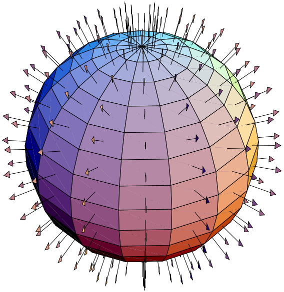
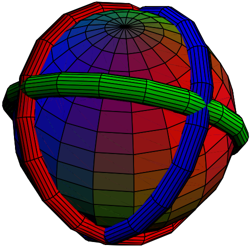
What do we know about the confinement mechanism?

Manfried Faber

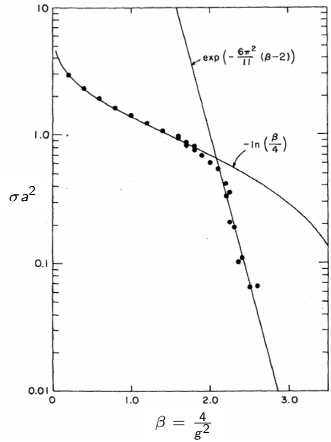




vortices or monopoles?

No proof, but verification of confinement

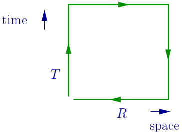
first verification 1980 by Mike Creutz on the lattice,
gluon string between color charges, constant force = string tension σ



M.Creutz: Phys.Rev.D 21 (1980) 2308

string tension σ

from Wilson loops



continuum:

$$W(\mathcal{C}_{R\times T}) = \langle \mathrm{Tr}\,\mathcal{P}\exp^{\mathrm{i}\oint_C A_\mu(x)\mathrm{d}x_\mu}\rangle$$

lattice:

$$W(R,T) = \langle \mathrm{Tr}\,\mathcal{P}\prod_{l\in C} U_l\rangle$$

$$\ln W(\mathcal{C}_{R\times T}) = -RT\sigma + \mathcal{O}(R+T)$$

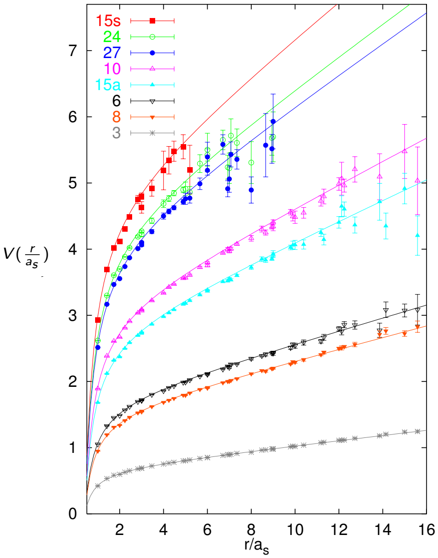
area law for Wilson loops

$$\sigma = -\ln\left(\frac{W(R+1,T+1)W(R,T)}{W(R,T+1)W(R+1,T)}\right)$$ Creutz ratio

asymptotic freedom prediction

$$a^2\sigma \propto \exp\{-\frac{6\pi^2}{11}(\beta-2)\}$$

Flux tubes



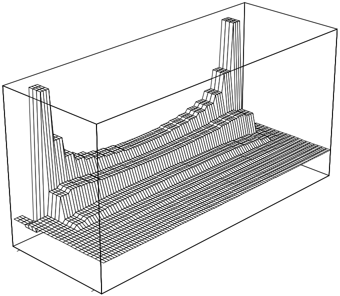
Static potentials in SU(3),

between various sources

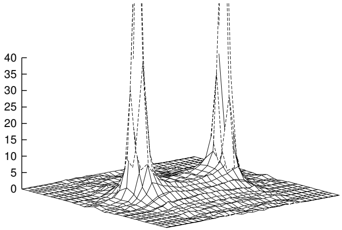
From: Gunnar S. Bali

hep-lat/9908021

Action density of flux tubes



in U(1), $R = 22a$



From: Gunnar S. Bali
hep-lat/9409005:
in SU(2), $\beta = 2.5$, $R = 16a$

Notions of confinement

- Confinement means that the spectrum consists only of color singlet states.
- Confinement means that no color-nonsinglet asymptotic states exist.
- In some cases, for theories with a center symmetry, confinement means that Wilson loops have an area law;
  the center symmetry is unbroken.

Only in the last case do we have a true order parameter for confinement, the expectation value of a Polyakov loop, $\langle L \rangle$,

$$L(\vec{x}) = \prod_{t=1}^{N_t} U_4(\vec{x}, t) \ , \qquad \text{world-line of static quark}$$

Under center transformations: $L(\vec{x}) \to zL(\vec{x})$, with $z \in Z_N$.

why interests in vacuum?

disorder in vacuum expells homogeneous electric flux

T.D.Lee (Cern 2007): Is the physical vacuum a medium?

- a state without matter,
- but with energy fluctuations,
- a complex condensate that can violate symmetry, therefore not aether
- like superconductor, can undergo phase transitions

vacuum fluctuations should explain the non-perturbative properties of QCD

- confinement
- chiral symmetry breaking
- anomalies, e.g. violation of scale invariance

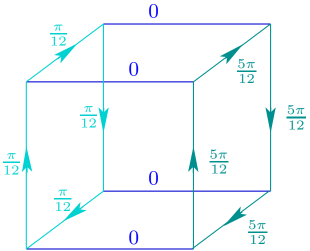
Models of the QCD vacuum

- Savvidy vacuum (1977): Infrared instability of the vacuum
- dual superconductor picture:
  Nielsen and Olesen (1973), Nambu and Creutz (1974), 't Hooft, Parisi, Jevicki and Senjanovic (1975), Mandelstam (1976)
  magnetic monopoles detected by Abelian Projection:
  Kronfeld, Laursen, Schierholz, Wiese
  Bornyakov,Boyko,Polikarpov,Zakharov
- instanton-dyons (1998) invented by Kraan, van Baal, Lee, Lu
  nonzero electric and magnetic charges, sources of Abelian gluons
  instanton-dyon ensemble
  Diakonov,Petrov,Shuryak,Schäfer
  V.G. Bornyakov, E.-M. Ilgenfritz, B.V. Martemyanov
- center vortex condensation: 't Hooft, Vinciarelli, Yoneya (1978), Cornwall, Mack, Petkova (1979)
  vortices detected by Center Projection $\longrightarrow$ P-vortices

do models lead to non-vanishing gluon and quark condensate ?

Fluctuations: Dirac's Magnetic monopoles

identify by singular gauge fields
lattice: non-trivial cubes: $\mathrm{div}\vec{B} \neq 0$



$U_\square = \frac{\pi}{3} = 60^\circ$

$\sum_\square U_\square = 2\pi$

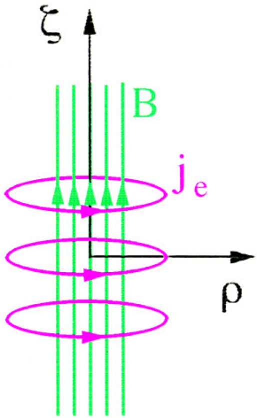
$\longrightarrow$ in a U(1) subgroup of SU(2) or SU(3)

$$U_\mu(x) = \underbrace{\begin{pmatrix} \sqrt{1-|c_\mu(x)|^2} & c_\mu(x) \\ -c_\mu^\star(x) & \sqrt{1-|c_\mu(x)|^2} \end{pmatrix}}_{\text{W-bosons}} \underbrace{\begin{pmatrix} \mathrm{e}^{\mathrm{i}\theta_\mu(x)} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\theta_\mu(x)} \end{pmatrix}}_{\in U(1)}$$

Maximal abelian gauge, abelian projection

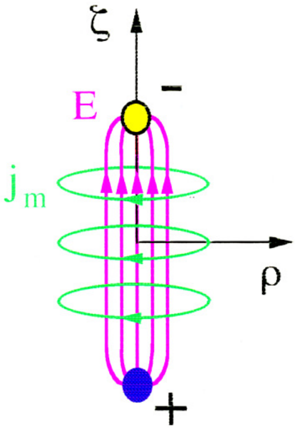
Confinement due to Magnetic Monopoles

type II superconductor



magnetic fluxoid quantisation

dual superconductor



electric fluxoid quantisation

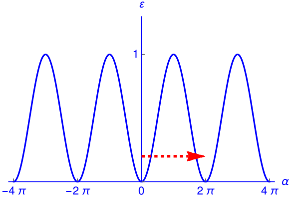
Fluctuations: Instantons

pure gauge fields: $\mathcal{A}_\mu := -\mathrm{i}\partial_\mu \Omega \Omega^\dagger$ with $\mathcal{A}_\mu =: \frac{\vec{\sigma}}{2}\vec{A}_\mu$

gauge function with windings: $\Omega(x) = \mathrm{e}^{\mathrm{i}\frac{\vec{\sigma}}{2}\vec{\omega}(x)} \in SU(2)$,

results in $\mathcal{F}_{\mu\nu} := \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu - \mathrm{i}[\mathcal{A}_\mu, \mathcal{A}_\nu] = 0$,

an infinite set of topological different vacua:



$\mathcal{A}_\mu := -\mathrm{i}\, f(R)\, \partial_\mu \Omega \Omega^\dagger, \quad R = \sqrt{x_\mu x_\mu}$

$f(R) = \frac{R^2}{R^2 + R_0^2}, \quad f(0) = 0, \; f(\infty) = 1$

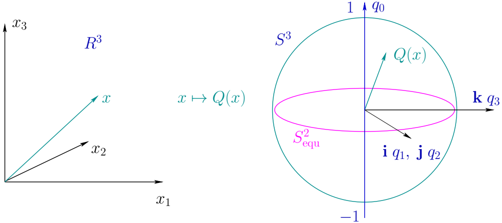
$Q_{\mathrm{top}} := \frac{1}{32\pi^2} \int_{\mathcal{B}} \mathrm{d}^4x \, F^a_{\mu\nu} \, {}^\star F^a_{\mu\nu} = \pm 1$

minima of action: $S = \frac{8\pi^2}{g^2}$

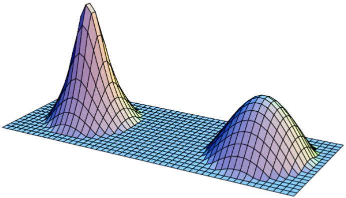
instantaneous transition between vacua

Instantons at finite temperature $\rightarrow$ dyons

a field of Polyakov loop matrices $Q(\vec{x})$, $\quad L(\vec{x}) = \mathrm{Tr}\, Q(\vec{x})$
covering $S^3 \cong SU(2) \cong$ unit quaternions: $\quad R^3 \rightarrow S^3$



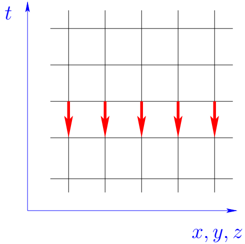
vacuum with broken symmetry
e.g. $Q(\infty) = -\mathrm{i}\sigma_3$
from:
Thomas C Kraan and Pierre van Baal:
Nuclear Physics B 533 (1998) 627–659

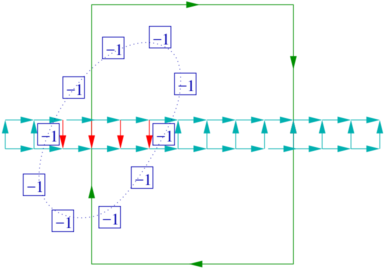


action density in R^3

$\rightarrow$ details calorons

Vortices





multiply all links
in one time-slice
with a center element.

Vortex as surface of Dirac volume,
low action - high entropy.

center symmetry of S

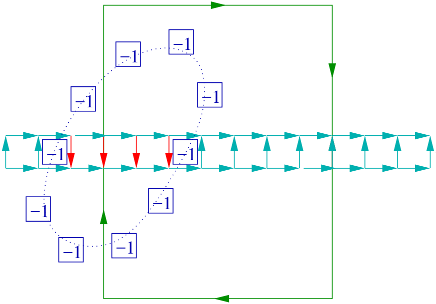
thin vortex $\rightarrow$ thick vortex.

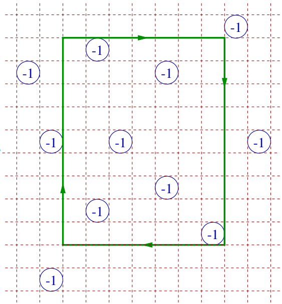
Polyakov loop $L = \mathrm{Tr} \prod_{t=1}^{T} U_4(t)$ sensitive to t-links.

Area law for center projected Wilson loops

Vortices are closed surfaces

only surface contribution to action



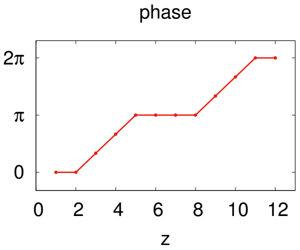


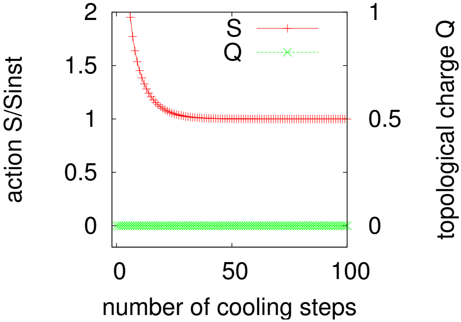
denote f the probability that a plaquette has the value -1

$$\langle W(A)\rangle \quad = [(-1)f + (+1)(1-f)]^A = \exp[\underbrace{\ln(1-2f)}_{-\sigma} A] =$$

$$= \exp[-\sigma \overbrace{R \times T}^{A}], \qquad \sigma \equiv -\ln(1-2f) \approx 2f$$

Vortex pair





smooth xy-vortex pair,
t-links vary in z-direction.

After cooling the action approaches the value $S_{\text{inst}} = \frac{8\pi^2}{g^2}$,
the topological charge is trivial.

a background field, a 3D topological object

Preference by action or "entropy"

monopoles: by entropy

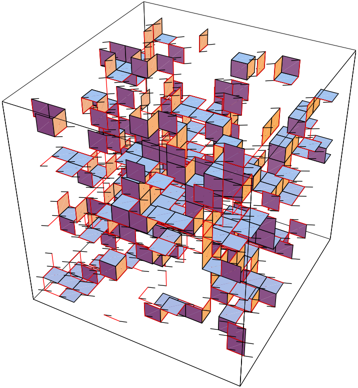
instantons: by local minima of the action: $S_{\rm inst} = \frac{8\pi^2}{g^2}$

vortices: center symmetry and entropy

Shapes of projected vortices

3-dimensional cuts through dual lattices

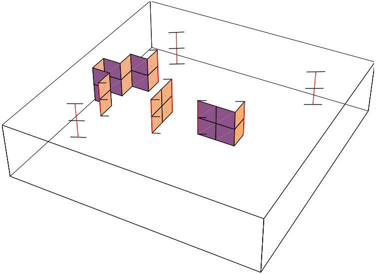
zero temperature



12^4-lattice

vortices percolate

finite temperature
above phase transition



2×12^3-lattice

constant in time $\rightarrow$ cylinders

area law for spatial Wilson loops

some Vortex properties

- form closed surfaces in dual space,
- vortices have a thick core,
- percolating in all directions
- deconfinement transition a de-percolation transition,
- in deconfinement: percolation in spatial directions only,
- scaling of the P-vortex density.

$\rightarrow$ Center vortex dominance

Vortices are colorful

quantised magnetic flux tubes evolving in time $\rightarrow$ closed surfaces

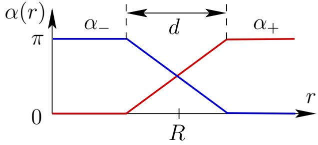
3D pictures

Colors are gauge dependent
In Abelian projection we use a color filter and find monopoles,
Monopoles are an indication of the color structure

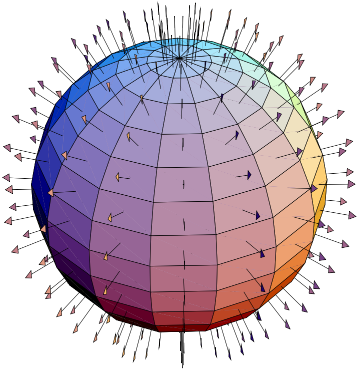
Monopoles as hint of color structure of vortices

Colorfull spherical vortex → *Höllwieser et al. 2012*

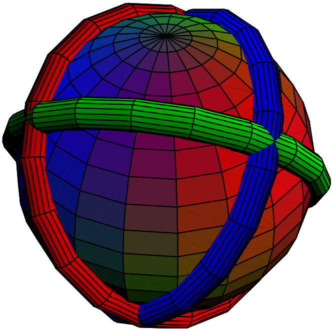
$$U_\mu(x) = \begin{cases} \exp\{\mathrm{i}\alpha(r)\,\vec{e}_r\cdot\vec{\sigma}\} & t=1,\mu=4 \\ \mathbb{1} & \text{elsewhere} \end{cases}$$



