Increase of the Coalescence Coefficient in Diffraction Processes

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Study of inclusive processes in diffraction and central regions

$d+d \rightarrow d+X$ n=2 **NICA SPD**

$^3He+^3He \rightarrow ^3He+X$ n=3 **proton production at the quark level**

$^4He+^4He \rightarrow ^4He+X$ n=4

Coalescence Coefficient

$$\kappa_2(\mathbf{k}) \equiv \frac{f_d(2\mathbf{k})}{f_p(\mathbf{k})f_n(\mathbf{k})} \approx \frac{f_d(2\mathbf{k})}{f_p^2(\mathbf{k})}$$

$$f(\mathbf{k}) \equiv \frac{k_0\, d^3\sigma}{d^3\mathbf{k}}$$

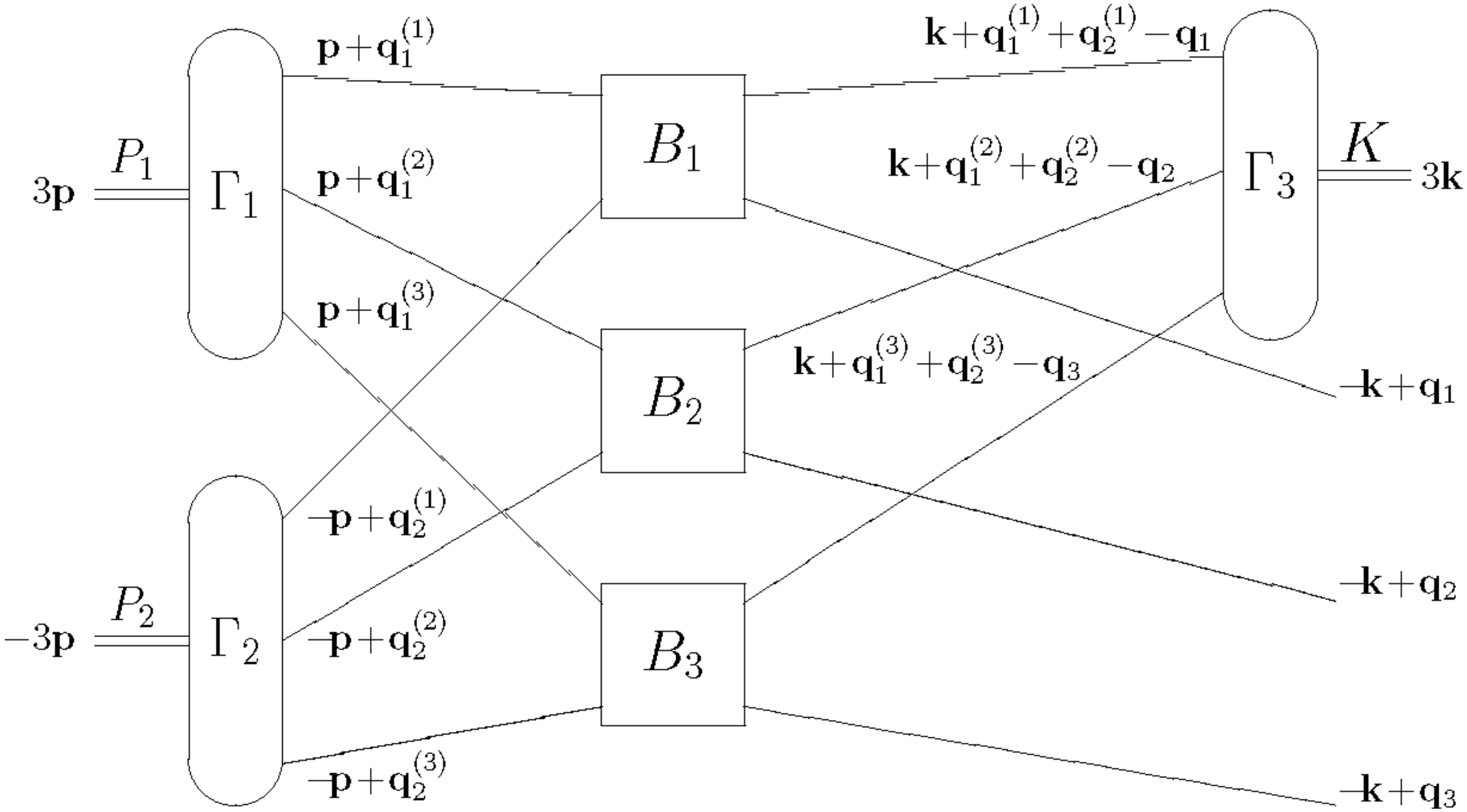
$d+d \rightarrow p+X$

Coherent Coalescence (n=3)

Butler S.T., Pearson C.A.
Phys. Rev. 129 (1963) 863.

Braun M.A., Vechernin V.V.,
Yad.Fiz. 36 (1982) 614; 44 (1986) 784; 47 (1988) 1452;
J. Phys. G 16 (1990) 1615.

$$T(n\mathbf{p}; n\mathbf{k}, \mathbf{q}_i) =$$



$$\sum_{i=1}^{3} \mathbf{q}_1^{(i)} = 0$$

$$\sum_{i=1}^{3} \mathbf{q}_2^{(i)} = 0$$

$$\sum_{i=1}^{3} \mathbf{q}_i = 0$$

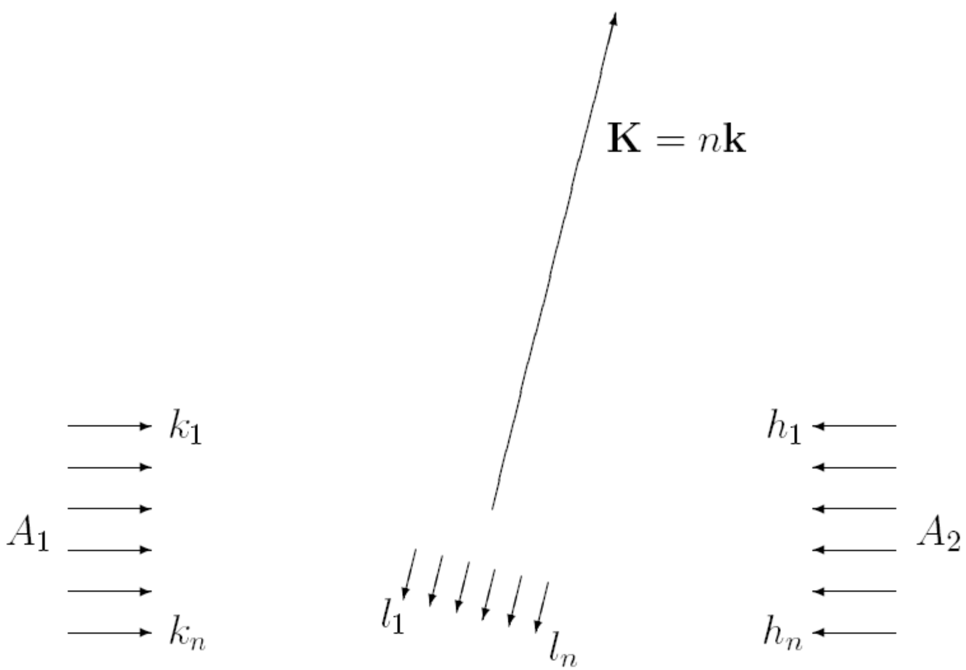
Inclusive cross-section

$$(2\pi)^3\, 2K_0 \frac{d^3\sigma}{d^3\mathbf{K}} = \frac{1}{J_{in}} \int |T(n\mathbf{p}; n\mathbf{k}, \mathbf{l}_i)|^2\, d\tau_n$$

$$d\tau_n \equiv (2\pi)^4 \delta^4(P_1 + P_2 - K - \sum_{i=1}^{n} l_i) \prod_{i=1}^{n} \frac{d^3\mathbf{l}_i}{2l_{i0}(2\pi)^3}$$

$$J_{in} \equiv 2A_1 A_2 \sqrt{s(s - 4m_N^2)} \approx 2s\, A^2$$

Kinematic region



First small parameter: $\frac{m_N}{p_N} = \frac{2m_N}{\sqrt{s_{NN}}} \ll 1$

Second small parameter: $1 - \frac{K}{K_{max}} \ll 1$

near the threshold: $K \to K_{max}$

=>

$$M_X{}^2 = (\sum_{i=1}^{n} l_i)^2 \to M_{Xmin}{}^2$$

$$l_1 = l_2 = \ ... = l_n \to \frac{K_{max}}{n}$$

$$\mathbf{q}_i \to 0$$

$^3He + ^3He \to ^3He + p + p + n$

$d + d \to d + p + n$

Weakly coupled systems

Third small parameter: $\alpha = \sqrt{\frac{\varepsilon}{m}} \ll 1$

$$M = n(m - \varepsilon)$$

$$\left\langle \frac{\mathbf{q}^{*2}}{2m} \right\rangle \simeq \varepsilon$$

$$|\mathbf{q}^*| \simeq \sqrt{m\varepsilon} = m\,\alpha$$

- in the rest frame of the nucleus

$$|\mathbf{q}_\perp| \simeq \sqrt{m\varepsilon} = m\,\alpha$$

$$|q_z| \simeq \gamma |q_z^*| \simeq \gamma \sqrt{m\varepsilon} = \frac{E_p}{m}\sqrt{m\varepsilon} = E_p\,\alpha$$

$$E(\mathbf{p}+\mathbf{q}) \equiv \sqrt{(\mathbf{p}+\mathbf{q})^2 + m^2}$$

$$E(\mathbf{p}+\mathbf{q}) = E_p \left\{ 1 + \frac{(\mathbf{qp})}{E_p^2} + \frac{1}{2E_p^2}\left[\mathbf{q}^2 - \frac{(\mathbf{qp})^2}{E_p^2} \right] + O(\alpha^3) \right\}$$

