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Exploring the Enigmatic Chiral Phase Transition of QCD at Finite Temperature

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Quantum Chromodynamics (QCD) is the fundamental theory describing the strong interactions between quarks and gluons, providing the essential framework for understanding high-energy physics, cosmology, and nuclear physics. A key aspect of QCD is the chiral phase transition, which occurs when quark interactions undergo significant changes at finite temperatures. At zero temperature, QCD respects chiral symmetry. However, as temperature increases, this symmetry is disrupted, leading to spontaneous chiral symmetry breaking. This breaking is characterized by quark condensates in the vacuum, which diminish and vanish above a critical temperature (T_c), where chiral symmetry is restored.

Experimental investigations of the chiral phase transition are challenging due to the extreme conditions required. Heavy-ion collision experiments at the Large Hadron Collider (LHC) and the Relativistic Heavy Ion Collider (RHIC) are crucial for probing this transition. These experiments analyze particle spectra and collective flow to gain insights into the behavior of matter under such conditions. Understanding the chiral phase transition not only deepens our knowledge of fundamental matter but also illuminates the early evolution of the universe and the formation of quark-gluon plasma shortly after the Big Bang.

1. **QCD Effective Theories Near Critical Point**
   - **QCD Symmetries:** For massless quarks with N_f flavors:

$$G = SU_L(N_f) \times SU_R(N_f) \times U_A(1) \times U_B(1) \times SU_c(3)$$

   - **Order Parameter for Chiral Symmetry:**

$$\Phi_{ij} \sim \frac{1}{2}\bar{q}^j(1-\gamma_5)q^i = \bar{q}_R^j q_L^i$$

2. **Symmetry Transformations**
   - **Transformation of Φ_{ij}:**

$$\Phi_{ij} \to e^{i\alpha V_L}\Phi e^{i\alpha V_R^\dagger}$$

   - **Quark Transformations:**

$$q_L \to e^{-i\alpha/2}V_L q_L, \quad q_R \to e^{i\alpha/2}V_R q_R$$

3. **Chiral Symmetry Breaking**
   - **If Broken:** The thermal average of Φ is nonzero.
   - **Decomposition of Φ:**

$$\Phi = \sum_{a=0}^{N_f^2-1} \Phi^a \frac{\lambda^a}{\sqrt{2}}$$

$$\Phi^a = S^a + iP^a$$

1. **Construction of Landau Functional**
   - **Effective Action:**

$$S_{\text{eff}} = \int d^d x\, \mathcal{L}_{\text{eff}}$$

   - **Effective Lagrangian:**

$$\begin{aligned}\mathcal{L}_{\text{eff}} = &\frac{1}{2}\text{tr}(\Phi^\dagger \Phi) + \frac{a}{2}\text{tr}(\Phi^\dagger \Phi) \\ &+ \frac{b_1}{4!}(\text{tr}(\Phi^\dagger \Phi))^2 + \frac{b_2}{4!}(\text{tr}(\Phi^\dagger \Phi))^2 \\ &- \frac{c}{2}(\det \Phi + \det \Phi^\dagger) - \frac{1}{2}\text{tr}(h(\Phi + \Phi^\dagger))\end{aligned}$$

2. **Symmetries and Terms**
   - **Symmetry:**
     - First four terms: $SU_L(N_f) \times SU_R(N_f) \times U_A(1)$ symmetry
     - Fifth term: Breaks $U_A(1)$ symmetry (axial anomaly)
     - Final term: Breaks $SU_L(N_f) \times SU_R(N_f)$ and $U_A(1)$ symmetries (quark masses)

3. **Phase Transition**

- **Soft Modes:**
  - Near the critical point, soft modes with divergent correlation lengths appear.
  - Finite correlation length modes are integrated out, affecting coefficients a, b_1, b_2, c, and h.
  - Complex issue with soft modes like vector mesons.

Anomalous Massless QCD with Axial Anomaly

Massless QCD without Axial Anomaly

- **Effective Lagrangian:**

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^2 - h\sigma^2$$

- **Phase Transition:**
  - **For** $N_f = 1$:
    - Fixed point $g^* = 3\epsilon/5$ is infrared stable (second-order transition).
  - **For** $N_f \geq 2$:
    - Two solutions: $g^* = (0,0)$ and $g^* = (3\epsilon/(N_f^2+4), 0)$.