P-vortex

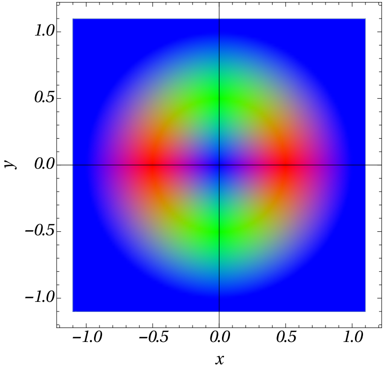


Abelian projection



Colorful plain vortex

plain xy-vortex: for $t = 1$ t-links vary in z-direction
from 1 to -1 in $|z - z_v| \leq d$



$$U_i(x) = 1$$

$$U_4(x) = \begin{cases} U_4'(\vec{x}) & \text{for} \quad t = 1 \\ 1 & \text{else} \end{cases}$$

where for $|z - z_v| \leq d$

$$U_4'(\vec{x}) = \begin{cases} \mathrm{e}^{\mathrm{i}\alpha(z)\sigma_n}, & \rho \leq R \\ \mathrm{e}^{\mathrm{i}\alpha(z)\sigma_3} & \text{else} \end{cases}$$

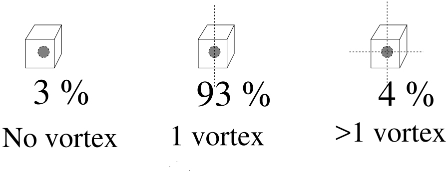
$$\sigma_n = \sigma_1 \sin\theta(\rho)\cos\phi \; + \\ \sigma_2 \sin\theta(\rho)\sin\phi \; + \\ \sigma_3 \cos\theta(\rho)$$

→ Continuum Form

Monopoles and Vortices

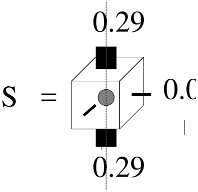
➜ *Greensite et al. (1997)*

Almost all monopole cubes are pierced by exactly one, P-vortex

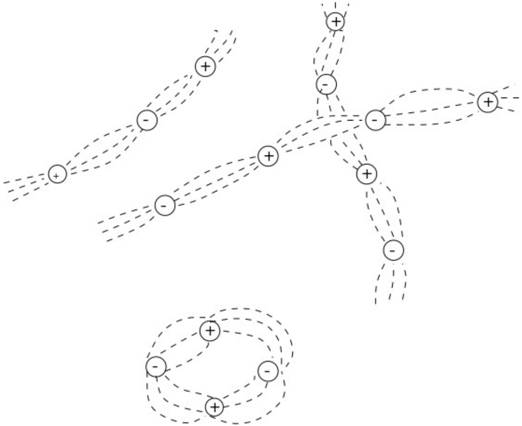


Monopole action is highly asymmetric:

Plaquette action
$S = (1 - \frac{1}{2}\mathrm{Tr}[U_\square]) - S_0$
mainly oriented in P-vortex direction



W-bosons change the field distribution



Monopoles arranged in monopole–antimonopole chains = Vortices

➜ *Ambjorm, Giedt, Greensite, 2000*

Vortices generate topological charge

Recall that the topological charge density is defined as

$$q(x) = \frac{1}{16\pi^2}\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}_{\mu\nu}\right) = \frac{1}{4\pi^2}\vec{E}\cdot\vec{B}\,, \qquad \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}\,.$$

We need flux in all four directions.
A vortex has flux perpendicular to its world sheet.

Generate topological charge by:

- intersecting vortices,
- vortex "writhing," *i.e.*, twisting around itself
- Color structure

P-vortices need an orientation
regions of different orientation are separated by monopole lines

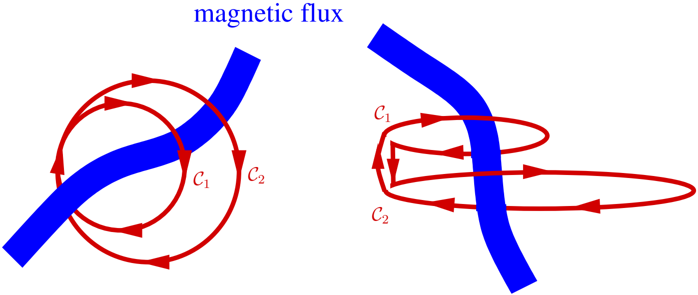
➜ *Engelhardt, Reinhardt (2000)*

details: top.charge   details: chiral symmetry breaking

Abelian or Center degrees of freedom

Double-winding Wilson loops

➜ *Greensite, Höllwieser*



Spherical symmetric monopole flux is spreading with $1/A$ and may lead to small contributions to Wilson loops

$$W_{\mathcal{C}_1+\mathcal{C}_2} = \langle \exp\{\mathrm{i}\frac{\sigma_3}{2}(\alpha_{\mathcal{C}_1} + \alpha_{\mathcal{C}_2})\}\rangle \approx \alpha_a \exp[-\sigma(A_1 + A_2) - \mu P]$$

Center vortex flux doesn't spread

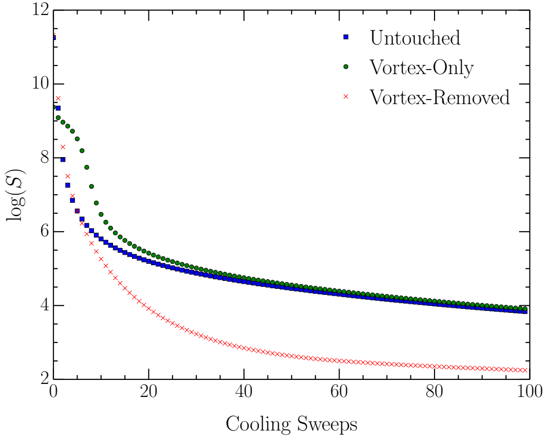
$$W_{\mathcal{C}_1+\mathcal{C}_2} = \langle (-1)^{n_{\mathcal{C}_1}+n_{\mathcal{C}_2}}\rangle = \langle (-1)^{|n_{\mathcal{C}_1}-n_{\mathcal{C}_2}|}\rangle \approx \alpha_c \exp[-\sigma|A_1 - A_2|]$$

details: doubleWinding

some SU(3) results

of Adelaide group: Trewartha, Kamleh, Leinweber

Average action in SU(3) by cooling



→ further Adelaide results

Vortex model explains

- non-trivial vacuum $\rightarrow$ gluon condensate
- area law of Wilson loops
- Casimir scaling of heavy-quark potential
- double winding Wilson loops
- finite temperature phase transition $\rightarrow$ Polyakov loops
- orders of phase transitions in SU(2) and SU(3)
- area law for spatial Wilson loops
- topological charge
- chiral symmetry breaking $\rightarrow$ quark condensate
- monopole picture of confinement
  $\rightarrow$ dual superconductor model
- color structure of vortices $\rightarrow$ instantons

Methods of vortex detection, problems

Laplacian center gauge: absence of scaling of P-vortex density
de Forcrand, D'Elia, Alexandrou and Langfeld, Reinhardt, Schäfke

Maximal center gauge = adjoint Landau gauge

$$R_{\mathrm{MCG}} = \sum_x \sum_\mu |\mathrm{Tr}[U_\mu(x)]|^2 \quad \rightarrow \quad \text{Maximum}$$

+ center projection

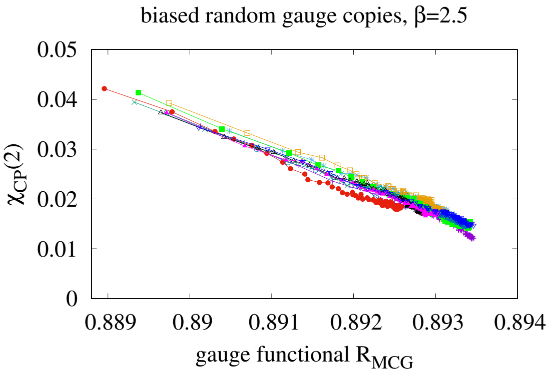
$$U_\mu(x) \quad \rightarrow \quad Z_\mu(x) \equiv \mathrm{sign}\ \mathrm{Tr}[U_\mu(x)]$$

Problems:

