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Crab Waist Collision

Very tentative estimation of the Z-factory Crab Waist e+ecollider with 45.6 GeV beam energy and 20.8 km orbit length. Configuration, magnetic lattice, parameters.

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Motivation

- On 2006 Pantaleo Raimondi proposed a novel collision technology Crab Waist, which theoretically promises luminosity of e+e- collider 1-2 order higher then reached before.
- Since that time all new circular e+e- colliders exploit the CW method including SuperB and Super C-Tau (Italy), Super C-Tau (Novosibirsk and China), FCC-ee (CERN), CEPC (China), etc.
- The method was tested at DAΦNE (partially) and Super KEKB.
- Collaboration in fundamental physics between Russia and Europe/US/Japan is suspended.
- Russian Government has started new mega-science program on particle physics.
- There is a ready circular UNK tunnel in Protvino with circumference enough for Z-factory.



МИНИСТЕРСТВО НАУКИ И ВЫСШЕГО ОБРАЗОВАНИЯ РОССИЙСКОЙ ФЕДЕРАЦИИ (МИНОБРНАУКИ РОССИИ)

протокол

совещания по привлечению российских ученых и специалистов, участвующих в экспериментах ЦЕРН, к работе в проектах на территории Российской Федерации свойств материи

от «4» апреля 2024 г.

04.04.2024 _№ КМ/8-пр

Москва

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4. В целях формирования единого пространства для развития фундаментальных исследований рекомендовать департаментам Минобрнауки России (Даутов Р. М., Киреев В. В., Канукоев А.С., Шашкин А.П., Швед К.А.) проработать предложение НИЦ «Курчатовский институт» о создании совместно с заинтересованными российскими научными и образовательными организациями Перспективной программы развития фундаментальных ядерных и нейтринных исследований в Российской Федерации.

Председатель:

К.И. Могилевский

Goal

The goal is to study (very preliminary!) parameters of the e+e- collider in the UNK tunnel with the maximum beam energy 45.6 GeV ($M_z = 91.1876$ GeV). To reach high luminosity we apply the CW collision.

As a fundamental limitation, we put ≤50 MW SR power per beam (like in FCC-ee and CEPC).

For less SR power limit, we can reduce the bunch number and luminosity drops linearly.

FCC-ee baseline (100 km)

Parameter	Z
beam energy [GeV]	45
beam current [mA]	1280
number bunches/beam	10000
bunch intensity [10 ¹¹]	2.43
SR energy loss / turn [GeV]	0.0391
total RF voltage 400/800 MHz [GV]	0.120/0
long. damping time [turns]	1170
horizontal beta* [m]	0.1
vertical beta* [mm]	0.8
horizontal geometric emittance [nm]	0.71
vertical geom. emittance [pm]	1.42
horizontal rms IP spot size [μm]	8
vertical rms IP spot size [nm]	34
luminosity per IP [10 ³⁴ cm ⁻² s ⁻¹]	182
total integrated luminosity / year [ab ⁻¹ /yr] 4 IPs	87
beam lifetime (rad Bhabha + BS+lattice)	8
· · · · ·	4 years

References

LEP

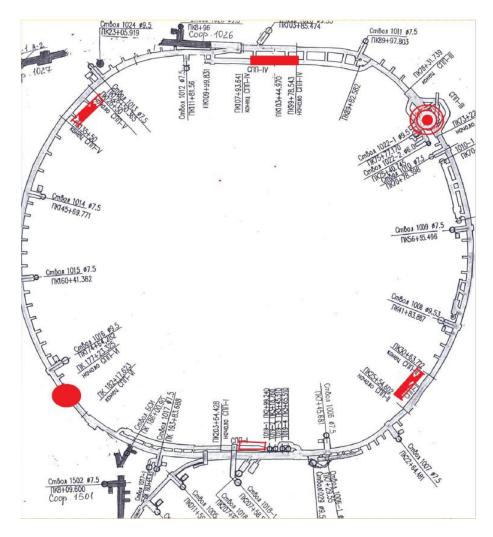
	(LEP1)	(LEP2)
Circumference [km]	26.7	26.7
Bending radius [km]	3.1	3.1
Beam energy [GeV]	45.4	104
Beam current [mA]	2.6	3.04
Bunches / beam	12	4
Bunch population [1011]	1.8	4.2
Transverse emittance ε		
- Horizontal [nm]	20	22
- Vertical [pm]	400	250
Momentum comp. [10 ⁻⁵]	18.6	14
Betatron function at IP β*		
- Horizontal [m]	2	1.2
- Vertical [mm]	50	50
Beam size at IP σ* [μm]		
- Horizontal	224	182
- Vertical	4.5	3.2
Bunch length [mm]		
 Synchrotron radiation 	8.6	11.5
- Total	8.6	11.5
Energy loss / turn [GeV]	0.12 ⁽¹⁾	3.34
SR power / beam [MW]	0.3(1)	11
Total RF voltage [GV]	0.24	3.5
RF frequency [MHz]	352	352
Longitudinal damping time τ_E [turns]	371	31
Energy acceptance RF [%]	1.7	0.8
Synchrotron tune Qs	0.065	0.083
Polarization time τ_p [min]	252	4
Hourglass factor H	1	1
Luminosity/IP [10 ³⁴ cm ⁻² s ⁻¹]	0.002	0.012
Beam-beam parameter		
- Horizontal	0.044	0.040
- Vertical	0.044	0.040
Luminosity lifetime [min]	1750	434

Infrastructure

Alexandre Zaitsev gave us sketches of the UNK tunnel, which we used to accommodate the collider (many thanks!).

We don't have tunnel blueprints, therefore results are very tentative.

Длина 20.772 км Прямолинейные промежутки СПП-1, 4 800 м СПП-2, 3, 5. 6 490 м 8 дуг 40° длина 1.84 км каждая (радиус кривизны 2.6 км) Глубина от 24 до 67 м Стволов доступа 27



