Polarization and kinematic properties of the fragmentation functions $q \to W^{\pm} + q'$ and $q \to Z^0 + q$

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Objectives

We consider the processes $q \to W + q'$ and $q \to Z + q$ and derive the respective fragmentation functions as functions of two kinematic variables: the longitudinal momentum fraction z and transverse momentum p_T of the produced bosons with respect to the parent quark. We take into account phase space restrictions connected with nonzero masses of the gauge bosons and with limited initial energy. We separately consider three different polarization states of the bosons.

Introduction

The structure of the proton, when probed at increasing energies, reveals increasing complexity of its composition. Not only light quarks and gluons can be found among the proton's constituents, but also heavy quarks and, maybe, even the electroweak bosons W and Z. Our note focuses on the presenting bosons in the real form as final state quanta. The emission of the final state quanta can be conveniently described in terms of quark fragmentation functions $q \to W^{\pm} + q'$ and $q \to Z^0 + q$. This issue has already been addressed in the pioneering works[1,2] and later in Refs.[3,4,5,6]. We have, however, introduced three innovations that were absent in the previous calculations known to the authors.

First, we consider the fragmentation function as a function of two (rather than one) kinematic variables, z and p_T.
Second, we take into account phase space limitations connected with nonzero W and Z masses and nonzero p_T. This may be especially important for particle event generators[7,8] running at the energies of real colliders (and not at $\sqrt{s} \to \infty$).
Third, we make distinction between two transverse polarizations (the polarization vector may either lie in the boson production plane or be perpendicular to this plane).

Calculation

To calculate the quark to W fragmentation function, we start with the process

$$e^+e^- \to \gamma^* \to \bar{q} + q + W \tag{1}$$

considered in the virtual photon rest frame.

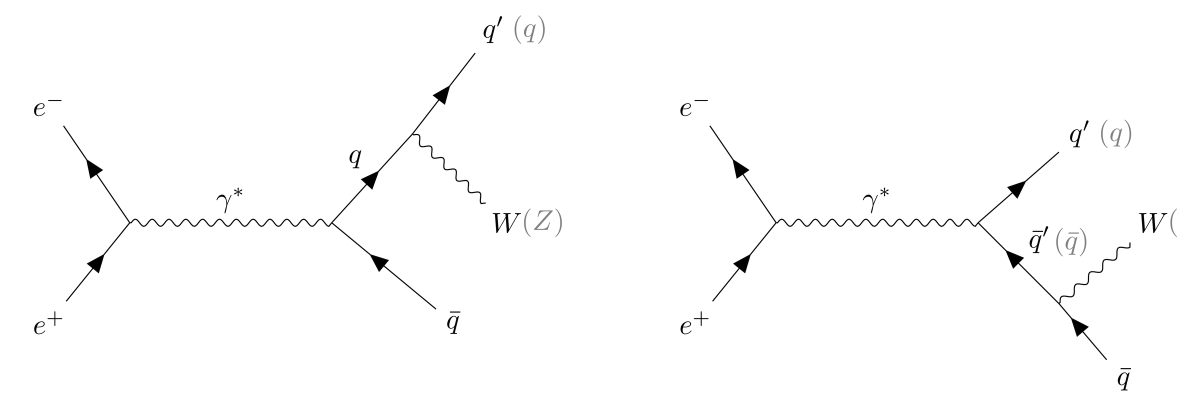

Figure 1: Feynman diagrams considered in the derivation of the $q \to Wq'$ or $q \to Zq$ fragmentation functions.