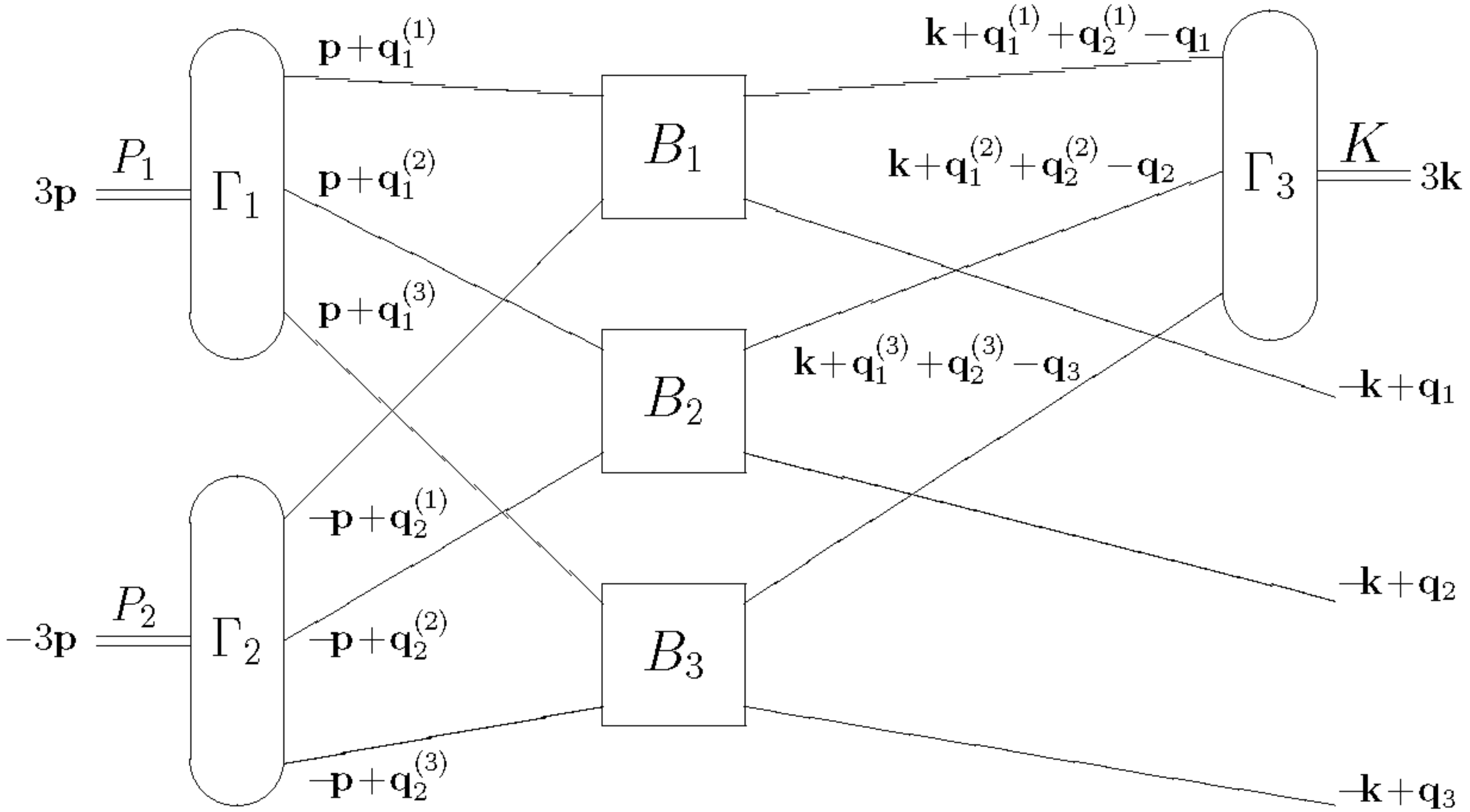
$$E_p \equiv E(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}\ , \ \ E_k \equiv E(\mathbf{k})$$

Valid both in the rest frame of the nucleus (p=0**) and**
in the frame in which the nucleus moves with relativistic momentum *n*p or *n*k
(|p|>>*m*, |k|>>*m*)

$$\frac{(\mathbf{qp})}{E_p^2} = \frac{q_z\,|\mathbf{p}|}{E_p^2} \simeq \frac{E_p\,\alpha\,|\mathbf{p}|}{E_p^2} = \frac{|\mathbf{p}|}{E_p}\alpha$$

Separation of hard blocks (n=3)

$$T(n\mathbf{p}; n\mathbf{k}, \mathbf{q}_i) =$$



$$\sum_{i=1}^{3} \mathbf{q}_1^{(i)} = 0$$

$$\sum_{i=1}^{3} \mathbf{q}_2^{(i)} = 0$$

$$\sum_{i=1}^{3} \mathbf{q}_i = 0$$

$$T(n\mathbf{p}; n\mathbf{k}, \mathbf{l}_i) = J(n\mathbf{p}; n\mathbf{k}, \mathbf{q}_i) \prod_{j=1}^{n} B_j(\mathbf{p}; \mathbf{k})$$

Used approximation

$$l^2 - m^2 + \mathrm{i}\epsilon = [l_0 + E(\mathbf{l})][l_0 - E(\mathbf{l}) + \mathrm{i}\epsilon] \approx 2E(\mathbf{l})[l_0 - E(\mathbf{l}) + \mathrm{i}\epsilon]$$

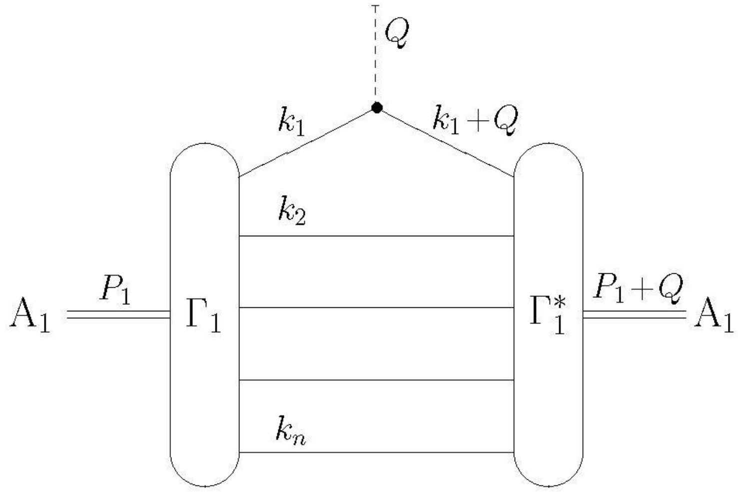
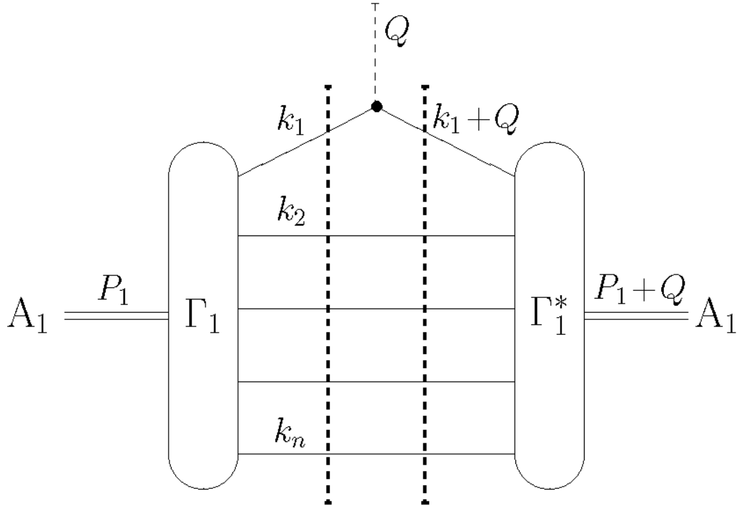
Integration over zero components of momenta

It leads to Heitler's (“old fashioned”) perturbation theory:

1) **All particles are considered on mass shell**
2) **Inclusion of "energy denominators" between interactions**
3) **Permutation of interactions in time where possible**
4) **Additional factor 1/[2*E*(l)] from each propagator**

$$\frac{1}{[\sum_i E(\mathbf{l}_i) - E_{init} - \mathrm{i}\epsilon]}$$

Connection of vertices Γ_i with wave functions. Normalization.

$F(Q^2) =$  $=$ 

$$Q^2 \to 0 \quad \Rightarrow \quad F(Q^2) \to 1$$

$$\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)}) = \frac{\Gamma_{n\mathbf{p}}(\mathbf{q}^{(i)})}{\sqrt{n}2E_p\left[\sum_{i=1}^{n} E(\mathbf{p}+\mathbf{q}^{(i)}) - E_{init} - \mathrm{i}\epsilon\right]}$$

$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=1}^{n-1} \frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3} = 1$$

Connection of vertices Γ_i with wave functions. Normalization.

