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Pomeron weights in QCD processes at high energy and the *S*-matrix unitarity constraint

Rami Oueslati

AGO Department
Liège University

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Outline

Context of the paper

Reference: Rami Oueslati, arXiv:2412.17267 [hep-ph] (2024).

- Data vs. model discrepancies $\Longrightarrow$ Need for improved hadronic interaction model
- Scattering amplitude can not be fully calculated using perturbative QCD $\Longrightarrow$ to phenomenological and non-perturbative approaches
- Testing the hypotheses that are central to their construction

Context of the paper

- Most of these models use the eikonal approximation.
- Direct and indirect evidence : this approximation is not optimum for dealing with composite particles, such as hadrons, which consist of bound quarks and gluons.
- Existing data remain insufficient to clearly distinguish the most appropriate unitarization scheme.
- Alternative approximations should be considered $\Longrightarrow$ raising several fundamental questions:
- which one is the most appropriate.
- Does the choice of unitarization scheme merely a matter of convenience or convention ?
- What is the fundamental nature of the pomeron exchange within these different frameworks, even though they all satisfy unitarity?

Unitarisation schemes

- Unitarity demands that $|S(b)|^2 \leq 1$ $\Rightarrow$ The physical amplitude lies within the unitarity circle
- one can map the upper complex plane into a circle via a complex exponential

$$S(s,b) = \exp(iz(s,b)) \qquad (1)$$

- It is also possible to use a one-to-one map through a Mobius transform

$$S(s,b) = \frac{1 + iz'(s,b)}{1 - iz'(s,b)}. \qquad (2)$$

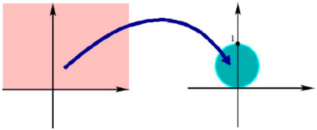


Figure 1: Unitarity circle.
Source: **Jean-René Cudell** (13 November 2017). *Elastic scattering, total cross sections and ρ parameters at the LHC*. Paper presented at the ATLAS Collaboration Meeting, Geneva, Switzerland. https://hdl.handle.net/2268/215693

Generalization

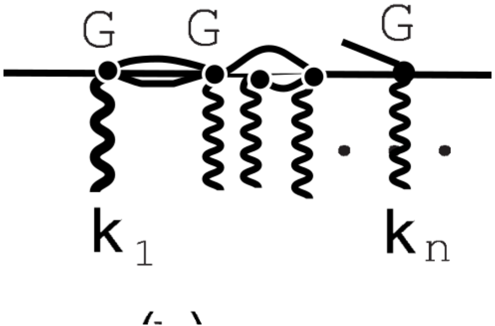
$$F(s,t) = \sum_{n=1}^{\infty} F_n(s,t), \tag{3}$$

with the n reggeon exchange amplitude given by :

$$F_n(s, t \simeq -k_\perp^2) = \frac{-i}{n n!} \int N_n^2(k_{\perp i}) \cdot \prod_{i=1}^{n} \frac{d^2 k_{\perp i}}{(2\pi)^2} \cdot D(s, k_{\perp i}) \delta^2 \left(k_\perp - \sum k_{\perp i} \right), \tag{4}$$

Generalization

- Kancheli's idea (arXiv:1309.5860v2): take into account the contribution from diffraction production in the weights of multi-pomeron exchange.



Generalization

$$S(s,b) = \sum_{n=0}^{\infty} \frac{\beta_n^2}{n!} \left(i\,\chi(s,b)\right)^n \tag{5}$$

$$S(s,b) \equiv S[\chi] = \int_0^{\infty} d\tau\, \rho(\tau)\, e^{i\tau\chi(s,b)} \tag{6}$$

where $\rho(\tau)$ functions as a weight.
The constraints $\beta_0 = \beta_1 = 1$ are imposed by the normalization condition for $S[\chi]$ and $w(\nu)$. This results in the following relations:

$$\int_0^{\infty} d\tau\, \rho(\tau) = \int_0^{\infty} d\tau\, \tau\rho(\tau) = 1 \tag{7}$$

$$N_n(k_i) = \prod_{i=1}^{n} g(k_i), \tag{8}$$

where $g(k) = G_{11}(p, p+k)$.

Some Observables

- The total cross-section of diffraction generation : single σ_{sd} and double σ_{dd}

$$\begin{aligned} \sigma_{dif}(s,b) &= \sigma_{in} - \sigma_{\text{in, cut}} = 2\sigma_{sd} + \sigma_{dd} \\ &= S[2iIm(\chi)] - \mid S[\chi] \mid^{2} \end{aligned} \quad (9)$$

$$\sigma_{\text{in, cut}}(s,b) = 1 - S[2i\ Im(\chi)] \quad (10)$$

- the pomeron topological x-section

$$\sigma_n(s,b) = \int_0^{\infty} d\tau \rho(\tau)\ \frac{(2\tau Im(\chi))^n}{n!}\ e^{-2\tau Im(\chi)} \quad (11)$$

where $\rho(\tau)$ is a spectral density.

