



XXXVI International Workshop on High Energy Physics

***Strong Interactions:
Experiment, Theory, Phenomenology***

Pomeron in QCD

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NRC Kurchatov Institute, Gatchina**

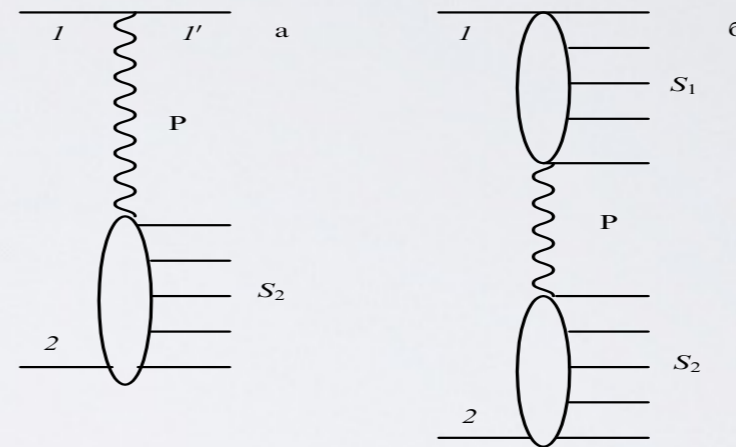


Outline:

- **Introduction & Motivation**
- **Pomeron before QCD**
- **Pomeron in perturbative QCD**
- **Pomeron in nonperturbative QCD**
- **Pomeron beyond QCD**

Pomeron at high energies is responsible for:

- **elastic scattering**
- **diffractive scattering**
- **inelastic scattering**
- **total x-section**





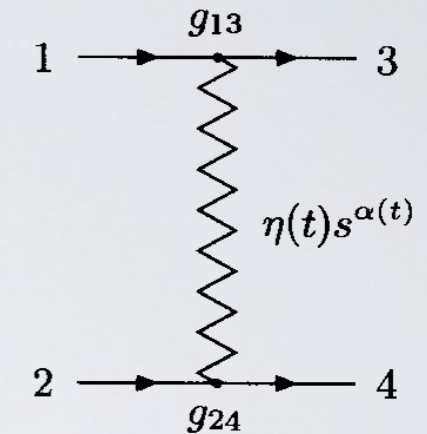
asymptotic theorem:

particle and antiparticle x-section equality

I. Pomeranchuk 34 (1958) 725

non-relativistic scattering: Regge poles

T. Regge (1959, 1960)



relativistic scattering: Regge poles

V. Gribov Nucl. Phys. 22 (1961) 249

Gribov-Froissart representation

M. Froissart Phys. Rev. 123 (1961) 1053

Pomeron: vacuum pole and trajectory $\alpha_{IP}(t) = \alpha_{IP}(0) + \alpha'_{IP} \cdot t$

V. Gribov ZhETP 41 (1961) 667 [JETP 14 (1962) 472]

G. Chew, S. Frautschi PRL 7 (1961) 394

$$A(s, t) = \beta(t) \eta(t) s^{\alpha(t)}$$

$$\frac{d\sigma_{el}}{dt} = F(t) s^{2\alpha(0)-2} e^{-2\alpha' |t| \ln s}$$

➡ **elastic & diffractive cone shrinkage**

➡ **x-section: constant with energy** IHEP (Protvino) U70 data since 1967

$$\sigma_{tot} \underset{s \rightarrow \infty}{\simeq} \frac{1}{s} \text{Im} A(s, t=0) \underset{s \rightarrow \infty}{\sim} s^{\alpha(0)-1}$$



Unitarity condition:

Froissart-Martin x-section asymptotic bound $\leq \log^2(s)$

M. Foissart Phys. Rev. 123 (1961) 1053

Reggeon field theory

V.N. Gribov (1967)

multi-Pomeron exchanges

V.N. Gribov, A.A. Migdal (1968-1970)

K.A. Ter-Martirosyan, A.A. Migdal, A.M. Polyakov 1972-1975

A.B. Kaidalov K.A. Ter-Martirosyan 1973-1979

supercritical Pomeron $\alpha_{IP}(0) > 1$

V.N. Gribov, A.A. Migdal, A.M. Polyakov 1970-1975

strongly-interacting supercritical Pomeron

V.N. Gribov, A.A. Migdal, A.M. Polyakov 1969



Born approximation: two-gluon Pomeron

F.E. Low, Phys. Rev. D12 (1975) 163

S. Nussinov, Phys. Rev. Lett. 34 (1975) 1286

Leading logarithmic approximation: LL BFKL Pomeron

V.S. Fadin, E.A. Kuraev, L.N. Lipatov, Phys. Lett. B 60 (1975) 50

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, ZhETF 71 (1976) 840 [JETP 45 (1977) 79]

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, ZhETF 72 (1977) 377 [JETP 45 (1977) 79]

I.I. Balitsky, L.N. Lipatov, Yad. Fiz. 28 (1978) 1597

Next-to-leading logarithmic approximation: NLL BFKL Pomeron

V.S. Fadin, L.N. Lipatov, Phys. Lett. B 429 (1998) 127

E.A. Camici, L.N. Ciafaloni, Phys. Lett. (1998)

S.J. Brodsky V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov, Pisma ZhETF 70 (1999) 161 (BFKLP)



- Large-angle scattering (hard processes):

QCD in Bjorken limit

- GLAPD: V. Gribov & L. Lipatov (71-72); L. Lipatov (74);
G. Altarelli & G. Parisi (77); Yu. Dokshitzer (77)

- Small-angle scattering (“semi-hard” processes):

QED in Gribov-Regge limit

- V. Gribov, V. Gorshkov, L. Lipatov & G. Frolov (67-70)
H. Cheng & T. Wu (66-70)

QCD in Gribov-Regge limit

- BFKL: V. Fadin, E. Kuraev & L. Lipatov (75-78)
I. Balitsky & L. Lipatov (78)



High-energy QCD asymptotics: GLAPD and BFKL



$$s=(p_1+p_2)^2$$

$$t=(p_1-p_3)^2$$

$$Q^2=-t$$

Scattering in the Standard Model (QCD) at high energies:

Large logarithms: as $\log(s)$, as $\log(Q^2)$

Bjorken limit (large-angle scattering):

$$s \sim Q^2 \gg m^2$$

$$Q^2/s = x \sim 1$$

Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (GLAPD):

$(a_s \log(Q^2))^n$ resummation

Inclusive cross section $\sim 1/Q^4$

Gribov-Regge limit (small-angle scattering):

$$s \gg Q^2 \gg m^2$$

$$Q^2/s = x \Rightarrow 0$$

Balitsky-Fadin-Kuraev-Lipatov (BFKL):

$(a_s \log(s))^n$ resummation

Total cross section $\sim s^{(a_P-1)}$

a_P – Pomeron intercept

soft scattering data: $a_P = 1.1$



Bjorken limit (GLAPD):

$$s \sim Q^2 \gg m^2$$

$$Q^2/s = x \sim 1$$

Large-angle (large-x) scattering

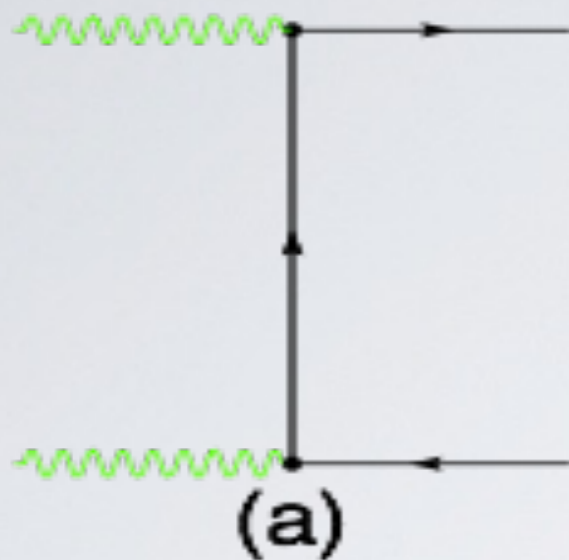
Gribov-Regge limit (BFKL):