In the rest frame of the nucleus (p=0):

$$\varphi_0(\mathbf{q}^{*(i)}) = \frac{\Gamma_0(\mathbf{q}^{*(i)})}{\sqrt{n}2m\left[\sum_{i=1}^{n}\frac{\mathbf{q}^{*(i)2}}{2m} - n\varepsilon - \mathrm{i}\epsilon\right]} = \frac{\Gamma_0(\mathbf{q}^{*(i)})}{\sqrt{n}\left[\sum_{i=1}^{n}\mathbf{q}^{*(i)2} - n2m\varepsilon - \mathrm{i}\epsilon\right]}$$

$$\int |\varphi_0(\mathbf{q}^{*(i)})|^2 \prod_{i=1}^{n-1}\frac{d^3\mathbf{q}^{*(i)}}{2m(2\pi)^3} = 1$$

In the frame in which the nucleus moves with relativistic momentum *n*p :

$$\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)}) = \frac{\Gamma_{n\mathbf{p}}(\mathbf{q}^{(i)})}{\sqrt{n}\left[\sum_{i=1}^{n}\mathbf{q}_{\perp}^{(i)2} + (q_z^{(i)}/\gamma)^2 - n2m\varepsilon - \mathrm{i}\epsilon\right]}$$

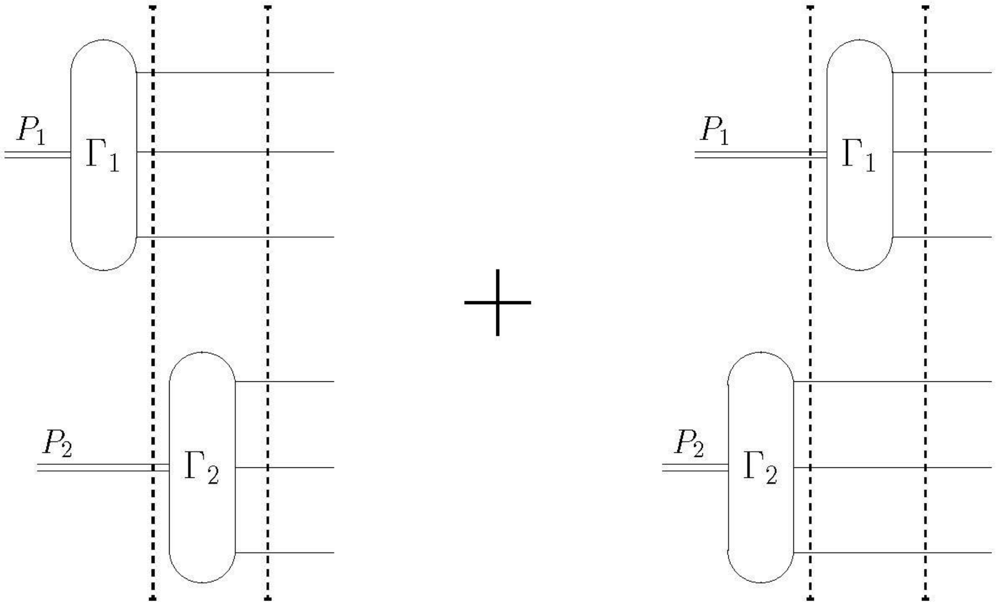
$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=1}^{n-1}\frac{d^3\mathbf{q}^{(i)}}{2m\gamma(2\pi)^3} = 1$$

$$\gamma = \frac{E_p}{m}$$

$$\mathbf{q}_{\perp}^{(i)} = \mathbf{q}_{\perp}^{*(i)} \simeq \sqrt{m\varepsilon} = m\,\alpha$$
$$q_z^{(i)} = \gamma q_z^{*(i)} \simeq \gamma\sqrt{m\varepsilon} = E_p\,\alpha$$

$$\alpha = \sqrt{\frac{\varepsilon}{m}}$$

Permutations of Γ_i vertices (n=3)



$$\varphi_{3\mathbf{p}}(\mathbf{q}_1^{(i)}) \equiv \frac{\Gamma_1}{[E_1 - E_1^{init}]2E_p\sqrt{3}} ,$$

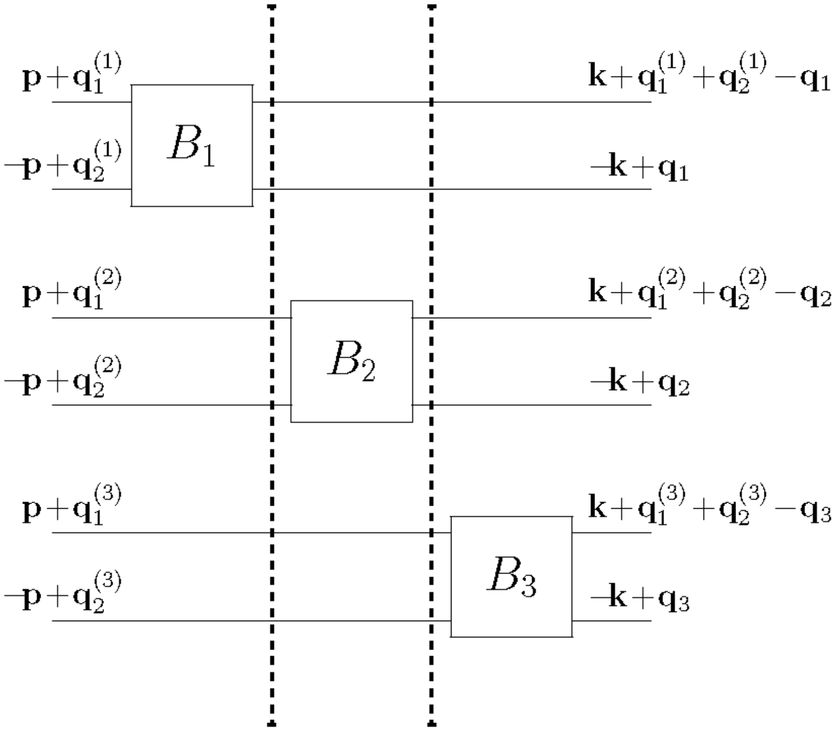
$$\varphi_{-3\mathbf{p}}(\mathbf{q}_2^{(i)}) \equiv \frac{\Gamma_2}{[E_2 - E_2^{init}]2E_p\sqrt{3}}$$

$$\left\{\frac{1}{[E_1 - E_1^{init} - \mathrm{i}\epsilon]} + \frac{1}{[E_2 - E_2^{init} - \mathrm{i}\epsilon]}\right\}\frac{1}{[E_1 + E_2 - E_1^{init} + E_2^{init} - \mathrm{i}\epsilon]} =$$

$$= \frac{1}{[E_1 - E_1^{init}][E_2 - E_2^{init}]}$$

$$E_1 \equiv \sum_{i=1}^{3} E(\mathbf{p} + \mathbf{q}_1^{(i)}) \; , \quad E_2 \equiv \sum_{i=1}^{3} E(-\mathbf{p} + \mathbf{q}_2^{(i)}) = \sum_{i=1}^{3} E(\mathbf{p} - \mathbf{q}_2^{(i)})$$

Permutations of B blocks (n=3)



$$\frac{E_p^2}{[-a_1 - i\epsilon][a_3 - i\epsilon]}$$

$$a_i \equiv (\mathbf{p},\, \mathbf{q}_1^{(i)} - \mathbf{q}_2^{(i)}) - (\mathbf{k},\, \mathbf{q}_1^{(i)} + \mathbf{q}_2^{(i)} - 2\mathbf{q}_i)$$

$$\sum_{i=1}^{3} a_i = 0$$

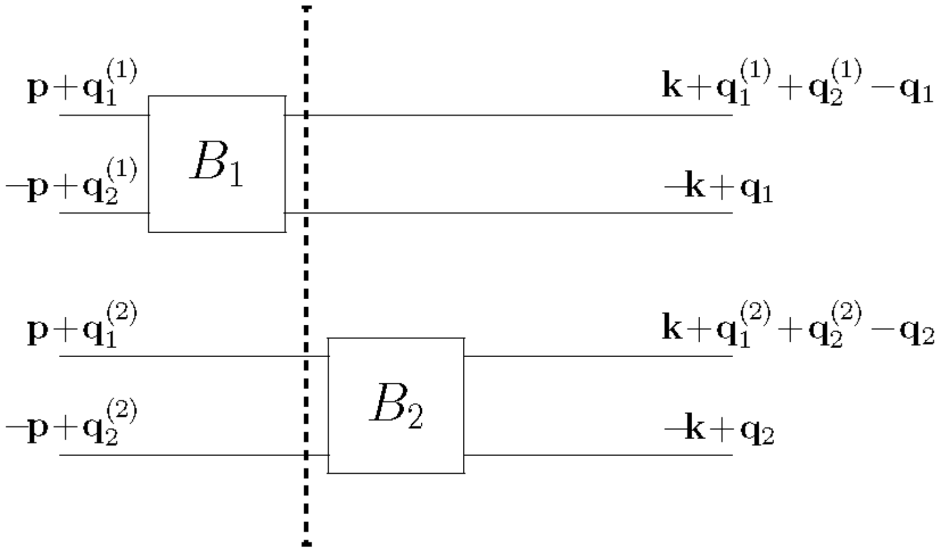
$$\frac{1}{b - i\epsilon} = \frac{1}{b} + i\pi\delta(b)$$

$$\frac{-E_p^2}{[a_1 + i\epsilon][a_3 - i\epsilon]} + \frac{-E_p^2}{[a_3 + i\epsilon][a_1 - i\epsilon]} = -\frac{2E_p^2}{a_1 a_3} - 2\pi^2 E_p^2 \delta(a_1)\delta(a_3)$$

$$\frac{1}{a_1 a_3} + \frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} = \frac{a_1 + a_2 + a_3}{a_1 a_2 a_3} = 0$$

$$D(a_1, a_2, a_3) = -2\pi^2 E_p^2 [\delta(a_1)\delta(a_2) + \delta(a_2)\delta(a_3) + \delta(a_1)\delta(a_3)]$$

