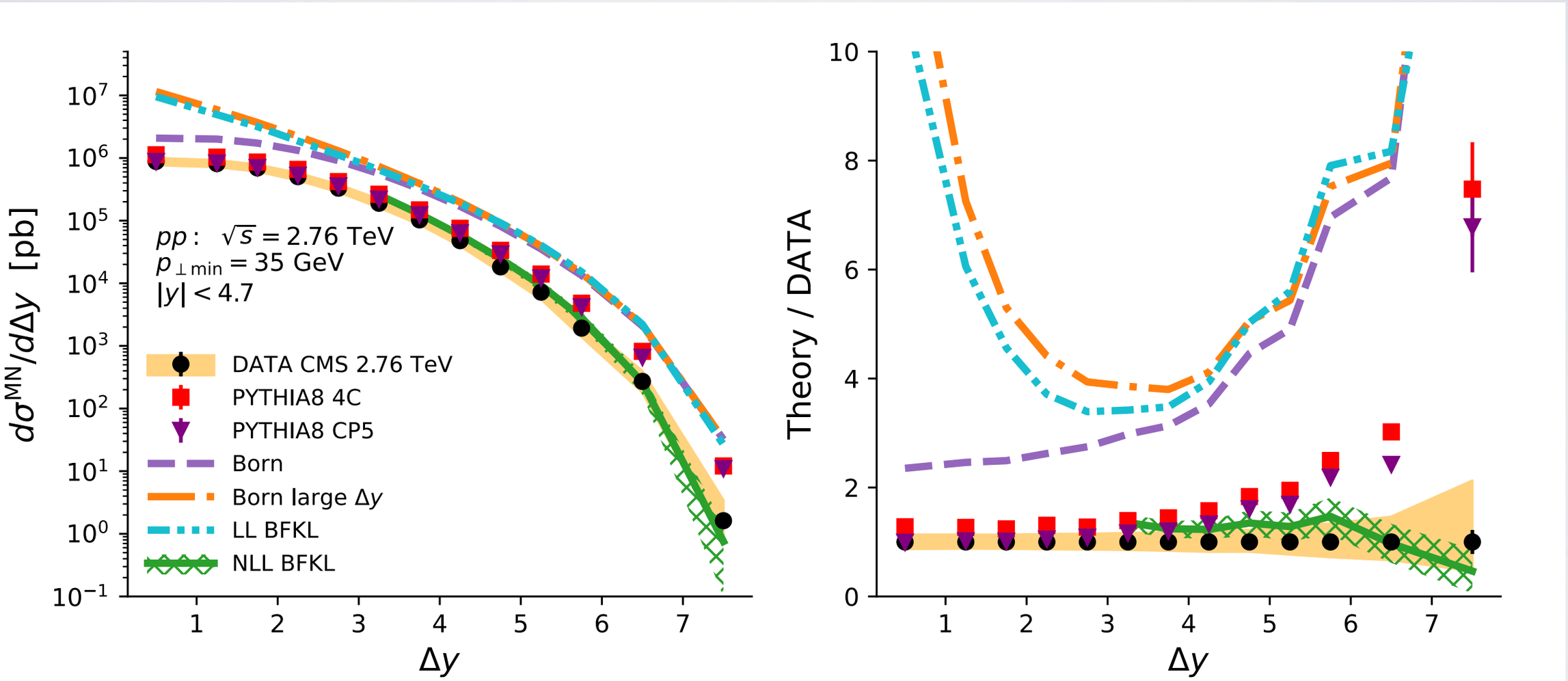


CMS (2022)
2.76 TeV, pT_min = 35 GeV

Odderon trajectory in perturbative QCD

C=-1

Odderon

L. Lukaszuk, B. Nicolescu, Lett. Nuov. Cim. Soc. Ital. Fis. 8 (1973) 405

Odderon at Born level: 3 gluons

M. Fukugita, J. Kwiecinski, Phys. Lett. 83 B (1979) 119
P. Gauron, B. Nicolescu, E. Leader, Phys. Lett. 238 B (1990) 406

Odderon in LLA: 3 reggeized gluon bound state

LL (Bartels-Kwiecinski-Praszalowicz) BKP equation

J. Bartels, Nucl. Phys. B 175 (1980) 365
J. Kwiecinski, M. Praszalowicz, Phys. Lett. 94 B (1980) 413
P. Gauron, L.N. Lipatov, B. Nicolescu, Phys.Lett. 304 B (1993) 334
P. Gauron, L.N. Lipatov, B. Nicolescu, Z.Phys.C63 (1994) 253 $a_{Odd}(0) \simeq 1.07$ $(a_{PBFKL}(0) \simeq 1.45)$
R.A. Janik, J. Wosiek, Phys. Rev. Lett. 82 (1999) 1092 $a_{Odd}(0) = 0.71$
J. Bartels, L.N. Lipatov, G. Vacca, Phys. Lett. 477 B (1999) 178 $a_{Odd}(0) = 1.00$