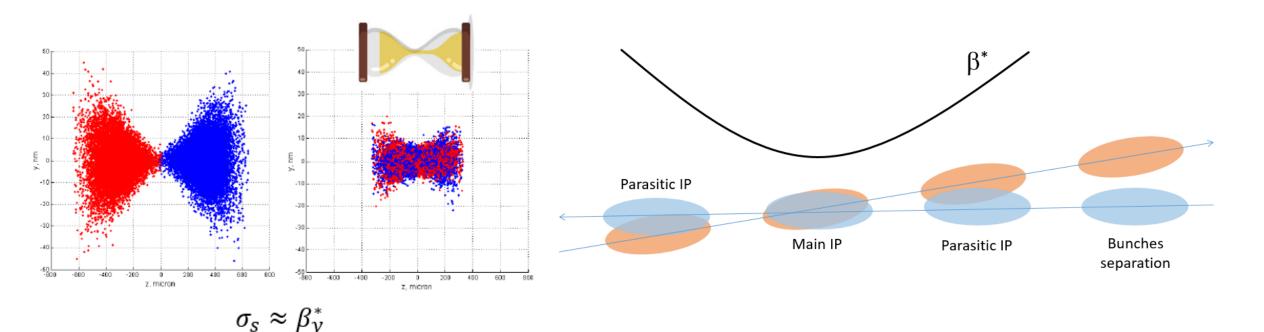




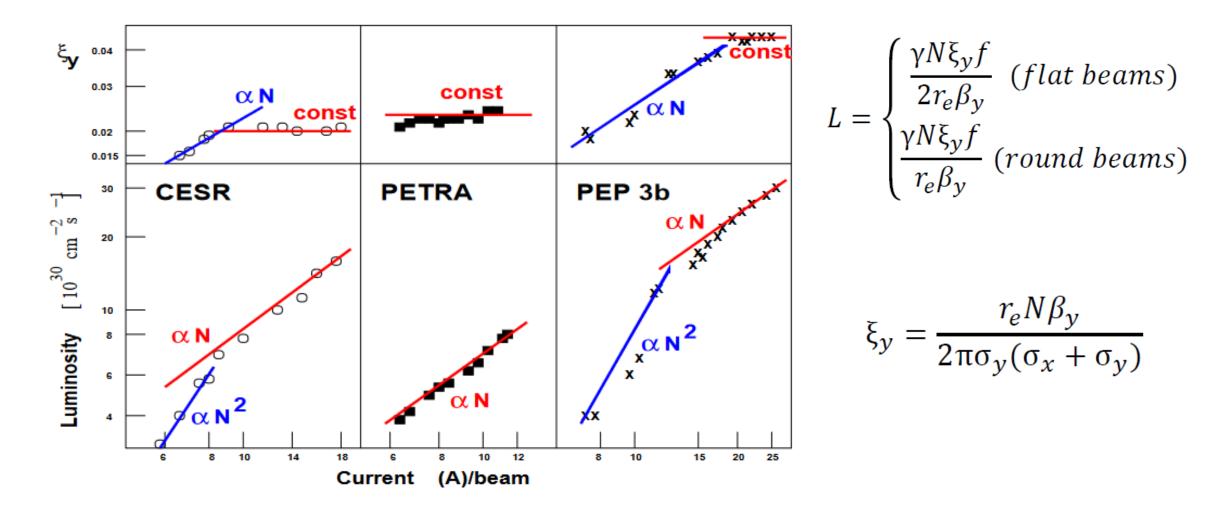


Luminosity limitation in head on collision

- Nonlinear force of the incident beam limits $\xi \le 0.07$ (≤ 0.13 for round beams).
- Hour-glass effect limits $\beta^* \approx \sigma_s$ while high current and collective effects prevent the bunch length shortening, single bunch luminosity drops down.
- "Parasitic" IPs reduce collision frequency and total luminosity.



Experimental limitation of $\xi_v \leq 0.07$ (flat beam)

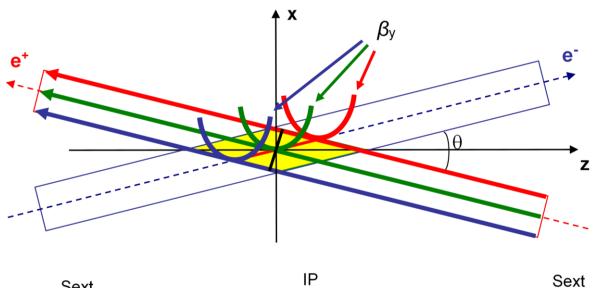


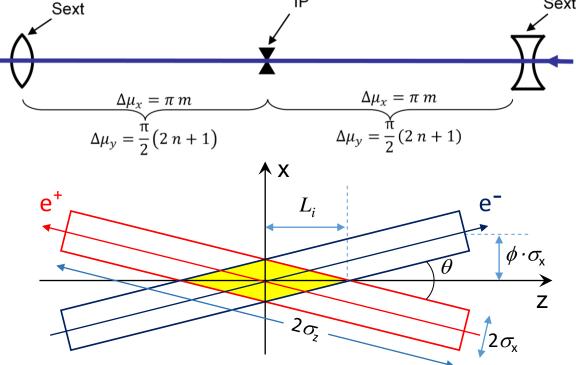
Luminosisty
$$L = \frac{\gamma}{2er_e} \cdot \frac{I_{tot}\xi_y}{\beta_y^*} R_H$$

Large Piwinski angle
$$\phi = \frac{\sigma_z}{\sigma_x} \tan\left(\frac{\theta}{2}\right) \gg 1$$

- 1. No parasitic crossings
- 2. Interaction area $L_i \ll \sigma_z$
 - $\beta_y^* \approx L_i \ll \sigma_z$ No hour-glass
- 3. CRAB waist (CRAB sextupoles) suppress coupling betatron and synchrobetatron resonances

