Doubly-Heavy Tetra- and Pentaquarks

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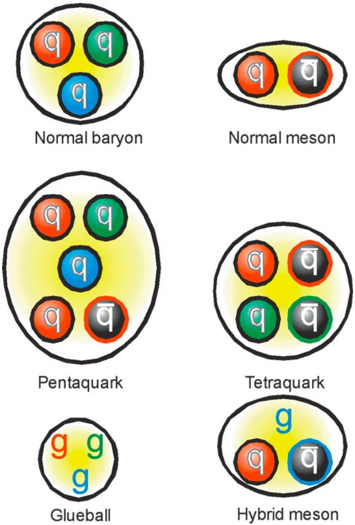
Introduction



Introduction

- In 2003, the first exotic hidden-charm state $X(3872)$ was observed by the Belle Collaboration
- This state was confirmed by BaBar, CDF, D0, BESII, and all the LHC collaborations
- Soon after, many other mesons with masses above the $D\bar{D}$ threshold have been observed
- Searches of new exotic states is one of the main topics of BESIII and LHCb collaborations at present
- Observation of charged hidden-charm and hidden-bottom mesons was the direct manifestation of tetraquarks
- In addition, LHCb observed a few narrow baryons which have got the interpretation as hidden-charm pentaquarks

Introduction

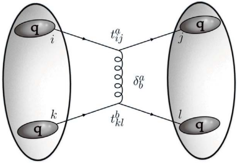
Reviews on exotic hadrons:

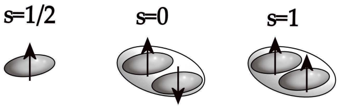
1. A. Esposito, A. Pilloni and A. D. Polosa, “Multiquark Resonances,” Phys. Rept. **668**, 1 (2016).
2. H. X. Chen, W. Chen, X. Liu and S. L. Zhu, “The hidden-charm pentaquark and tetraquark states,” Phys. Rept. **639**, 1 (2016).
3. R. F. Lebed, R. E. Mitchell and E. S. Swanson, “Heavy-Quark QCD Exotica,” Prog. Part. Nucl. Phys. **93**, 143 (2017).
4. A. Ali, J. S. Lange and S. Stone, “Exotics: Heavy Pentaquarks and Tetraquarks,” Prog. Part. Nucl. Phys. **97**, 123 (2017).
5. F. K. Guo, C. Hanhart, U. G. Meissner, Q. Wang, Q. Zhao and B. S. Zou, “Hadronic molecules,” Rev. Mod. Phys. **90**, 015004 (2018).
6. S. L. Olsen, T. Skwarnicki and D. Zieminska, “Non-Standard Heavy Mesons and Baryons, an Experimental Review,” Rev. Mod. Phys. **90**, 015003 (2018).
7. A. Ali, L. Maiani and A. D. Polosa, “Multiquark Hadrons,” Cambridge University Press, Cambridge, 2019.
8. Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, “Pentaquark and Tetraquark states,” Prog. Part. Nucl. Phys. **107**, 237-320 (2019).
9. N. Brambilla et al., “The *XYZ* states: experimental and theoretical status and perspectives,” Phys. Rept. **873**, 1 (2020).
10. H. X. Chen, W. Chen, X. Liu, Y. R. Liu and S. L. Zhu, “An updated review of the new hadron states,” Rept. Prog. Phys. **86**, 026201 (2023).

Quark-Diquark Model of Hadrons

- Quarks q_i^α and diquarks $\mathcal{Q}_{i\alpha}$ are building blocks of baryons
- α is the $SU(3)_C$ index and i is the $SU(3)_F$ index
- Color repres.: $3 \otimes 3 = \bar{3} \oplus 6$; only $\bar{3}$ is attractive

$$t^a_{ij} t^a_{kl} = -\frac{2}{3} \underbrace{(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } \bar{3}} + \frac{1}{3} \underbrace{(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } 6}$$





- Interpolating diquark operators for the two spin states

$$\text{Scalar:} \quad 0^+ \quad \mathcal{Q}_{i\alpha} = \epsilon_{\alpha\beta\gamma}\left(\bar{c}_c^\beta \gamma_5 q_i^\gamma - \bar{q}_{ic}^\beta \gamma_5 c^\gamma\right)$$

$$\text{Axial-Vector:} \quad 1^+ \quad \vec{\mathcal{Q}}_{i\alpha} = \epsilon_{\alpha\beta\gamma}\left(\bar{c}_c^\beta \vec{\gamma} q_i^\gamma + \bar{q}_{ic}^\beta \vec{\gamma} c^\gamma\right)$$

