Production of bound states of quarks and leptons in rare Higgs boson decays

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Introduction

After the discovery of the Higgs boson, the direction of research of various processes in the Higgs sector was finally formed.

A separate group of reactions consists of rare exclusive decays of the Higgs boson with the production of bound states of quarks and leptons. Our interest in these reactions is purely theoretical, which is connected with the relativistic approach we are developing. We will talk about the description of the listed reactions:

- $H \to J/\Psi, \Upsilon + J/\Psi, \Upsilon$
- $H \to Z, \gamma + (e^+e^-)$ $\quad H \to Z, \gamma + (\mu^+\mu^-)$ $\quad H \to Z, \gamma + (\tau^+\tau^-)$
- $H \to (e^+e^-) + (e^+e^-)$ $\quad H \to (\mu^+\mu^-) + (\mu^+\mu^-)$ $\quad H \to (\tau^+\tau^-) + (\tau^+\tau^-)$

The experimental study of rare processes seems possible in future colliders in $p-p$ collisions, in $e^+ - e^-$ annihilation when a sufficiently large number of Higgs bosons will be produced.

- CEPC *CEPC Study Group Collaboration, M. Dong et al., "CEPC Conceptual Design Report: Volume 2 - Physics & Detector," arXiv:1811.10545 [hep-ex].*
- FCC-ee *FCC Collaboration, A. Abada et al., "FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2," Eur. Phys. J. ST 228 (2019) 261–623.*
- FCC-hh *FCC Collaboration, A. Abada et al., "FCC-hh: The Hadron Collider: Future Circular Collider Conceptual Design Report Volume 3," Eur. Phys. J. ST 228 (2019) 755–1107.*

Introduction

We study rare two-particle decays of the Higgs boson, which produce bound states of heavy quarks and leptons (positronium, dimuonium, ditauonium). Note that ditauonium and dimuonium have not yet been observed experimentally, so estimates of the probability of their production in different reactions are important.

- *d'Enterria D., Le V. D. Rare and exclusive few-body decays of the Higgs, Z, W bosons, and the top quark //arXiv preprint arXiv:2312.11211. – 2023.*

Motivation for studying rare decays of the Higgs boson:

- Determination of the parameters of the Standard Model in the Higgs sector.
- Testing the theory of the decay of the Higgs boson with the formation of bound states of particles, identifying leading decay mechanisms, taking into account leading corrections.
- Exploring physics beyond the Standard Model (BSM)

- *Martynenko A. P., Martynenko F. A. Paired Double Heavy Baryons Production in Decays of the Higgs Boson //Symmetry. – 2023. – V. 15. – №. 10. – P. 1944.*
- *Martynenko A. P., Martynenko F. A. Relativistic Corrections to the Higgs Boson Decay into a Pair of Vector Quarkonia //Symmetry. – 2023. – V. 15. – №. 2. – P. 448.*
- *Faustov R. N., Martynenko A. P., Martynenko F. A. Relativistic corrections to paired production of charmonium and bottomonium in decays of the Higgs boson //Physical Review D. – 2023. – V. 107. – №. 5. – P. 056002.*
- *Faustov R. N., Martynenko F. A., Martynenko A. P. Higgs boson decay to the pair of S-and P-wave B c mesons //The European Physical Journal A. – 2022. – V. 58. – №. 1. – P. 4.*

Aim of the work

The work is devoted to calculating the widths of Higgs boson decays with pair production of charmonium, bottomonium, B_c mesons, with single and pair production of bound ortho - states of leptons (positronium (e^+e^-), dimuonium $(\mu^+\mu^-)$, ditauonium $(\tau^+\tau^-)$).
Research method: relativistic quasipotential approach to describing the production of bound states of particles.

- $H \to J/\Psi + J/\Psi, \Upsilon + \Upsilon, B_c + B_c^*$
- $H \to Z, \gamma + (e^+e^-)$ $\quad$ $H \to Z, \gamma + (\mu^+\mu^-)$ $\quad$ $H \to Z, \gamma + (\tau^+\tau^-)$
- $H \to (e^+e^-) + (e^+e^-)$ $\quad$ $H \to (\mu^+\mu^-) + (\mu^+\mu^-)$ $\quad$ $H \to (\tau^+\tau^-) + (\tau^+\tau^-)$

There are several production mechanisms which we study. Corresponding production amplitudes are shown in diagrams.
Let us consider the basics of the formalism using an example of the quark-gluon pair production mechanism. First, the Higgs boson turns into a quark-antiquark pair. The quark or antiquark can then emit a gluon, which also turns into a quark-antiquark pair.

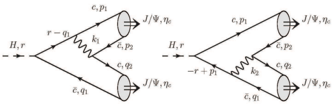


Рис.: Quark-gluon mechanism of pair production.

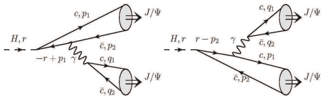


Рис.: Quark-photon mechanism of pair production.

General formalism

There are two stages of pair production of quarkonia in the leading order of perturbation theory:

- Production of two free quarks and two free antiquarks in Higgs boson decay.
- Formation quark bound states.

Let us consider the basics of the calculation method using the example of the quark-gluon mechanism of production of bound states.
In the quasipotential approach the decay amplitude can be presented as a convolution of a perturbative production amplitude of two c-quark and two $\bar{c}$-antiquark and the quasipotential wave functions of the final mesons. Therefore, the amplitude of pair production is determined by two momentum integrals.

$$\mathcal{M}^{(1)}_{\mathcal{V}\mathcal{V}}(P,Q)=-i(\sqrt{2}G_F)^{\frac{1}{2}}\frac{2\pi}{3}M_{\mathcal{Q}\bar{\mathcal{Q}}}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\times$$

$$\times Tr\left\{\Psi^{\mathcal{V}}(p,P)\Gamma_1^{\nu}(p,q,P,Q)\Psi^{\mathcal{V}}(q,Q)\gamma_\nu+\Psi^{\mathcal{V}}(q,Q)\Gamma_2^{\nu}(p,q,P,Q)\Psi^{\mathcal{V}}(p,P)\gamma_\nu\right\},$$

$M_{\mathcal{Q}\bar{\mathcal{Q}}}$ is quarkonium mass. Vertex functions $\Gamma_{1,2}$ are determined by a specific decay mechanism.