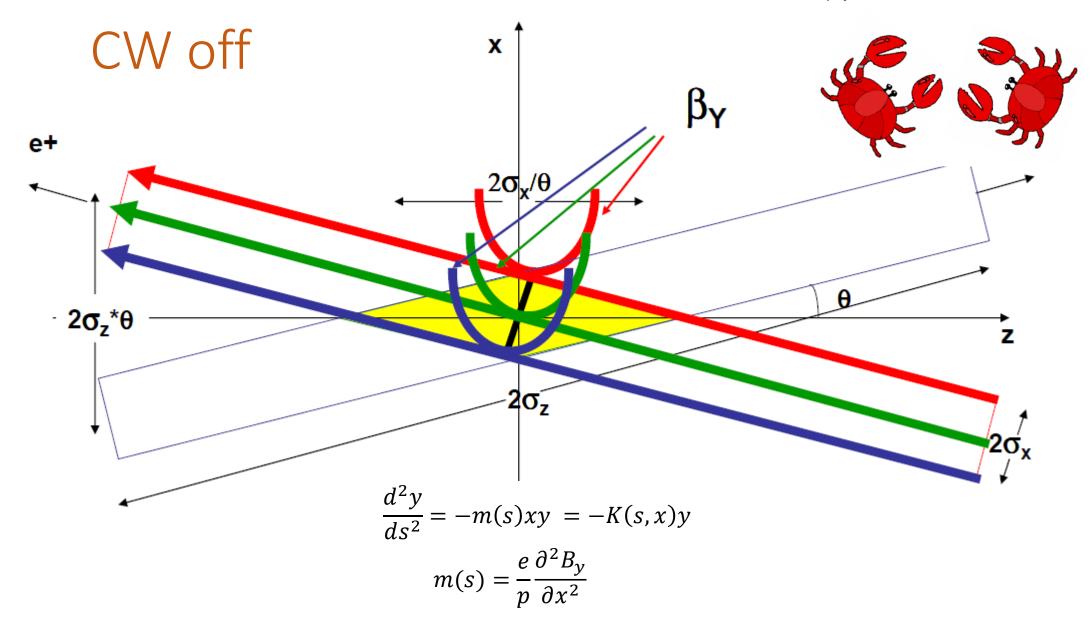
•
$$\xi_y \sim 0.2$$





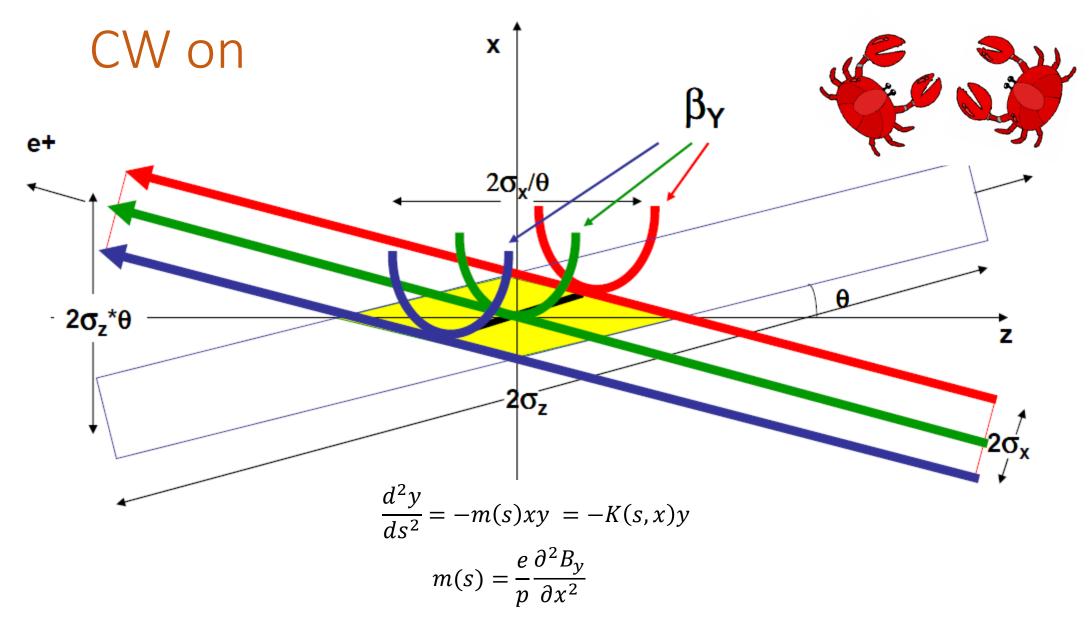
1. *P.Raimondi,* 2°*SuperB Workshop, March* 2006

2. *P.Raimondi, D.Shatilov, M.Zobov, physics/0702033*

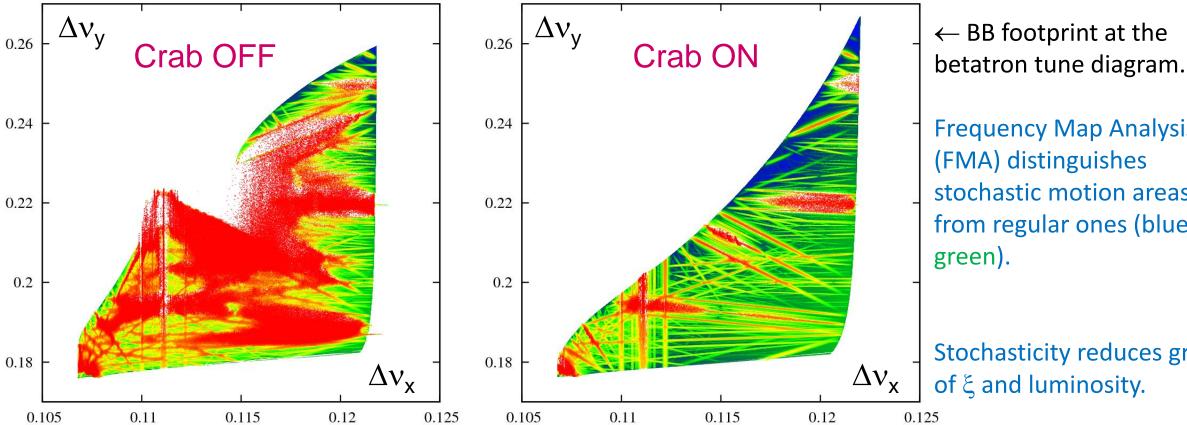


1. *P.Raimondi,* 2°*SuperB Workshop, March* 2006

2. *P.Raimondi, D.Shatilov, M.Zobov, physics/0702033*



Crab Waist suppresses coupling resonances in the nonlinear interaction force.



Frequency Map Analysis (FMA) distinguishes stochastic motion areas (red) from regular ones (blue,

Stochasticity reduces growth of ξ and luminosity.

E.Levichev, D.Shatilov and E.Simonov, e-Print: arXiV:1002.3733, also IPAC10, THPE075

- Simulation gives the CW beam-beam parameter of $\xi \approx 0.18.$
- No Hour-glass effect; due to the short crossing area we can have at the IP β^* = 1 mm and even less.
- No parasitic crossings.

On paper, CW yields 50-100 times luminosity increase compare to the head-on collision. BUT...

- Low emittance lattice reduces dynamic aperture.
- Strong Crab Sextuples reduces dynamic aperture.
- Low emittance, short bunch length and high current led to the strong Intrabeam Scattering and short beam lifetime (for low energy collider only!).
- Very low IP beta ⇒ very high beta at the FF quads ⇒ FF quads nonlinearities suppress the DA.
- Very strong SC FF quads should be placed inside the detector as near as possible to the IP
 ⇒ technical challenge.
- High current, collective instabilities.

50 MW SR power limit: what follows?

 $P_{SR} = I \cdot U_0 \le 50 \text{ MW} - \text{total SR power/ring}$

 $U_0 = \frac{C_{\gamma}}{2\pi} E^4 \oint \frac{ds}{\rho^2} \propto \frac{E^4}{\rho}$ SR loss per turn depends on the ring size (2 $\pi\rho$)