- Colorless combination with the quark results into the baryon

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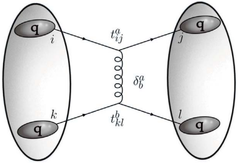

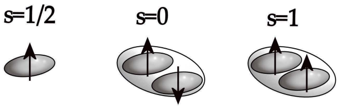

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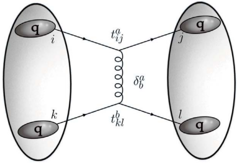

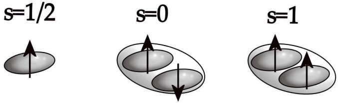

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- (Anti)diquark spin $S_{\mathcal{Q}\,(\bar{\mathcal{Q}})}$, total angular mom. $J \Longrightarrow \left|S_{\mathcal{Q}}, S_{\bar{\mathcal{Q}}};\ J\right\rangle$
- Tetraquarks:

$$\left|0_{\mathcal{Q}}, 0_{\bar{\mathcal{Q}}};\ 0_J\right\rangle = \Gamma^0 \otimes \Gamma^0, \qquad \left|1_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}};\ 0_J\right\rangle = \frac{1}{\sqrt{3}}\,\Gamma^i \otimes \Gamma_i$$

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- Review on Diquarks: M.Yu. Barabanov et al. Diquark correlations in hadron physics: Origin, impact and evidence. Prog. Part. Nucl. Phys. 116 (2021) 103835.

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Doubly Heavy Tetraquarks

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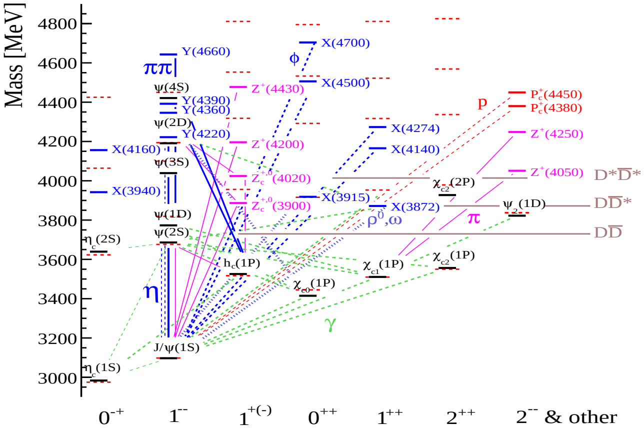
X, Y, Z, P_c and Charmonium States

[S. L. Olsen, T. Skwarnicki, D. Zieminska, Rev. Mod. Phys. 90 (2018) 015003]



Approaches for Tetraquarks

Quarkonium Tetraquarks:

1. Compact tetraquarks
2. Meson molecule
3. Hadro-quarkonium
4. quarkonium-adjoint meson

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Masses of Hidden-Charm Tetraquarks

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Hamiltonian for tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

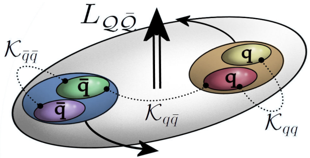
$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

with

$$H_{SS}^{(qq)} = 2\mathcal{K}_{cq}\left[(\mathbf{S}_c\cdot\mathbf{S}_q) + (\mathbf{S}_{\bar{c}}\cdot\mathbf{S}_{\bar{q}})\right]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})\left[(\mathbf{S}_c\cdot\mathbf{S}_{\bar{q}}) + (\mathbf{S}_{\bar{c}}\cdot\mathbf{S}_q)\right] + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c\cdot\mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}})$$

$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}}\cdot\mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}}\cdot\mathbf{L}) = 2A_{\mathcal{Q}}\,(\mathbf{S}\cdot\mathbf{L}$$



$$H_{LL} = \frac{1}{2}\,B_{\mathcal{Q}}\,L_{\mathcal{Q}\bar{\mathcal{Q}}}\,(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)$$

$$H_T = \frac{1}{4}\,b_Y\,S_{12} = b_Y\left[3\,(\mathbf{S}_{\mathcal{Q}}\cdot\mathbf{n})\,(\mathbf{S}_{\bar{\mathcal{Q}}}\cdot\mathbf{n}) - (\mathbf{S}_{\mathcal{Q}}\cdot\mathbf{S}_{\bar{\mathcal{Q}}})\right], \quad (\mathbf{n} = \text{unit vector})$$

- In the following, we neglect quark-antiquark couplings $\mathcal{K}_{q\bar{q}'} \simeq 0$