- The pomeron topological x-section resembles a superposition of Poisson distributions,

Results

- Pomeron topological x-section in each scheme :
- eikonal :

$$\sigma_n(s,b) = \frac{(2\text{Im}(\chi(s,b)))^n}{n!} e^{-2\text{Im}(\chi(s,b))} \tag{12}$$

Pomeron multiplicity distribution in b space $\Longrightarrow$ poison distribution

- *U*-matrix :

$$\sigma_n(s,b) = 2\frac{(\text{Im}(\chi(s,b)))^n}{(1+\text{Im}(\chi(s,b))^{1+n}} \tag{13}$$

Pomeron multiplicity distribution in b space $\Longrightarrow$ A geometric distribution

Pomeron multiplicity distribution

- The pomeron multiplicity distribution $W_n(s)$: the probability of n pomerons exchanged in an inelastic collision at the energy s :

$$W_n = \frac{\sigma_n}{\sum_{n'} \sigma_{n'}} \qquad (14)$$

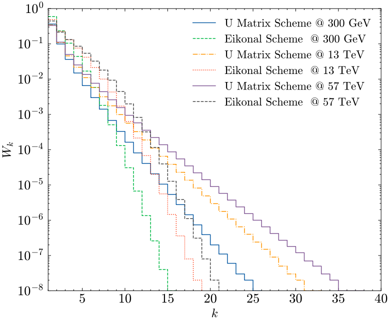


Figure 2: Pomeron multiplicity distribution in both cases, eikonal and U-matrix.

Mean and Variance

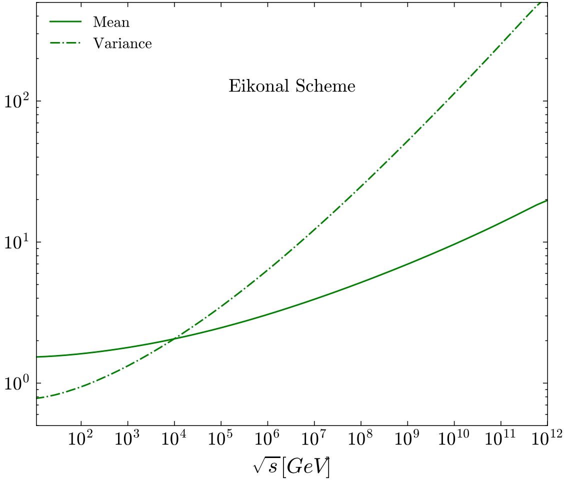


Figure 3: Mean and variance of the number of pomerons in the eikonal case

Mean and Variance

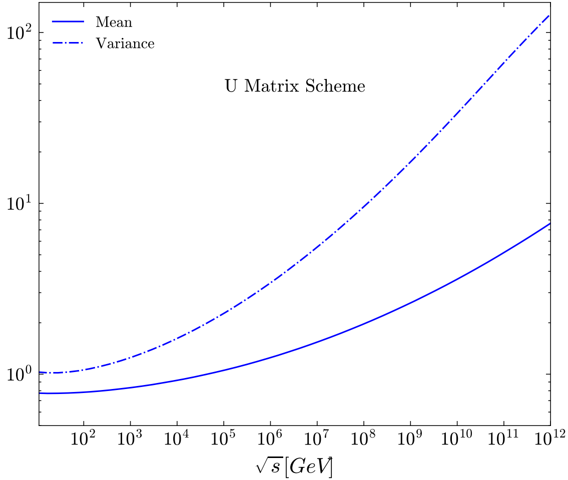


Figure 4: Mean and variance of the number of pomerons in U-matrix case

Factorial moment

- Normalized factorial moment of rank q :
- eikonal : $\forall$ e and $b \Rightarrow F_q = 1$
- U-matrix :

$$\mathrm{F}_q(s) = \frac{p \ \mathrm{Li}_{-q}(1-p)}{(\frac{1-p}{p})^q} \tag{15}$$

and p defined by

$$p = \frac{1}{1+\gamma} \tag{16}$$

$\gamma = Im(\chi(s, b))$.

Pomerons correlation

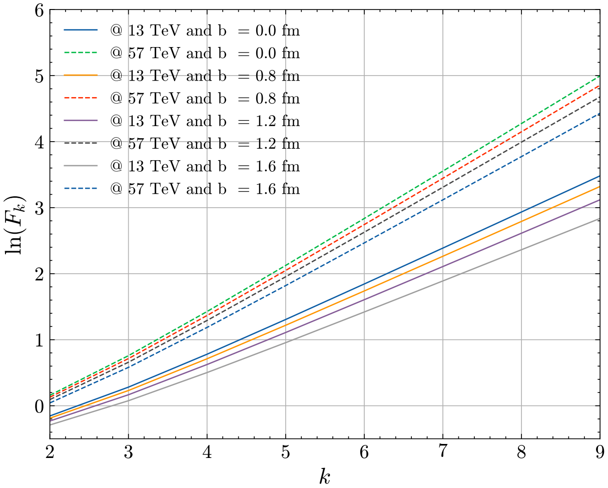


Figure 5: F_q of as a function of the rank q and for different impact parameter b values with the U-matrix scheme

Conclusion

- The U-matrix inherently incorporates a larger amount of diffraction production into the multi-pomeron vertices $\Rightarrow$ a larger variability in pomeron exchanges across all energy ranges.
- In eikonal scheme, such fluctuations only become significant beyond a specific high-energy threshold.
- Within the U-matrix scheme, an increase in exchanged pomerons results in more pronounced pomeron correlations.
- The choice of the scheme is no longer a matter of taste but is dictated by a fundamental reason.

Thank you!