- cooled or RG-smoothed configurations, Kovacs-Tomboulis:
  string tension is drastically reduced after only a few cooling steps,
  why: vortex cores expand considerably,
  every region of the lattice is part of a vortex core,
  fits fail badly near the middle of the vortex.
- Gribov ambiguity: local maxima versus global maxima,
  extensive simulated annealing: Bornyakov, Komarov, Polikarpov, Veselov → loss of vortex finding property

towards global maxima of R_{MCG}

Do (biased) gauge transformations Ω with $\mathrm{tr}\Omega > 0$ only

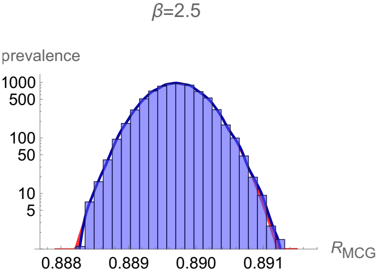


from Z.Dehghan et al., Universe 8 (2023) 387,

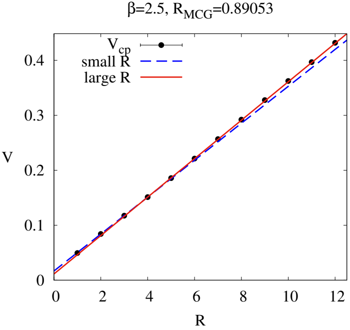
compare $\chi_{\mathrm{CP}}(2) \approx \sigma = 0.0350(8)$ by Bali,Schilling,Schlichter

local maxxima of R_{MCG}-values

from Z.Dehghan et al., PhysRevD.110.014501 (2024)
for 100 gauge copies for 200 independent gauge fields = 20.000 fields

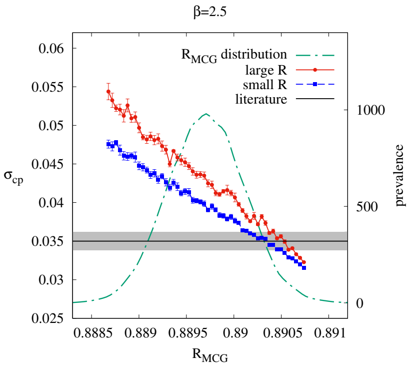


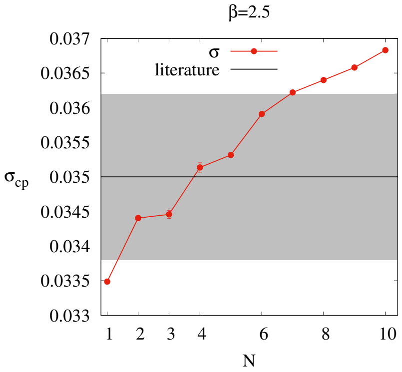
	Statistic	P-Value
Cramér-von Mises	0.0411769	0.658597



small R [2:5] σ=0.033596 +/- 0.0001541 (0.4588%)
large R [4:10] σ=0.034968 +/- 0.0001435 (0.4103%)
compare $\sigma = 0.0350(8)$ by Bali,Schilling,Schlichter

important region is high tip of distribution





Conclusion

many successes of vortex model

- explains confinement
- explains finite temperature phase transition
- explains topological charge
- explains chiral symmetry breaking
- explains success of abelian monopoles

"the standard model of Physics is too complex to be the last truth": Gerard 't Hooft:

"El modelo estándar de la Física es demasiado complejo para ser la última verdad" in: https://www.lainformacion.com/asuntos-sociales/investigadores-del-csic-calculan

-que-el-neutrino-tiene-una-masa-dos-millones-de-veces-inferior-a-la-del-electron_almiguhsb2gxcblwppz0g7/-1/

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Is QCD the final theory of strong interaction?

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Is QCD the final theory of strong interaction?

Pro:

- many excellent predictions
- no free parameters besides m_q and Λ_{QCD}

Contra:

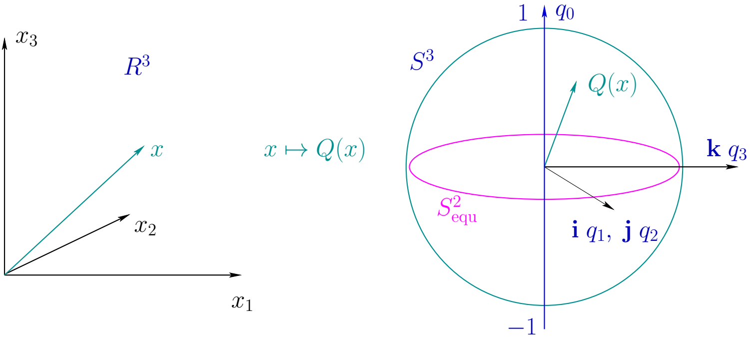
- infinite number of vacua
- vacuum is not empty, densely packed by gluon fields
- opposite to statistical mechanics:
  low temperature phase is disordered,
  high temperature phase is ordered
- no glueballs found

Thanks you for your attention!

Questions?

Calorons in SU(2)

a field of Polyakov loop matrices $Q(\vec{x})$, $\quad L(\vec{x}) = \mathrm{Tr}\, Q(\vec{x})$
covering $S^3 \cong SU(2) \cong$ unit quaternions



$$R^3 \to S^3, \quad x \mapsto Q(\vec{x}) = q_0 + \mathrm{i}q_1 + \mathrm{j}q_2 + \mathrm{k}q_3 = q_0 - \mathrm{i}\vec{q}\vec{\sigma}$$

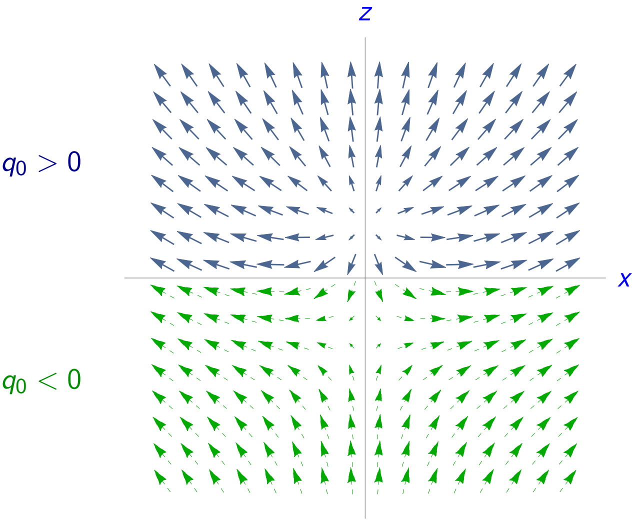
$$\mathrm{i} = -\mathrm{i}\sigma_1,\ \mathrm{j} = -\mathrm{i}\sigma_2,\ \mathrm{k} = -\mathrm{i}\sigma_3$$

(an)holonomy = vacuum with broken symmetry, e.g. $Q(\infty) = -\mathrm{i}\sigma_3$

$Q(\vec{x}) = q_0(\vec{x}) - \mathrm{i}\vec{q}(\vec{x})\vec{\sigma}$

$q_0^2 + \vec{q}^2 = 1$

$\vec{q}(\vec{x})$-field

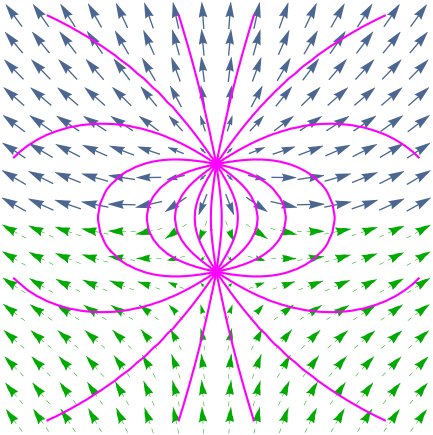


$$Q(\vec{x}) = q_0(\vec{x}) - \mathrm{i}\vec{q}(\vec{x})\vec{\sigma} = \cos\alpha(\vec{x}) - \mathrm{i}\vec{\sigma}\vec{n}(\vec{x})\sin\alpha(\vec{x})$$

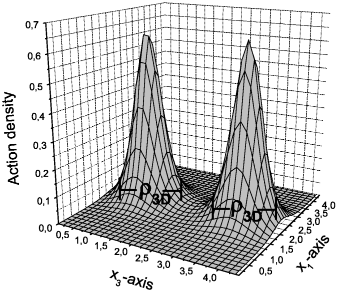
connect points with $\vec{n} = \text{const.}$