Four-momenta p_1 and p_2 of c-quark and $\bar{c}$-antiquark in the pair forming the first $(\mathcal{Q}\bar{\mathcal{Q}})$ meson, and 4-momenta q_1 and q_2 for quark and antiquark in the second meson are expressed in terms of relative and total four momenta as follows:

$$p_{1,2}=\frac{1}{2}P\pm p,\quad (pP)=0;\qquad q_{1,2}=\frac{1}{2}Q\pm q,\quad (qQ)=0,$$

General formalism

The relativistic wave functions of the bound quarks accounting for the transformation from the rest frame to the moving one with four momenta P, and Q have the form:

$$\Psi^{\mathcal{V}}(p,P)=\frac{\Psi_0(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}\left[\frac{\hat{v}_1-1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}-\frac{\hat{p}}{2m}\right]\hat{\varepsilon}(P,S_z)$$

$$(1+\hat{v}_1)\left[\frac{\hat{v}_1+1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}+\frac{\hat{p}}{2m}\right],$$

$$\Psi^{\mathcal{V}}(q,Q)=\frac{\Psi_0(\mathbf{q})}{\left[\frac{\epsilon(q)}{m}\frac{\epsilon(q)+m}{2m}\right]}\left[\frac{\hat{v}_2-1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}+\frac{\hat{q}}{2m}\right]\hat{\varepsilon}(Q,S_z)$$

$$(1+\hat{v}_2)\left[\frac{\hat{v}_2+1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}-\frac{\hat{q}}{2m}\right],$$

where $v_1=P/M_{Q\bar{Q}}$, $v_2=Q/M_{Q\bar{Q}}$; $\epsilon(p)=\sqrt{m^2+\mathbf{p}^2}$, m is $c(b)$-quark mass. $\varepsilon^{\lambda}(P,S_z)$ is the polarization vector of the $J/\Psi(\Upsilon)$ meson.

Relativistic wave functions are dependent on relative momenta $\mathbf{p}$, $\mathbf{q}$ including the bound state wave function in the rest frame $\Psi_0(\mathbf{p})$. The color part of the meson wave function in the amplitudes is taken as $\delta_{ij}/\sqrt{3}$ (color indexes $i,j,k=1,2,3$). The general structure of relativistic wave functions allows us to say that they are the product of the wave functions of mesons in the rest frame and special projection operators resulting from the transformation from the moving reference frame to the reference frame in which the meson is at rest. Relativistic wave functions make it possible to correctly take into account the relativistic corrections connected with the relative momenta of heavy quarks.

General formalism

Total amplitude of the Higgs boson decay in the case of quark-gluon mechanism in the leading order in strong coupling constant α_s can be presented in the form:

$$\mathcal{M}^{(1)}_{\mathcal{V}\mathcal{V}} = \frac{4\pi}{3} M_{\mathcal{Q}\bar{\mathcal{Q}}}\alpha_s \Gamma_Q \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \mathrm{Tr}\{\mathcal{T}_{12} + \mathcal{T}_{34}\},$$

$$\mathcal{T}_{12} = \Psi^{\mathcal{V}}(p,P)\left[\frac{\hat{p}_1 - \hat{r} + m}{(r-p_1)^2 - m^2}\gamma_\mu + \gamma_\mu \frac{\hat{r} - \hat{q}_1 + m}{(r-q_1)^2 - m^2}\right] D^{\mu\nu}(k_2)\Psi^{\mathcal{V}}(q,Q)\gamma_\nu,$$

$$\mathcal{T}_{34} = \Psi^{\mathcal{V}}(q,Q)\left[\frac{\hat{p}_2 - \hat{r} + m}{(r-p_2)^2 - m^2}\gamma_\mu + \gamma_\mu \frac{\hat{r} - \hat{q}_2 + m}{(r-q_2)^2 - m^2}\right] D^{\mu\nu}(k_1)\Psi^{\mathcal{V}}(p,P)\gamma_\nu,$$

where we introduce the designations $\alpha_s = \alpha_s\left(\frac{M_H^2}{4\Lambda^2}\right)$, $\Gamma_Q = m(\sqrt{2}G_F)^{\frac{1}{2}}$.

Four-momentum of Higgs boson squared $r^2 = M_H^2 = (P+Q)^2 = 2M^2_{\mathcal{Q}\bar{\mathcal{Q}}} + 2PQ$. The gluon four-momenta are $k_1 = p_1 + q_1$, $k_2 = p_2 + q_2$. Relative momenta p, q of heavy quarks enter in the gluon propagators $D_{\mu\nu}(k_{1,2})$ and quark propagators as well as in relativistic wave functions. Accounting for the small ratio of relative quark momenta p and q to the mass of the Higgs boson M_H, we can simplify the denominators of quark and gluon propagators as follows:

$$\frac{1}{(p_1+q_1)^2} \approx \frac{1}{(p_2+q_2)^2} = \frac{4}{M_H^2}, \quad \frac{1}{(r-q_1)^2 - m_1^2} = \frac{2}{M_H^2}.$$

We completely neglect corrections of the form $|\mathbf{p}|/M_H$, $|\mathbf{q}|/M_H$. At the same time, we keep in the decay amplitudes the second-order correction for small ratios $|\mathbf{p}|/m$, $|\mathbf{q}|/m$ relative to the leading order result.

General formalism

Calculating the trace in obtained expression, we find relativistic amplitudes of the paired meson production in the form:

$$\mathcal{M}_{\mathcal{V}\mathcal{V}}{}^{(1)} = \frac{256\pi}{3M_H^4}(\sqrt{2}G_F)^{\frac{1}{2}} m M_{\mathcal{Q}\bar{\mathcal{Q}}} \alpha_s \varepsilon_1^{\lambda} \varepsilon_2^{\sigma} F_{1,\mathcal{V}\mathcal{V}}^{\lambda\sigma} |\tilde{\Psi}_{\mathcal{V}}(0)|^2,$$

where ε_1^{λ}, ε_2^{σ} are the polarization vectors of spin 1 mesons. The superscript in amplitude and subscript in tensor function $F_{\mathcal{V}\mathcal{V}}$ denote the contribution of the quark-gluon mechanism.

The contribution of the quark-photon amplitudes must also be taken into account. Despite the replacement $\alpha_s \to \alpha$ these amplitudes contain in the denominator the mass of the meson instead of the mass of the Higgs boson. The expression for the production amplitudes of the pair J/Ψ has a similar structure with slight changes in the common factors:

$$\mathcal{M}_{\mathcal{V}\mathcal{V}}{}^{(2)} = \frac{288\pi}{M_H^2 M_{\mathcal{Q}\bar{\mathcal{Q}}}}(\sqrt{2}G_F)^{\frac{1}{2}} m e_Q^2 \alpha \varepsilon_1^{\lambda} \varepsilon_2^{\sigma} F_{2,\mathcal{V}\mathcal{V}}^{\lambda\sigma} |\tilde{\Psi}_{\mathcal{V}}(0)|^2.$$

We also considered the contributions of other mechanisms of paired meson production, which we call the W-boson loop, quark loop, and ZZ-boson. The names are related to the structure of the interaction amplitudes. They contain different degrees of different parameters α_s, α, $\frac{M_H}{M}$, $\frac{m}{M}$, $\frac{M_Z}{M}$ and others and can give similar numerical contributions to the decay width.

General formalism

We also take into account the contribution of the W-bosonic loop, quark loop and ZZ-mechanisms.

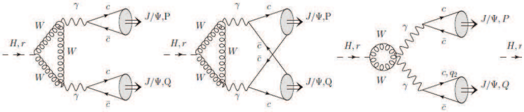


Рис.: W-boson loop mechanism.