Renormalization Group Flows:

$$\beta_1 = -\epsilon g_1 + \frac{(N_f^2+4)}{3}g_1^2 + \frac{4N_f}{3}g_1 g_2 + g_2^2$$
$$\beta_2 = -\epsilon g_2 + 2g_1 g_2 + \frac{2N_f}{3}g_2^2$$

Massless QCD with Axial Anomaly ($c \neq 0,\ h = 0$)

- **Critical Orders:**

Таблица: Critical Orders for Massless Quarks at $h = 0$

	Without Axial Anomaly ($c = 0,\ h = 0$)	**With Axial Anomaly ($c \neq 0,\ h = 0$)**
$N_f = 1$	Second-order [O(2)]	No phase transition
$N_f = 2$	First-order	Second-order [O(2)]
$N_f = 3$	First-order	First-order
$N_f \geq 4$	First-order	First-order

Special Cases:

- **For** $N_f = 1$:

$$-c/2\,\mathrm{det}(\Phi + \Phi^{\dagger}) = -c\sigma$$

- **For** $N_f = 2$:

$$\Phi = \sqrt{2}(\sigma + i\eta + \delta \cdot \tau + i\pi \cdot \tau)$$

$$L_{\mathrm{eff}} = \frac{1}{2}(\nabla\phi)^2 + \frac{(a-c)}{2}\phi^2 + \frac{(b_1 + b_2/2)}{4!}(\phi^2)^2$$

- **For** $N_f = 3$:

$$-c/2\,\mathrm{det}(\Phi + \Phi^{\dagger}) = -c/(3\sqrt{3}) + \sigma^3$$

Anomalous Massless QCD with Axial Anomaly (continued)

Renormalization Group Flow:

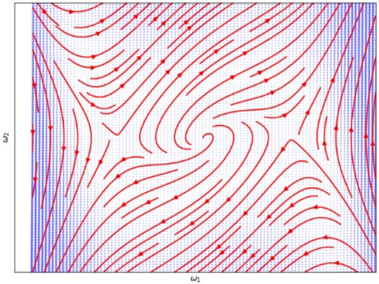

Рис.: RG Flow of Symmetry Model for $SU_L(N_f) \times SU_R(N_f) \times U_A(1)$

Effects of Light Quark Mass

Introduction

- Zero quark mass assumption previously discussed.
- Nonzero u, d, and s quark masses: $c \neq 0$ and $h \neq 0$.

Phase Transition in (m_{ud}, m_s) Plane

$$(m_{ud}, m_s) = \begin{cases} (\infty, \infty), & N_f = 0 \\ (\infty, 0), & N_f = 1 \\ (0, \infty), & N_f = 2 \\ (0, 0), & N_f = 3 \end{cases}$$

Phase Transition Regions

- First-order regions separated by continuous transition regions.
- $Z(2)$ symmetry of the Ising model for boundaries.

Effective Landau Functional

$$L_{\text{eff}} = \frac{1}{2}(\partial\vec{\phi})^2 + \frac{a(m_s,T)}{2}\vec{\phi}^2 + \frac{b(m_s,T)}{4!}(\vec{\phi}^2)^2 + \frac{c}{6!}(\vec{\phi}^2)^3 - h\phi_0$$

where $h \propto m_{ud}$.

Effects of Light Quark Mass (conclusion)

Phase Transitions

- Leading-order behavior: $m_{ud} \sim (m_s^{\text{tri}} - m_s)^{5/2}$.
- Position of physical quark mass in (m_{ud}, m_s) plane uncertain.

Таблица: Secondary QCD phase transitions for different flavor numbers when $N_f = 3$.

N_f	0	2	$2+1$	3
m_{ud}	∞	0	~ 5 MeV	0
m_s	∞	∞	~ 100 MeV	0
Order	First	Second	First or Transition	First
Symmetry	$Z(3)$	$O(4)$	$SU_L(3) \times SU_R(3)$	$SU_L(3) \times SU_R(3)$
T_C (Lattice)	~ 270 MeV	~ 170 MeV	–	~ 150 MeV

Conclusion

Finite Chemical Potential Effects

- Non-zero baryon chemical potential.
- QCD phase diagram in (T, μ, m_{ud}) space.
- Effective Landau functional with $O(4)$ symmetry.

Technological Advancements

- Advancements in understanding the chiral phase transition critical point.
- Potential future directions and emerging technologies.

Thank you for the attention!

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