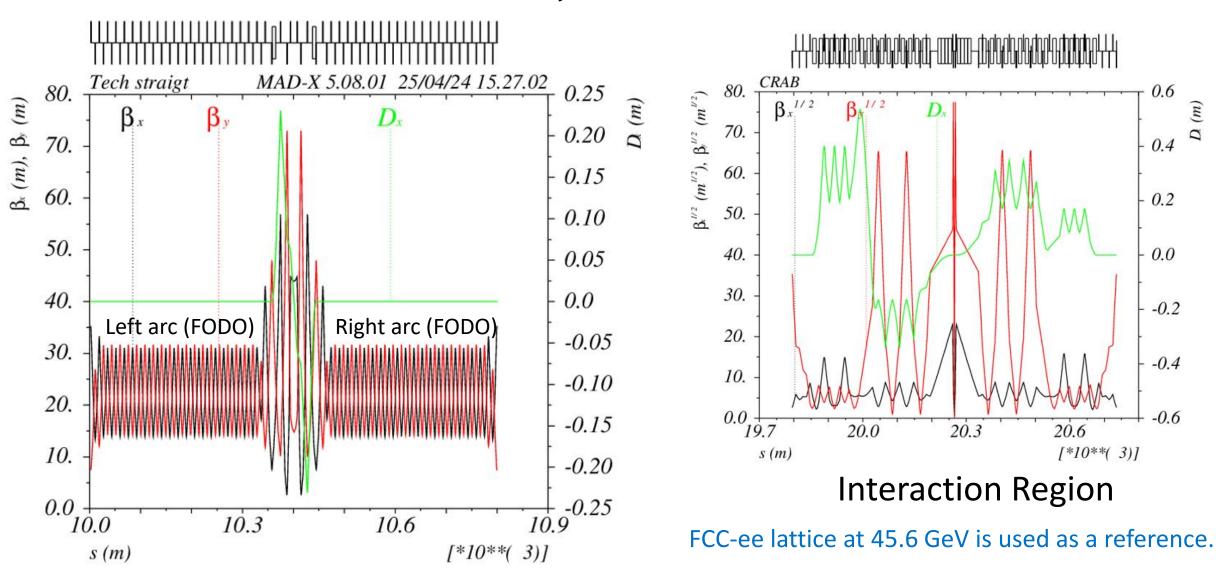
$$L = \frac{\gamma}{2er_e} I \cdot \frac{\xi_y}{\beta_y^*} \propto P_{SR} \frac{\rho}{E^3} \frac{\xi_y}{\beta_y^*}$$

For the same SR power, energy, beam-beam parameter and beta at the IP, the UNK luminosity will be

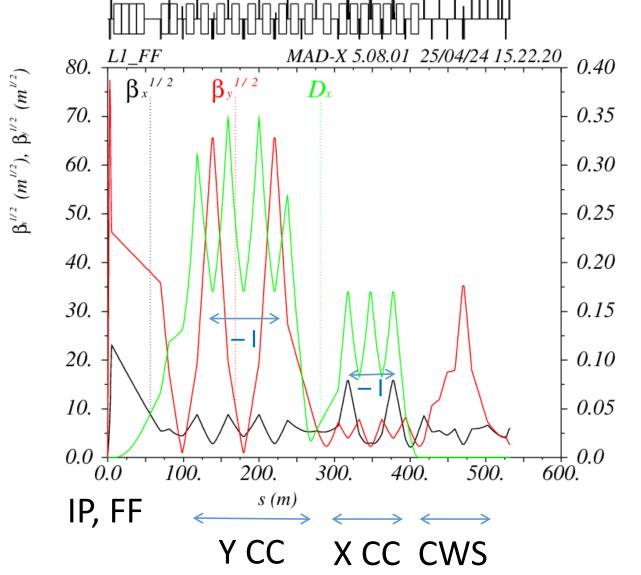
$$\frac{C_{FCC}}{C_{UNK}} \approx \frac{100 \text{ km}}{20 \text{ km}} = 5$$
 less than for FCC-ee.

ZUNK lattice

CRAB: $\beta_x = 7.29810791104132$, $\beta_y = 1247.10540609207$, $K2_{crab} = -4.99815236728314$



ZUNK lattice, IR



$$\beta_{y}^{*} = 1 \text{ mm at the IP}$$

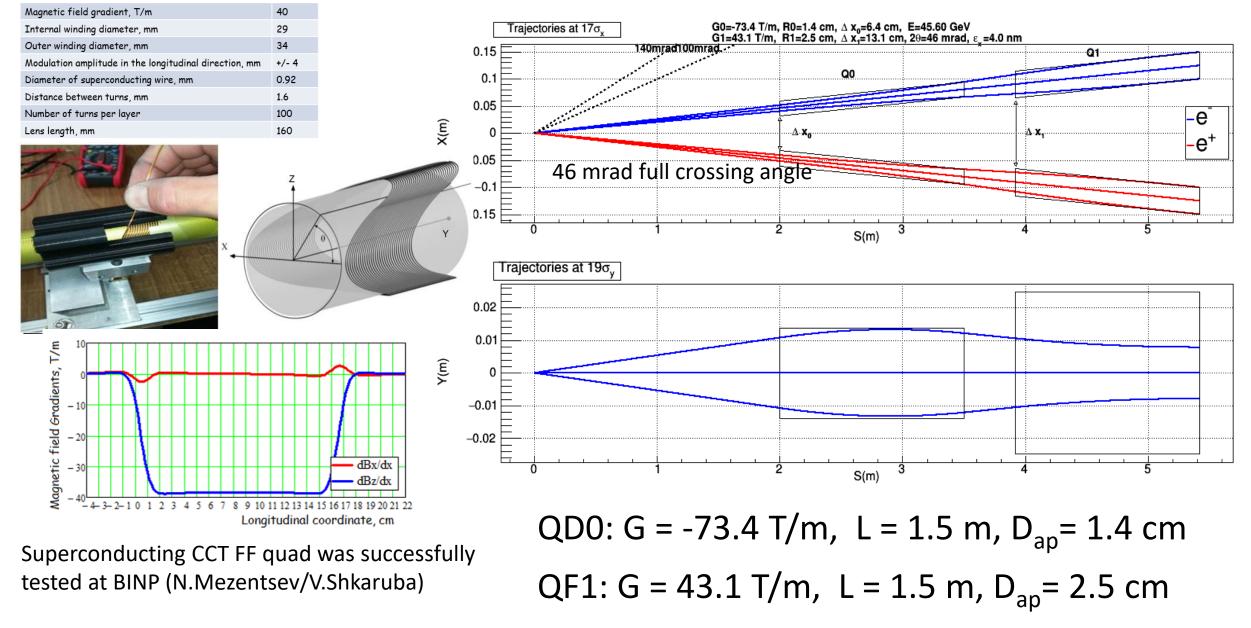
$$\beta_{FF} = 6 \text{ km at the QDO}$$

