On the fine structure of the massless PT QCD series representations

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Strong Interactions: Experiment, Theory, Phenomenology - 2024

Based on Kataev, Molokoedov; Phys.Part.Nucl. (23); arXv:2211.10242 ;
Phys.Rev. (23); arXv: 2309.03994;
Baikov, Mikhailov ; JHEP (22-23) ; arXv:2206.14063
Kataev, Talks at RAS Session, Dubna (03.04.24) and HSFI-20214, Gatchina (09.07.24)
Mikhailov; arXiV: 2406.1501 (24)
Di Guistino, Brodsky, Ratcliffe, Wu, Wang;
BLM (83) related reviewn Prog.Part.Nucl.Phys (24);
arXiV:2307.03951

IHEP, Protvino
24 July 2024

Plan

- Large N_f expansion and $O(1/N_f^k)$ effects ; generalization of BLM to high order PT QCD for RG-invariant quantities;
- $\{\beta\}$ decomposed representations and PMC /BLM properly β-expanded QCD and *extended* QCD related diagrammatically supported realizations
- relation of large N_f and β-expansions and ambiguities (model dependence)
- Comments on PMC/BLM reconsiderations (2023) of the e^+e^- to hadrons Adler D-function and Bjorken polarized sum rule : in favour of theory but not phenomenology applications
- Comments on analogy with Adler (1972) clarification on status of Finite *quenched* QED Program by Johnson, Baker , Willey et al (63 up to 70s)
- PMC/BLM related consequencies for possible futher studies

Basis for e^+e^- to hadrons Adler function

$$D(a_s(Q^2)) = -\frac{d\Pi(a_s)}{d\ln Q^2} = Q^2\int_0^\infty ds \frac{R^{th}_{e^+e^-}(a_s(s))}{(s+Q^2)^2} \to Q^2\int_0^\infty ds \frac{R^{exp+th}_{e^+e^-}(s)}{(s+Q^2)^2},$$

$$R^{th}_{e^+e^-}(a_s) = \sigma_{tot}^{e^+e^-\to hadrons}(a_s)/\sigma_0(e^+e^-\to\mu^+\mu^-)$$

$$\left(\frac{\partial}{\partial\ln\mu^2} + \beta(a_s)\frac{\partial}{\partial a_s}\right)D(a_s) = 0,$$

$$\frac{\partial a_s}{\partial\ln\mu^2} = \beta(a_s) = -\sum_{n\geq 0}\beta_n a_s^{n+2}.$$

$$D\left(a_s(Q^2)\right) = \left(\sum_i q_i^2\right) D^{ns}\left(a_s(Q^2)\right) + \left(\sum_i q_i\right)^2 D^{si}\left(a_s(Q^2)\right)$$

The a_s^4 Baikov,Chetyrkinand Kuhn (2010+...) BChK group ;

The $\overline{MS}$-scheme large N_f BLM generalization

In the $\overline{MS}$-scheme the expansions read:

$$D^{ns}(a_s) = 1 + d_{10}a_s + (d_{20} + d_{21}N_f)a_s^2 + (d_{30} + d_{31}N_f + d_{32}N_f^2)a_s^3$$
$$+(d_{40} + d_{41}N_f + d_{42}N_f^2 + d_{43}N_f^3)a_s^4$$

Grunberg,Kataev (92); Beneke, Braun (95) ; Brodsky,Wu (2012)
d_{n0}- scale-invariant contributions ; After absorbing all N_f dependence into the BLM related scales (Grunberg-Kataev generalization of BLM)
$d_{10} = +1$; $d_{20} = \frac{1}{12} \approx 0.085$**;** (BLM) $d_{30} \approx= -23.227$**;** (GK-92)
$d_{40} = +82.344$ (Brodsky-Wu (2012) (Sign ! ; Not small !)
As shown by Goriachuk, K., Molokoedov (22) agree with β-expanded model (see next page) and Brodsky, Wu et al (12)
R_δ procedure $a_s(\mu^2) = a_s(\mu_\delta) + \sum_{n\geq 1} \frac{1}{n!}\frac{d^n a_s(\mu_\delta^2)}{dln(\mu_\delta^2)}(-\delta)^n$ with
$\delta = ln(\mu_\delta^2/\mu^2)$

