Multiplicity distribution and the reflective scattering

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Overview

Abstract

Transition to the reflective scattering mode which has emerged at the highest LHC energy of $\sqrt{s} = 13$ TeV results in a relative shrinkage with the energy of the impact parameter region responsible for the inelastic hadron collisions. Respective increasing role of the multiplicity fluctuations of quantum origin is emphasized.

Historical remarks

- The term anishadowing has been introduced in under consideration of the “black disc limit” exceeding, 1993. This term means that reduction of the inelastic interactions contribution with the energy results in the elastic amplitude increase.
  S.M. Troshin, N.E. Tyurin, Phys. Lett. B **316**, *175 (1993)*
- The respective scattering mode can take place in the limited range (dependent on the collision energy) of the impact parameter variation when $|f| > 1/2$ only, i.e. in the region where the amplitude is beyond the so called black disc limit. This mode differs from the shadow case when both the elastic and inelastic components of the unitarity equation grow up with the energy increase, and $|f| < 1/2$. (f is a partial or an impact parametr elastic scattering amplitude).
- Interpretation of antishadowing as a reflective scattering was proposed. Analogy with optics, 2007.
  Troshin S.M., Tyurin N.E., Int. J. Mod. Phys. A **22** *(2007) 4437.*
  Positive reflective ability observation, the LHC results. This is not a unique interpretation of antishadowing.

More remarks

Transition from the shadow to reflective scattering is developing with increase of the elastic scattering amplitude in the impact parameter representation. This evolution is connected with formation of a peripheral impact parameter profile of the inelastic overlap function (black ring formation). The process of black ring formation starts at small impact parameter values. It is expected that this will manifest itself in the elastic scattering differential cross–section behavior at large transferred momenta. Here we consider further consequences of the reflective scattering mode presence for the multiplicity distribution emphasizing the role of the impact parameter–dependent mean multiplicity and inelastic overlap function. The consideration has a qualitative nature, it concerns mainly the asymptotic energy region which is at least beyond the presently available energies. But the presented conclusions are in correspondence with the observed tendencies.

Peripheral form of the inelastic overlap function

Unitarity equation for the elastic scattering amplitude $F(s,t)$ has the form

$$\mathrm{Im}F(s,t) = H_{el}(s,t) + H_{inel}(s,t), \tag{1}$$

where $H_{el}(s,t)$ is the two–particle intermidiate state contribution and $H_{inel}(s,t)$ is the sum of the contributions from the multi–particle intermidiate states. For the forward scattering when $-t = 0$ Eq. (1) turns into

$$\sigma_{tot}(s) = \sigma_{el}(s) + \sigma_{inel}(s), \tag{2}$$

where $\sigma_i(s)$ are the respective cross–sections. High–energy elastic scattering amplitude is a predominantly imaginary and is given by the sum, Eq. (1). In the impact parameter representation (i.e. in the framework of quasiclassical geometrical picture, Fig. 1) the elastic and inelastic overlap functions $h_{el}(s,b)$ and $h_{inel}(s,b)$ have different profiles at high energies.

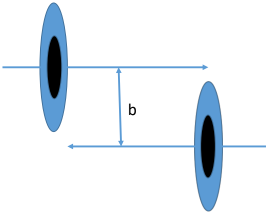

Figure: Schematic form of hadron scattering geometry.

The unitarity equation, Eq. (1), for the elastic scattering amplitude in the impact parameter representation, $f(s,b)$, has a diagonal form, i.e.:

$$\mathrm{Im}f(s,b)[1-\mathrm{Im}f(s,b)] = [\mathrm{Re}f(s,b)]^2 + h_{inel}(s,b). \tag{3}$$

It is evident that $\mathrm{Re}f \to 0$ when $\mathrm{Im}f \to 1$ and under assumption of the vanishing real part the following relation takes place ($f \to if$) for the inelastic overlap function $h_{inel}(s,b)$:

$$h_{inel}(s,b) = f(s,b)[1-f(s,b)]. \tag{4}$$

The impact parameter representation provides a geometric, semiclassical picture for hadron interactions. The elastic overlap function preserves central profile when the energy increases. Contrary, the inelastic overlap function becomes peripheral when $f > 1/2$. Indeed, for s and b values where $f(s,b) > 1/2$, the inelastic overlap function, Eq. (4), decreases with the energy growth and acqures a peripheral profile (Fig. 2).