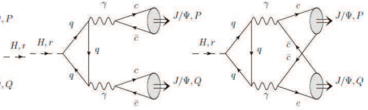


Рис.: Quark-loop mechanism.

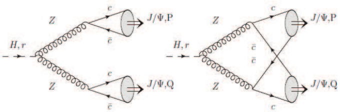


Рис.: ZZ-mechanism of pair production.

The difference between direct and crossed diagrams is that they have different mass factors in the denominator.

The tensor corresponding to the quark or W-boson loops in decay amplitudes has the following structure:

$$T^{\mu\nu}_{Q,W} = A_{Q,W}(t)(g^{\mu\nu}(v_1 v_2) - v_1^{\nu} v_2^{\mu}) + B_{Q,W}(t)[v_2^{\mu} - v_1^{\mu}(v_1 v_2)][v_1^{\nu} - v_2^{\nu}(v_1 v_2)],$$

$t = \frac{M_H^2}{4m_Q^2}$ or $t = \frac{M_H^2}{4m_W^2}$. The structure functions $A_{Q,W}(t)$, $B_{Q,W}(t)$ can be obtained using an

General formalism

Decay amplitudes of other mechanisms on these figures have the similar structure:

W-boson loop mechanism:

$$\mathcal{M}_{\mathcal{V}\mathcal{V}}{}^{(3)} = \frac{2052\pi^2}{m_Q M_{\mathcal{Q}\bar{\mathcal{Q}}}} (\sqrt{2}G_F)^{\frac{1}{2}} e_q^2 e_Q^2 \alpha^2 \varepsilon_1^{\lambda} \varepsilon_2^{\sigma} F^{\lambda\sigma}_{3,\mathcal{V}\mathcal{V}} |\tilde{\Psi}_{\mathcal{V}}(0)|^2,$$

Quark loop mechanism:

$$\mathcal{M}_{\mathcal{V}\mathcal{V}}{}^{(4)} = \frac{48\pi^2 M_Z M_W}{M^4_{\mathcal{Q}\bar{\mathcal{Q}}}} (\sqrt{2}G_F)^{\frac{1}{2}} e_q^2 \alpha^2 \cos\theta_W \varepsilon_1^{\lambda} \varepsilon_2^{\sigma} F^{\lambda\sigma}_{4,\mathcal{V}\mathcal{V}} |\tilde{\Psi}_{\mathcal{V}}(0)|^2,$$

ZZ-boson mechanism:

$$\mathcal{M}_{\mathcal{V}\mathcal{V}}{}^{(5)} = \frac{48\pi\alpha}{M_Z^2 \sin^2 2\theta_W} (\sqrt{2}G_F)^{\frac{1}{2}} \varepsilon_1^{\lambda} \varepsilon_2^{\sigma} F^{\lambda\sigma}_{5,\mathcal{V}\mathcal{V}} |\tilde{\Psi}_{\mathcal{V}}(0)|^2,$$

m is the mass of heavy quark c, b, produced in the vertex of Higgs boson decay, m_Q is the heavy quark mass in quark loop, e_q is the charge (in units e) of heavy quark (c or b) entering in final mesons, e_Q is the charge (in units e) of quark (c or b) in the quark loop.

It is appropriate to emphasize here that there are a number of parameters that determine the order of contribution of different quarkonium production amplitudes: α_s, α, $\frac{M_H}{M}$, $\frac{m}{M}$, $\frac{M_Z}{M}$ and others. The amplitude may have a higher order in α, but the appearance of different masses in the denominators of propagators for different decay mechanisms leads to compensation for these smallness factors.

General formalism

Tensor functions can be expressed in terms of two other functions g_1, g_2:

$$F^{\alpha\beta}_{i,\mathcal{V}\mathcal{V}} = g_1^{(i)} v_1^{\alpha} v_2^{\beta} + g_2^{(i)} g^{\alpha\beta}, \quad g_1^{(1)} = -2 + \frac{2}{9}\omega_1^2,$$

$$g_2^{(1)} = -1 - 2r_2 + r_1^2 + \frac{4}{3} r_2\omega_1 + \frac{1}{9}\omega_1^2 + \frac{2}{3} r_2\omega_1^2 - \frac{1}{9} r_1^2\omega_1^2,$$

$$g_1^{(2)} = 4 - \frac{4}{9}\omega_1^2, \;\; g_2^{(2)} = 2 + 4r_2 - 2r_1^2 - \frac{8}{3} r_2\omega_1 - \frac{2}{9}\omega_1^2 - \frac{4}{3} r\omega_1^2 + \frac{2}{9} r_1^2\omega_1^2,$$

$$g_{1,Q}^{(3)} = -A_Q(t)(1 + \frac{2}{3}\omega_1 + \frac{1}{9}\omega_1^2) + B_Q(t)(1 + \frac{2}{3}\omega_1 + \frac{1}{9}\omega_1^2),$$

$$g_{2,Q}^{(3)} = A_Q(t)\left(-1 - \frac{2}{3}\omega_1 - \frac{1}{9}\omega_1^2 + \frac{1}{2} r_1^2 + \frac{1}{3}\omega_1 r_1^2 + \frac{1}{18}\omega_1^2 r_1^2\right),$$

$$g_1^{(4)} = -A_W(t)\left(1 + \frac{2}{3}\omega_1 + \frac{1}{9}\omega_1^2\right) + B_W(t)\left(1 + \frac{2}{3}\omega_1 + \frac{1}{9}\omega_1^2\right),$$

$$g_2^{(4)} = A_W(t)\left(-1 - \frac{2}{3}\omega_1 - \frac{1}{9}\omega_1^2 + \frac{1}{2} r_1^2 + \frac{1}{3}\omega_1 r_1^2 + \frac{1}{18}\omega_1^2 r_1^2\right),$$

$$g_2^{(5)} = \left(1 + \frac{1}{3}\omega_1\right)^2 \left(\frac{1}{2} - a_z\right)^2 - \frac{M_z^4}{3\left(\frac{M_H^2}{4} - M_Z^2\right)^2}\Big(-\frac{1}{4} - \frac{1}{6}\omega_1 - \frac{1}{36}\omega_1^2 +$$

$$+\frac{1}{2} a_z + \frac{1}{3}\omega_1 a_z + \frac{1}{18}\omega_1^2 a_z - \frac{1}{2} a_z^2 - \frac{1}{3}\omega_1 a_z^2 - \frac{1}{18}\omega_1^2 a_z^2\Big), \quad g_1^{(5)} = 0,$$

where the mass ratios are: $r_1 = \frac{M_H}{M_{\mathcal{Q}\bar{\mathcal{Q}}}}$, $r_2 = \frac{m}{M_{\mathcal{Q}\bar{\mathcal{Q}}}}$, $a_z = 2|e_Q|\sin^2\theta_W$.