$$\beta_{YCC} = 4.3 \text{ km at the Y}$$
chromaticity
correction section

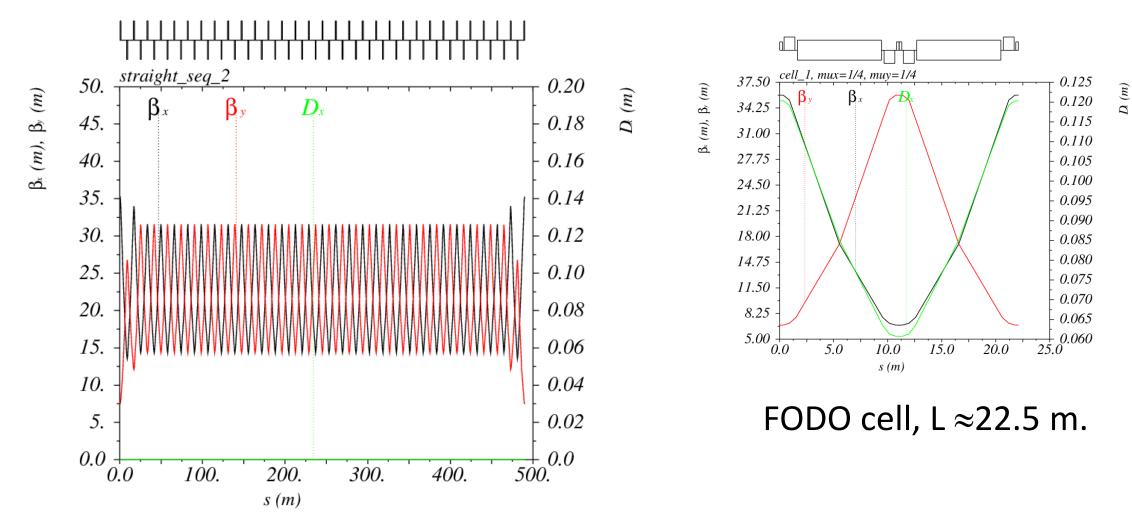
$$\beta_{CWS} = 1.2 \text{ km at theCrab}$$
Sextupoles

FF trajectory at E=45.6 GeV

Parameters of CCT quadrupole lens prototype:

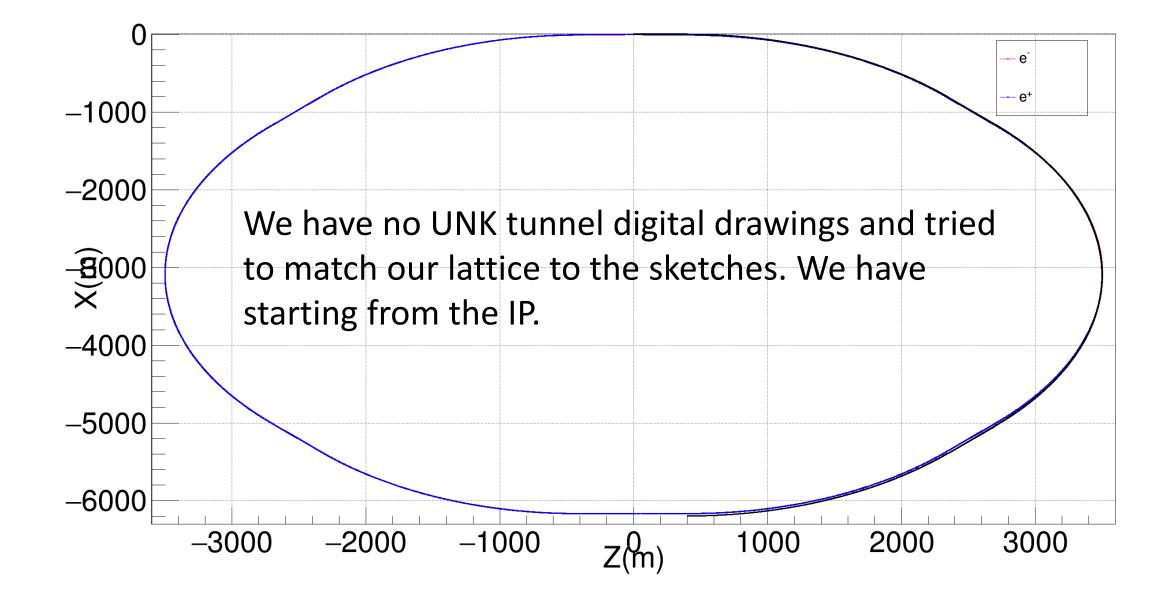


ZUNK lattice. Arcs

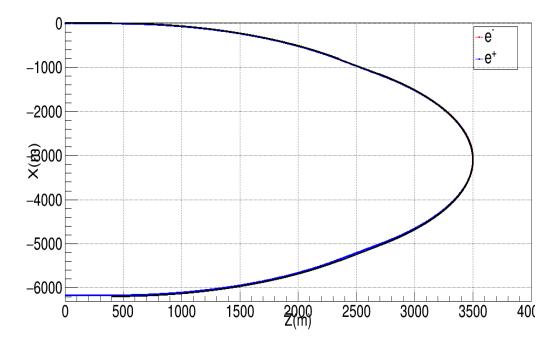


Arc lattice, simple FODO.

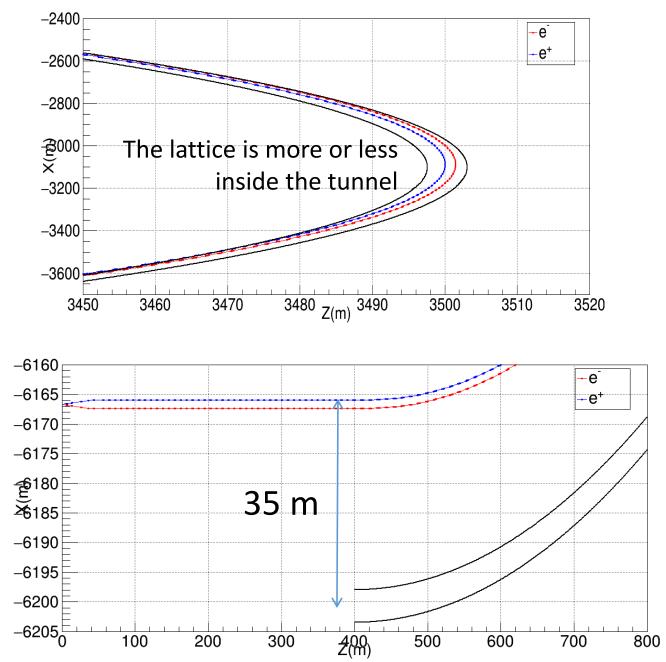
ZUNK Layout



ZUNK Layout details



The maximum discrepancy between the lattice and the tunnel is 35 m. There is no sense to adjust it more precisely at the moment.