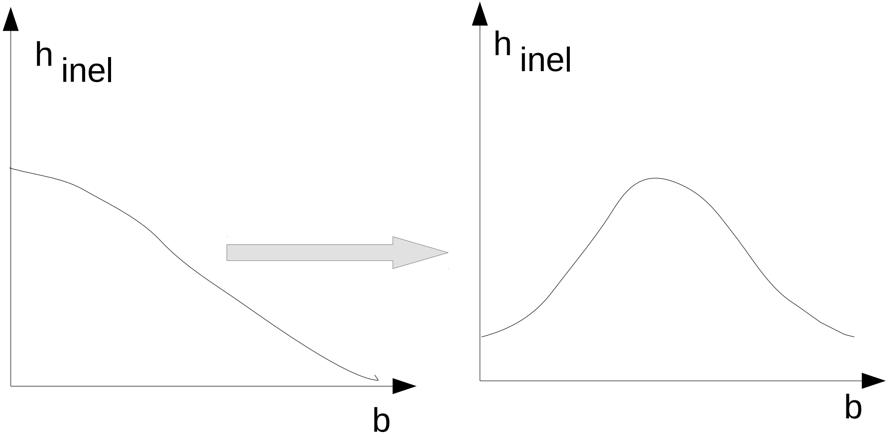

Figure: Schematic forms of inelastic overal function dependening on impact parameter in the shadow (left) and reflective (right) scattering modes.

Form of $h_{inel}(s,b)$ becomes relatively more narrow when s increases and this concentrates our attention on the impact parameter values close to position of the inelatic overlap function peak which we denote by $R(s)$. One should note that $R(s) \sim \ln s$ at $s \to \infty$. Thus, the probability of an inelastic processes under hadron collision at the impact parameter b is

$$\sigma_{inel}(s,b) \equiv 4h_{inel}(s,b). \tag{5}$$

with maximum at $b = R(s)$.
In what follows we use the function

$$P_n(s,b) \equiv \sigma_n(s,b)/\sigma_{inel}(s,b) \tag{6}$$

for the multiplicity distribution at the energy s and impact parameter b. In Eq. (6), $\sigma_n(s,b)$ is the production cross–section of n particles ($n \geq 3$) .
Influence of a peripheral form of the inelastic overlap function $h_{inel}(s,b)$ on the multiplicity distribution $P_n(s,b)$.

$P_n(s,b)$ is to be used for the calculations of the final states entropy and other thermodynamic quantities in hadron interactions. Their impact parameter dependence – an importance of the spatial proton's structure and is a replacement of Q^2–dependence of the entropy under the deep–inelastic scattering:
Z. Tu, D.E. Kharzeev and T. Ullrich, Phys. Rev .Lett. **124**, *062001 (2020).*

$$P_n(s) = \int_0^\infty P_n(s,b)\sigma_{inel}(s,b)bdb / \int_0^\infty \sigma_{inel}(s,b)bdb \tag{7}$$

and

$$\langle n\rangle(s) \equiv \sum_n nP_n(s) = \int_0^\infty \langle n\rangle(s,b)\sigma_{inel}(s,b)bdb / \int_0^\infty \sigma_{inel}(s,b)bdb, \tag{8}$$

and $\langle n\rangle(s,b) \equiv \sum_n nP_n(s,b)$. The averaging corresponds to smoothing the quantum fluctuations of multiplicity. Eq. (8): further averaging, now over classical fluctuations of b. Pb+Pb collisions at the LHC:
E. Roubertie, M. Verdan, A. Kirchner and J.-Y. Ollitrault, arXiv: 2503.17035v1.

Quantum fluctuations are smoothed out in the function $\langle n\rangle(s,b)$ and it makes this quantity relevant for a quasiclassical modelling in the framework of impact parameter picture. In this regard it should be noted that commutator of the impact parameter operator with the Hamiltonian is vanishing at very high energies and the impact parameter itself becomes a quasiclassical quantity.
B.R. Webber, Nucl. Phys. B **87**, *269 (1975).*

$$P_n(s) \simeq P_n(s,b)|_{b=R(s)}. \tag{9}$$

$$P_n(s,b)|_{b=R(s)} = \sigma_n(s,b)|_{b=R(s)} \tag{10}$$

and the mean multiplicity $\langle n\rangle(s)$:

$$\langle n\rangle(s) \simeq \langle n\rangle(s,b)|_{b=R(s)} \tag{11}$$

$P_n(s,b)$ receives contributions from the two sources of different origins. These are the characteristic b-dependence of $P_n(s,b)$ associated with the varying b–values and the quantum fluctuations over n at fixed values of b. Quantum fluctuations of multiplicity become more significant gaining an extra weight with the energy increase.