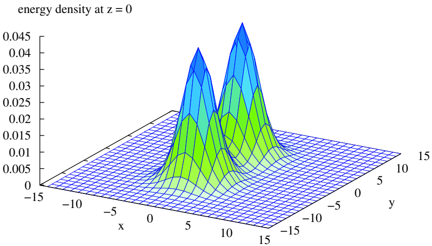
$q_0 > 0$

$q_0 < 0$



Action density





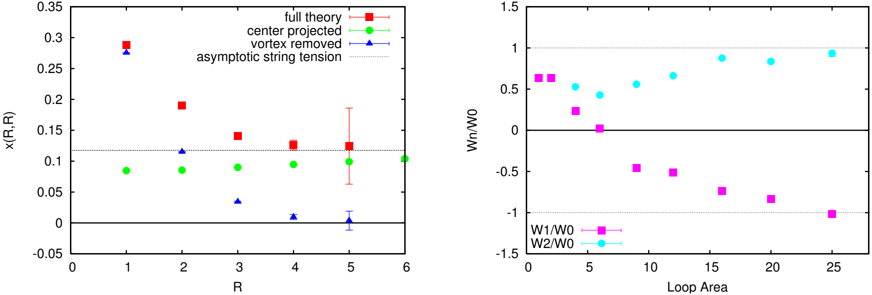
Action density for caloron,
zero anholonomy,
Gerhold, Ilgenfritz, Müller-Preussker (2007)

The energy density
for a particle anti-particle solution

$$S = \frac{8\pi^2}{g^2}.$$

$\longrightarrow$ back

Center vortex dominance



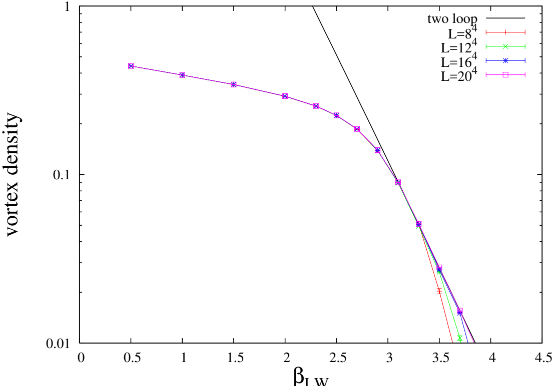
From: Höllwieser et al.:PhysRevD.78.054508.

Left: Creutz ratios for full, center-projected, and vortex-removed gauge fields for $\beta_{LW} = 3.3$.

Right: Wilson loop pierced by n P-vortices W_n.
Expect $W_n \to (-1)^n W_0$ as area is increased.

Cancellations lead to area-law of confinement.

Center vortex dominance



P-vortex
surface density
vs. β_{LW}

From: Höllwieser et al.:PhysRevD.78.054508.

"Two-loop" line is scaling prediction with $\sqrt{\rho_v/6\Lambda^2} = 50$.

Scaling shows the vortex density is a physical quantity, with a well defined continuum limit.

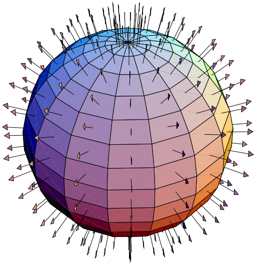
$\longrightarrow$ back

Continuum Form of colorful spherical vortex

after time-dependent gauge transformation $\Omega(\vec{r}, t)$

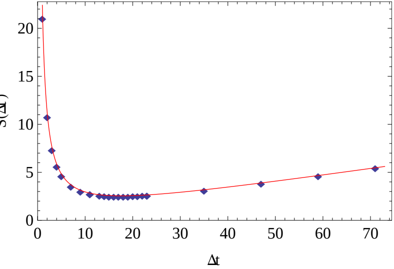
vortex $\equiv$ vacuum - vacuum transition

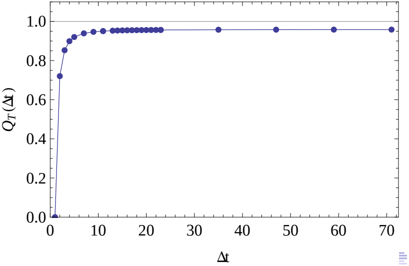
$$\left.\begin{array}{l} t = 1 \\ t = 2 \end{array}\right\} \begin{array}{c} \text{vacuum} \\ \text{pure gauge} \end{array} \left\{\begin{array}{ll} R^3 \mapsto 1 & \text{no winding} \\ R^3 \mapsto S^3 & \text{winding} \end{array}\right.$$



smoothing possible ➜ Schweigler, 2013

distribute to several time-slices Δt $\quad \Rightarrow \mathcal{A}_\mu = \mathrm{i} f(t) \partial_\mu g^\dagger g$





Continuum Form of Colorful plain vortices

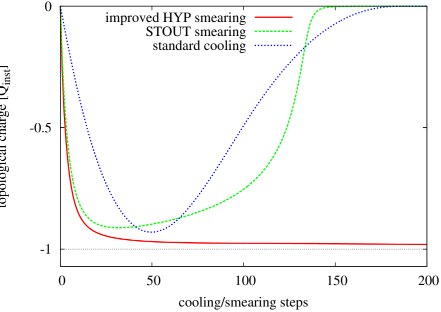
gauge transformation:

rotate time-links to $U_4(x) = \mathbb{1}$

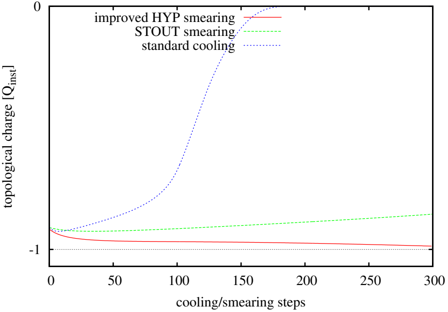
distribute transition over Δt time slices

topological charge during cooling for $R = d = 7$ on $28^3 \times 40$

$\Delta t = 1$

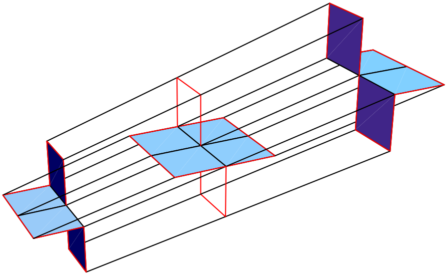


$\Delta t = 11$



$\longrightarrow$ back

Topological charge from intersections and writhing points



➜ *Bruckmann, Engelhardt (2003)*

Intersections and writhing points contribute to the topological charge of a P-vortex surface

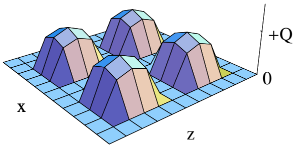
- intersections $Q = \pm\frac{1}{2}$
- writhing points $Q = \pm\frac{1}{8}$

H. Reinhardt, NPB628 (2002) 133 [hep-th/0112215], hep-th/0204194

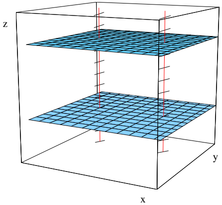
Intersecting plane vortices

Intersecting two orthogonal pairs of plane vortices we can generate topology. A *xy* vortex generates a chromo-electric field, E_z, and a *zt* vortex a chromo-magnetic field, B_z. Each intersection point contributes $Q = \pm 1/2$ to the total topological charge.

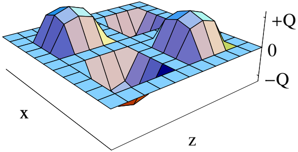
Parallel Vortices



Geometry



Antiparallel Vortices



So we can get $Q = 2$ with parallel intersecting vortices and $Q = 0$ with antiparallel intersecting vortices.