The $\{\beta\}$-expansion PT approach for the RG-invariant quantities

Consider the PT expansion

$$D^{ns}(a_s) = 1 + d_1 a_s + d_2 a_s^2 + d_3 a_s^3 + d_4 a_s^4 + O(a_s^5)$$

In the MS-like schemes β-expansion prescription is:

$$\begin{gathered} d_1 = d_1[0] \\ d_2 = \beta_0 d_2[1] + \mathbf{d_2[0]} - \textbf{ the Basis of BLM procedure} \\ d_3 = \beta_0^2 d_3[2] + \beta_1 d_3[0,1] + \beta_0 d_3[1] + \mathbf{d_3[0]}, \\ d_4 = \beta_0^3 d_4[3] + \beta_2 d_4[0,0,1] + \beta_1\beta_0 d_4[1,1] + \beta_0^2 d_4[2] + \beta_1 d_4[0,1] \\ +\beta_0 d_4[1] + \mathbf{d_4[0]}; \ldots \end{gathered}$$

Suggested by Mikhailov (Quarks2004, JHEP(07)) Further on Bakulev,Mikhailov, Stefanis(10) ; Kataev, Mikhalov M(12-16); Brodsky,Wu, Mojaza et al(12-23); Cvetic,Kataev(16); Kataev,Molokoedov (22,23) ; Baikov, Mikhailov (22-23) ; Mikhailov (24)

Theory ambiguity in terms of the $\{\beta\}$-expansion. Why ? Where ?

The problem appears starting from N^2LO QCD:

$d_3 = d_{32}n_f^2 + d_{31}n_f + d_{30} \to \beta_0^2\ d_3[2] + \beta_1 d_3[0,1] + \beta_0 d_3[1,0] + d_3[0]$,

where $\beta_0 = \beta_{00} + \beta_{10}n_f$, $\beta_1 = \beta_{10} + \beta_{11}n_f$. How to get from single n_f- term two terms $\beta_1 d_3[0,1] + \beta_0 d_3[1]$. Mikhailov(07): Add to QCD **additional degree of freedom**, i.e. $n_{\tilde{g}}$ flavour number of multiplet of *MSSM gluino* . **Not seBLM** (Mikhailiov - *It is the fraction represntation* -07) but **PMC/BLM-type** (K,Mikhailov-15 and further) There $\beta_0 = \beta_0(n_f, n_{\tilde{g}})$, $\beta_1 = \beta_0(n_f, n_{\tilde{g}})$ (Clavelli,Surguladze(97) and $d_3(n_f, n_{\tilde{g}})$ (Chetyrkin (97)) are known analytically. In extended QCD (eQCD) D- and β-function are evaluated analytically by Chetyrkin(22); Zoller (2016) and β-expansion has solution; though model dependence exist Cvetic, K (16); K Molokoedov (23) Bednyakov (24, in private) and Mikhailov (22,24) who gives $d_{20} = \frac{1}{12} \approx 0.085$; $d_{30} \approx= -35.87(model)$; $d_{40} \approx -98$ (sign !);

Non-diagrammatic representations not only for the D^{ns} in not only QCD

Whether expansion in powers of conformal anomaly $\beta(a_s)/a_s$, where $\beta(a_s) = -\sum_{j\geq 0}\beta_j a_s^{j+2}$ is valid for the D^{ns}? Cvetic, Kataev (16); K,Mikhailov (09-12) motivated; Valid say for static potential as well K, Molokoedov (23) - CrBK related expansion

$$D^{ns}(a_s) = 1 + \sum_{n\geq 0}\left(\frac{\beta(a_s)}{a_s}\right)^n D_n(a_s)$$

$$D_n(a_s) = \sum_{r=1}^{4-n} a_s^r \sum_{k=1}^{r} D_n^{(r)}[k, r-k] C_F^k C_A^{r-k} + a_s^4 \delta_{n0} \times$$

$$\left(D_0^{(4)}[F,A]\frac{d_F^{abcd}d_A^{abcd}}{d_R} + D_0^{(4)}[F,F]\frac{d_F^{abcd}d_F^{abcd}}{d_R}n_f\right) + O(a_s^5)$$

with $D_0^{(4)}[F,A]$ and $D_1^{(4)}[F,F]$ and $D_n^{(r)}[k,r-k]$ analytical expressions In useful R_δ based comment in Appendix of Shen,Wu, Ma,Brodsky (16) β-expansion of $\gamma^{ns}(a_s)$ related to $\gamma_{ph}(a_s)$ in RG equation for Π (see e.g. K, Mikhailov (15)) is not taken inton account (and not only there starting from 2012 (!))