General formalism

The decay widths of the Higgs boson into a pair of vector quarkonia states are determined by the following expression:

$$\Gamma_{\mathcal{V}\mathcal{V}} = \frac{2^{14}\sqrt{2}\pi\alpha_s^2 m^2 G_F |\tilde{\Psi}_{\mathcal{V}}(0)|^4 \sqrt{\frac{r_1^2}{4}-1}}{9M_H^5 r_1^5} \sum_{pol} |\varepsilon_1^{\lambda}\varepsilon_2^{\sigma} F_{\mathcal{V}\mathcal{V}}^{\lambda\sigma}|^2,$$

The tensor function consists of five parts corresponding to five production mechanisms:

$$F_{\mathcal{V}\mathcal{V}}^{\lambda\sigma} = \Big[g_1^{(1)} + \frac{9}{16} r_1^2 \frac{e_q^2\alpha}{\alpha_s} g_1^{(2)} + \sum_Q \frac{27\pi}{8} r_1^4 \frac{e_Q^2 e_q^2 \alpha^2 m_Q^2}{\alpha_s m M_{\mathcal{Q}\bar{\mathcal{Q}}}} g_{1,Q}^{(3)} + \frac{9\pi e_q^2 \alpha^2 r_1^4 M_Z M_W}{64\alpha_s m M_{\mathcal{Q}\bar{\mathcal{Q}}}} g_1^{(4)} +$$

$$\frac{9M_H^4\alpha}{16 M_Z^2 m M_{\mathcal{Q}\bar{\mathcal{Q}}}\alpha_s} \frac{\left(\frac{1}{2} - 2|e_q|\sin^2\theta_W\right)^2}{\sin^2 2\theta_W} g_1^{(5)}\Big] v_1^{\sigma} v_2^{\lambda} + \Big[g_2^{(1)} + \frac{9}{16} r_1^2 \frac{e_q^2\alpha}{\alpha_s} g_2^{(2)} +$$

$$\sum_Q \frac{27\pi}{8} r_1^4 \frac{e_q^2 e_Q^2 \alpha^2 m_Q^2}{\alpha_s m M_{\mathcal{Q}\bar{\mathcal{Q}}}} g_{2,Q}^{(3)} + \frac{9\pi e_q^2\alpha^2 r_1^4 M_Z M_W}{64\alpha_s m M_{\mathcal{Q}\bar{\mathcal{Q}}}} \cos\theta_W g_2^{(4)} + \frac{9M_H^4\alpha}{16M_Z^2 m M_{\mathcal{Q}\bar{\mathcal{Q}}}\alpha_s} \frac{1}{\sin^2 2\theta_W} g_2^{(5)}\Big] g^{\lambda\sigma}.$$

We found it convenient to separate in square brackets the coefficients denoting the relative contribution of different decay mechanisms with respect to the quark-gluon mechanism. The common factor corresponds to the amplitudes of the quark-gluon decay mechanism.

The coefficients for the functions g_1, g_2 differ in different degrees of α, α_s and mass factors.

Numerical results

To calculate ω_n, we assume that the dynamics of quark-antiquark pairs is determined by the QCD generalization of the standard Breit Hamiltonian in the center-of-mass reference frame (Despite the existing differences in quark models, such terms of the Hamiltonian are used by many authors):

$$H = H_0 + \Delta U_1 + \Delta U_2, \quad H_0 = 2\sqrt{\mathbf{p}^2 + m^2} - 2m - \frac{C_F \tilde{\alpha}_s}{r} + Ar + B,$$

$$\Delta U_1(r) = -\frac{C_F \alpha_s^2}{4\pi r}\left[2\beta_0 \ln(\mu r) + a_1 + 2\gamma_E \beta_0\right], \quad a_1 = \frac{31}{3} - \frac{10}{9} n_f, \quad \beta_0 = 11 - \frac{2}{3} n_f,$$

$$\Delta U_2(r) = -\frac{C_F \alpha_s}{2m^2 r}\left[\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right] + \frac{\pi C_F \alpha_s}{m^2}\delta(\mathbf{r}) + \frac{3 C_F \alpha_s}{2m^2 r^3}(\mathbf{S}\mathbf{L}) -$$

$$-\frac{C_F \alpha_s}{2m^2}\left[\frac{\mathbf{S}^2}{r^3} - 3\frac{(\mathbf{S}\mathbf{r})^2}{r^5} - \frac{4\pi}{3}(2\mathbf{S}^2 - 3)\delta(\mathbf{r})\right] - \frac{C_A C_F \alpha_s^2}{2mr^2},$$

where $\mathbf{L} = [\mathbf{r} \times \mathbf{p}]$, $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$, n_f is the number of flavors, $C_A = 3$ and $C_F = 4/3$ are the color factors of the SU(3) color group, $\gamma_E \approx 0.577216$ is the Euler constant. To describe the hyperfine structure of the energy spectrum, the following confinement potential is usually added:

$$\Delta V_{conf}^{hfs}(r) = f_V\left[\frac{A}{2m^2 r}\left(1 + \frac{8}{3}\mathbf{S}_1\mathbf{S}_2\right) + \frac{3A}{2m^2 r}\mathbf{L}\mathbf{S} + \frac{A}{3m^2 r}\left(\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - \mathbf{S}_1\mathbf{S}_2\right)\right] -$$

$$-(1 - f_V)\frac{A}{2m^2 r}\mathbf{L}\mathbf{S},$$

Using such a Hamiltonian, it is possible to describe the mass spectrum of charmonium and bottommonium with an accuracy of 0.1 percent.

Numerical results

Using this Hamiltonian, we construct an effective model of the interaction of quarks in a bound state of the Schrödinger type. The numerical solution of the Schrödinger equation, taking into account various corrections in the potential, makes it possible to find the wave function of the bound state and the relativistic parameters. We thus have a self-consistent method for describing the decay processes of the Higgs boson, which allows us to obtain not only general expressions for the decay amplitudes, but also to calculate numerous parameters while being inside the model.

In the case of S-states parameters ω_n are determined by the momentum integrals I_n in the form:

$$I_n^{\mathcal{P},\mathcal{V}} = \int_0^\infty p^2 R^{\mathcal{P},\mathcal{V}}(p) \frac{(\varepsilon(p)+m)}{2\varepsilon(p)} \left(\frac{\varepsilon(p)-m}{\varepsilon(p)+m}\right)^n dp,$$

$$\tilde{R}(0) = \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^\infty \frac{(\varepsilon(p)+m)}{2\varepsilon(p)} p^2 R(p) dp, \quad \omega_1^{\mathcal{P},\mathcal{V}} = \frac{I_1^{\mathcal{P},\mathcal{V}}}{I_0^{\mathcal{P},\mathcal{V}}}, \quad \omega_2^{\mathcal{P},\mathcal{V}} = \frac{I_2^{\mathcal{P},\mathcal{V}}}{I_0^{\mathcal{P},\mathcal{V}}},$$

Таблица: Numerical results for the decay widths in the nonrelativistic approximation and with the account for relativistic corrections (in GeV).