$$s \gg Q^2 \gg m^2$$

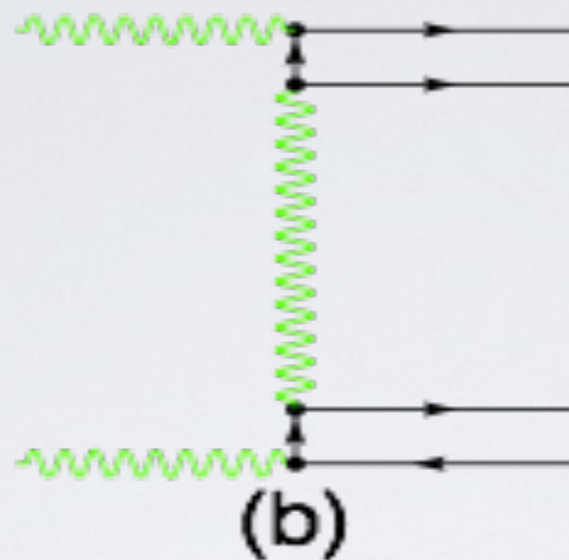
$$Q^2/s = x \rightarrow 0$$

Small-angle (small-x) scattering

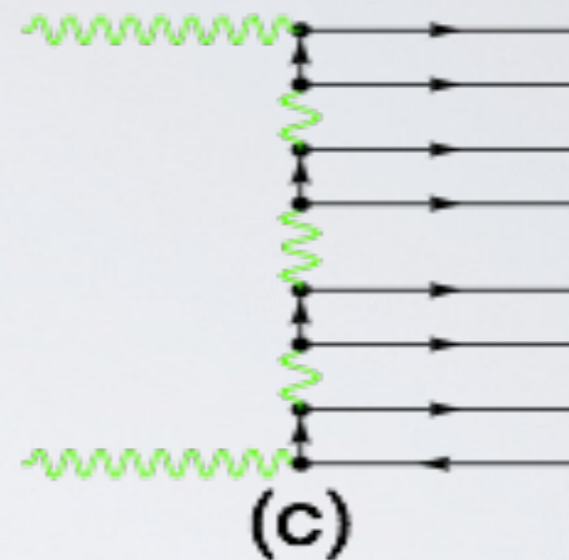
Asymptotics of QED cross sections



$$\sigma \sim (\alpha_{\text{QED}})^2 \log(s)/s$$



$$\sigma \sim (\alpha_{\text{QED}})^4 \text{const}(s)$$



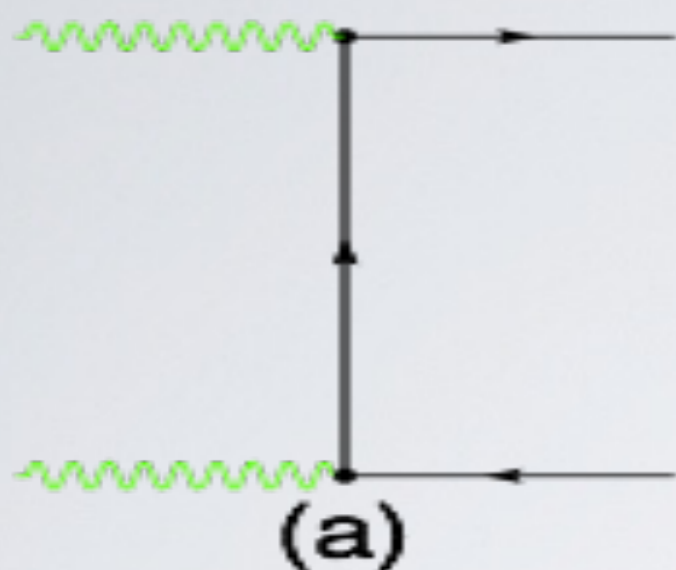
All orders: V.N. Gribov, L.N. Lipatov, G.V. Frolov & V.G. Gorshkov (69-71)
H. Cheng & T.T. Wu (69-70)

Cross section at $s \rightarrow \infty$: $\sim (\alpha_{\text{QED}})^4 (S/S_0)^{(\alpha_P - 1)}$

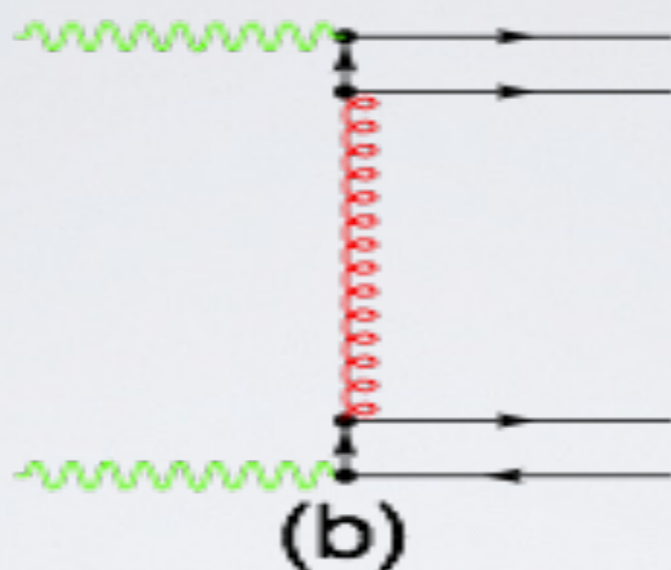
$$\alpha_P = 1 + C (\alpha_{\text{QED}})^2 \approx 1.002$$

photon: no reggeization!

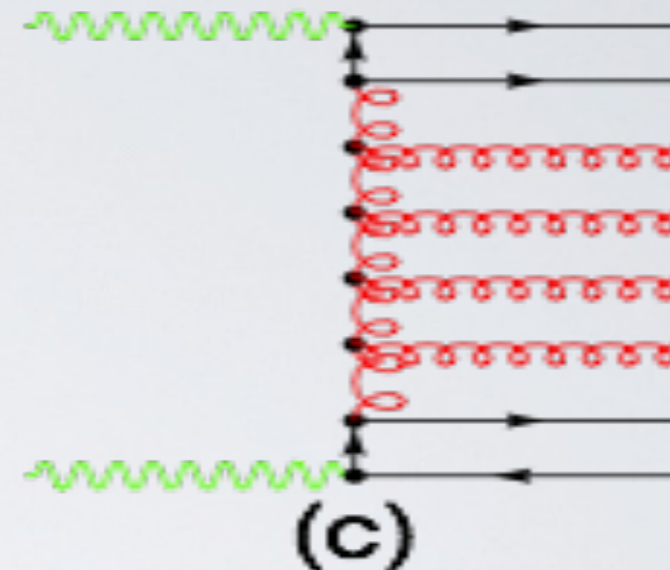
High-energy limit pQCD as LL BFKL: $\gamma\gamma$



$$\sigma \sim (\alpha_{\text{QED}})^2 \log(s)/s$$



$$\sigma \sim (\alpha_{\text{QED}})^2 (\alpha_s)^2 \text{const}(s)$$



Resummation of all leading logarithms: **LL BFKL**

gluon: reggeization!

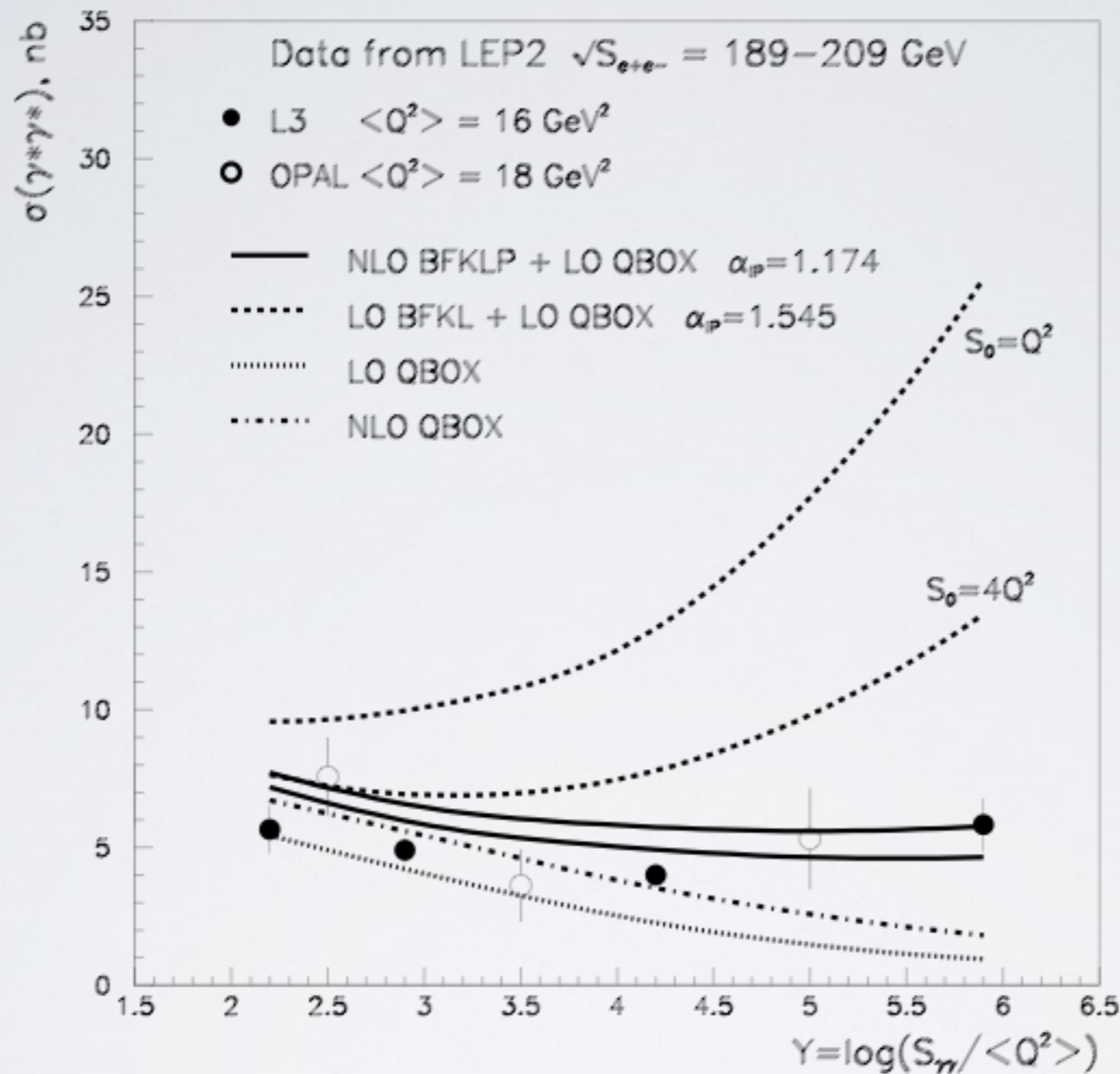
Cross section at $s \rightarrow \infty$: $\sim (\alpha_{\text{QED}})^2 (\alpha_s)^2 (S/S_0)^{(\alpha_P-1)}$