The $\{\beta\}$ expanded QCD terms for D^{ns} in $SU(N_c)$ non-diagrammatic and diagarammatic (!) differences

Using the $\overline{MS}$-scheme factorized representation, Cvetic,Kataev(16). The results differs from QCD+gluino theory (Mikhailov (07))

$$d_1[0] = \frac{3}{4}C_F \;\; d_2[0] = \left(-\frac{3}{32}C_F^2 + \frac{1}{16}C_F C_A\right) \;\; d_2[1] = \left(\frac{33}{8} - 3\zeta_3\right)C_F$$

$$d_3[0] = -\frac{69}{128}C_F^3 - \left(\frac{101}{256} - \frac{33}{16}\zeta_3\right) \neq +\frac{\mathbf{71}}{\mathbf{64}} \;\; C_F^2 C_A$$

$$-\left(\frac{53}{192} + \frac{33}{16}\zeta_3\right) \neq +\left(\frac{\mathbf{523}}{\mathbf{768}} - \frac{\mathbf{27}}{\mathbf{8}}\boldsymbol{\zeta_3}\right) \;\; C_F C_A^2$$

As the result one has $d_3[0] = -23.227 \neq -\mathbf{35.87}$, $d_4[0] = +83.344 \neq -\mathbf{98}$ (Cvetic,Kataev (16) $\neq$ K, Mikhailov (15) and Baikov,Mikhailov (22,23) eQCD related)

The $\{\beta\}$ expansion QCD expression for d_4 was also obtained

We present model dependent one from Cvetic, K (2016)

$$d_4[0] = \left(\frac{3}{16} - \frac{1}{4}\zeta_3 - \frac{5}{4}\zeta_5\right)\frac{d_F^{abcd}d_A^{abcd}}{d_R} - \left(\frac{13}{16} + \zeta_3 - \frac{5}{2}\zeta_5\right)\frac{d_F^{abcd}d_F^{abcd}}{d_R}n_f$$

$$+\left(\frac{4157}{2048} + \frac{3}{8}\zeta_3\right)C_F^4$$

$$-\left(\frac{3509}{1536} + \frac{73}{128}\zeta_3 + \frac{165}{32}\zeta_5\right) \neq -\left(\mathbf{\frac{2409}{512}} + \mathbf{\frac{27}{16}}\zeta_{\mathbf{3}}\right) \quad C_F^3 C_A$$

$$+\left(\frac{9181}{4608} + \frac{299}{128}\zeta_3 + \frac{165}{64}\zeta_5\right) \neq -\left(\mathbf{\frac{3105}{1024}} + \mathbf{\frac{81}{32}}\zeta_{\mathbf{3}}\right) \quad C_F^2 C_A^2$$

$$\left(-\frac{30863}{36864} - \frac{147}{128}\zeta_3 + \frac{165}{64}\zeta_5\right) \neq \left(\mathbf{\frac{68047}{12288}} + \mathbf{\frac{8113}{512}}\zeta_{\mathbf{3}} - \mathbf{\frac{3555}{128}}\zeta_{\mathbf{5}}\right)C_F C_A^3$$

The difference is from diagrammatic related expression of Mikhailov (22-24) which is closer to Ball, Beneke, Braun (95) . Not clear is it possible to get theory relation between the results (!!)