Final state	Nonrelativistic decay width	Relativistic decay width	D.-N. Gao and X. Gong PLB 2022	I.N. Belov et al. PRD 2023
$J/\Psi + J/\Psi$ $1^3S_1 + 1^3S_1$	$3.29 \cdot 10^{-12}$	$0.69 \cdot 10^{-12}$	$1.9 \cdot 10^{-12}$	$23.72 \cdot 10^{-12}$
$\Upsilon + \Upsilon$ $1^3S_1 + 1^3S_1$	$0.63 \cdot 10^{-12}$	$0.74 \cdot 10^{-12}$	$1.4 \cdot 10^{-12}$	$3.57 \cdot 10^{-12}$

One-loop corrections to decay rate are calculated by I.N. Belov, A.V. Berezhnoy, E. A. Leshchenko, A.K. Likhoded.

Numerical results

Accounting for relativistic corrections in this work shows that such contributions lead to a significant change in nonrelativistic results. The main parameter that greatly reduces the nonrelativistic results is $\tilde{R}(0)$, which enters in the decay width to the fourth power. Therefore, the difference between the relativistic and nonrelativistic results in Table turns out to be more significant in the case of pair production of charmonium.

In Table, we present separately the numerical values of the contributions from different decay mechanisms with an accuracy of two significant figures after the decimal point. The leading contribution is the contribution from the quark-photon mechanism in Fig.2 and the ZZ-boson mechanism in Fig.5, which are one order of magnitude greater than the other contributions in the case of charmonium production. Its value is due to the structure of the decay amplitudes, in which small denominators $1/M^2$ appear from the photon propagators in contrast to other amplitudes in which there is a factor $1/M_H^2$. Theoretical error 10% is determined by higher order relativistic corrections.

Таблица: The contributions of different mechanisms to the Higgs boson decay widths in GeV.

The contribution accounting for relativistic corrections		
Contribution	$H \to J/\psi\ J/\psi$	$H \to \Upsilon\ \Upsilon$
Fig.1	$0.36 \cdot 10^{-15}$	$0.10 \cdot 10^{-12}$
Fig.2	$0.80 \cdot 10^{-12}$	$0.16 \cdot 10^{-12}$
Fig.3	$0.70 \cdot 10^{-13}$	$0.37 \cdot 10^{-12}$
Fig.4	$0.74 \cdot 10^{-13}$	$0.68 \cdot 10^{-13}$
Fig.5	$0.22 \cdot 10^{-12}$	$1.45 \cdot 10^{-12}$
Total contribution	$0.69 \cdot 10^{-12}$	$0.74 \cdot 10^{-12}$

Production of leptonic bound states

Another part of the work is devoted to calculating the widths of Higgs boson decays with single and pair production of bound ortho - states of leptons (positronium (e^+e^-), dimuonium $(\mu^+\mu^-)$, ditauonium $(\tau^+\tau^-)$).
For such reactions we also use our calculation method. The difference between these reactions and the production of mesons is that in this case all calculations can be performed analytically, since the explicit form of the wave function of the bound state of leptons is known.

Reactions of single leptonium production.

- $H \to Z, \gamma + (e^+e^-)$ $\quad H \to Z, \gamma + (\mu^+\mu^-)$ $\quad H \to Z, \gamma + (\tau^+\tau^-)$

Reactions of double leptonium production.

- $H \to (e^+e^-) + (e^+e^-)$ $\quad H \to (\mu^+\mu^-) + (\mu^+\mu^-)$ $\quad H \to (\tau^+\tau^-) + (\tau^+\tau^-)$

Various mechanisms for the production of bound states of leptons are considered.

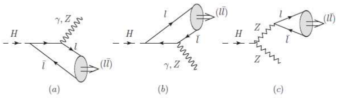


Рис.: Lepton - photon and lepton - Z - boson mechanisms of single production of leptonium.

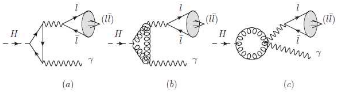


Рис.: Quark loop and W - boson loop mechanisms of single leptonium production.

Research method

Decay widths are calculated using the relativistic quasipotential approach

The amplitude of the production of a bound state of leptons is represented by the convolution of the amplitude of the production of free leptons with the wave function of the bound state. For the case of a single production $(H \to Z, \gamma + (l^+ l^-))$:

$$d\Gamma = \frac{|\mathbf{P}|}{32\pi^2 M_H^2} \overline{|M|^2} d\Omega, \quad \mathcal{M}(k,P) = \int \frac{d\mathbf{p}}{(2\pi)^3} \mathcal{M}_{H\to Z,\gamma+l+\bar{l}}(k,P,\mathbf{p}) \Psi_{(l\bar{l})}(\mathbf{p})$$

$\mathbf{P}$ is the total momentum of the bound state, $\mathbf{p}$ is the relative momentum of leptons.
In the case of lepton-photon production mechanism the amplitude has the form:

$$\mathcal{M}_1(k,P) = \frac{4\pi\alpha m}{\sin 2\theta_W M_Z} \int \frac{d\mathbf{p}}{(2\pi)^3} Tr\Big\{ \Psi^{\mathcal{V}}(P,p) \Big(\hat{\varepsilon}_\gamma \frac{(\hat{r} - \hat{p}_2 + m)}{(r-p_2)^2 - m^2} + \frac{(\hat{p}_1 - \hat{r} + m)}{(p_1 - r)^2 - m^2} \hat{\varepsilon}_\gamma \Big) \Big\}.$$

$p_{1,2} = \frac{1}{2}P \pm p$ are four-momenta of lepton and anti-lepton.

The amplitude contains the relativistic wave function of the bound state, which is obtained as a result of the Lorentz transformation from a rest frame to a moving frame:

$$\Psi^{\mathcal{V}}(P,p) = \frac{\Psi_0(\mathbf{p})}{\frac{\varepsilon}{m}\frac{(\varepsilon+m)}{2m}} \left[\frac{\hat{v}-1}{2} - \hat{v}\frac{p^2}{2m(\varepsilon+m)} - \frac{\hat{p}}{2m} \right] \hat{\varepsilon}_{l\bar{l}} \frac{(\hat{v}+1)}{2\sqrt{2}} \left[\frac{\hat{v}+1}{2} - \hat{v}\frac{p^2}{2m(\varepsilon+m)} + \frac{\hat{p}}{2m} \right].$$

$v = \frac{P}{M}$, M is the bound state mass, $\varepsilon = \sqrt{\mathbf{p}^2 + m^2}$, m is the lepton mass.