The diagrams constitute a gauge invariant subset of all lowest order diagrams. When the input energy becomes large, the entire process can be factorized.
The fully differential cross section for the process (1) reads

$$d\sigma(e^+e^- \to W\, q\, \bar{q}) = \frac{1}{2s} \frac{1}{(2\pi)^5} \frac{ds_1\, ds_2\, d\phi\, d\psi\, d\cos\theta}{32s} \times \left| \mathcal{M}(ee \to \gamma^* \to W q\, \bar{q}) \right|^2, \tag{2}$$

where $s_1 = (p_1 + p_W)^2$, $s_2 = (p_2 + p_W)^2$, and ϕ, ψ. θ are three Euler's angles.
The differential cross section of the 'prequel' process is

$$d\sigma(e^+e^- \to q\, \bar{q}) = \frac{1}{2s} \frac{1}{(2\pi)^2} \left| \mathcal{M}(ee \to \gamma^* \to q\, \bar{q}) \right|^2 \times \frac{\lambda^{1/2}(s, p^{*2}, m_q^2)}{8s}\, d\Omega\,, \tag{3}$$

where p^* is the momentum of the parent quark.
After dividing Eq.(2) by Eq.(3) we obtain

$$dD\,(q^* \to W q) = \frac{1}{(2\pi)^3} \frac{1}{4\, \lambda^{1/2}(s, p^{*2}, m_q^2)} \frac{\left| \mathcal{M}(\gamma^* \to W\, q\, \bar{q}) \right|^2}{\left| \mathcal{M}(\gamma^* \to q\, \bar{q}) \right|^2} \times ds_1 ds_2\, d\phi\, d\psi\; d\cos\theta. \tag{4}$$

By introducing the light-cone variable $z = p_W^+/p^{*+} = (E_W + p_{W,||})/(E^* + |p^*|)$ and integrating over all other variables Eq.(4) can be reduced to the conventional fragmentation function:

$$D_{q/W}(z) = \int D\,(q^* \to W\, q)\; \delta(z - p_W^+/p^{*+}) \times ds_1\, ds_2\; d\phi\, d\psi\; d\cos\theta. \tag{5}$$

We also consider fragmentation function with unintegrated p_T dependence: $D\,(z, p_T; s)$.
The phase space limitations make the fragmentation function scale dependent. At low s, the scale dependence mostly comes from a requirement that the quark energy be large enough to produce a heavy boson. At much higher energies, the scale dependense is dominated by radiative corrections (not considered in this note).

Numerical results

To be definite, we show the results obtained for quark to W fragmentation, $q \to Wq'$. To come from W to Z one only has to change the boson mass and an overall normalizing factor.
The plots show strong dependence on the input energy s, which roughly determines the energy of the radiating quark: $E^* \simeq |p^*| \simeq s/2$.
– At low s, the lack of phase space pushes the fragmentation function towards large z and makes the overall probability small.
– At larger s, the fragmentation function tends to smaller z and the overall normalization increases.
– Finally, at very high s, it restores the shape of Weizsäcker-Williams approximation in full consistency with the results[1,2,7] (gray solid curves in Fig.2).

We separately show three different polarization states of the bosons.
– **Dashed** curves, the transverse polarization vector lies in the Wq production plane;
– **Dotted** curves, the transverse polarization perpendicular to the production plane;
– **Dash-dotted** curves, longitudinal polarization;
– **Solid** curves, the sum of all contributions.