$\alpha_P = 1 + C \alpha_s \approx 1.5$ LL BFKL S. Brodsky & F. Hautmann (96)

$\alpha_P = 1 + C \alpha_s \approx 1.2$ NLL BFKL

S. Brodsky, V Fadin, VK, L. Lipatov, G. Pivovarov (2001-02)

Highly virtual photon scattering at LEP-2



S.J Brodsky, VK, L.N. Lipatov, V.S. Fadin & G.B. Pivovarov (2002)

BFKLP: NLL BFKL + generalized BLM

LO Impact factor

Full NLL BFKL calculations: require extra studies



LL BFKL: problems



**LL BFKL: designed for infinite collision energies
multi-Regge-kinematics**

LL BFKL problems (at finite energies):

- **fixed (non-running) coupling α_s**
- **energy-momentum conservation**
- **transverse momentum conservation**

Cross section in LL BFKL:

$$\sigma = \sigma_0 (S/S_0)^{(\alpha_P - 1)}$$

$$\alpha_P = 1 + C \alpha_s \approx \mathbf{1.5-1.6}$$

Data: $\alpha_P \approx 1.2-1.3$



BFKL: next-to-leading logs (NLL) improved by running a_s

**next-to-leading log approximation (NLL) BFKL
MSbar-renormalization scheme: large corrections**

V.S. Fadin & L.N. Lipatov (89-98)

C. Camici & M. Ciafaloni (96-98)

**BFKLP: NLL BFKL + resummation of running coupling a_s
generalized for the case with non-Abelian LO**

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov (98-99) BFKLP

→ BLM approach Brodsky, Lepage & Mackenzie – 1983

→ works only (!) for the case with Abelian LO

S.Brodsky, P. Lepage, P.Mackenzie (1983) **BLM**

$$\beta_0 = \frac{11}{3}N_C - \frac{2}{3}n_F$$

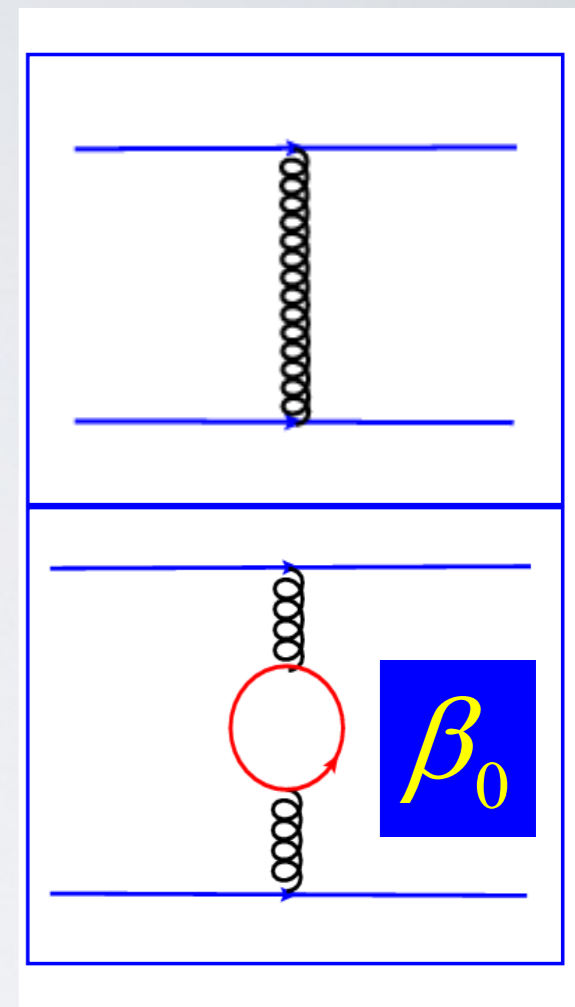
$$\rho = C_0 \alpha_{\overline{\text{MS}}}(Q) \left[1 + \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \left(-\frac{3}{2}\beta_0 A_{\text{VP}} + \frac{33}{2}A_{\text{VP}} + B \right) \right]$$

$$\mu^2 \frac{da_s}{d\mu^2} = \beta(a_s) = - \sum_{i \geq 0} \beta_i(n_f) a_s^{i+2}$$

$$\rho = C_0 \alpha_{\overline{\text{MS}}}(Q^*) \left[1 + \frac{\alpha_{\overline{\text{MS}}}(Q^*)}{\pi} C_1^* + \dots \right]$$

$$Q^* = Q \exp(3A_{\text{VP}})$$

$$C_1^* = \frac{33}{2}A_{\text{VP}} + B .$$



LO Abelian -> LO non-Abelian

MSbar-scheme -> MOM scheme 3g-vertex

S.Brodsky, V.Fadin, VK, L.Lipatov, G. Pivovarov(99) **BFKLP**



BFKLP (generalized BLM) works for non-Abelian cases **NLL BFKL and $Y \rightarrow ggg$ decay**

Naïve BLM application does not work (!):

- **NLL BFKL in \overline{MS} scheme**
- **Upsilon $\rightarrow ggg$ decay in NLO in \overline{MS} scheme**

\overline{MS} -scheme: nonphysical RG scheme (!)

numerically close to V-scheme (heavy quark potential) – Abelian in LO

physical RG scheme: MOM scheme (gauge dependent)

- **NLL BFKL** **<- non-Abelian in LO**
- **Upsilon $\rightarrow ggg$ decay** **<- non-Abelian in LO**

one can use MOM-scheme based on ggg-vertex non-Abelian in LO

BLM generalized for non-Abelian case:

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP

BFKLP: NLL BFKL + resummation of running coupling as

BLM resummation depends on non-Abelian structure in LO



BFKLP: NLL BFKL within generalized BLM



S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov(98-99) BFKLP

$$\sigma \sim S^{\alpha_{IP} - 1} = S^{\omega^{\max}}$$

$$\omega_{\overline{MS}}(Q_1^2, \nu) = \int d^2 Q_2 K_{\overline{MS}}(\mathbf{Q}_1, \mathbf{Q}_2) \left(\frac{Q_2^2}{Q_1^2} \right)^{-\frac{1}{2} + i\nu}$$

$$= N_C \chi_L(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \left[1 + r_{\overline{MS}}(\nu) \frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi} \right],$$

$$\chi_L(\nu) = 2\psi(1) - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)$$

$$r_{\overline{MS}}(\nu) = r_{\overline{MS}}^{\beta}(\nu) + r_{\overline{MS}}^{\text{conf}}(\nu)$$