Relativistic corrections to the interaction amplitude

Relativistic corrections to the decay width are divided into three types:

- relativistic corrections to the decay amplitude
- relativistic corrections in the law of transformation of the wave function of a bound state
- relativistic corrections to the wave function of the bound state of leptons in the rest frame

Relativistic corrections in the interaction amplitude are determined by the powers of the relative momentum $\mathbf{p}/m$, $|\mathbf{p}| \sim W = \frac{m}{2}\alpha$. They are determined by special relativistic parameters ω_i, which are expressed by momentum integrals:

$$\frac{|\mathbf{p}|}{2m} = \sum_{n=1}^{\infty} \left(\frac{\varepsilon(p) - m}{\varepsilon(p) + m} \right)^{n+\frac{1}{2}}, \quad \psi_{1S}^{C}(p) = \frac{8\sqrt{\pi} W^{5/2}}{(p^2 + W^2)^2}, \quad W = \frac{m}{2}\alpha,$$

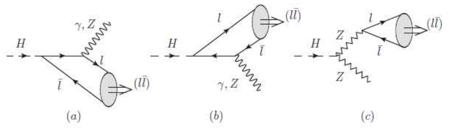
$$I^{(i)} = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{\varepsilon(p) + m}{2\varepsilon(p)} \psi_{1S}^{C}(\mathbf{p}) \left(\frac{\varepsilon(p) - m}{\varepsilon(p) + m} \right)^{i}, \quad \omega_i = \frac{I^{(i)}}{I^{(0)}}.$$

The integrals $I^{(i)}$ can be calculated analytically. We take into account second-order relativistic corrections $\sim \alpha^2$ in the interaction amplitude (residue at the pole of the wave function):

$$I^{(0)} = \int \frac{d\mathbf{p}}{(2\pi)^3} \psi_{1S}^{C}(\mathbf{p}) \left[1 - \frac{\mathbf{p}^2}{4m^2} \right] = \psi_{1S}^{C}(0) \left[1 + \frac{3}{4}\alpha^2 \right],$$

$$I^{(1)} = \int \frac{d\mathbf{p}}{(2\pi)^3} \psi_{1S}^{C}(\mathbf{p}) \frac{\mathbf{p}^2}{4m^2} = -\frac{3}{4}\alpha^2 \psi_{1S}^{C}(0), \quad \omega_1 = -\frac{3}{4}\alpha^2.$$

Single leptonium production. Analytical result.



Total decay width $H \to \gamma + (l\bar{l})$ (diagrams (a) and (b)):

$$\Gamma_{\gamma(l\bar{l})} = \frac{512\pi\alpha^2 r_1^2}{r_2^3 \sin^2 2\theta_W M_Z^2 (r_2^2 - 1)} \times \left[\frac{1}{2} r_2^2 g_2^2 + 3g_1^2 - \frac{1}{4} g_2^2 - \frac{1}{4} r_2^4 g_2^2\right] |\tilde{\psi}(0)|^2.$$

$$\tilde{\psi}(0) = I^{(0)} = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{\varepsilon(p)+m}{2\varepsilon(p)} \psi_{1S}^C(\mathbf{p}),\ r_1 = m/M,\ r_2 = M_H/M.$$

Functions g_1, g_2 are obtained after the trace calculation in the amplitude numerator in FORM package:

$$N = g_1(\varepsilon_{l\bar{l}}\varepsilon_\gamma) - g_2(v\varepsilon_\gamma)(v_\gamma\varepsilon_{l\bar{l}}),\quad g_1 = \left[r_2^2\left(\frac{1}{2} + \frac{5}{6}\omega_1\right) - r_1 - r_1\omega_1\right],\quad g_2 = \left[1 + \frac{5}{3}\omega_1\right].$$

The Higgs boson decay width with the production of leptonium contains the leptonium wave function at zero:

$$\psi_{nS}^C(\mathbf{r} = 0) = \frac{(m\alpha)^3}{8\pi n^3}$$

Due to the smallness of both the leptonic mass and the constant α, the decay width in the case of leptonic bound states will be strongly suppressed even with a single production of leptonic bound states.

Single leptonium production. Analytical result.

Total decay width $H \to Z + (l\bar{l})$ (diagrams (a) and (b)):

$$\Gamma_{Z(l\bar{l})} = \frac{32\pi\alpha^2\sqrt{[(r_2+1)^2 - r_3^2][(r_2-1)^2 - r_3^2]}}{r_2^3 \sin^4 2\theta_W M_Z^2 (r_2^2-1)^2} |\tilde{\psi}(0)|^2 \times \Big[g_{2\ Z}^2 (\frac{3}{8} + \frac{1}{16} r_3^{-4} - \frac{1}{4} r_3^{-2} - \frac{1}{4} r_3^2 + \frac{1}{16} r_3^4 -$$

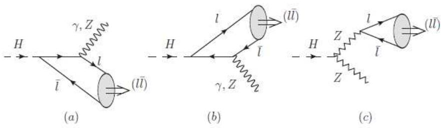
$$\frac{1}{4} r_2^2 r_3^{-4} + \frac{1}{4} r_2^2 r_3^{-2} + \frac{1}{4} r_2^2 - \frac{1}{4} r_2^2 r_3^2 + \frac{3}{8} r_2^4 r_3^{-4} + \frac{1}{4} r_2^4 r_3^{-2} + \frac{3}{8} r_2^4 - \frac{1}{4} r_2^6 r_3^{-4} - \frac{1}{4} r_2^6 r_3^{-2} + \frac{1}{16} r_2^8 r_3^{-4}) +$$

$$g_{1\ Z} g_{2\ Z} (\frac{1}{4} r_3^{-3} - \frac{1}{4} r_3^{-1} - \frac{1}{4} r_3 + \frac{1}{4} r_3^3 - \frac{3}{4} r_2^2 r_3^{-4}) + g_{1\ Z}^2 (\frac{5}{2} - \frac{1}{4} r_3^{-2} + \frac{1}{4} r_3^2 - \frac{1}{2} r_2^2 r_3^{-2} - \frac{1}{2} r_2^2 + \frac{1}{4} r_2^4 r_3^{-2}) \Big],$$

$$g_{1\ Z} = (r_2^2 - r_3^2)(\frac{1}{8} - \frac{1}{4} a_z + \frac{5}{24}\omega_1 - \frac{5}{12}\omega_1 a_z) + r_1(-\frac{1}{4} + \frac{1}{2} a_z - \frac{1}{4}\omega_1 + \frac{1}{2}\omega_1 a_z),$$

$$g_{2\ Z} = (\frac{1}{4} - \frac{1}{2} a_z + \frac{5}{12}\omega_1 - \frac{5}{6}\omega_1 a_z),$$

where additional mass ratio is introduced: $r_3 = M_Z/M$, $a_z = 2\sin^2(\theta_W)$.



Total decay width $H \to Z + (l\bar{l})$ (diagram (c)) can be obtained with the replacement:

$$g_{1\ Z} \to g_{1\ Z} + \frac{r_2^2}{r_1} g_{1\ ZZ},$$

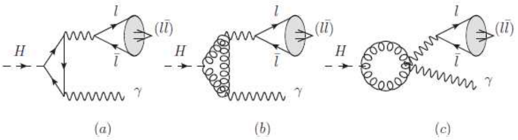
$$g_{1\ ZZ} = \frac{1}{4} + \frac{1}{2} a_z + \frac{1}{12}\omega_1 + \frac{1}{6}\omega_1 a_z.$$

Single leptonium production. Loop mechanism.