ZUNK parameters

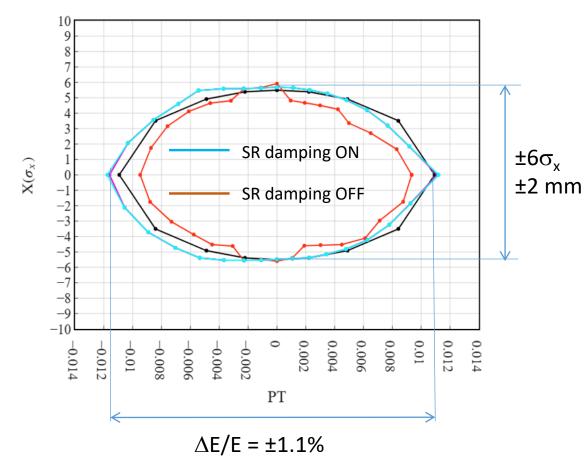
Parameter	LEP	FCC-ee	CEPC	ZUNK
Circumference (km)	26.7	91.1	100	20.8
Beam current (mA)	2.6	1280	803	232
Bunch No	12	10000	11951	332
Bunch intensity (10 ¹¹)	1.8	2.43	1.4	3
SR energy loss/turn (MeV)	120	39.1	37	216
RF voltage (MV)	240	120	120	250
Bunch length (mm)	8.6	12	8.7	11
Horizontal beta [*] (m)	2	0.1	0.13	0.15
Vertical beta [*] (mm)	50	0.8	0.9	1
Horizontal emittance (nm)	20	0.71	0.27	4
Vertical emittance (pm)	400	1.42	1.4	40
Horizontal spot size [*] (µm)	224	8	6	24.4
Vertical spot size [*] (µm)	4.5	0.034	0.035	0.2
Long.damping time (turns)	371	1170	340	211
Beam-beam parameter	0.044	0.12	0.127	0.06
Luminosity/IP (10 ³⁴ cm ⁻² s ⁻¹)	0.002	152	115	11.5

- 50 MW full SR power/ring.
- BB parameter ≤ 0.07
- Beta_y at IP 1 mm
- Emittances are not too low.
- Coupling is not too small.

Dynamic Issues (radiation ON, $f_0 \tau_x = 423$)

Very first test of dynamic aperture: Crab Sextupoles, Chromatic Sextupoles, Quad fringe field. No optimization.

6d-DA, $y_0 = \sigma_y, \sigma_x = 3.74e - 04m, \sigma_e = 9.94e - 04$



At the azimuth of the DA plot

$$\beta_x$$
 = 35.2 m, β_y = 7.5 m, No dispersion.
 σ_x = 373.7 µm, σ_y = 17.2 µm

Strong SR damping slightly increase the DA.

$$A_x = \pm 6\sigma_x \quad \Delta E/E = \pm 1.1\%$$

Not so bad for the first run.

Main magnets

Arc dipoles:	L=7.788 m,	B=0.08 T,	ϕ =4.1 mrad
Arc quadrupoles:	L=1 m,	G=15.2 T/m,	
Arc quadrupoles:	L=1 m,	G=-15.1 T/m,	
Arc sextupoles:	L=0.5 m,	S=-800 T/m^2	
Arc sextupoles:	L=0.5 m,	S=400 T/m^2	
Crab sextupoles:	L=0.5 m,	S=760 T/m^2	
RCSY sextupole:	L=0.5 m,	S=740 T/m^2	
LCSY sextupole:	L=0.5 m,	S=-711 T/m^2	
RCSX sextupole:	L=0.5 m,	S=288 T/m^2	
LCSX sextupole:	L=0.5 m,	S=611 T/m^2	

All magnets are simple and easy.

Injection

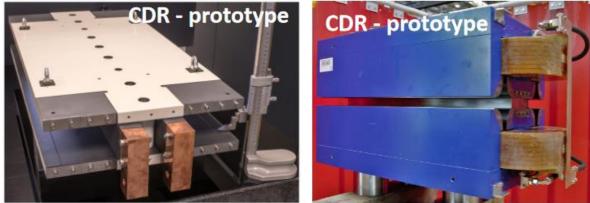
- 6 GeV linac system (like for the SILA light source).
- Full energy booster synchrotron inside the UNK tunnel.

Conclusion

- Zero approximation of the e+e- Z-factory in the UNK tunnel is presented.
- A Crab Waist solution is matched the UNK tunnel more or less well.
- At 45.6 GeV estimation shows ~10³⁵cm⁻²s⁻¹ peak luminosity with moderate beam parameters (×2 with aggressive ones).

Back up slides.

Cost (order of magnitude)



FCC (2021): FCC-ee and FCC-INT cost estimates

Double aperture single block FCC-ee magnets.

Domain	Cost in MCHF	
Stage 1 - Civil Engineering	5,400	──→ Tunnels are expensive
Stage 1 - Technical Infrastructure	2,200	
Stage 1 - FCC-ee Machine and Injector Complex	4,000	\longrightarrow ZUNK = 1/4 X FCC-ee = 100 BRUB??

Total construction cost FCC-ee (Z, W, H) amounts to 10.5 BCHF + 1.1 BCHF (tt)

- Associated to a total project duration of ~20 years (2025 – 2045)

Chinese CEPC costs about half of FCC-ee: 36.4 BRMB or US\$5B.

Cost (order of magnitude)

 $CostUNK = Cost \frac{20 \text{ km} \times (1.5 \text{ collider} + 1 \text{ injector})}{\text{Length}}$ $CostUNK = Cost \frac{20 \text{ km}}{\text{Length}}$

For single ring light source.

For double ring collider + injector.

Project	Length (km)	Cost (BRUB)	Cost ZUNC
SKIF	0.5	8	800
ESRF EBS	0.8	15	940
Super C-Tau ¹⁾	1	33	660
FCC-ee	91	400	88(?)

¹⁾ +Detector cost ≈20 BRUB

Замечания

- Светимость ограничена мощностью
- Необходимо удлинять пучок
 - Резонатор 3-ей гармоники (UNKZ5)
 - Меньшая частота ВЧ
 - UNKZ3 и UNKZ5 отличаются гармоникой ВЧ, сравнение