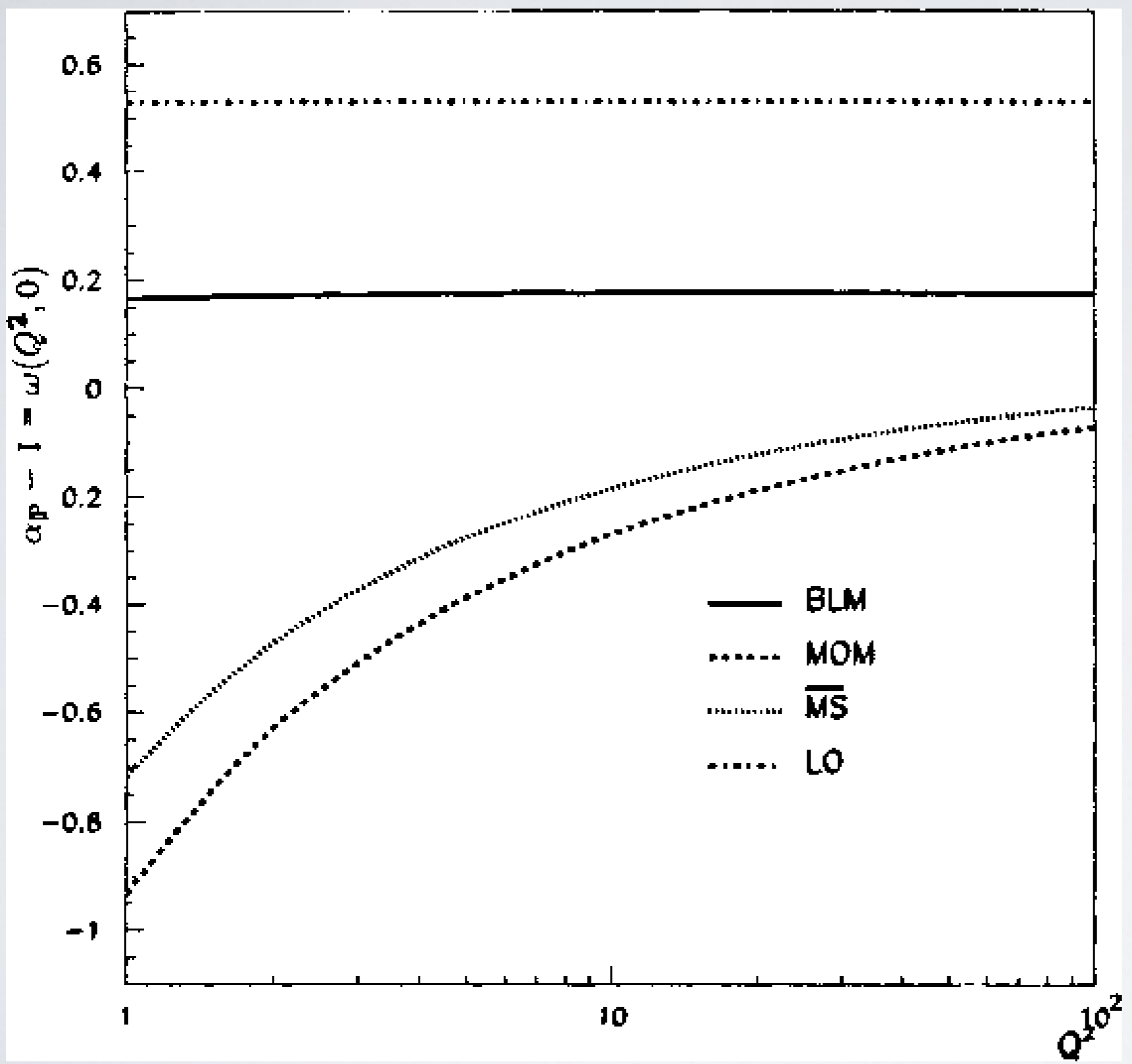
$$r_{\overline{MS}}^{\beta}(\nu) = -\frac{\beta_0}{4} \left[\frac{1}{2} \chi_L(\nu) - \frac{5}{3} \right]$$

$$r_{\overline{MS}}^{\text{conf}}(\nu) = -\frac{N_C}{4\chi_L(\nu)} \left[\frac{\pi^2 \sinh(\pi\nu)}{2\nu \cosh^2(\pi\nu)} \left(3 + \left(1 + \frac{N_F}{N_C^3} \right) \frac{11 + 12\nu^2}{16(1 + \nu^2)} \right) - \chi_L''(\nu) + \frac{\pi^2 - 4}{3} \chi_L(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} - 6\zeta(3) + 4\varphi(\nu) \right]$$

QCD N=4 A.V. Kotikov, L.N. Lipatov (2000)



BFKLP: NLL BFKL within generalized BLM



$$\sigma \sim S^{\alpha_{IP} - 1} = S^{\omega^{\max}}$$



V.S. Fadin & L.N. Lipatov (89-98)

C. Camici & M. Ciafaloni (96-98)

next-to-leading log approximation (NLL) BFKL

\overline{MS} -renormalization scheme: large corrections

S.J. Brodsky, V.S. Fadin, VK, L.N. Lipatov, G.B. Pivovarov (98-99) BFKLP

BFKLP: NLL BFKL + resummation of running coupling as

in physical renormalization scheme

BFKLP: Conformal BFKL kernel in NLL \rightarrow SUSY N=4

Pomeron intercept: $a_P = 1.2 - 1.3$

Cross section: $\sigma_0 (S/S_0)^{(a_P-1)}$ $a_P = 1 + C \alpha_s$

L.N. Lipatov, A.V. Kotikov et al. (2000-06)

SUSY N=4 BFKL Pomeron

Anomalous dimensions: test of AdS/CFT



BFKL observables

Heavy quark production

I.I. Balitsky, L.N. Lipatov (1978)

Inclusive jet

M.G. Ryskin (1980)

Lepton pair production

M.G. Ryskin, E.M. Levin (1981)

**Deep inelastic processes -> small-x physics
unitarization -> small-x shadowing**

L.V. Gribov, M.G. Ryskin, E.M. Levin (1981-83)

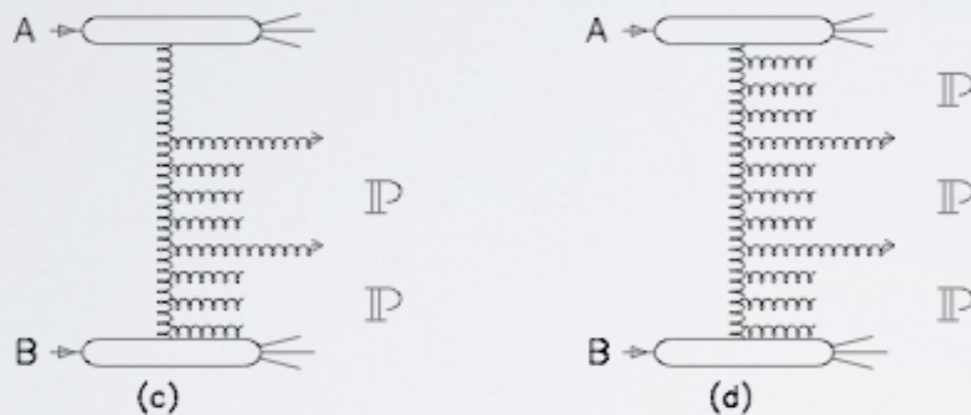
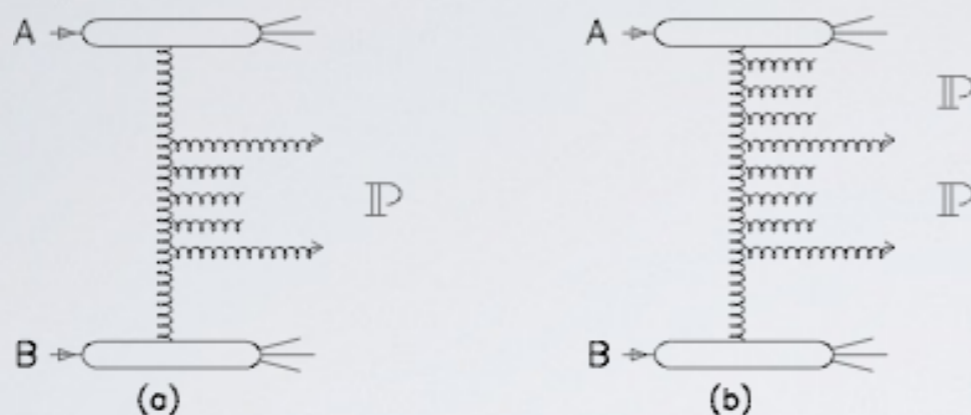
Most forward/backward (Mueller-Navelet) dijets:

x-section $\sim \exp(|\Delta|y)$

A. Mueller & H. Navelet, Nucl. Phys. B (1987)



BFKL direct observable: dijet with large rapidity separation between jets



Jet production

**GLAPD: ordering on κT
 y – no ordering**

**BFKL: ordering on y
 κT – no ordering**

Most forward/backward (Mueller-Navelet) dijets: x-section $\sim \exp(|\Delta|y)$

A. Mueller & H. Navelet, Nucl. Phys. B (1987)

Most forward/backward (Mueller-Navelet) dijets: azimuthal decorrelations

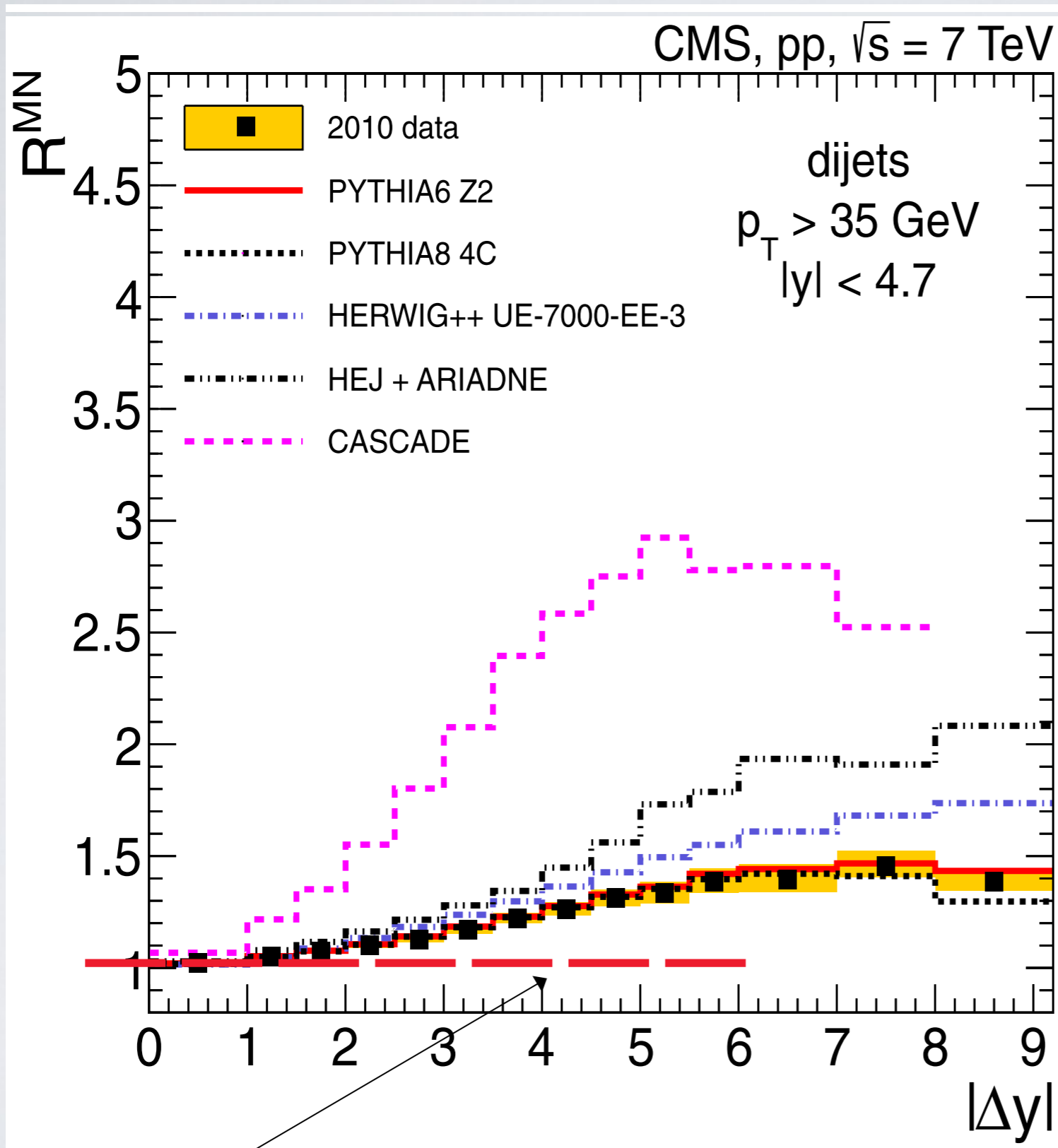
V. Del Duca & C. Schmidt, Phys. Rev. D (1994)

W.J. Stirling, Nucl. Phys. B (1994)

Inclusive dijets

VK & G.B. Pivovarov, Phys. Rev. D (1996)

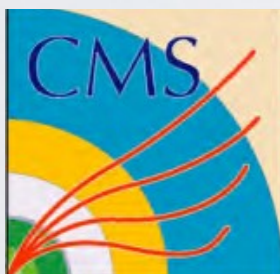
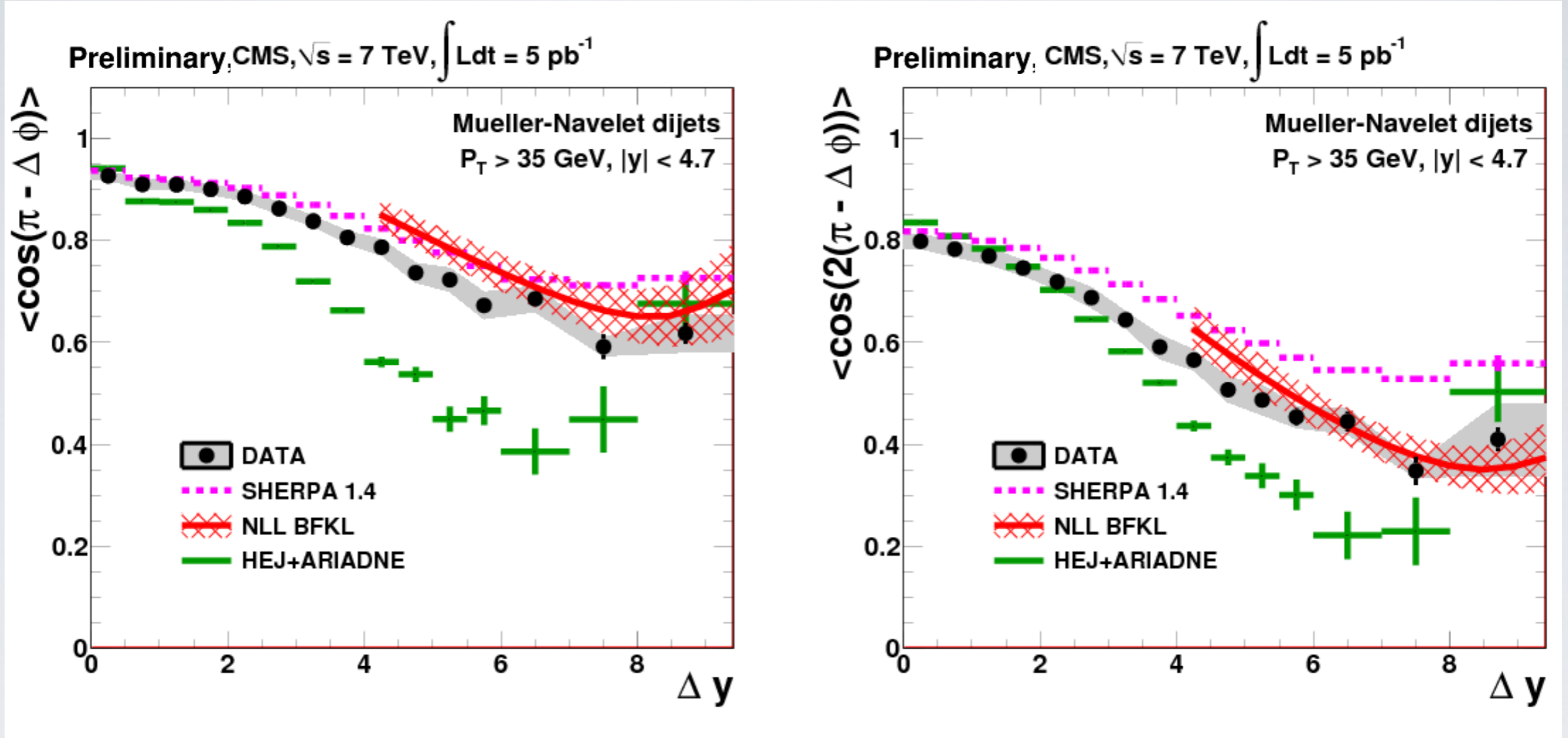
CMS: dijet "K-factor"