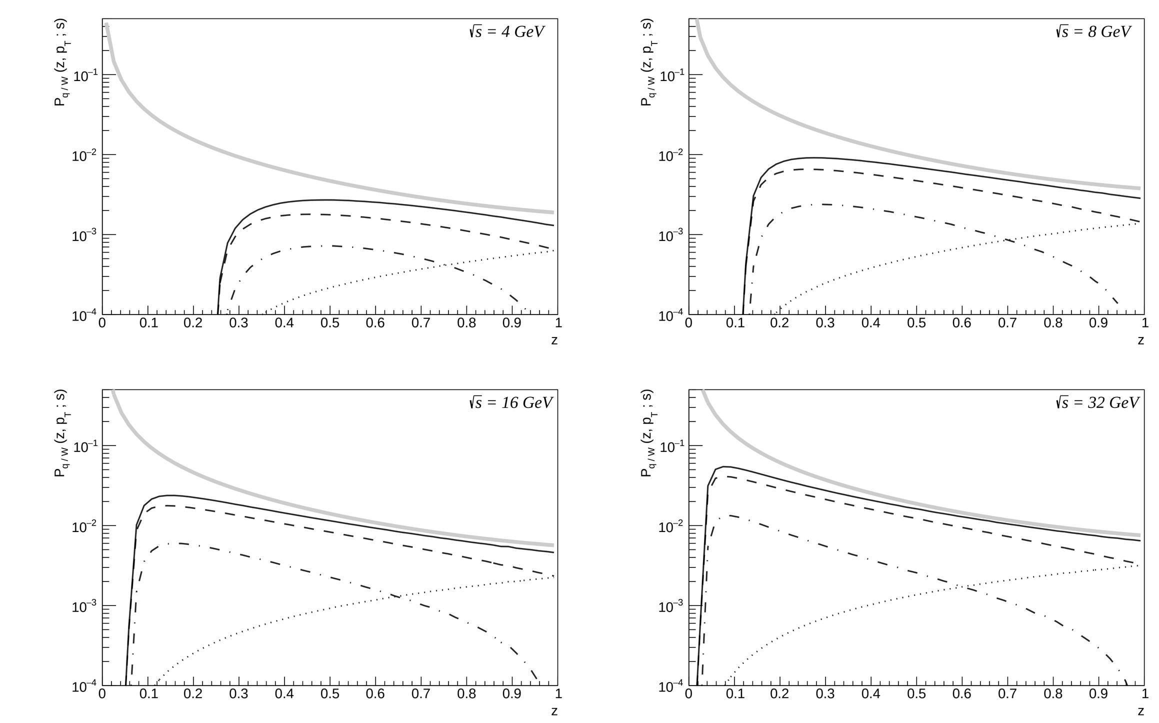

Figure 2: Behavior of $D_{q/W}(z, p_T; s)$ as function of z.