LILLE	UNKZI	UNKZZ	UNKZO	UNKZ4	UNK25. 505	UNKZO
E, Gev	45.6	45.6	45.6	45.6	45.6	45.6
Π,m	20797.7	20797.7	20797.7	20797.7	20797.7	20797.7
∂, rad	0.023	0.023	0.023	0.023	0.023	0.023
f0,Hz	14414.7	14414.7	14414.7	14414.7	14414.7	14414.7
I,A	1.49985	0.229999	0.231616	0.461846	0.228614	0.231154
Ν	3.×10 ¹¹	3.×10 ¹¹	1.003×10^{11}	1.×10 ¹¹	1.5×10 ¹²	1.43×10^{11}
Nb	2165.	332.	1000.	2000.	66.	700.
q	27749.	27749.	27749.	27749.	27749.	13874.
Frf, Hz	3.99993×10 ⁸	3.99993×10 ⁸	3.99993×10 ⁸	3.99993×10 ⁸	3.99993×10 ⁸	1.99989×10^{8}
U0, GeV/turn	0.216234	0.216234	0.216234	0.216234	0.216234	0.216234
Vrf, GeV	0.25	0.25	0.25	0.25	0.236	0.25
φs°	120.125	120.125	120.125	120.125	113.617	120.125
η	0.0000264039	0.0000264039	0.0000264039	0.0000264039	0.0000264039	0.0000264039
γs	0.0179125	0.0179125	0.0179125	0.0179125	0.0155491	0.0126658
δrf	0.0149823	0.0149823	0.0149823	0.0149823	0.0101587	0.0211886
σe	0.0011138	0.0011138	0.00100089	0.00100083	0.00122764	0.00100115
σs	0.00543445	0.00543445	0.00488355	0.00488324	0.020701	0.00690826
σχ	0.0000244175	0.0000244175	0.0000244175	0.0000244175	0.0000244175	0.0000244175
σγ	1.99368×10 ⁻⁷	1.99368×10-7	1.99368×10-7	1.99368×10-7	1.99368×10 ⁻⁷	1.99368×10-7
σ x'	0.000162784	0.000162784	0.000162784	0.000162784	0.000162784	0.000162784
σу'	0.000199368	0.000199368	0.000199368	0.000199368	0.000199368	0.000199368
∈x,m rad	3.97477×10 ⁻⁹	3.97477×10 ⁻⁹	3.97477×10 ⁻⁹	3.97477×10 ⁻⁹	3.97477×10 ⁻⁹	3.97477×10 ⁻⁹
∈y,m rad	3.97477×10 ⁻¹¹	3.97477×10 ⁻¹¹	3.97477×10 ⁻¹¹	3.97477×10 ⁻¹¹	3.97477×10 ⁻¹¹	3.97477×10 ⁻¹
ey/ex	0.01	0.01	0.01	0.01	0.01	0.01
βx,m	0.15	0.15	0.15	0.15	0.15	0.15
βy,m	0.001	0.001	0.001	0.001	0.001	0.001
L _{int} , m	0.00130565	0.00130565	0.00129997	0.00129997	0.00132858	0.00131489
$L1/IP, cm^{-2}s^{-1}$	7.50267×10 ³⁵	1.15053×10 ³⁵	4.2959×10^{34}	8.54099×10^{34}	1.52164×10 ³⁵	4.35982×10^{34}
L0/IP, cm ⁻² s ⁻¹	8.80136×10 ³⁵	1.34968×10 ³⁵	5.03475×10 ³⁴	1.001 × 10 ³⁵	1.79184×10^{35}	5.12235×10^{34}
L1/L0	0.852445	0.852445	0.853249	0.85325	0.849206	0.851138
NTP	1.	1.	1.	1.	1.	1.
ξX	0.0139175	0.0139175	0.00571109	0.0056947	0.00497138	0.00416501
ξy	0.0593721	0.0593721	0.0219932	0.0219288	0.0793007	0.0224206
φ	5.11986	5.11986	4.60085	4.60056	19.5026	6.50836
vs/ξx	1.28705	1.28705	3.13644	3.14547	3.12772	3.04101
τε , turn	210.882	210.882	210.882	210.882	210.882	210.882
æ	0.01	0.01	0.01	0.01	0.01	0.01
τ b, s	2.29367×10^{7}	2.29367×10 ⁷	5.57648×10 ²³	6.63566×10 ²³	90121.2	3.4733×10 ²³
τ _L , s						13369.
	5040.77	5040.77	13595.1	13635.	3788.4	
u _c /En	5040.77 0.000465708	5040.77 0.000465708	13595.1 0.000173266	13635. 0.000172759	3788.4 0.000611291	
	0.000465708	0.000465708			0.000611291	0.000174628
u _c ∕ En ℕ	0.000465708 3.02457×10 ⁻¹²	0.000465708 3.02457×10 ⁻¹²	0.000173266 1.24404×10 ⁻²⁸	$\frac{0.000172759}{1.04547 \times 10^{-28}}$	0.000611291 7.69783×10 ⁻¹⁰	0.000174628 1.99734×10 ⁻²
u _c ∕En ℕ ₽, m	0.000465708 3.02457×10 ⁻¹² 9.90453	0.000465708 3.02457×10 ⁻¹² 9.90453	0.000173266 1.24404×10 ⁻²⁸ 26.6216	0.000172759 1.04547×10 ⁻²⁸ 26.6998	0.000611291 7.69783×10 ⁻¹⁰ 7.5457	0.000174628 1.99734×10 ⁻² 26.4139
u _c /En N 0, m I2, m ⁻¹	0.000465708 3.02457×10 ⁻¹² 9.90453 0.0035516	$\begin{array}{c} 0.000465708\\ 3.02457 \times 10^{-12}\\ 9.90453\\ 0.0035516\end{array}$	0.000173266 1.24404×10 ⁻²⁸ 26.6216 0.0035516	0.000172759 1.04547×10 ⁻²⁸ 26.6998 0.0035516	0.000611291 7.69783×10 ⁻¹⁰ 7.5457 0.0035516	0.000174628 1.99734×10 ⁻² 26.4139 0.0035516
u _c /En N P, M I2, m ⁻¹ I3, m ⁻²	$\begin{array}{c} 0.000465708\\ 3.02457 \times 10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881 \times 10^{-6} \end{array}$	$\begin{array}{c} 0.000465708\\ 3.02457\times 10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881\times 10^{-6} \end{array}$	$\begin{array}{c} 0.000173266\\ 1.24404 \times 10^{-28}\\ 26.6216\\ 0.0035516\\ 2.29881 \times 10^{-6} \end{array}$	$\begin{array}{c} \textbf{0.000172759} \\ \textbf{1.04547} \times \textbf{10}^{-28} \\ \textbf{26.6998} \\ \textbf{0.0035516} \\ \textbf{2.29881} \times \textbf{10}^{-6} \end{array}$	$\begin{array}{c} 0.000611291 \\ \overline{7.69783 \times 10^{-10}} \\ \overline{7.5457} \\ 0.0035516 \\ 2.29881 \times 10^{-6} \end{array}$	0.000174628 1.99734×10 ⁻² 26.4139 0.0035516 2.29881×10 ⁻⁶
u _c /En N P, M I2, m ⁻¹ I3, m ⁻² △I2, m ⁻¹	$\begin{array}{c} 0.000465708\\ 3.02457 \times 10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881 \times 10^{-6}\\ 0.0000133094 \end{array}$	$\begin{array}{c} 0.000465708\\ 3.02457\times 10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881\times 10^{-6}\\ 0.0000133094 \end{array}$	$\begin{array}{c} 0.000173266\\ 1.24404 \times 10^{-28}\\ 26.6216\\ 0.0035516\\ 2.29881 \times 10^{-6}\\ 1.83428 \times 10^{-6} \end{array}$	$\begin{array}{c} 0.000172759\\ 1.04547\times 10^{-28}\\ 26.6998\\ 0.0035516\\ 2.29881\times 10^{-6}\\ 1.82355\times 10^{-6}\\ \end{array}$	$\begin{array}{c} 0.000611291 \\ \overline{7.69783 \times 10^{-10}} \\ \overline{7.5457} \\ 0.0035516 \\ 2.29881 \times 10^{-6} \\ 0.000023334 \end{array}$	0.000174628 1.99734×10 ⁻² 26.4139 0.0035516 2.29881×10 ⁻¹ 1.88463×10 ⁻¹