Permutations of B blocks (n=2)



$$\frac{E_p}{[-a_1 - \mathrm{i}\epsilon]} + \frac{E_p}{[-a_2 - \mathrm{i}\epsilon]}$$

$$a_i \equiv (\mathbf{p},\, \mathbf{q}_1^{(i)} - \mathbf{q}_2^{(i)}) - (\mathbf{k},\, \mathbf{q}_1^{(i)} + \mathbf{q}_2^{(i)} - 2\mathbf{q}_i)$$

$$a_1 + a_2 = 0$$

$$\frac{-E_p}{[a_1 + \mathrm{i}\epsilon]} + \frac{-E_p}{[a_2 + \mathrm{i}\epsilon]} = i\pi E_p[\delta(a_1) + \delta(a_2)]$$

$$\frac{1}{b - \mathrm{i}\epsilon} = \frac{1}{b} + i\pi\delta(b)$$

$$D(a_1, a_2) = i\pi E_p[\delta(a_1) + \delta(a_2)] = 2\pi i E_p\, \delta(a_1)$$

Inclusive amplitude (n=3)

$$(2\pi)^3\, 2K_0 \frac{d^3\sigma}{d^3\mathbf{K}} = \frac{1}{J_{in}} \int |T(\mathbf{p};\mathbf{k},\mathbf{q}_i)|^2\, d\tau_n$$

$$T(\mathbf{p};\mathbf{k},\mathbf{q}_i) = J(\mathbf{p};\mathbf{k},\mathbf{q}_i) \prod_{j=1}^{3} B_j(\mathbf{p};\mathbf{k})$$

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_i) = \int \prod_{i=1}^{2} \frac{d^3\mathbf{q}_1^{(i)}}{2E_p(2\pi)^3} \frac{d^3\mathbf{q}_2^{(i)}}{2E_p(2\pi)^3} \times$$

$$\times \varphi_{3\mathbf{p}}(\mathbf{q}_1^{(i)})\varphi_{-3\mathbf{p}}(\mathbf{q}_2^{(i)}) D(a_1,a_2,a_3)\varphi^*_{3\mathbf{k}}(\mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-\mathbf{q}_i)$$

$$D(a_1,a_2,a_3) = -2\pi^2 E_p^2[\delta(a_1)\delta(a_2) + \delta(a_2)\delta(a_3) + \delta(a_1)\delta(a_3)]$$

$$a_i \equiv (\mathbf{p},\, \mathbf{q}_1^{(i)} - \mathbf{q}_2^{(i)}) - (\mathbf{k},\, \mathbf{q}_1^{(i)} + \mathbf{q}_2^{(i)} - 2\mathbf{q}_i)$$

$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=1}^{n-1} \frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3} = 1$$

Inclusive amplitude (n=2) d+d → d+p+n

$$(2\pi)^3\, 2K_0 \frac{d^3\sigma}{d^3\mathbf{K}} = \frac{1}{J_{in}} \int |T(\mathbf{p};\mathbf{k},\mathbf{q}_1)|^2\, d\tau_2$$

$$T(\mathbf{p};\mathbf{k},\mathbf{q}_1) = J(\mathbf{p};\mathbf{k},\mathbf{q}_1) \prod_{j=1}^{2} B_j(\mathbf{p};\mathbf{k})$$

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_1) = \int \frac{d^3\mathbf{q}_1^{(1)}}{2E_p(2\pi)^3} \frac{d^3\mathbf{q}_2^{(1)}}{2E_p(2\pi)^3} \times$$

$$\times 2\pi i E_p\, \delta(a_1)\, \varphi_{2\mathbf{p}}(\mathbf{q}_1^{(1)}) \varphi_{-2\mathbf{p}}(\mathbf{q}_2^{(1)}) \varphi^*_{2\mathbf{k}}(\mathbf{q}_1^{(1)} + \mathbf{q}_2^{(1)} - \mathbf{q}_1)$$

$$a_1 \equiv (\mathbf{p},\, \mathbf{q}_1^{(1)} - \mathbf{q}_2^{(1)}) - (\mathbf{k},\, \mathbf{q}_1^{(1)} + \mathbf{q}_2^{(1)} - 2\mathbf{q}_1)$$

$$\int |\varphi_{2\mathbf{p}}(\mathbf{q})|^2 \frac{d^3\mathbf{q}}{2E_p(2\pi)^3} = 1 \qquad \int |\varphi_{2\mathbf{k}}(\mathbf{q})|^2 \frac{d^3\mathbf{q}}{2E_k(2\pi)^3} = 1$$

Fourier transform of Inclusive amplitude (n=2) d+d → d+p+n

$$\mathbf{q} = \mathbf{q}_1^{(1)} + \mathbf{q}_2^{(1)}$$

$$\widetilde{\mathbf{q}} = (\mathbf{q}_1^{(1)} - \mathbf{q}_2^{(1)})/2$$

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_1) \sim \int d\mathbf{q}\, d\widetilde{\mathbf{q}}\, \varphi_{2\mathbf{p}}(\mathbf{q}/2+\widetilde{\mathbf{q}})\varphi_{-2\mathbf{p}}(\mathbf{q}/2-\widetilde{\mathbf{q}})\varphi^*_{2\mathbf{k}}(\mathbf{q}-\mathbf{q}_1)\delta(a_1)$$

$$a_1 = (2\mathbf{p},\, \widetilde{\mathbf{q}}) - (\mathbf{k},\, \mathbf{q}-2\mathbf{q}_1) = 2p\,\widetilde{q}_z - (\mathbf{k},\, \mathbf{q}-2\mathbf{q}_1)$$

$$\int |\varphi_{2\mathbf{p}}(\mathbf{q})|^2 \frac{d\mathbf{q}}{(2\pi)^3} = 1$$

$$\int |\varphi_{2\mathbf{k}}(\mathbf{q})|^2 \frac{d\mathbf{q}}{(2\pi)^3} = 1$$

$$\int |\psi_{2\mathbf{p}}(\mathbf{r})|^2 d\mathbf{r} = 1$$

$$\int |\psi_{2\mathbf{k}}(\mathbf{r})|^2 d\mathbf{r} = 1$$

$$\psi_{2\mathbf{p}}(\mathbf{r}_1) \equiv \int \varphi_{2\mathbf{p}}(\mathbf{q}) e^{-i\mathbf{r}_1\mathbf{q}} \frac{d\mathbf{q}}{(2\pi)^3}$$

$$\psi_{-2\mathbf{p}}(\mathbf{r}_2) \equiv \int \varphi_{-2\mathbf{p}}(\mathbf{q}) e^{-i\mathbf{r}_2\mathbf{q}} \frac{d\mathbf{q}}{(2\pi)^3}$$

$$\psi_{2\mathbf{k}}(\mathbf{r}) \equiv \int \varphi_{2\mathbf{k}}(\mathbf{q}) e^{-i\mathbf{r}\mathbf{q}} \frac{d\mathbf{q}}{(2\pi)^3}$$

Inclusive amplitude in coordinate space (n=2) d+d → d+p+n

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_1) \sim \int d\mathbf{r}_\perp dz_1 dz_2\, \psi_{2\mathbf{p}}(z_1,\mathbf{r}_\perp)\psi_{-2\mathbf{p}}(z_2,\mathbf{r}_\perp)\psi^*_{2\mathbf{k}}(\mathbf{r}_K) \times e^{-i\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}-\frac{z_1+z_2}{2}\frac{\mathbf{k}}{p},\mathbf{q}_1\right)}$$

$$\mathbf{r}_K = \frac{\mathbf{r}_1+\mathbf{r}_2}{2} + \frac{z_1+z_2}{2}\frac{\mathbf{k}}{p}$$

$$\mathbf{r}_{1\perp} = \mathbf{r}_{2\perp} = \mathbf{r}_\perp$$

The importance of taking into account the coherence of the coalescence process

$$T(\mathbf{p};\mathbf{k},\mathbf{q}_1) = J(\mathbf{p};\mathbf{k},\mathbf{q}_1)\prod_{j=1}^{2} B_j(\mathbf{p};\mathbf{k})$$

$$(2\pi)^3\, 2K_0 \frac{d^3\sigma}{d^3\mathbf{K}} = \frac{1}{J_{in}}\int |T(\mathbf{p};\mathbf{k},\mathbf{q}_1)|^2\, d\tau_2$$