The general structure of the tensor corresponding to the loop in the case of two external virtual photons has the form:

$$T^{\mu\nu}_{Q,W} = A_{Q,W}(t)(g^{\mu\nu}(v_1v_2) - v_1^{\nu}v_2^{\mu}) + B_{Q,W}(t)[v_1^2v_2^{\mu} - v_1^{\mu}(v_1v_2)][v_1^{\nu}v_2^2 - v_2^{\nu}(v_1v_2)],$$

where $t = \frac{M_H^2}{4m_Q^2}$ or $t = \frac{M_H^2}{4m_W^2}$ for quark loop and W - boson loop.



When considering a single leptonium production, one of the photons is real, and only the contribution of the structure function $A_{Q,W}(t)$ takes place. The function $A_{Q,W}(t)$ is calculated from its imaginary part using the dispersion relation.

$$A_W(t) = A_W(0) + \frac{t}{\pi}\int_1^{\infty}\frac{ImA(t')dt'}{t'(t'-t+i0)}.$$

To calculate its imaginary part, the Mandelstam-Cutkosky rule is used.

Single leptonium production. Loop mechanism.

The imaginary part in the case of a W - bosonic loop is equal to:

$$ImA_W = \frac{r_4^2}{64\pi}\frac{1}{t(4t-r_4^2)^2}\Bigg[r_4^2\sqrt{t(t-1)}(r_4^2(2t+1)-4t-6)+$$

$$4t(6-12t+r_4^2(2t+3)-r_4^4)arcsh(\sqrt{t-1})\Bigg], \quad r_4 = \frac{M}{M_W}.$$

The imaginary part in the case of a quark loop is equal to:

$$ImA_Q = \frac{r_5^2}{32\pi}\frac{1}{t(4t-r_5^2)^3}\Bigg[3r_5^2\sqrt{t(t-1)}(4t-r_5^2)+$$

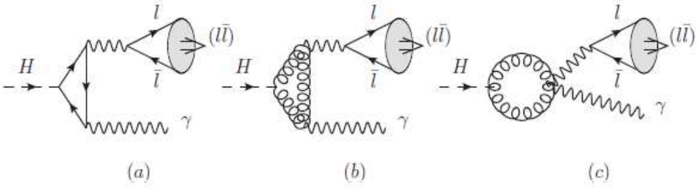
$$4t(r_5^4+r_5^2(2-4t)+8(t-1)t)arcsh(\sqrt{t-1})\Bigg], \quad r_5 = \frac{M}{m_Q}.$$

After expansion in r_4 and r_5, respectively, $(M_W \gg M,\ m_Q \gg M)$ in leading order, the structure function $A_{Q,W}(t)$ is determined by the following expression:

$$A_W(t) = \frac{r_4^2}{16\pi^2}\left[2+\frac{3}{t}+\frac{3}{t^2}(2t-1)f^2(t)\right], \quad f(t) = \begin{cases} \arcsin\sqrt{t}, & t \le 1, \\ \frac{i}{2}\left[\ln\frac{1-\sqrt{1-t^{-1}}}{1+\sqrt{1-t^{-1}}} - i\pi\right], & t > 1. \end{cases}$$

$$A_Q(t) = \frac{r_5^2}{16\pi^2}\left[\frac{1}{t}+\frac{(t-1)}{t^2}f^2(t)\right].$$

Single leptonium production. Loop mechanism.



Higgs boson decay width from amplitude with W - boson loop is equal to

$$\Gamma = \frac{8\pi^3\alpha^4(r_2^2-1)ctg^2(\theta_W)M_Z^2}{r_2^3 M_{l\bar{l}}^4}|\tilde{\psi}(0)|^2 A_W^2[3g_{1\ W}^2 - \frac{1}{4}(r_2^2-1)^2 g_{2\ W}^2],$$

$$g_{1\ W} = (r_2^2-1)(1+\frac{7}{3}\omega_1+\frac{11}{3}\omega_2),\ g_{2\ W} = 2(1+\frac{7}{3}\omega_1+\frac{11}{3}\omega_2).$$

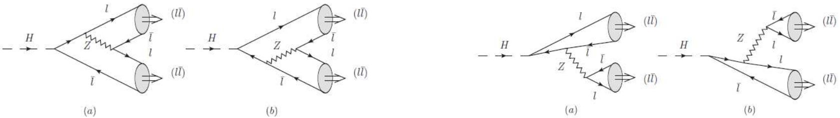
Higgs boson decay width from amplitude with quark loop has a similar structure and can be obtained by substitution:

$$g_{1,2\ W} \to g_{1,2\ W}\left(1+\sum_{Q=c,b,t}\frac{24 q_Q^2 m_Q^2}{\cos^2\theta_W M_Z^2}\frac{|A_Q|}{A_W}\right).$$

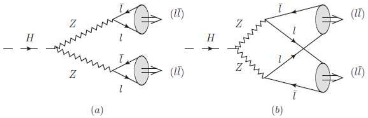
Pair leptonium production

Lepton - photon mechanism

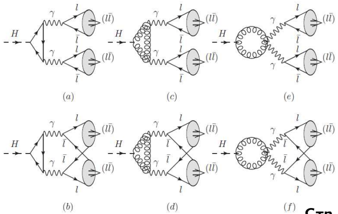
Lepton - Z - boson mechanism



ZZ-boson mechanism: $H \to Z + Z \to (l\bar{l}) + (l\bar{l})$



Quark loop and W - boson loop mechanisms



Relativistic corrections to the bound state wave function.

In the nonrelativistic approximation, the wave function of the bound state of leptons is obtained from solving the Schrödinger equation with a Coulomb potential.

To calculate relativistic corrections to the wave function at zero, we use the expression:

$$\psi^{(1)}_{1S}(0) = \int \tilde{G}_{1S}(0, \mathbf{r}) \Delta V(\mathbf{r}) \psi^{C}_{1S}(\mathbf{r}) d\mathbf{r},$$

where $\tilde{G}_{1S}$ is the reduced Coulomb Green function, ΔV is the perturbation potential.

The Green's function with one zero argument is known analytically:

$$\tilde{G}_{1S}(\mathbf{r}, 0) = \frac{\mu W}{4\pi} \frac{e^{-x}}{x} (4x(\ln(2x) + \gamma_E) + 4x^2 - 10x - 2), \quad x = Wr$$

As a perturbation we use the following operators:

Terms from the Breit potential:

$$\Delta V_1 = \frac{\pi\alpha}{m^2}\delta(\mathbf{r}) - \frac{\mathbf{p}^4}{4m^3},$$

$$\Delta V_2 = -\frac{\alpha}{2m^2}\frac{1}{r}\left(\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right),$$

$$\Delta V_3 = \frac{\pi\alpha}{m^2}\left[\frac{7}{3}\mathbf{S}^2 - 2\right]\delta(\mathbf{r}).$$

Vacuum polarization potential:

$$\Delta V_{vp}(r) = -\frac{\alpha^2}{3\pi}\int_1^\infty \rho(\xi) d\xi \frac{1}{r} e^{-2m_e \xi r},$$

$$\rho(\xi) = \frac{\sqrt{\xi^2 - 1}(2\xi^2 + 1)}{\xi^4}.$$

Relativistic corrections to the bound state wave function.