u _c /En N ρ, m I2, m ⁻¹ I3, m ⁻² ΔI2, m ⁻¹ ΔI3, m ⁻²	$\begin{array}{c} 0.000465708\\ 3.02457 \times 10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881 \times 10^{-6}\\ 0.0000133094\\ 1.34377 \times 10^{-6} \end{array}$	$\begin{array}{c} 0.000465708\\ 3.02457\times10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881\times10^{-6}\\ 0.0000133094\\ 1.34377\times10^{-6} \end{array}$	$\begin{array}{c} 0.000173266\\ 1.24404 \times 10^{-28}\\ 26.6216\\ 0.0035516\\ 2.29881 \times 10^{-6}\\ 1.83428 \times 10^{-6}\\ 6.89019 \times 10^{-8} \end{array}$	$\begin{array}{c} 0.000172759\\ 1.04547 \times 10^{-28}\\ 26.6998\\ 0.0035516\\ 2.29881 \times 10^{-6}\\ 1.82355 \times 10^{-6}\\ 6.82982 \times 10^{-8} \end{array}$	$\begin{array}{c} 0.000611291 \\ \overline{7.69783 \times 10^{-10}} \\ \overline{7.5457} \\ 0.0035516 \\ 2.29881 \times 10^{-6} \\ 0.000023334 \\ \overline{3.09235 \times 10^{-6}} \end{array}$	0.000174628 1.99734×10 ⁻² 26.4139 0.0035516 2.29881×10 ⁻¹ 1.88463×10 ⁻¹ 7.13498×10 ⁻¹
u _c /En N ρ, m I2, m ⁻¹ I3, m ⁻² ΔI2, m ⁻¹ ΔI3, m ⁻² ΔI2/I2	$\begin{array}{c} 0.000465708\\ 3.02457\times10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881\times10^{-6}\\ 0.0000133094\\ 1.34377\times10^{-5}\\ 0.00374693 \end{array}$	$\begin{array}{c} 0.000465708\\ 3.02457\times10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881\times10^{-6}\\ 0.0000133094\\ 1.34377\times10^{-6}\\ 0.00374693 \end{array}$	$\begin{array}{c} 0.000173266\\ 1.24404 \times 10^{-28}\\ 26.6216\\ 0.0035516\\ 2.29881 \times 10^{-6}\\ 1.83428 \times 10^{-6}\\ 6.89019 \times 10^{-8}\\ 0.000516394 \end{array}$	$\begin{array}{c} 0.000172759\\ 1.04547\times 10^{-28}\\ 26.6998\\ 0.0035516\\ 2.29881\times 10^{-6}\\ 1.82355\times 10^{-6}\\ 6.82982\times 10^{-8}\\ 0.000513373\\ \end{array}$	$\begin{array}{c} 0.000611291 \\ \overline{7.69783 \times 10^{-10}} \\ \overline{7.5457} \\ 0.0035516 \\ 2.29881 \times 10^{-6} \\ 0.000023334 \\ \overline{3.09235 \times 10^{-6}} \\ 0.00656907 \end{array}$	0.000174628 1.99734×10 ⁻² 26.4139 0.0035516 2.29881×10 ⁻¹ 1.88463×10 ⁻¹ 7.13498×10 ⁻¹ 0.000530568
u _c /En N ρ, m I2, m ⁻¹ I3, m ⁻² ΔI2, m ⁻¹ ΔI3, m ⁻² ΔI2/I2 ΔI3/I3	$\begin{array}{c} 0.000465708\\ 3.02457 \times 10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881 \times 10^{-6}\\ 0.0000133094\\ 1.34377 \times 10^{-5}\\ 0.00374693\\ 0.46527\\ \end{array}$	$\begin{array}{c} 0.000465708\\ 3.02457\times10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881\times10^{-6}\\ 0.0000133094\\ 1.34377\times10^{-6}\\ 0.00374693\\ 0.46527\\ \end{array}$	$\begin{array}{c} 0.000173266\\ 1.24404 \times 10^{-28}\\ 26.6216\\ 0.0035516\\ 2.29881 \times 10^{-6}\\ 1.83428 \times 10^{-6}\\ 6.89019 \times 10^{-8}\\ 0.000516394\\ 0.0295428\\ \end{array}$	$\begin{array}{c} 0.000172759\\ 1.04547 \times 10^{-28}\\ 26.6998\\ 0.0035516\\ 2.29881 \times 10^{-6}\\ 1.82355 \times 10^{-6}\\ 6.82982 \times 10^{-8}\\ 0.000513373\\ 0.0292876\\ \end{array}$	$\begin{array}{c} 0.000611291\\ \overline{}7.69783\times10^{-10}\\ \overline{}7.5457\\ \overline{}0.0035516\\ 2.29881\times10^{-6}\\ \overline{}0.00023334\\ 3.09235\times10^{-6}\\ \overline{}0.00656907\\ \overline{}.881337\end{array}$	0.000174628 1.99734×10 ⁻² 26.4139 0.0035516 2.29881×10 ⁻¹ 1.88463×10 ⁻¹ 7.13498×10 ⁻¹ 0.000530568 0.0305768
u _c /En N ρ, m I2, m ⁻¹ I3, m ⁻² ΔI2, m ⁻¹ ΔI3, m ⁻² ΔI2/I2	$\begin{array}{c} 0.000465708\\ 3.02457\times10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881\times10^{-6}\\ 0.0000133094\\ 1.34377\times10^{-5}\\ 0.00374693 \end{array}$	$\begin{array}{c} 0.000465708\\ 3.02457\times10^{-12}\\ 9.90453\\ 0.0035516\\ 2.29881\times10^{-6}\\ 0.0000133094\\ 1.34377\times10^{-6}\\ 0.00374693 \end{array}$	$\begin{array}{c} 0.000173266\\ 1.24404 \times 10^{-28}\\ 26.6216\\ 0.0035516\\ 2.29881 \times 10^{-6}\\ 1.83428 \times 10^{-6}\\ 6.89019 \times 10^{-8}\\ 0.000516394 \end{array}$	$\begin{array}{c} 0.000172759\\ 1.04547\times 10^{-28}\\ 26.6998\\ 0.0035516\\ 2.29881\times 10^{-6}\\ 1.82355\times 10^{-6}\\ 6.82982\times 10^{-8}\\ 0.000513373\\ \end{array}$	$\begin{array}{c} 0.000611291\\ \overline{}7.69783\times10^{-10}\\ \overline{}7.5457\\ 0.0035516\\ 2.29881\times10^{-6}\\ 0.000023334\\ \overline{}3.09235\times10^{-6}\\ 0.00056907 \end{array}$	0.000174628 1.99734×10 ⁻² 26.4139 0.0035516 2.29881×10 ⁻¹ 1.88463×10 ⁻¹ 7.13498×10 ⁻¹ 0.000530568