EPJ C 72 (2012) 2216
7 TeV, $p_{T_min} = 35$ GeV
 $\Delta y = | | < 9.4$

**MC generators:
 contain terms
 beyond GLAPD**

GLAPD



CMS (2016)
7 TeV, $p_{T_min} = 35 \text{ GeV}$
 $\Delta y = | | < 9.4$

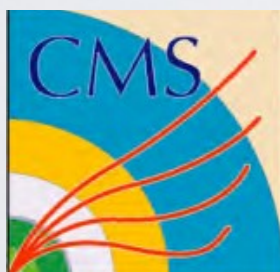
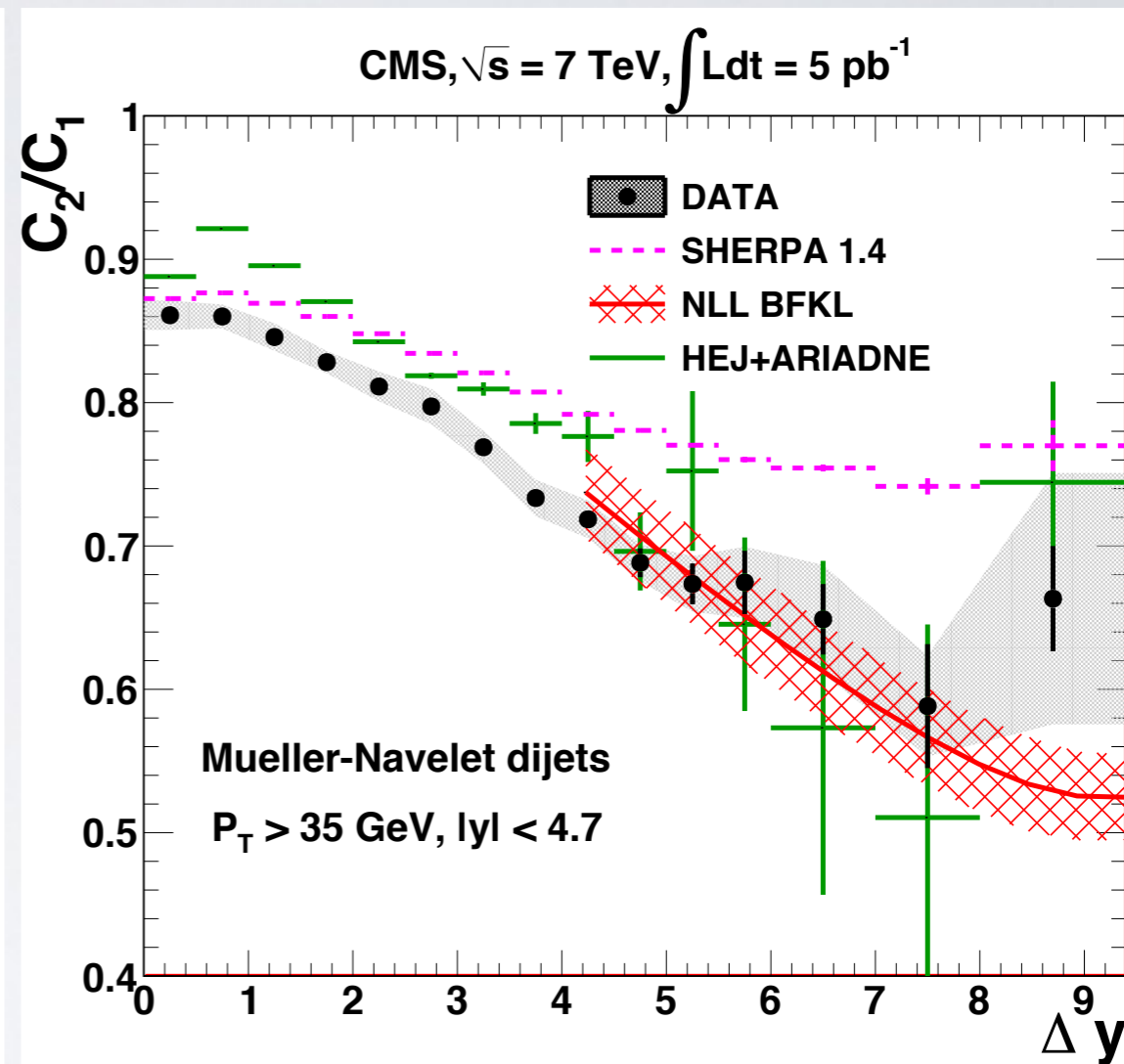
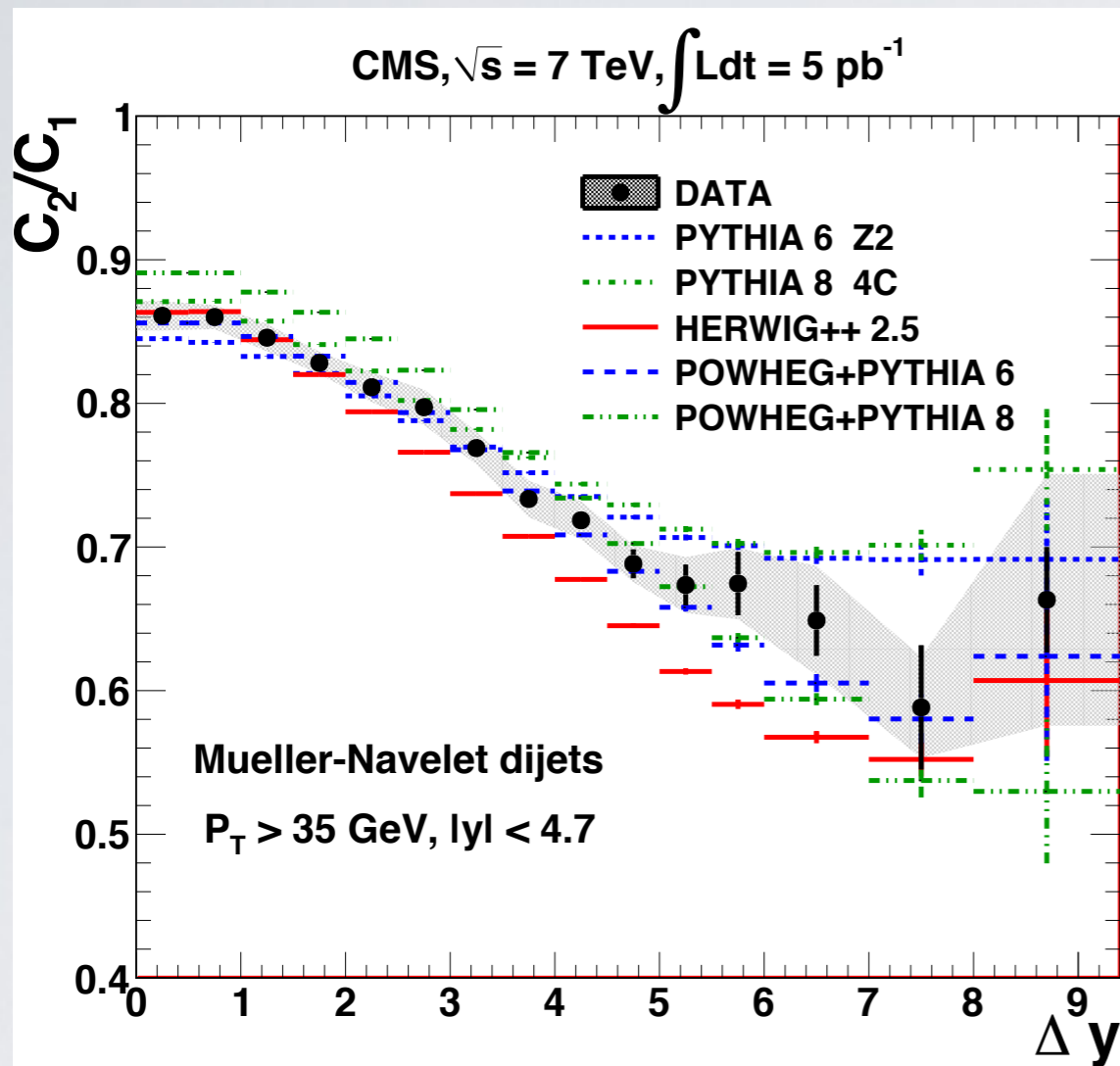
NLL BFKL + BFKLP (2014)
B. Ducloue, L. Szymanowski & S. Wallon



Dijets: $\langle \cos 2\phi \rangle / \langle \cos \phi \rangle$ vs NLL BFKL + BFKLP