Gaussian approximation for wave functions

$$\psi_{2\mathbf{p}}(z,\mathbf{r}_\perp) = C_p \exp -\frac{(z\gamma_p)^2 + \mathbf{r}_\perp^2}{2R^2}$$

$$\gamma_p = \frac{E_p}{m}$$

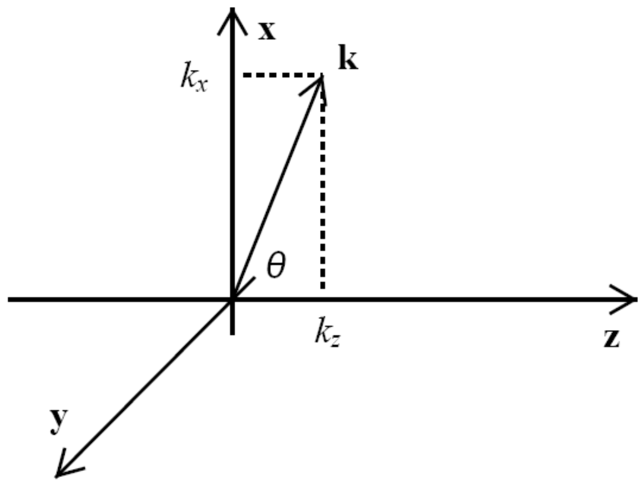
$$\mathbf{n} = \frac{\mathbf{k}}{|\mathbf{k}|}$$

$$\gamma_k = \frac{E_k}{m}$$

$$\psi_{2\mathbf{k}}(\mathbf{r}) = C_k \exp -\frac{[(\mathbf{nr})\gamma_k]^2 + [\mathbf{r}-\mathbf{n}(\mathbf{nr})]^2}{2R^2} = C_k \exp -\frac{(\mathbf{nr})^2[\gamma_k^2 - 1] + \mathbf{r}^2}{2R^2}$$

$$\mathbf{k} = (k_x, 0, k_z) \quad k_x = |\mathbf{k}_\perp|$$

$$(\mathbf{nr}) = x n_x + z n_z = x\sin\theta + z\cos\theta$$

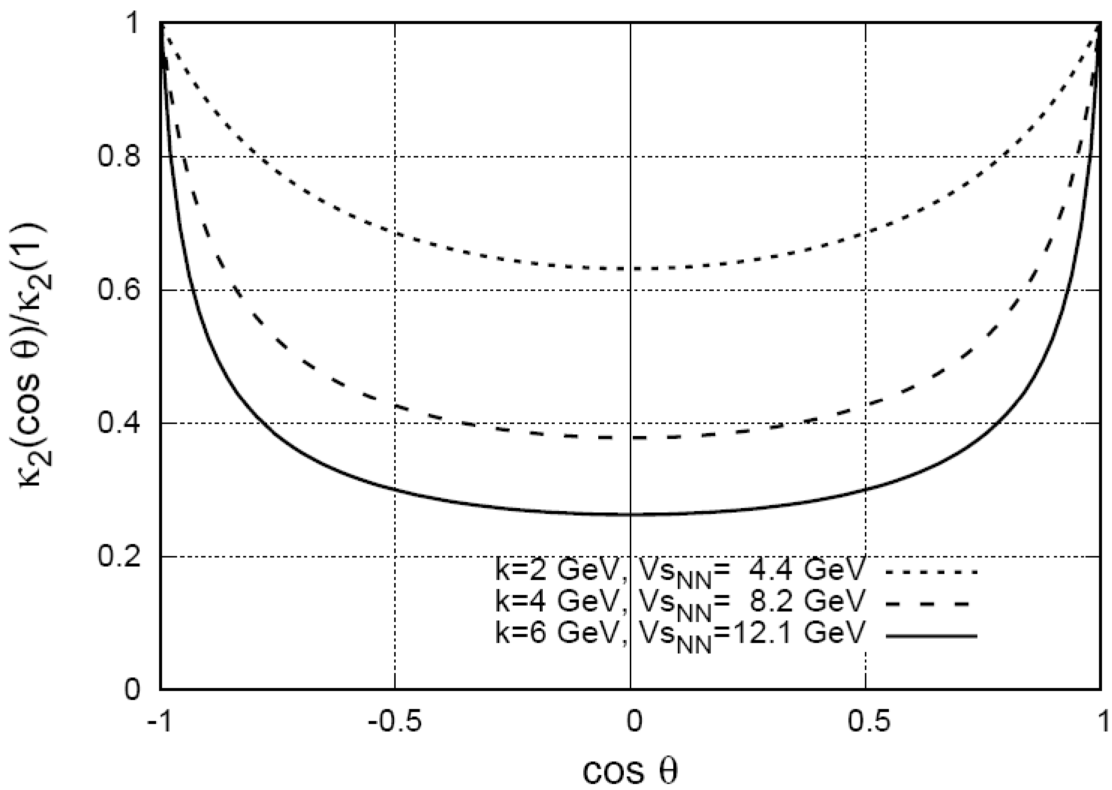


Angular dependence of the Coalescence Coefficient for d+d → d+p+n

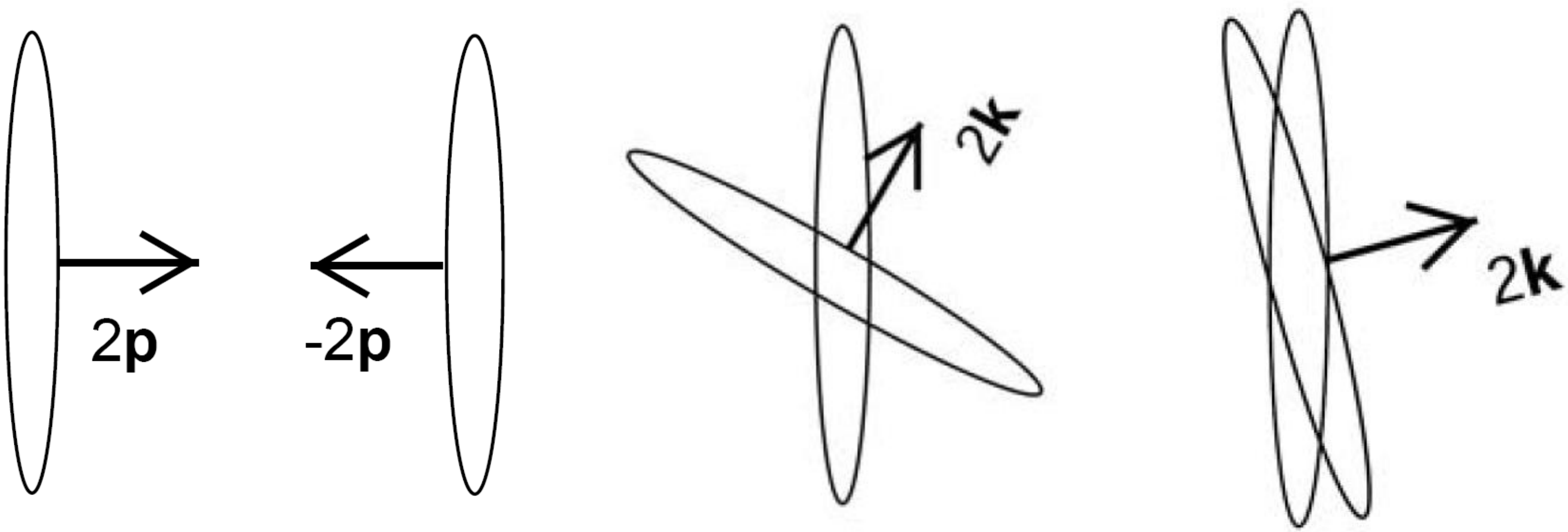
$$\frac{\kappa_2(\cos\theta)}{\kappa_2(1)} = \frac{\sqrt{3}}{\sqrt{\sin^2\theta[\gamma_k^2 - 1] + 3}} = \frac{\sqrt{3m^2}}{\sqrt{\mathbf{k}_\perp^2 + 3m^2}}$$

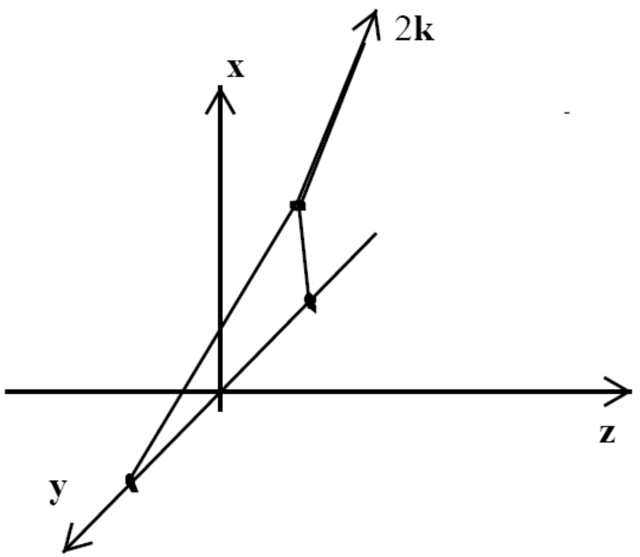
$$\sin\theta \leq \frac{m}{|\mathbf{k}|}$$

Increase in the Coalescence Coefficient in the diffraction region for d+d → d+p+n



Physical interpretation in the center of mass system





Historical note

Coherent Coalescence Mechanism at nucleon level:

Braun M.A., Vechernin V.V., Yad.Fiz. 47 (1988) 1452; J. Phys. G 16 (1990) 1615.