PMC/BLM vs massless $\overline{MS}$: K,Molokoedov PRD(23): in Adler function $\gamma_{ph}(a_s)$ corrctly β expanded as K, Mikhailov (15) ; Salinas-Arzimendi ,Schmidt (2210.01851)

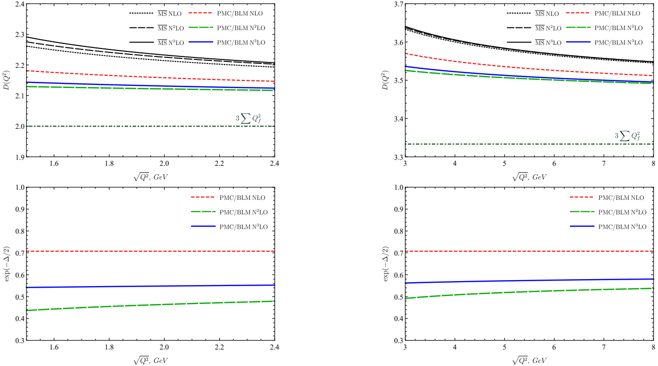
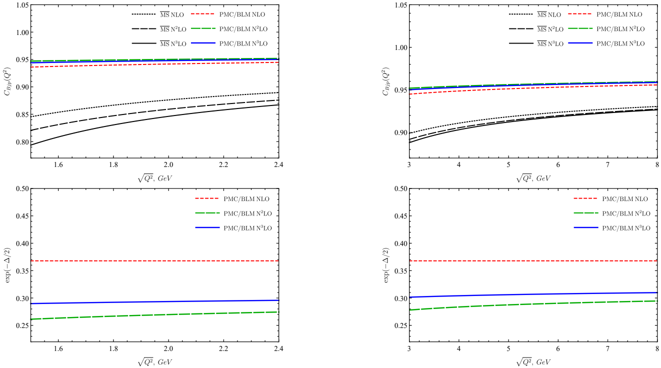


Figure: (1a) Adler function $D(Q^2)$ on $\sqrt{Q^2}$ at $n_f = 3, 4$ in the massless limit. (1b) PMC Factor $\exp(-\Delta/2)$ on $\sqrt{Q^2}$. Experimental data higher $\overline{MS}$ Eidelman, Jegerlehner, K, Veretin (98); Davier et al (23). Bad for PMC/BLM and in cases of SUSY QCD related effective model and eQCD as well .

PMC/BLM vs massless $\overline{MS}$: Bjorken polarized SR at n_f=3,4 $S_{Bjp}(Q^2) = \frac{1}{6}(g_A/g_V)C_{Bjp}(Q^2)$ by AK and Molokoedov drawn @ 23



Experriment lower than $\overline{MS}$ Deur et al (23) and Shirkov et al (08) (and Kotikov 24 talks) Effects of conformal symmetry violation by both PT and non-PT effects ARE NOT SEEN in PMC but ARE SEEN in NATURE (!) . Considerations see also D.Kortlorz,Mikhailiov,Teryaev,A.Kotlorz (19); D.Kotlorz,Mikhailov(19); Ayala, Pineda (22) and AK (05).

Conclusions

- PMC/BLM do not feel running of QCD coupling constant and is useful tool for study of CS limit results
- Analogy with Finite QED Program treatment by Adler.
- Masless PMC/BLM lower experimental data for Adler and higher for Bjorken polarized data ...
  Following Aristoteles let me say " You are respected , Brodsky, Lepage, MacKenzie , but truth is more friend" .
- Is it possible to understand better the existing model dependence in coefficients of β-expanded terms of PT series ?
- Leading renormalon chains desribe nicely effects of growth of PT coefficvients of Eucledian PT series
- Claim of α_s CERN Working group gided with participation of Michelangelo Mangano (2024). We should take into account in α_s extraction "scale systematics" or "missing higher order systematics" or "procedure dependence" .

Remained theory questions

- Why R_δ agree with multiple β-expasnion ? . Why it agrees with Cvetic-Valenzuela (08) approach ?
- What is the study of the eQCD-related study of this model for getting β-expansion for Adler and Bjorken polarized sum rule PT coefficient function ?
- Whether mutiple β-expansion and thus is R_δ are distingushed in $N = 1$ SUSY QCD NSVZ-related sacheme D-function considerations ? At next-to-leading order level yes (Aleshin,Kataev,Stepanyantz (19))
- MOM-scheme BLM considerations by Brodsky,Fadin,Kim, Lipatov,Pivovarov (99) and Ivanov,Papa et al(15). Landau gauge distingished. Gauge dependence in MOM is the delicate problem (Chetyrkin et al (17) and Gracey (24))
- Why β-function is factorized in the CSB PT QCD expression for of $\pi^0 \to \gamma\gamma$ formfactor in the gauge-invariant and definite MOM schemes ??? (Crewther -type relation).