BFKL conformal feature: cosine ratio

A. Sabio Vera et al (2007)



CMS (2016)
7 TeV, $p_{T_min} = 35$ GeV
 $\Delta y < 9.4$

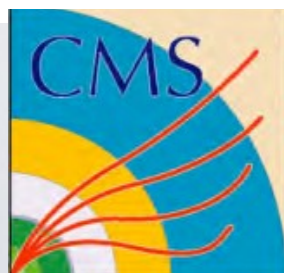
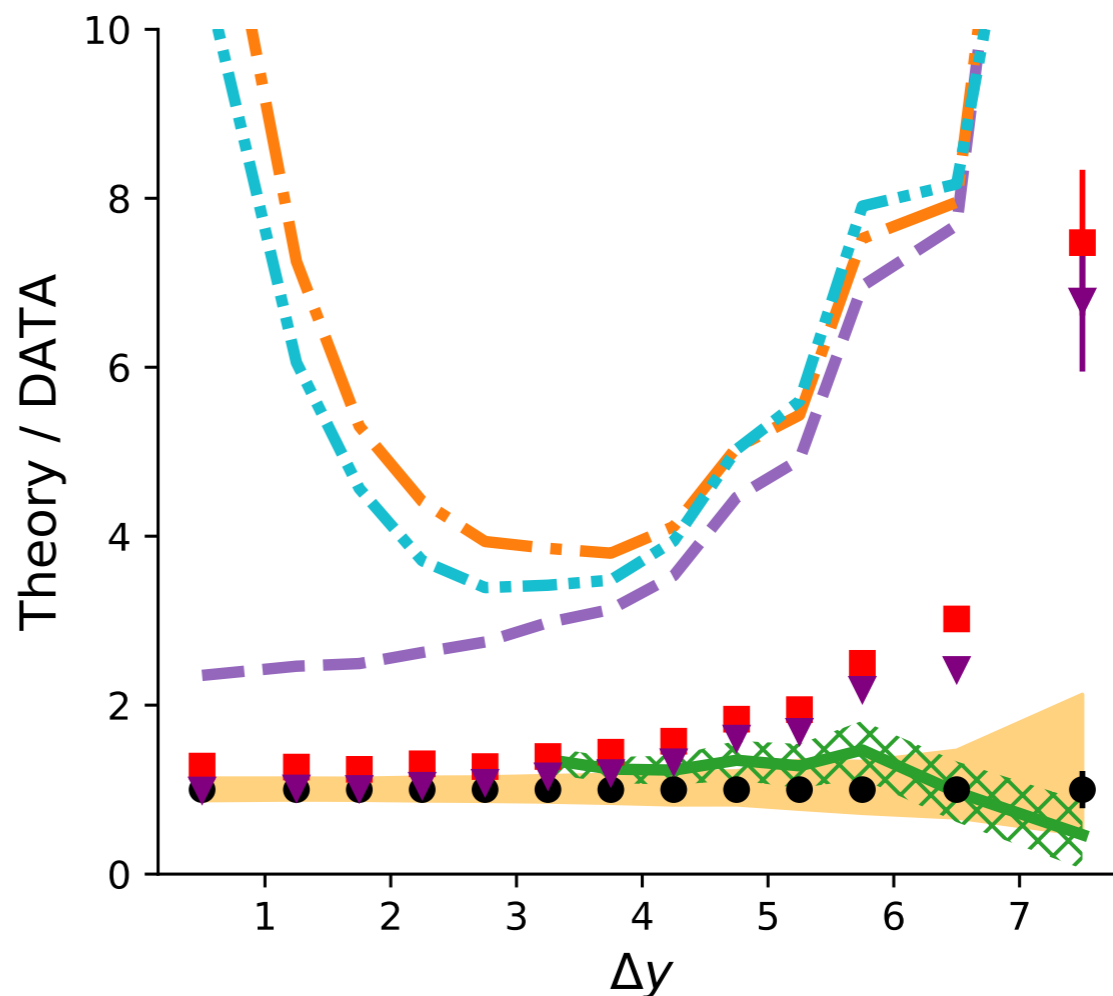
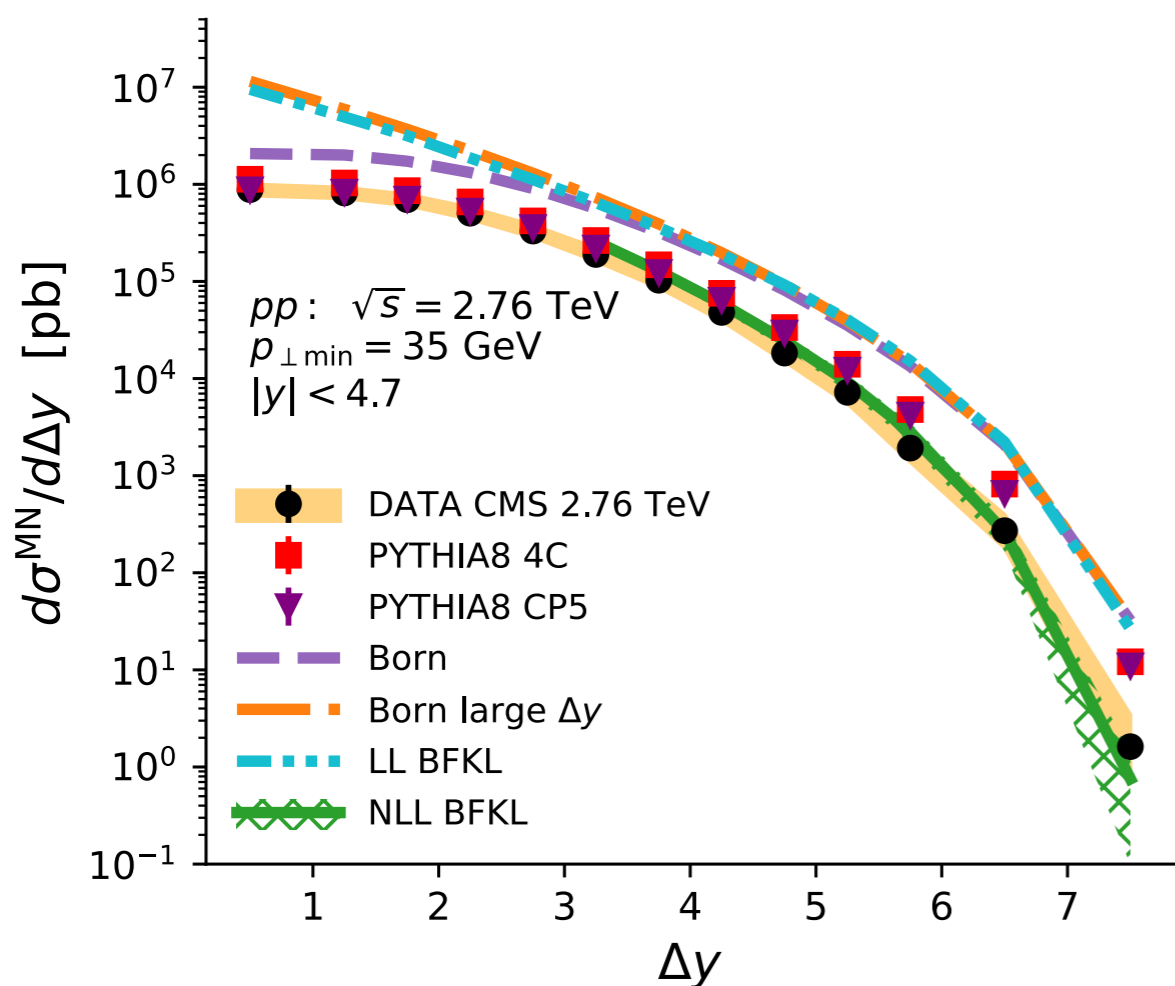
NLL BFKL + BFKLP (2014)
B. Ducloue, L. Szymanowski & S. Wallon



MN dijets within NLL BFKL improved by BFKLP

BFKL with BFKLP F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, *Phys. Rev. D* **92**, 034004 (2015)