$p+A \to d(Tr, {}^3He)+X$ - **in central and fragmentation regions**

- using the Feynman diagram technique
- taking into account the stretching of the wave function of the resulting fragment in momentum space

$$\kappa_2(\mathbf{k}) \sim \int d^2\mathbf{b}\, dz_1\, dz_2\; \rho_{n_1}(\mathbf{b}, z_1)\, \rho_{n_2}(\mathbf{b}, z_2)\, |\psi(y^*)|^2.$$

$$k \equiv |\mathbf{k}|$$

$$y^* \equiv |\mathbf{y}^*| = \frac{k_-}{m}|z_1 - z_2|$$

$$k_- = \sqrt{k^2 + m^2} - k\cos\theta$$

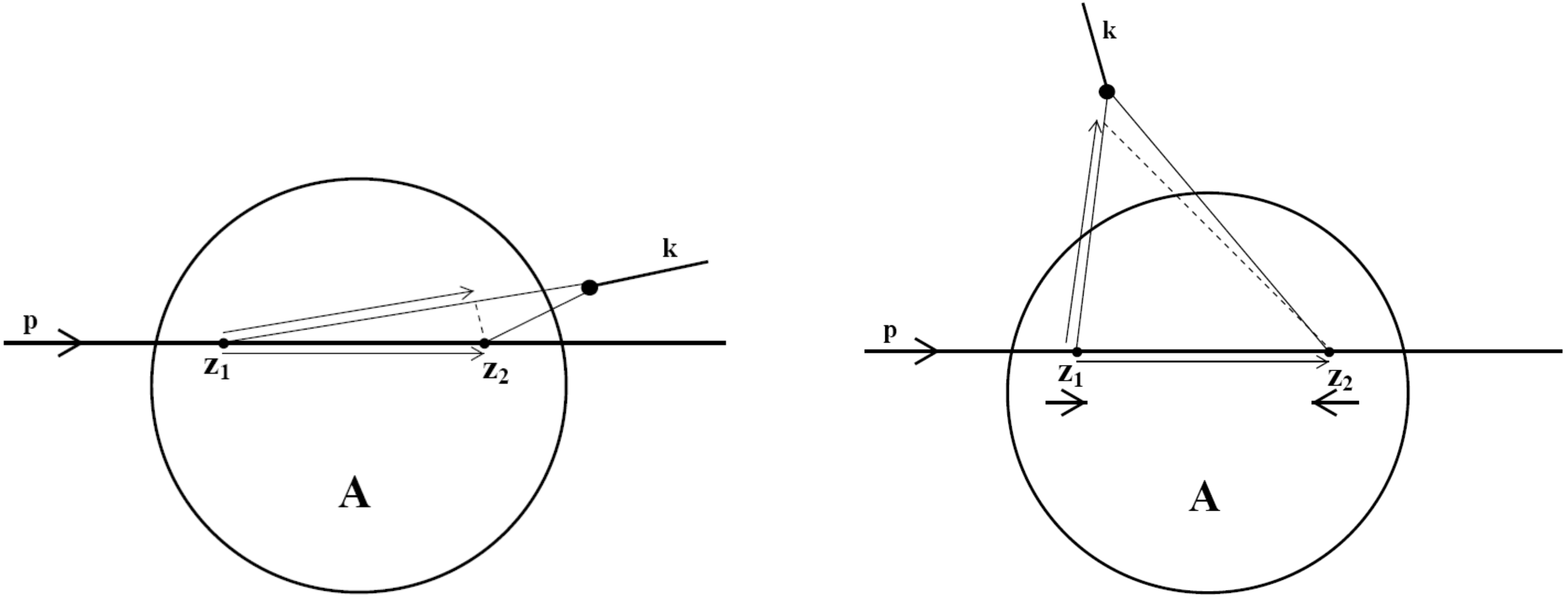
$$\kappa_2(\mathbf{k}) \sim \frac{1}{k_-}$$

$$\theta \simeq 0^\circ \quad k_- \to 0$$
$$\theta \simeq 90^\circ \quad k_- \simeq |\mathbf{k}|$$
$$\theta \simeq 180^\circ \quad k_- \simeq 2|\mathbf{k}|$$

- increase of the Coalescence Coefficient

$$z_1 \longrightarrow z_2$$

Physical interpretation in the laboratory frame



Gavrilov V.B., Kornienko N.L., Leksin G.A., Semenov S.V., Sov. J. Nucl. Phys. 41 (1985) 540;

Preprint ITEP-69 Moscow, 1985.

Comparison with low angle data

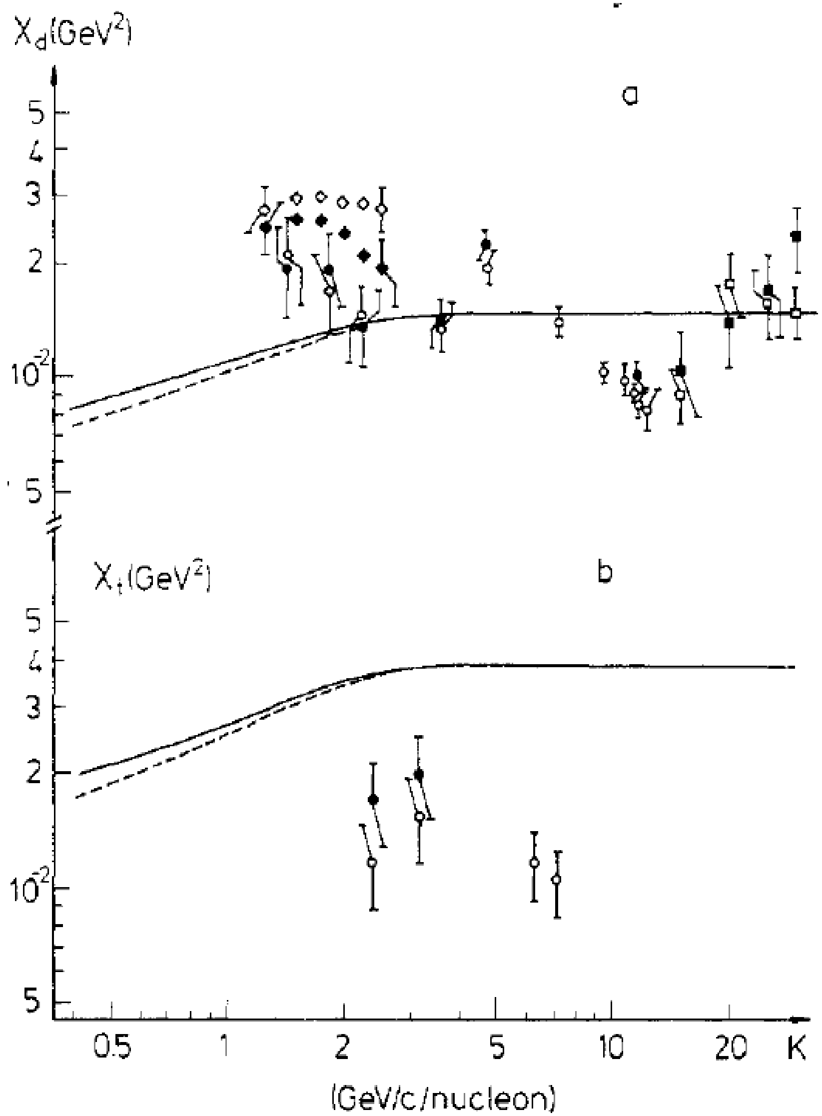


Figure 4. The coalescence coefficient χ_F (9) for the production of (*a*) deuterons and (*b*) tritium in the central kinematic region of the reaction (1), (i.e. for the small angles in the lab. system and with the high momenta *k* per nucleon). The full and broken curves are the results of our calculations with the formula (29) for the target nuclei Cu and Pb, respectively. The experimental data points correspond to the production of fragments in the following reactions: ● and ○, on Cu and Pb for the angle 9° at the initial energy 70 GeV [3]; ■ and □, on Ti and W for the angle 4.4° at the initial energy 300 GeV [7]; ◆ and ◇, on Cu and Ta for the angle 3.5° at the initial energy 9.2 GeV [4]. Here *k* is the momentum per one nucleon of fragment.

Summary

1. The importance of taking into account the coherence of the coalescence process is shown:

- the convolution of wave functions (not probabilities!)
- MC simulations cannot be applied for calculations
- usual factorization assumption is invalid
- calculation of Feynman graphs automatically leads to the correct space-time picture of the process

2. Taking into account the stretching of the wave function of the resulting relativistic fragment in momentum space is essential.

3. The probability of neutron and proton fusion into a high-energy deuteron occurs higher in the diffraction region than in the central rapidity region.

4. The physical interpretation of this phenomenon is presented.

The work was supported by the Russian Science Foundation grant 23-12-00042.

Backup slides

Connection of vertices Γ_i with wave functions. Light-cone variables.

Light-cone partonic wave function.