Odderon in NLLA: NLL BKP equation

J. Bartels, V.S. Fadin, L.N. Lipatov, G. Vacca, Nucl. Phys. B 867 (2013) 827

? $a_{Odd}(0) \simeq 1.0$ $(a_{P\ BFKLP}(0) \simeq 1.2)$

Non-perturbative QCD: dual parton expansion

1/N expansion

Planar diagrams

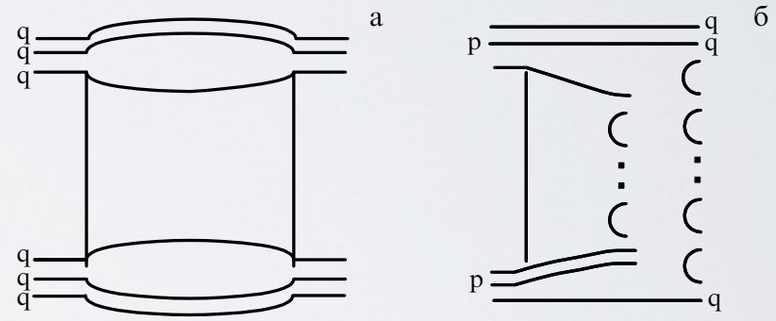
G. ‘t Hooft, Nucl. Phys. B 89 (1974) 461

Topological expansion for Pomeron: cylinder

G. Veneziano, Nucl. Phys. B 117 (1976) 519

Pomeron: high-energy limit -> only then (!) 1/N expansion

Dual parton unitarization expansion



Quark-Gluon String Model

A.B. Kaidalov K.A. Ter-Martirosyan, Phys. Lett. 117 B (1982) 247

Dual parton unitarization

A. Capella, J. Tran Tanh Van Z. Phys. C 23 (1984) 165

Pomeron in nonpertubative QCD

Unitarity with 1/N expansion for saturation limit

Balitsky-Kovchegov equation **with $\alpha_S \to 0$: reproduces BFKL**

I.I. Balitslky (1996) Yu. Kovchegov (1999, 2000)

Color Glass Condensate evolution for saturation limit

with $\alpha_S \to 0$: reproduces BFKL

L. McLerran, R. Venugopalan (1994)

J. Jalilian-Marian, A. Kovner, L.D. McLerran, H. Weigert, Phys. Rev. D 59 (1998) 014014

E. Iancu , A. Leonidov, L.D. McLerran, Phys. Lett. 510 B (2001) 138

F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan (2010)

Reggeons in nonpertubative QCD

Pomeron by quark and gluons condensates

P.V. Landshoff, O. Nachtmann Z. Phys. C 35 (1987) 405

O. Nachtmann, Ann. Phys. 209 (1991) 436

H.G. Dosch, E. Ferreira, A. Kramer, Phys. Rev. D 50 (1994) 1992

Lattice QCD

T. Biswas, M. Grisaru, W. Siegel Nucl. Phys. B 708 (2005) 317

No linearity (! ?) in Regge trajectories

$$a(t) = a(0) + a' |t|$$

$$a(0) = a_{non\text{-}Pert}(0) + a_{Pert}(0)$$

$$a' = a'_{non\text{-}Pert} + a'_{Pert}$$

Reggeons beyond QCD

AdS/CFT duality conjecture

J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231

Conformal field theory

1/N → 0

BFKL Pomeron in SUSY QCD N=4

A.V. Kotikov, L.N. Lipatov (2000)

Anomalous dimensions in BFKL Pomeron SUSY QCD N=4

L.N. Lipatov, A.V. Kotikov, A. Onischenko, V. Velizhanin (2002-2006)

confirming AdS/CFT duality conjecture

Gravition-Pomeron duality

L.N. Lipatov, Nucl. Phys. (2006)

C.I. Tan, C. Brower (2006)

Alvarez-Gaume et al. (2007)

Summary

- **Perturbative QCD (LL BFKL and NLL BFKL):**

- phenomenology successful

- reproduces main classical Pomeron properties

- bringing new remarkable features: conformality, integrability, AdS/CFT duality, holographic properties ...

- **Non-Perturbative QCD (QGSM, etc.) :**

- phenomenology successful

- **Beyond QCD (AdS/CFT holographic QCD approach):**

- under active developments