$\longrightarrow$ back

Vortices and chiral symmetry breaking
Atiyah-Singer index theorem

- zero-modes of fermionic matrix: $D[A]\psi(x) = 0$
- ψ has definite chirality:

$$\psi_{\substack{R\\L}} = \frac{1}{2}(1 \pm \gamma_5)\psi, \quad \Rightarrow \quad \gamma_5 \psi_{\substack{R\\L}} = \pm\psi_{\substack{R\\L}}$$

- Index theorem (wilson, overlap fermions):

$$n_-, n_+: \text{ number of left-/right-handed zeromodes}$$

$$\mathrm{ind} D[A] = n_- - n_+ = Q[A]$$

- (Asqtad) staggered fermions:

$$\mathrm{ind}\ D[A] = 2Q[A] \text{ (SU(2), double degeneracy)}$$

- Adjoint overlap fermions:

$$\mathrm{ind}\ D[A] = 2NQ[A] = 4Q[A] \text{ (real representation)}$$

➜ *Neuberger, Fukaya (1999)*

Banks-Casher relation

Chiral symmetry breaking $\Longrightarrow$

$\Longrightarrow$ Low-lying eigenmodes of Dirac operator

Dirac equation: $D[A]\,\psi_n = \mathrm{i}\lambda_n\psi_n,$
$\{\gamma_5,\gamma_\mu\} = 0, \quad D[A]\,\gamma_5\psi_n = -\mathrm{i}\lambda_n\gamma_5\psi_n$
Non-zero eigenvalues appear in imaginary pairs $\pm\mathrm{i}\lambda_n$.

$$\langle\bar\psi\psi\rangle = -\lim_{m\to 0}\lim_{V\to\infty}\left\langle\frac{1}{V}\sum_n\frac{1}{m+\mathrm{i}\lambda_n}\right\rangle =$$

$$= -\lim_{m\to 0}\lim_{V\to\infty}\left\langle\frac{1}{V}\int\mathrm{d}\lambda\,\rho_V(\lambda)\frac{1}{2}\left(\frac{1}{m+\mathrm{i}\lambda}+\frac{1}{m-\mathrm{i}\lambda}\right)\right\rangle$$

$$-\lim_{m\to 0}\frac{m}{m^2+\lambda^2} = \lim_{m\to 0}\frac{\mathrm{d}}{\mathrm{d}\lambda}\arctan\frac{m}{\lambda} \quad\longrightarrow\quad \pi\delta(0)$$

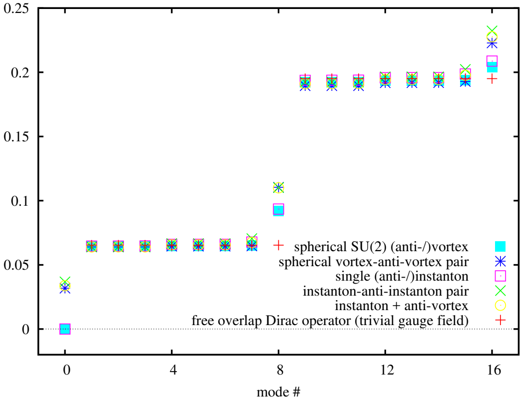
Chiral condensate $\Longrightarrow$ Density of Near-Zero-modes

$$\langle\bar\psi\psi\rangle = \frac{\pi\rho_V(0)}{V}$$

Banks, *Casher*(1980)

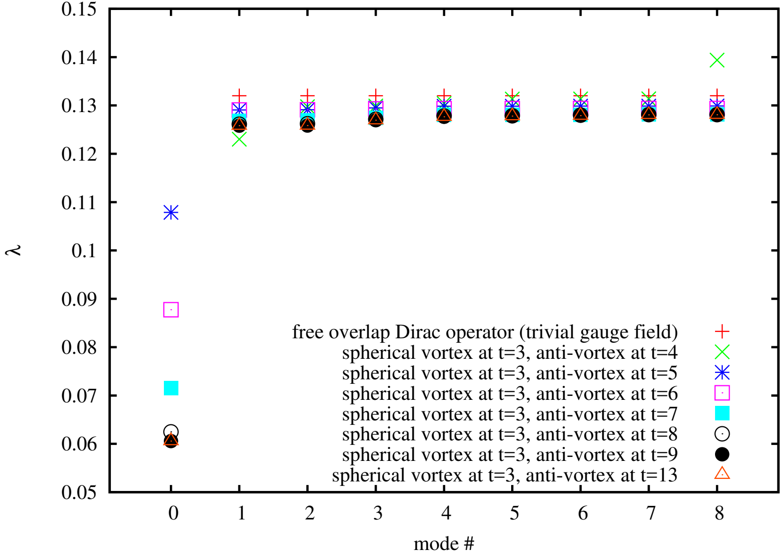
Dirac spectra, spherical vortices and instantons

The overlap Dirac eigenvalues, and even the eigenmodes, in the background of spherical vortices are very similar to those with instantons.



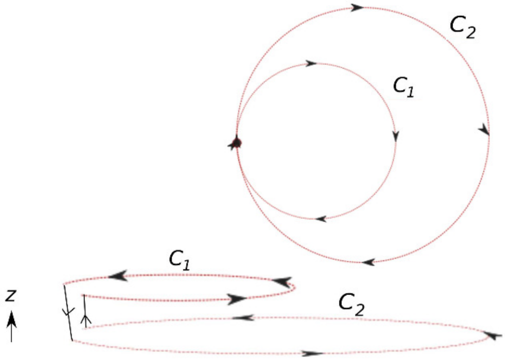
With objects of opposite topological charge, the would-be zero modes interact and become near-zero modes.

changing distance between Vortex and Anti-vortex



⟶ back

Double-winding Wilson loops



➜ *Greensite, Höllwieser*

check monopole and vortex picture in SU(2)

Double-winding Wilson loops $C = C_1 + C_2$

- Sum of areas behavior in Abelian models:

$$\begin{aligned} W(C) &= \frac{1}{2}\langle \mathrm{Tr} P \exp[i \oint_C dx^\mu A_\mu^a \frac{\sigma^a}{2}]\rangle \approx \langle \mathrm{Tr}\exp[\frac{i}{2}\oint_C dx^\mu A_\mu^3]\rangle \\ &= \langle \exp[\frac{i}{2}\oint_{C_1} dx^\mu A_\mu^3] \exp[\frac{i}{2}\oint_{C_2} dx^\mu A_\mu^3]\rangle \\ &\approx \langle \exp[\frac{i}{2}\oint_{C_1} dx^\mu A_\mu^3]\rangle\langle \exp[\frac{i}{2}\oint_{C_2} dx^\mu A_\mu^3]\rangle \\ &\approx \exp[-\sigma(A_1 + A_2) - \mu P] \end{aligned}$$

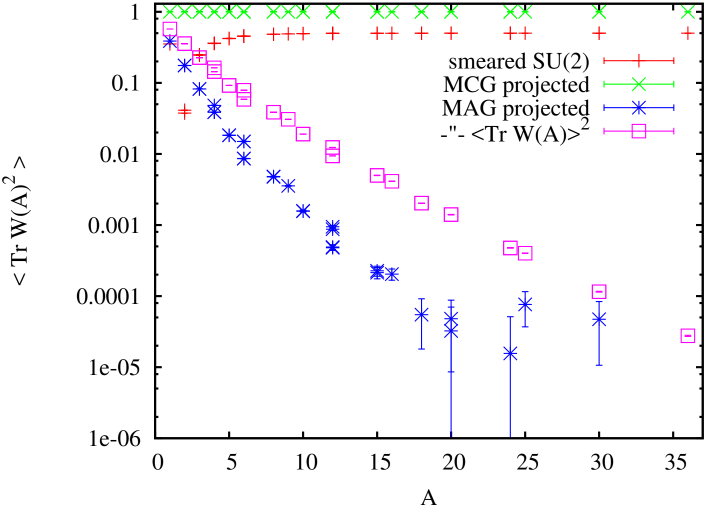
- vs. Difference of areas behavior in center vortex picture:

$$W(C) = \alpha \exp[-\sigma|A_1 - A_2|]$$

Winding around a vortex twice gives no contribution to $W(C)$:

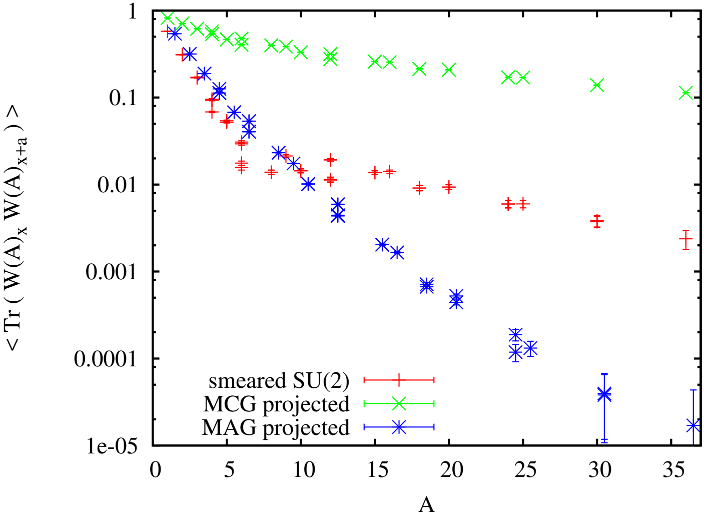
$$(-1)^2 = +1$$

Double-winding loops $C = C_1 = C_2$

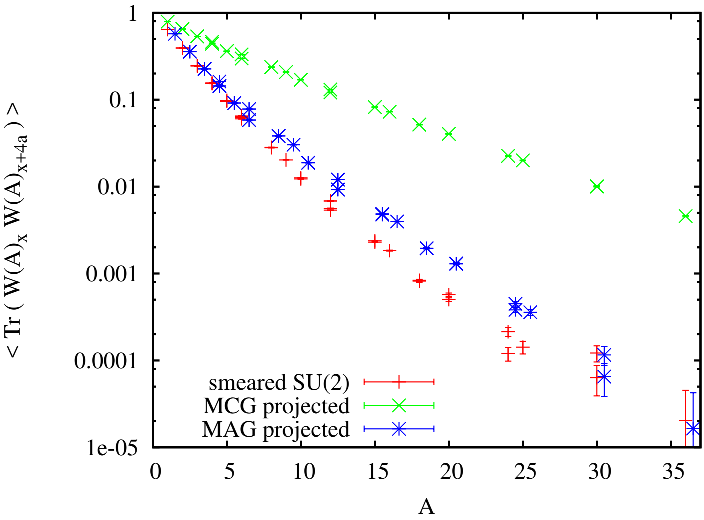


SU(2) group identity: $\mathrm{Tr}[U(C)U(C)] = 1 + \mathrm{Tr}_A U(C)$,
$\langle \mathrm{Tr}_A U(C) \rangle \ll 1 \Rightarrow W(C) \approx 1/2$

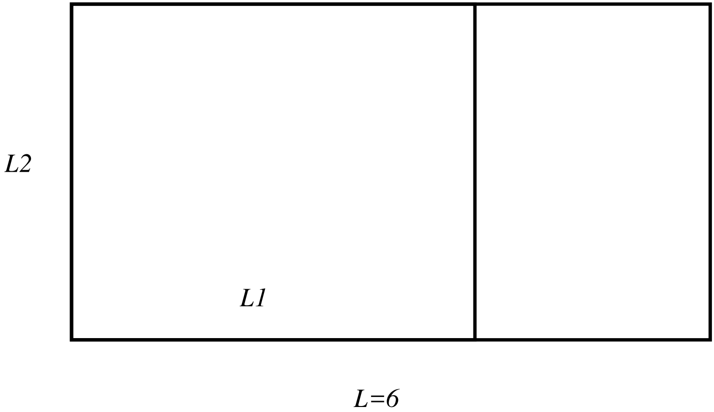
Double-winding loops $C = C_1 = C_2$



Double-winding loops $C = C_1 = C_2$

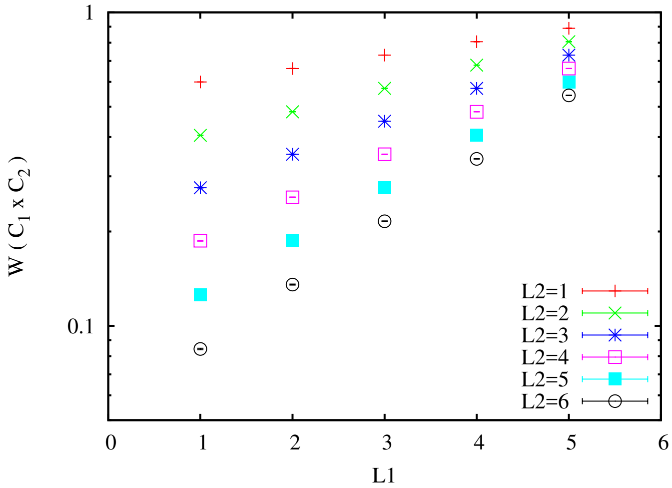


Double-winding loops



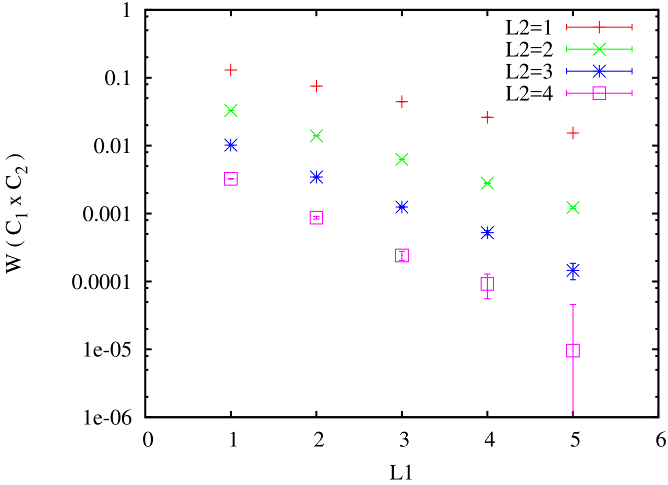
$$A_1 = 6L_2, \quad A_2 = L_1 L_2$$

Double-winding loops: Z(2)



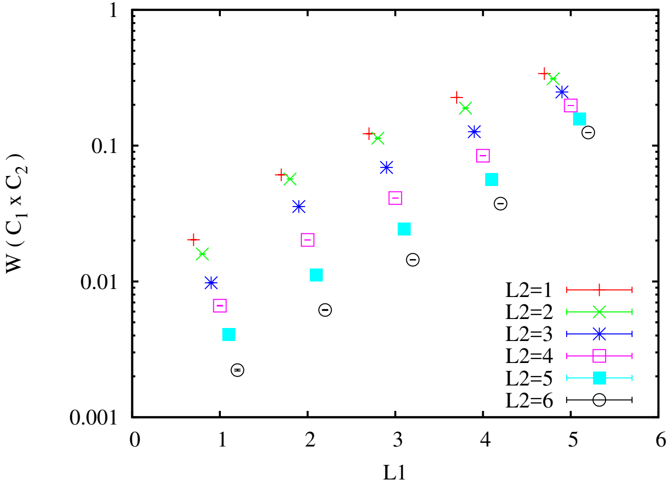
$$A_1 - A_2 = (6 - L_1)L_2$$

Double-winding Wilson loops: MAG



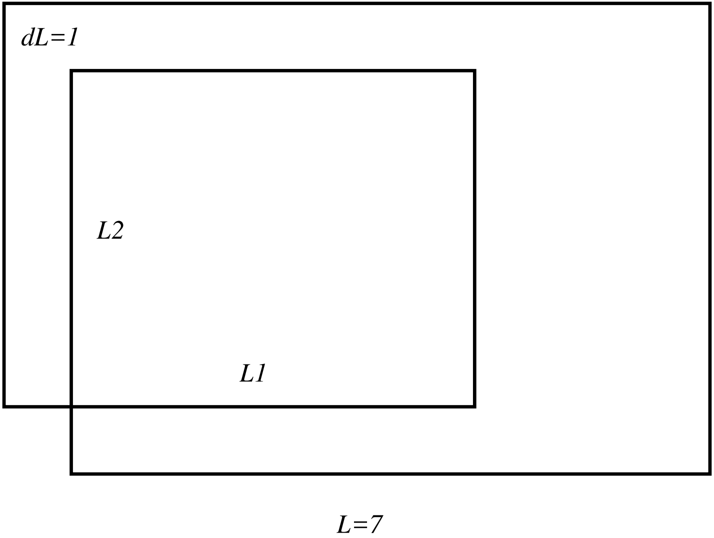
$$A_1 + A_2 = (6 + L_1)L_2$$

Double-winding Wilson loops: SU(2)



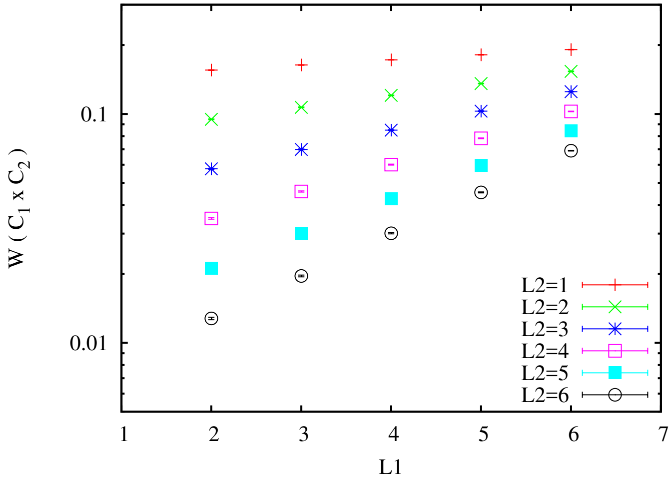
$$A_1 - A_2 = (6 - L_1)L_2 \quad \text{versus} \quad A_1 + A_2 = (6 + L_1)L_2$$