S.J. Brodsky, P. Hoyer, A. Mueller, W.-K. Tang, Nucl.Phys. B369 (1992) 519;
M.A. Braun, V.V. Vechernin, Nucl.Phys. B427 (1994) 614.

$$\varphi_{lc}(x_i, \mathbf{q}_\perp^{(i)}) = \frac{\Gamma_{lc}(x_i, \mathbf{q}_\perp^{(i)})}{\sqrt{Ax_0}\left[\sum_{i=1}^{n} \frac{m_q^2 + \mathbf{q}_\perp^{(i)2}}{x_i} - \frac{M_A^2}{A} - \mathrm{i}\epsilon\right]}$$

$$\int |\varphi_{lc}(x_i, \mathbf{q}_\perp^{(i)})|^2 \prod_{i=1}^{n-1} \frac{dx_i\, d^2\mathbf{q}_\perp^{(i)}}{2x_i(2\pi)^3} = 1$$

$$x_i = \frac{k_+^{(i)}}{p_+}$$

$$\sum_{i=1}^{n} \mathbf{q}_\perp^{(i)} = 0$$

$$\sum_{i=1}^{n} x_i = A$$

$$x_0 \equiv A/n$$

$$A = 1 \quad n = 2$$

$$\frac{m_q^2 + \mathbf{q}_\perp^2}{x_1} + \frac{m_q^2 + (-\mathbf{q}_\perp)^2}{x_2} = \frac{m_q^2 + \mathbf{q}_\perp^2}{x(1-x)}$$

Connection of vertices Γ_i with wave functions.
Light-cone variables.
Light-cone partonic wave function.

$$M_A^2/A = (nm_q - n\varepsilon_q)^2/A \approx \frac{n}{x_0}(m_q^2 - 2m_q\varepsilon_q)$$

$$x_i = \frac{k_+^{(i)}}{p_{N+}} \approx \frac{k_z^{(i)}}{p_{Nz}} = \frac{p_{Nz}A/n + q_z^{(i)}}{p_{Nz}} = x_0 + \frac{q_z^{(i)}}{p_{Nz}} = x_0 + \frac{q_z^{(i)}}{p_{Nz}}$$

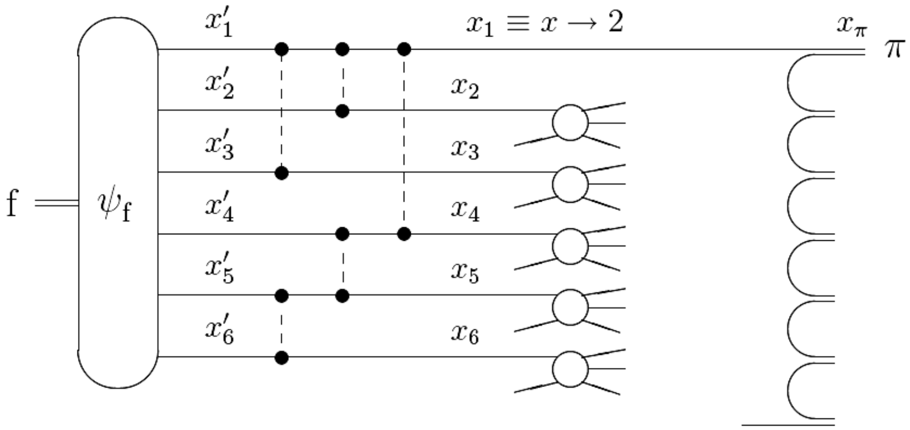
$$\gamma = \frac{E_q}{m_q} \approx \frac{p_{Nz}A/n}{m_q} = \frac{p_{Nz}x_0}{m_q}$$

$$\sum_{i=1}^{n} \frac{m_q^2 + \mathbf{q}_\perp^{(i)2}}{x_i} - \frac{M_A^2}{A} = \sum_{i=1}^{n} \frac{\mathbf{q}_\perp^{(i)2}}{x_0} + \frac{m_q^2}{x_0}\sum_{i=1}^{n}\left[1 - \frac{q_z^{(i)}}{x_0 p_{Nz}} + \frac{q_z^{(i)2}}{(x_0 p_{Nz})^2}\right] - \frac{n}{x_0}(m_q^2 - 2m_q\varepsilon_q)$$

$$\sum_{i=1}^{n} \frac{m_q^2 + \mathbf{q}_\perp^{(i)2}}{x_i} - \frac{M_A^2}{A} = \frac{1}{x_0}\sum_{i=1}^{n}\left[\mathbf{q}_\perp^{(i)2} + (q_z^{(i)}/\gamma)^2\right] - \frac{2nm_q\varepsilon_q}{x_0}$$

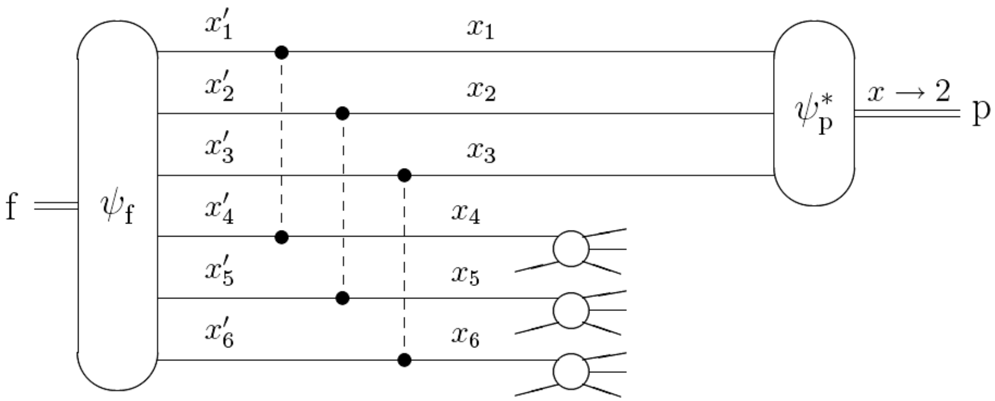
$$\varphi_{lc}(q_z^{(i)}, \mathbf{q}_\perp^{(i)}) = \frac{\Gamma_{lc}(q_z^{(i)}, \mathbf{q}_\perp^{(i)})}{\sqrt{n}\left[\sum_{i=1}^{n}[\mathbf{q}_\perp^{(i)2} + (q_z^{(i)}/\gamma)^2] - 2nm_q\varepsilon_q - i\epsilon\right]}$$

Coherent Quark Coalescence and Production of Cumulative Protons



- **the cumulative pion production by hadronization of one fast quark**

M.A. Braun, V.V. Vechernin, Nucl.Phys.B 427, 614 (1994); Phys.Atom.Nucl. 60, 432 (1997); 63, 1831 (2000); V.Vechernin,S.Yurchenko, Int.J.Mod.Phys.E 33, 2441022 (2024) S.Yurchenko, V.Vechernin, Phys.Atom.Nucl. 88, 349 (2025)



- **the cumulative proton production by coherent quark coalescence mechanism**

M.A. Braun, V.V. Vechernin, Nucl.Phys.B 92, 156 (2001); Theor.Math.Phys 139, 766 (2004); V.Vechernin, AIP Conf.Proc.1701, 06002 (2016) V.Vechernin, S.Belokurova, S.Yurchenko. Phys.Part.Nucl. 55, 889 (2024); Symmetry 16, 79 (2024)

The last **recalls** the few nucleon **short-range correlations** in a nucleus
L.L. Frankfurt, M.I. Strikmann, Phys. Rep. 76, 215 (1981); ibid 160, 235 (1988).
But instead of using the relativistic generalization of non-relativistic NN wave function
the microscopic analysis of the flucton fragmentation process near cumulative thresholds on the base of the intrinsic diagrams of QCD in light-cone gauge
Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl. Phys. ***B369*** *(1992) 519.*
was developed and applied.

Comparison with target nucleus fragmentation data

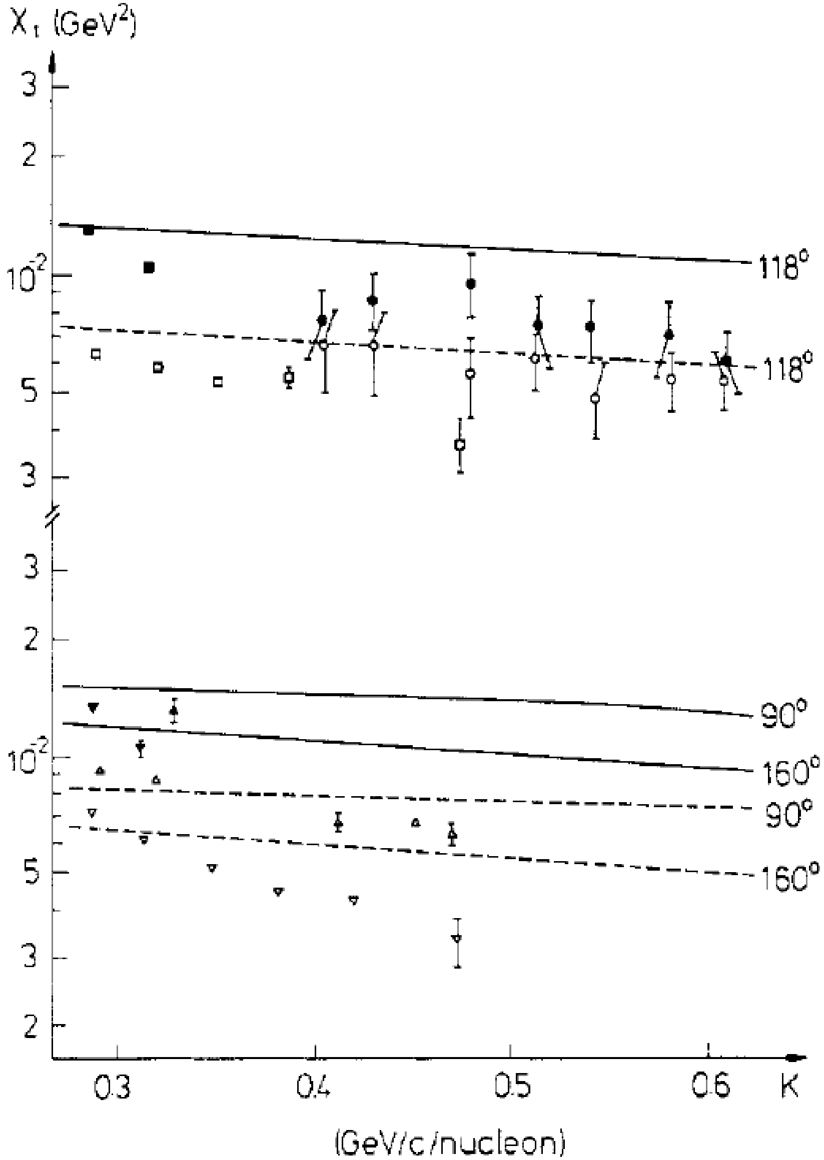


Figure 5. The coalescence coefficient for the emission of tritium in the backward hemisphere in the lab. system, (i.e. in the cumulative area of the target nucleus fragmentation region of the process (1)). The full and broken curves are the results of our calculations with a formula, which is similar to (29) but with $n = 2$, for the target nuclei Cu and Ta, respectively. The experimental data points correspond to the production of tritium in the following reactions: ▲, ■ and ▼, on Cu for the angles 90°, 118° and 160°, △, □ and ▽, on Ta for the same angles at the initial energy 400 GeV [10]; ● and ○, on Cu and Ta for the angle 119° at the initial energy 10 GeV [2].