Considering the perturbation potential $\sim \delta(\mathbf{r})$ from ΔV_1 we obtain the correction to the wave function:

$$\psi_{1S}^{(1)}(0) = \tilde{G}_{1S}(0,0)\frac{\pi\alpha}{m^2}\psi_{1S}^{C}(0),$$

in which $\tilde{G}_{1S}(0,0)$ represents a divergent quantity. Along with the coordinate representation, it is convenient to use the momentum representation. In the momentum representation, this correction is determined by a divergent integral in the form:

$$\psi_1^{(1)}(0) = \frac{\pi\alpha}{m^2}\psi_{1S}^{C}(0)\int \tilde{G}_{1S}(\mathbf{q},\mathbf{p})\frac{d\mathbf{q}}{(2\pi)^3}\frac{d\mathbf{p}}{(2\pi)^3},$$

$$\tilde{G}_{1S}(\mathbf{p},\mathbf{q}) = -\frac{64\pi}{\alpha W^4}\left[\frac{\pi^2 W^5\delta(\mathbf{p}-\mathbf{q})}{4(\mathbf{p}^2+W^2)} + \frac{W^6}{4(\mathbf{p}^2+W^2)(\mathbf{p}-\mathbf{q})^2(\mathbf{q}^2+W^2)} + R(\mathbf{p},\mathbf{q})\right] \quad .$$

If we take into account, together with this correction, the contribution from the second term $\Delta V_1 \sim \mathbf{p}^4$:

$$\psi_2^{(1)}(0) = -\frac{1}{4m^3}\int \tilde{G}_{1S}(0,\mathbf{r})\hat{\mathbf{p}}^4\psi(r)d\mathbf{r} = -\frac{\pi\alpha}{m^2}\psi_{1S}^{C}(0)\int \tilde{G}(\mathbf{q},\mathbf{p})\frac{d\mathbf{p}d\mathbf{q}}{(2\pi)^6}\left[1-\frac{W^2(2p^2+W^2)}{(p^2+W^2)^2}\right].$$

then, as a result, the divergent terms cancel each other and the contribution of the operator turns out to be finite. The remaining integrals are calculated analytically and lead to a correction to the wave function $\sim \alpha^2$:

$$\Delta\psi^{(1)}(0) = \psi_{1S}^{C}(0)[-\frac{63}{128}\alpha^2].$$

Relativistic corrections to the bound state wave function.

Corrections to the wave function from other perturbation potentials also can be calculated analytically. The results are the following:

- ΔV_2: $$\Delta\psi^{(2)}(0) = \psi^C_{1S}(0)\left[\frac{1}{2}\alpha^2\ln\alpha^{-1} + \frac{5}{8}\alpha^2\right]$$
- ΔV_3: $$\Delta\psi^{(3)}(0) = \psi^C_{1S}(0)\left[2 - \frac{7}{3}S(S+1)\right]\frac{1}{4}\alpha^2\ln\alpha^{-1}$$
- The vacuum polarization potential ΔV_{vp} leads to the α leading correction to the wave function $\sim\alpha$:

$$\Delta\psi^{(4)}(0) = \psi^C_{1S}(0)\left[a_{vp}\frac{\alpha}{\pi}\right],\ a_{vp} = \int_1^\infty \frac{\rho(\xi)d\xi}{6(1+r_4\xi)}\left[2r_4^2\xi^2 + 7r_4\xi + 2(1+r_4\xi)\ln(1+r_4\xi) + 3\right]$$

The total expression for the wave function at zero taking into account the calculated corrections is the following:

$$\psi(0) = \psi^C_{1S}(0)\left\{1 + a_{vp}\frac{\alpha}{\pi} + \left(2 - \frac{7}{6}S(S+1)\right)\frac{1}{2}\alpha^2\ln\alpha^{-1} - \frac{3}{128}\alpha^2\right\}\quad .$$

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Numerical results.

In Table we present the numerical results of calculating the decay widths of the Higgs boson with the production of single lepton bound states and a photon or Z-boson. The obtained values for Γ of these rare decays are small, and their observation is possible only at high-luminosity colliders being designed in the future, when the production of a significant number of Higgs bosons ($\sim 10^{10}$) will be possible.

Таблица: Numerical values of the relative widths (Γ_i/Γ_{tot}) of Higgs boson decay with single or pair production of bound states of leptons.

			Our result	D. d'Enterria and V.D. Le
		(e^+e^-)	$0.88 \cdot 10^{-10}$	$0.35 \cdot 10^{-11}$
	$\gamma +$	$(\mu^+\mu^-)$	$1.12 \cdot 10^{-11}$	$0.35 \cdot 10^{-11}$
		$(\tau^+\tau^-)$	$3.48 \cdot 10^{-12}$	$0.22 \cdot 10^{-11}$
		(e^+e^-)	$7.87 \cdot 10^{-13}$	$5.2 \cdot 10^{-13}$
H $\rightarrow$	$Z +$	$(\mu^+\mu^-)$	$9.85 \cdot 10^{-13}$	$5.7 \cdot 10^{-13}$
		$(\tau^+\tau^-)$	$5.68 \cdot 10^{-11}$	$1.4 \cdot 10^{-11}$
	$(e^+e^-) +$	(e^+e^-)	$2.05 \cdot 10^{-19}$	—
	$(\mu^+\mu^-) +$	$(\mu^+\mu^-)$	$1.13 \cdot 10^{-20}$	—
	$(\tau^+\tau^-) +$	$(\tau^+\tau^-)$	$1.09 \cdot 10^{-18}$	—

Conclusion

- The decay widths of the Higgs boson with the production of a pair of bound states of heavy quarks have been calculated within the framework of the relativistic quasipotential approach.
- The decay widths of the Higgs boson with the production of one or a pair of bound states of leptons have been calculated within the framework of the relativistic quasipotential approach.
- The calculations take into account relativistic corrections connected with the relative motion of quarks and leptons both in the interaction amplitude and in the wave function of the bound state.
- Various mechanisms of single and pair production of bound states of particles have been studied.

The results obtained for the production width of mesons and leptonia show that to observe such events it is necessary to increase the luminosity of the LHC. At future accelerators, in particular at $e^+ - e^-$ at FCC, it will be possible to search for such decays, for example, by their three-photon decays for lepton bound states.

The CMS collaboration began the search for rare Higgs boson decays into a pair of heavy vector quarkonia in 2019. The results of new upper limits on the branching fractions:

$$B(H \to J/\Psi, J/\Psi) < 3.8 \cdot 10^{-4},$$

$$B(H \to \Upsilon(1S), \Upsilon(1S)) < 1.7 \cdot 10^{-3}.$$

CMS Collaboration, arXiv:2206.03525v1 [hep-ex].

Thank you!