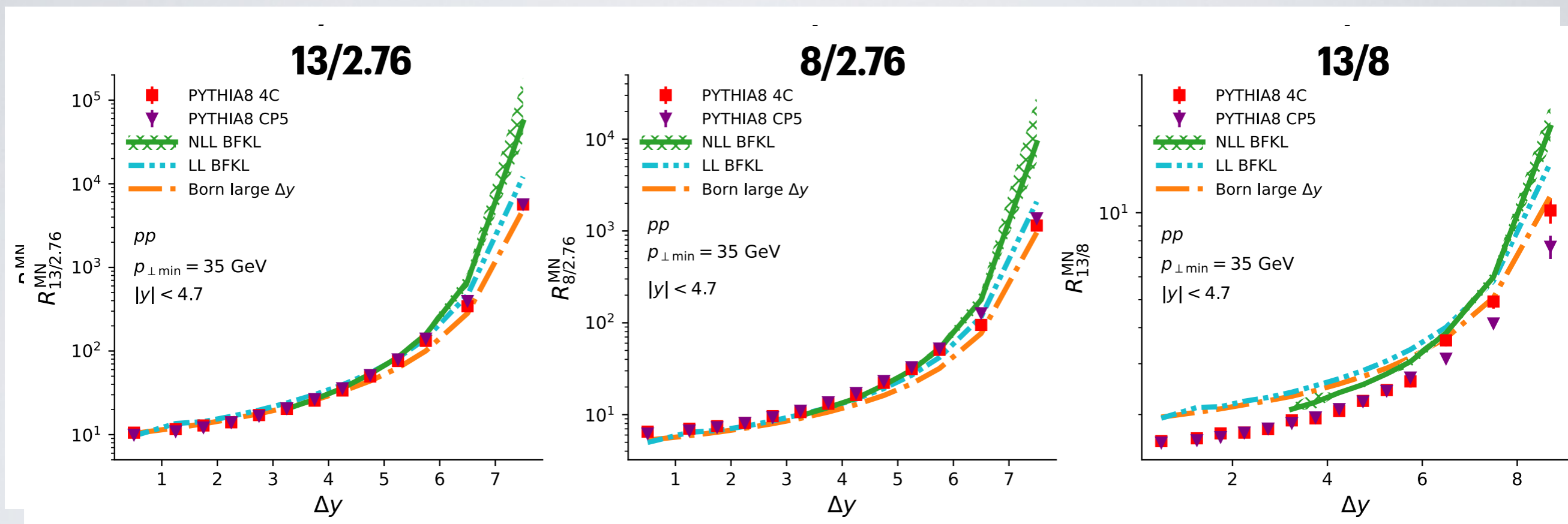
BFKL with BFKLP: 2.76 TeV dijet x-section A. Egorov & VK *Phys. Rev. D* **107**, 034004 (2023)



CMS (2022)
2.76 TeV, $p_{T_min} = 35$ GeV



MN dijet x-section ratio within NLL BFKL with BFKLP: collision energy dependence at LHC



A. Egorov & VK, Phys. Rev. D (2023)

NLL BFKL with BFKLP prediction: strong energy dependence



Direct NLL BFKL manifestation



CMS dijet production with large rapidity separation between jets A. Egorov & VK, Phys. Rev. D (2023)

- > **Some indication on BFKL in exclusive dijets production**
 - at LHC 13 TeV at CMS:
 - **Mueller-Tang (MT) dijets**
- > **Some indication with NLL BFKL (BFKLP improved) in Mueller-Navelet (MN) and inclusive dijet in x-section ratios and azimuthal decorrelations at LHC 7 TeV**
 - **MN and inclusive dijet**
- > **The new observation of NLL BFKL (BFKLP improved) in dijets**
 - in at LHC 2.26 TeV
 - **MN dijet x-sections**
 -
 - **Prediction for dijet observables:**
 - - MN dijet x-section energy ratios 8/2.76, 13/2.76 13/8
 - - K-factor with extra jet veto, number of extra jets, ... ?
 - **LHC Run 3 at 13.6 TeV ?!**
 -



Pomeron in pQCD: established NLL BFKL in dijets



New Physics:

- **new particles and interactions beyond SM**
- **new dynamics within SM**

New dynamics within SM:

- **phase transitions at dense baryon matter**
-

**NB. New Physics beyond SM should manifest above
new high energy SM dynamics!**



2D-conformal properties

BFKL(Schredinger eq) as "quantization" of RG-DGLAP (Euler-Lagrange eq)

L.N. Lipatov (1986)

Effective action for reggeized gluons

L.N. Lipatov (1995)

**LL BFKL 2D-conformal block symmetry:
Feynman-like rules for inclusive x-sections**

VK, G.B. Pivovarov (1997)

LL BFKL 2D-conformal block symmetry

H. Navelet, R. Peschanski (1998-1999)

Effective Regge QCD: gluon intercept as RG constant

VK, G.B. Pivovarov (1997)

Feynman rules for Reggeized gluons

E.N. Antonov, E.A. Kuraev, L.N. Lipatov, I. Cherednikov (2005)



LL BFKL Pomeron

2D conformal symmetry and $1/N$ expansion

↳ factorization into integrable theory

high-energy QCD -> integrable system!

L.N. Lipatov (1994)

L.D. Faddeev, G.P. Korchemsky (1994)

LL BFKL Pomeron with $1/N$ expansion

Dipole Pomeron

A.H. Mueller (1994)

N.N. Nikolaev, B.G. Zakharov (1994)

Reggeon field theory with BFKL Pomeron

E.M. Levin, A. Kovner, M. Lublinsky (2024)



kT-factorization

S. Catani, M. Ciafaloni, F. Hautmann (1991)

J.C. Collins, R.K. Ellis (1991)

E.M. Levin, M.G. Ryskin, Yu. Shabelski, M.G. Shuvaev (1991)

G. Salam, H. Jung, N. Raicevic

S.P. Baranov, A.V. Lipatov, M.A. Malyshev, N.P. Zotov, G.I. Lykasov,

V.A. Saleev, A. Shipilova, A. Nefedov, ...

CCFM evolution: interpolates with color coherence between LL BFKL and DGLAP

M. Ciafaloni (1988), S. Catani, F. Fiorani, G. Marchesini (1990)

KMR evolution: interpolates between LL BFKL and DGLAP

M.A. Kimber, A.D. Martin, M.G. Ryskin (1999)



**LL BFKL Pomeron for diffractive physics:
double-Pomeron exchange for dijet, Higgs boson production**
V.A. Khoze, M.G. Ryskin, A.D. Martin (1997-2006)

Pomeron and Reggeon calculus for various processes
V.A. Petrov, R.A. Ryutin, A.A. Godizov



SUSY N=2 NLL BFKL Pomeron

A.V. Kotikov, L.N. Lipatov (2000)

AdS/CFT-correspondence test with anomalous dimensions

A.V. Kotikov, L.N. Lipatov, A. Onischenko, V. Velizhanin (2002-2006)

Graviton-Pomeron duality

C.-I. Tan, C. Brower (2006)

L. Alvarez-Gaume et al. (2007)

Pomeron by quark and gluons condensates

P.V. Landshoff, O. Nachtmann (1987)

Non-planar Pomeron in QCD with $1/N$ expansion:

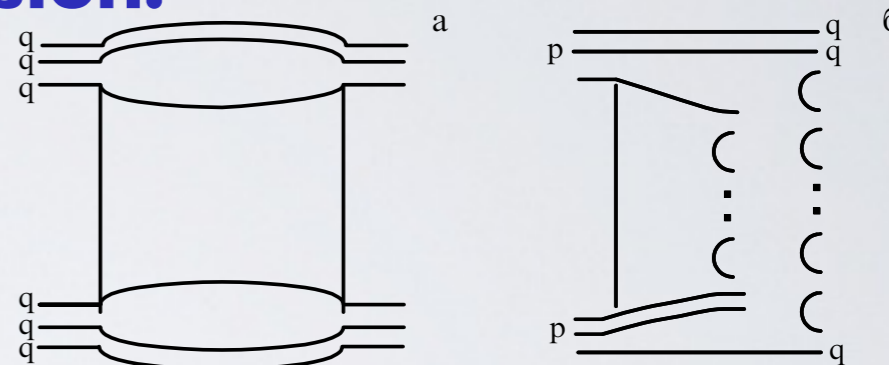
G.Veneziano (1977)

- dual parton model

A.Capella, J. Tran Thanh Van (1981)

- quark-gluon string model (QGSM)

A.B. Kaidalov, K.A. Ter-Martirosyan (1982)



Unitarity with $1/N$ expansion for saturation limit

Balitsky-Kovchegov equation

with $\alpha_s \rightarrow 0$: reproduces BFKL

I.I. Balitsky (1996) Yu. Kovchegov (1999, 2000)

Color Glass Condensate evolution for saturation limit

with $\alpha_s \rightarrow 0$: reproduces BFKL

L. McLerran, R. Venugopalan (1994) H. Weigert, A. Kovner, A. Leonidov (2001)

F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan (2010)



- **BFKL reproduces main classical Pomeron properties bringing new remarkable features: conformality, integrability, AdS/CFT duality, holographic properties ...**
- **NLL BFKL manifests in dijet production with large rapidity separation in CMS data at LHC 2.76 TeV**
- **New Physics beyond SM should manifest within BFKL: the new high energy SM dynamics!**