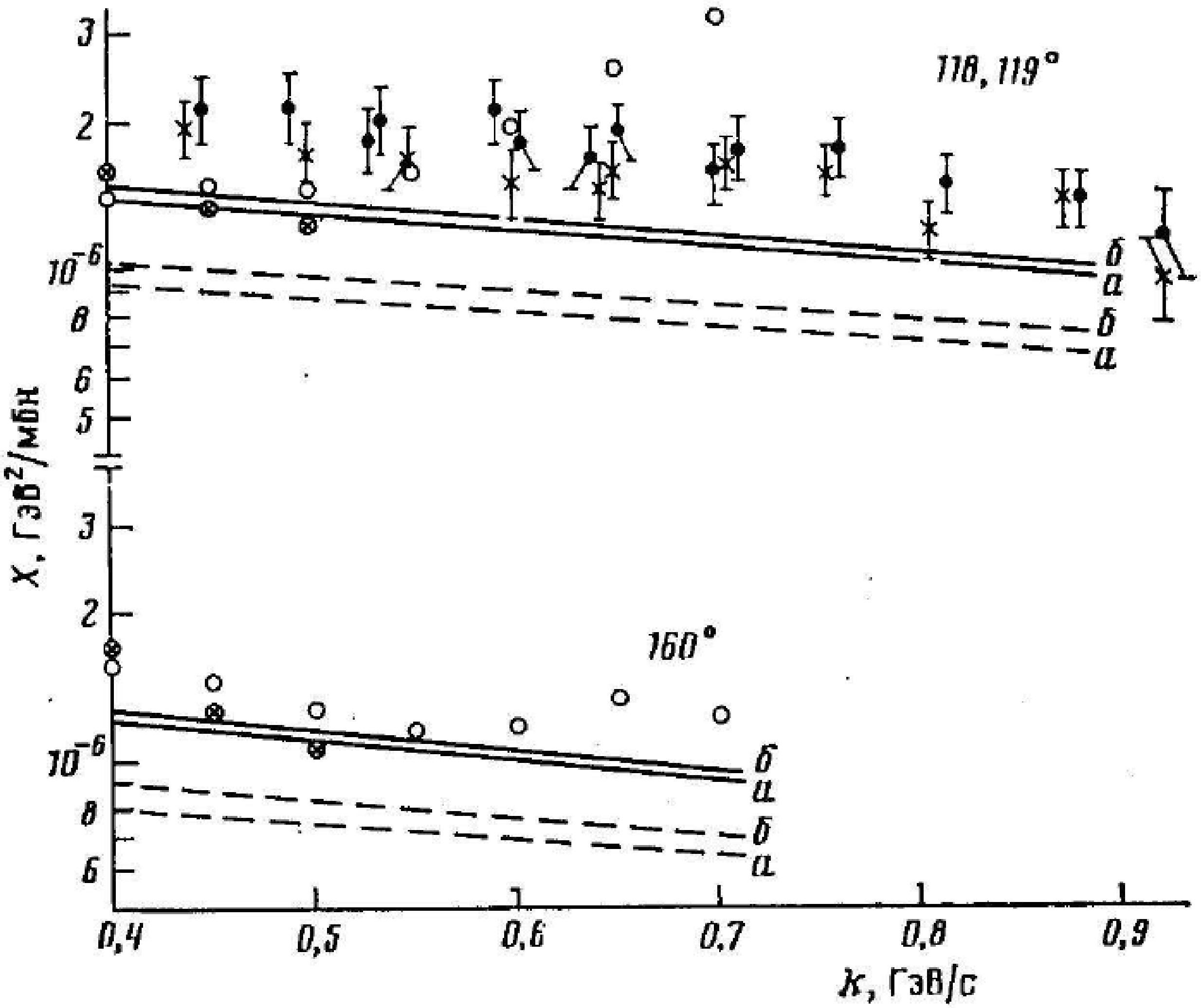


Рисунок 2.15: Сравнение в кумулятивной области теоретических расчетов величины $X = (I_A^{(d)}/A)/(I_A/A)^2$ по формуле (2.66) с экспериментальными данными [23], полученными при облучении ядер-мишеней (⊗ - Cu и ○ - Ta) протонами с начальным импульсом p=400 ГэВ/c и с экспериментальными данными [132], полученными при облучении тех же мишеней (× - Cu и ● - Ta) протонами с энергией 10 ГэВ

$$E_{init} = E_{fin} \quad \Rightarrow$$

$$6E_p = 3E_k + \sum_{i=1}^{3} E(\mathbf{k} - \mathbf{q}_i)$$

$$2n\, E_p = 2n\, E_k - \frac{1}{2E_k} \sum_{i=1}^{n} \left[\mathbf{q}_i^2 - \frac{(\mathbf{q}_i \mathbf{k})^2}{E_k^2} \right] \;, \quad n = 2, 3.$$

$$E_p = E_k - \frac{1}{4n\, E_k} \sum_{i=1}^{n} \left[\mathbf{q}_i^2 - \frac{(\mathbf{q}_i \mathbf{k})^2}{E_k^2} \right] \;, \quad n = 2, 3.$$

Calculation of Phase Volume

$$\tau_p = (2\pi)^{4-3p} \int \prod_{i=1}^{p} \frac{d^3 \mathbf{l}'_i}{2 l_{i0}} \delta^{(3)}\left(\sum_{i=1}^{p} \mathbf{l}'_i\right) \times$$

$$\times \delta\left(\sum_{i=1}^{p}\left[\sqrt{(\mathbf{k}/p + \mathbf{l}'_i)^2 + m^2} - \sqrt{(\mathbf{k}/p)^2 + m^2}\right] - \Delta\right)$$

$$\mathbf{l}'_i = -\mathbf{k}/p + \mathbf{l}_i$$

$$l_{i0} = \sqrt{(\mathbf{k}/p + \mathbf{l}'_i)^2 + m^2}$$

$$p = n_1 + n_2 - 1$$

$$\Delta = A\sqrt{s} - \sqrt{k^2 + m^2} - \sqrt{k^2 + p^2 m^2}$$

$k \rightarrow k_{max}$ => $\Delta \rightarrow 0$

$$\sqrt{k_{max}^2 + m^2} + \sqrt{k_{max}^2 + (p\,m)^2} = A\sqrt{s}$$

$$\tau_p = \frac{1}{2^p m^{p-1} p^{\frac{3}{2}}} \frac{\left(\frac{E_p}{2\pi}\Delta\right)^{\frac{3}{2}p - \frac{5}{2}}}{\left(\frac{3}{2}p - \frac{5}{2}\right)!}$$

$$E_p \equiv \sqrt{k^2/p^2 + m^2}$$

Relation with Cumulative Number

$$x\sqrt{s} = \sqrt{k^2 + m^2} + \sqrt{k^2 + [p(x)\, m]^2}\ .$$

$$p(x) = n_1 + n_2 - 1 = 3A_1 + 3A_2 - 1 = 6A - 1 = 6x - 1.$$

$$\Delta = (A - x)[\sqrt{s} + O(1/\sqrt{s})]$$

$$\tau_p = \frac{1}{2^{4p-5} p^{3p/2-1} m^{p-1}} \frac{\left[\frac{A}{\pi} s(A-x)\right]^{\frac{3}{2}p - \frac{5}{2}}}{\left(\frac{3}{2}p - \frac{5}{2}\right)!}$$

$$p = p(A)$$

$$I(x) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p - \frac{5}{2}}}{(m^2 R^3)^{p-1} s^{(p+3)/2}}$$

two (!)
small parameters:
$m/\sqrt{s} \ll 1$
$A - x \ll 1$

Can string junction carries the baryon number?

L. Montanet, G. C. Rossi, and G. Veneziano, “Baryonium Physics,”
Phys. Rept. 63, 149–222 (1980).

D. Kharzeev, “Can gluons trace baryon number?”
Phys.Lett. B 378, 238–246 (1996), arXiv:nucl-th/9602027.
Can be verified experimentally by studing of
baryon stopping in central pp and AA collisions.

Yu.M. Shabelski,
String Junction and Diffusion of Baryon Charge in Multiparticle Production Processes,
arXiv: 0705.0947 [hep-ph], (2007).
F. Bopp, Yu.M. Shabelski,
String junction effects for forward and central baryon production in hadron-nucleus collisions
Eur.Phys.J.A 28 (2006) 237-243

G.Pihan, A.Monnai, B.Schenke, Chun Shen,
Unveiling baryon charge carriers through charge stopping in isobar collisions
arXiv:2405.19439v1 [nucl-th] (2024).

Connection with diquarks:
Now B=1 corresponds to diquark