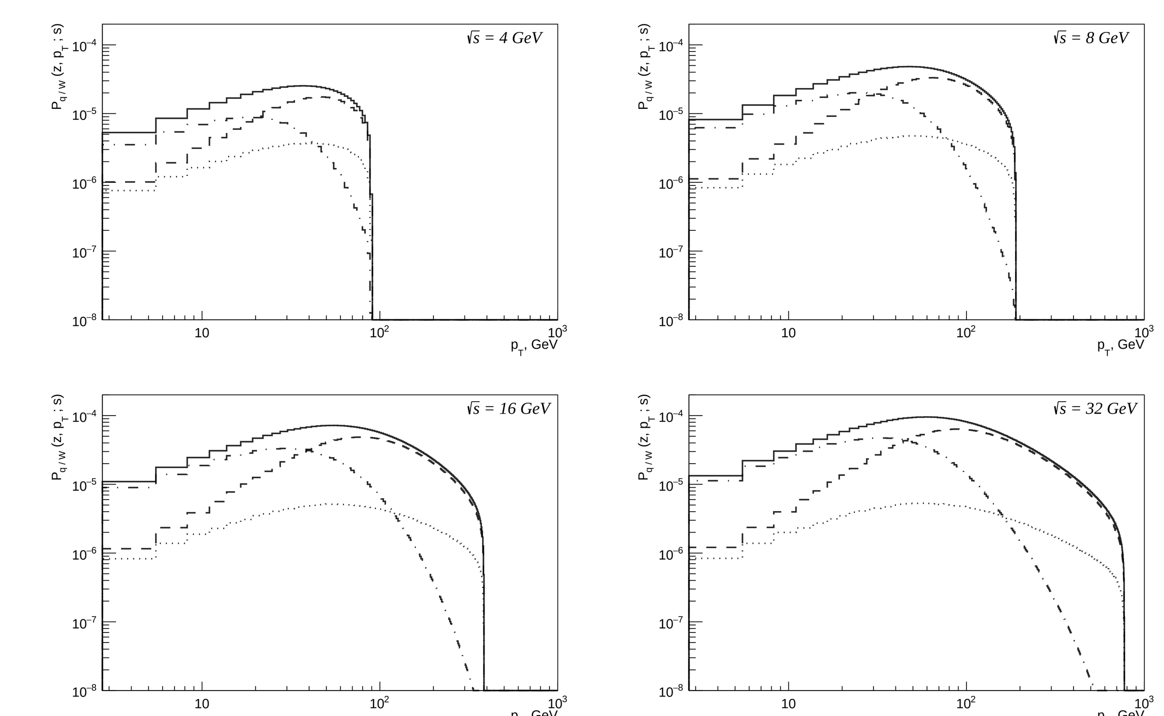

Figure 3: Transverse momentum distributions $D_{q/W}\,(z, p_T; s)$.

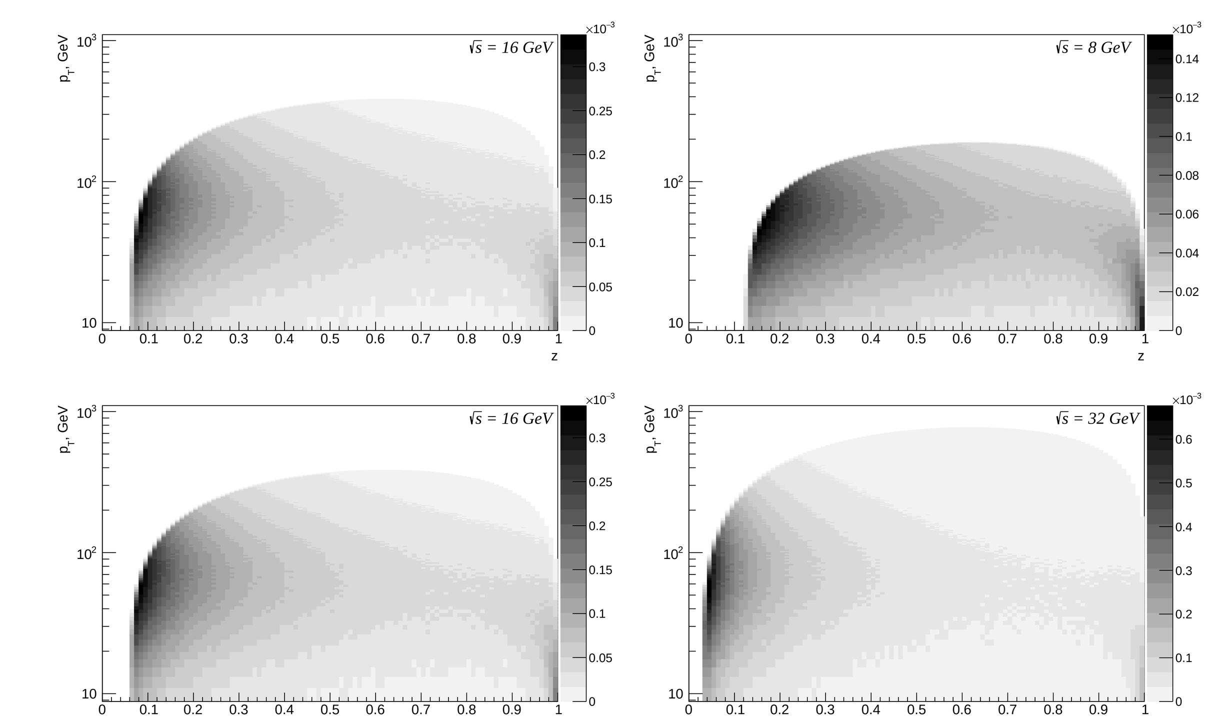

Figure 4: Double differential distributions for $D_{q/W}(z, p_T; s)$.

Interesting Outcomes

– The transverse momentum is by far not negligible and can cause a substantial deviation of the produced boson from the direction of the parent quark.
– The phase space restrictions do dramatically affect the shape and the overall normalization of the effective W and Z spectra.
– A significant difference between two transverse polarization states of the produced bosons is observed.

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