The amplitude value $f(s,0)$ moves from the region of $(0,1/2]$ into the region of $[1/2,1)$, increases and tends to unity at $s\to\infty$. The inelastic overlap function $h_{inel}(s,0)$ monotonically decreases and tends to 0 at $s\to\infty$. $\sigma_n(s,b)$ at $b=0$ should also decrease with energy for any $n\geq 3$.

$$\lim_{s\to\infty}\sigma_n(s,b)=0. \tag{12}$$

$$\langle\Delta b^2\rangle_{inel}/\langle b^2\rangle_{inel}\sim\sigma_{inel}(s)/\sigma_{tot}(s)\sim R^{-1}(s) \tag{13}$$

and

$$\sigma_{tot}(s)\sim\sigma_{el}(s)\sim R^2(s)\,\text{and}\,\langle b^2\rangle_{el}\sim\langle b^2\rangle_{inel}\sim R^2(s) \tag{14}$$

while

$$\sigma_{inel}(s)\simeq 8\pi R(s)\int_0^\infty dbh_{inel}(s,b) \tag{15}$$

Modelling the mean multiplicity $\langle n \rangle(s,b)$

Averaging contained in $\langle n \rangle(s,b)$ smooths the quantum fluctuations of multiplicity. Assumption:

$$\langle n \rangle(s,b) = \nu(s)\sigma_{inel}(s,b). \tag{16}$$

Integrated mean multiplicity $\langle n \rangle(s)$:

$$\langle n \rangle(s) = \nu(s)\int_0^\infty \sigma_{inel}^2(s,b)bdb/\int_0^\infty \sigma_{inel}(s,b)bdb. \tag{17}$$

The ratio of the integrals in Eq. (17) is limited by unity. and asymptotically $\nu(s) \to \langle n \rangle(s)$.

$\langle n \rangle(s,b)$ studies can be extracted from the inclusive overlap functions introduced in:

N. Sakai, Nuov. Cim. A **21**, *368 (1974)*

$$\langle n \rangle(s,b) = 4\nu(s)f(s,b)[1-f(s,b)]. \tag{18}$$

Invariance of $\langle n \rangle(s,b)$ under replacement $f \to 1-f$, the same average multiplicity value corresponding to both values: f and $1-f$.

Conclusion

Two scattering modes at high energies: the shadow scattering mode (SSM) and the antishadow one. An existence of the RSM is allowed if we do not introduce constraint $|f| \leq 1/2$. RSM existence becomes ultimate in view of the maximal strength principle by Chew and Frautchi. It is also implied by invariance of the inelastic overlap function under replacement $f \to 1 - f$.

Gradual transition to the RSM corresponds to the relative shrinkage of the impact parameter variation region effectively populated by the inelastic processes. No such effect in the case of a shadow scattering mode with flat dependence on impact parameter.

Two different sources of multiplicity fluctuations in hadron production at modern energies: one is due to variation of the collision impact parameter value and another one associated with quantum fluctuations of multiplicity at fixed impact parameters. Transition to the reflective scattering mode with the energy increase makes the quantum fluctuations a dominant mechanism associated with the multiplicity fluctuations.

Analogy with quantum optics can be useful for the modelling the particle distributions at fixed impact parameters. Use of the gamma–distribution

$$P_n(s,b) = \frac{k^k}{\langle n\rangle \Gamma(k)} z^{k-1} e^{-kz} \tag{19}$$

as a kernel is good for nuclei–nuclei and hadron–nuclei reactions. k is a parameter and $z = n/\langle n\rangle(s,b)$. It is relevant for the various systems. Extension for description of the small systems such as hadrons and their interactions is supported by the experimentally observed similarity of the observables in nuclear and hadron reactions, discovered the ridge and other collective effects under interactions of small systems. Application to hadron collisions is complimentary gaining advantage from validity of the unitarity condition in this case. Gamma–distribution has also been applied for modelling the eikonal treated as a stochastic quantity. Exponential form is tranformed into a rational representation of the scattering amplitude where an averaged eikonal function serves as an input.

But, it is difficult to expect relevance of gamma–distribution for the asymptotic energies with a dominance of the quantum fluctuations of multiplicity. How to separate quantum fluctuations of multiplicity at finite energies? Studies of the difference $\bar{z} \equiv n_F - n_B$ versus the sum $n \equiv n_F + n_B$.

T.T. Chou and C.N. Yang, Int. J. Mod. Phys. A, Vol. 2, No 6 (1987)1727-1753.

σ_n for fixed n is determined by the inelastic activity, i.e. the impact parameter responsible for inelastic collisions, studies of fluctuations of $\bar{z}$ at fixed values of n should be helpful.

Cumulative activity of inelastic events under hadron collisions

S.M. Troshin, N.E. Tyurin

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Analogy of particle production with quantum optics$\rightarrow$ distribution of photocounts is similar to multiplicity distribution.

Reflective scattering $\rightarrow$ ring-like form of incoming exitation light beam instead of its spot-like form

The End