Double-winding Wilson loops

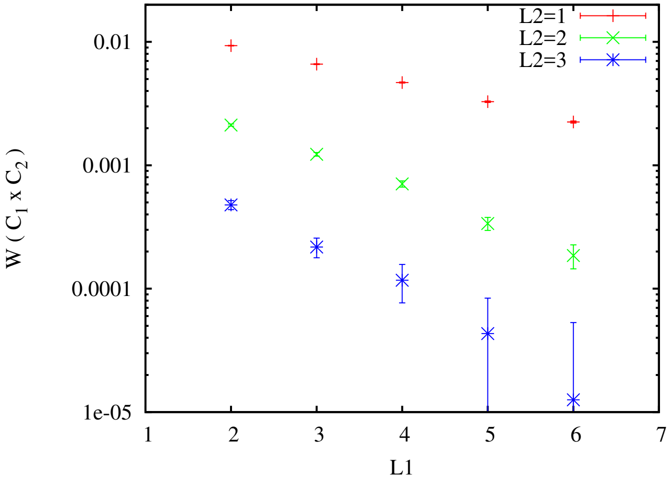


$$A_1 = 8(L_2 + 1) - 1, \quad A_2 = L_1 L_2$$

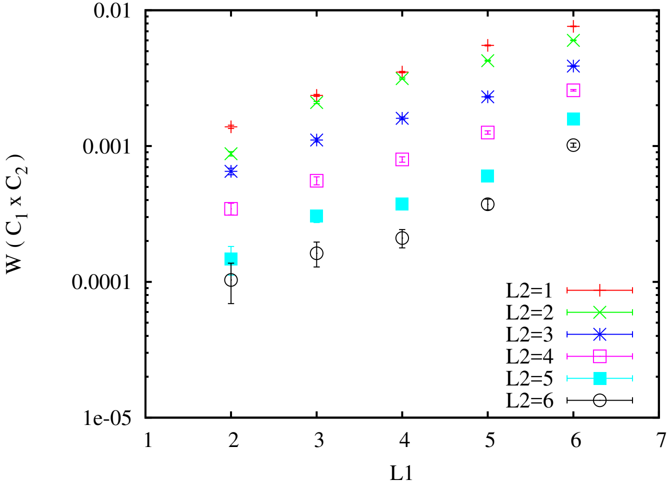
Double-winding Wilson loops: Z(2)



Double-winding Wilson loops: MAG



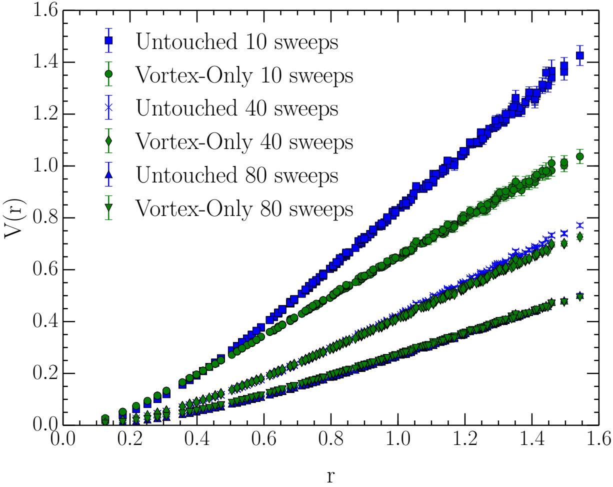
Double-winding Wilson loops: SU(2)



→ back

String tension in SU(3) by cooling

Trewartha et al.

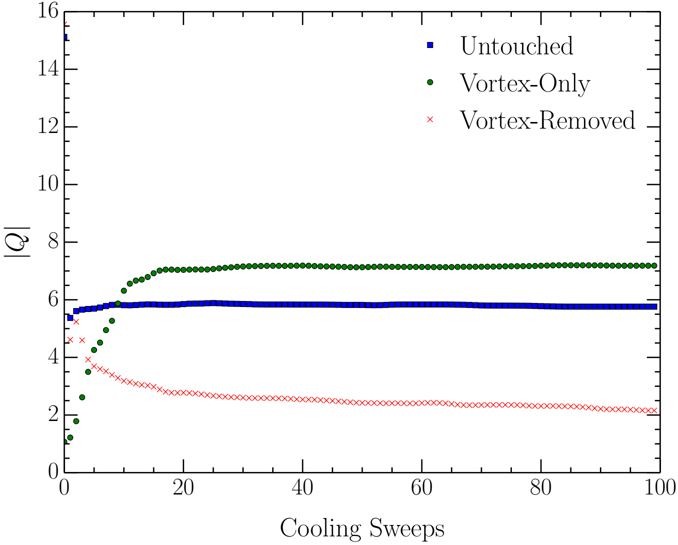


N	$\frac{\sigma_{\mathrm{VO}}}{\sigma_{\mathrm{UT}}}$
20	$0,67$
40	$0,64$
80	$0,97$

Examine instanton content in SU(3) by cooling

Trewartha et al.

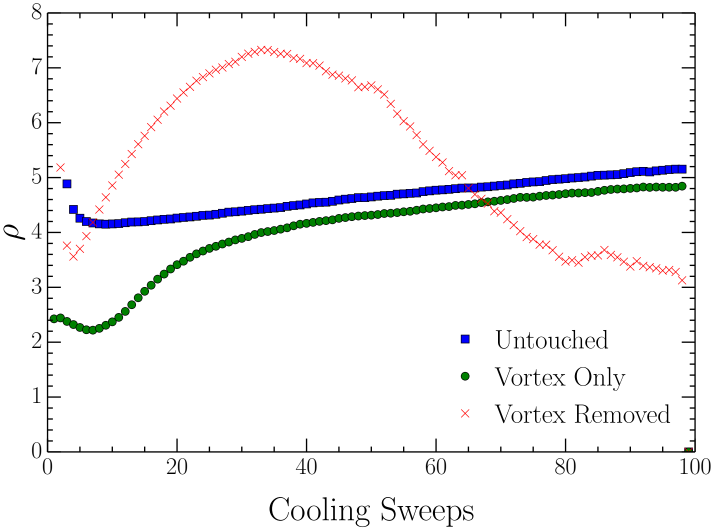
Average absolut value of topological charge



Examine instanton content in SU(3) by cooling

Trewartha et al.

Average radius ρ of instanton candidates



Examine instanton content in SU(3) by cooling

recent results of Adelaide group: Trewartha, Kamleh, Leinweber

same smoothing of
- original configurations
- vortex only configurations
- vortex removed configurations

- Vortex removal spoils and destabilizes instantons
- Spoiled instantons are removed via cooling
- Under cooling vortex only configurations produce background of instanton-like objects
- gauge field smoothing can restore agreement between untouched and vortex only configurations
- consistency with instanton model of dynamical mass generation

Support of hypothesis

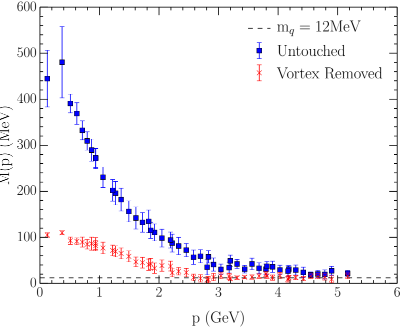
Center vortices are the fundamental long-range structures underpinning chiral symmetry breaking

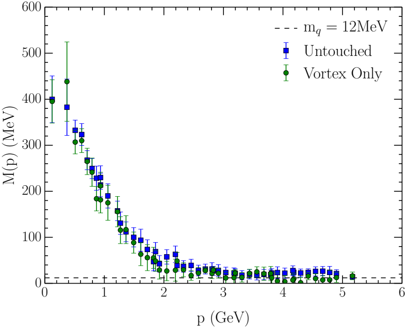
Landau gauge quark propagator in SU(3) by cooling

Trewartha et al.

Lattice quark propagator $S(p) = \frac{Z(p)}{i\not{q} + M(p)}$

nonp-perturbative mass function $M(p)$





after 10 cooling sweeps

Fermion results of Adelaide group

presence of dynamical fermions gives rise to

- increased abundance of centre vortices and branching points
- a single percolating cluster
- abundance of smaller clusters

→ back