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Hamiltonian for tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

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constituent diquark mass

with

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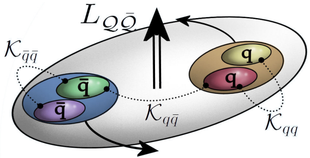

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with qq spin coupling $q\bar{q}$ spin coupling

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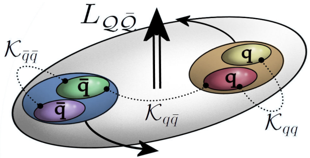

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LS coupling *LL* coupling

with

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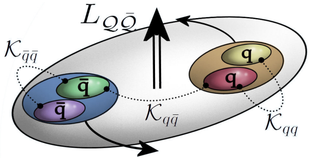

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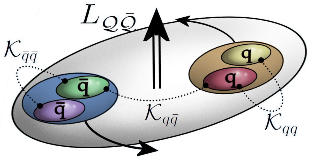


$$H_{SS}^{(qq)} = 2\mathcal{K}_{cq}\left[(\mathbf{S}_c\cdot\mathbf{S}_q) + (\mathbf{S}_{\bar{c}}\cdot\mathbf{S}_{\bar{q}})\right]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})\left[(\mathbf{S}_c\cdot\mathbf{S}_{\bar{q}}) + (\mathbf{S}_{\bar{c}}\cdot\mathbf{S}_q)\right] + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c\cdot\mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}})$$

$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}}\cdot\mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}}\cdot\mathbf{L}) = 2A_{\mathcal{Q}}\,(\mathbf{S}\cdot\mathbf{L}$$

$$H_{LL} = \frac{1}{2}\,B_{\mathcal{Q}}\,L_{\mathcal{Q}\bar{\mathcal{Q}}}\,(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)$$

$$H_T = \frac{1}{4}\,b_Y\,S_{12} = b_Y\left[3\,(\mathbf{S}_{\mathcal{Q}}\cdot\mathbf{n})\,(\mathbf{S}_{\bar{\mathcal{Q}}}\cdot\mathbf{n}) - (\mathbf{S}_{\mathcal{Q}}\cdot\mathbf{S}_{\bar{\mathcal{Q}}})\right], \quad (\mathbf{n} = \text{unit vector})$$

- In the following, we neglect quark-antiquark couplings $\mathcal{K}_{q\bar{q}'} \simeq 0$

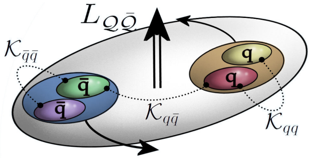
$$H_{\text{eff}}(X,Y,Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}\mathbf{L}^2 + 2A_Q\,(\mathbf{L}\cdot\mathbf{S}) + 2\mathcal{K}_{qQ}\left[(\mathbf{s}_q\cdot\mathbf{s}_Q) + (\mathbf{s}_{\bar{q}}\cdot\mathbf{s}_{\bar{Q}})\right] + b_Y\,\frac{S_{12}}{4}$$

Hamiltonian for tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

with



$$\begin{aligned}
H_{SS}^{(qq)} &= 2\mathcal{K}_{cq}\left[(\mathbf{S}_c\cdot\mathbf{S}_q) + (\mathbf{S}_{\bar{c}}\cdot\mathbf{S}_{\bar{q}})\right] \\
H_{SS}^{(q\bar{q})} &= 2(\mathcal{K}_{c\bar{q}})\left[(\mathbf{S}_c\cdot\mathbf{S}_{\bar{q}}) + (\mathbf{S}_{\bar{c}}\cdot\mathbf{S}_q)\right] \\
&\quad + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c\cdot\mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}}) \\
H_{SL} &= 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}}\cdot\mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}}\cdot\mathbf{L}) = 2A_{\mathcal{Q}}\,(\mathbf{S}\cdot\mathbf{L}) \\
H_{LL} &= \frac{1}{2}\,B_{\mathcal{Q}}\,L_{\mathcal{Q}\bar{\mathcal{Q}}}\,(L_{\mathcal{Q}\bar{\mathcal{Q}}}+1) \\
H_T &= \frac{1}{4}\,b_Y\,S_{12} = b_Y\left[3\,(\mathbf{S}_{\mathcal{Q}}\cdot\mathbf{n})\,(\mathbf{S}_{\bar{\mathcal{Q}}}\cdot\mathbf{n}) - (\mathbf{S}_{\mathcal{Q}}\cdot\mathbf{S}_{\bar{\mathcal{Q}}})\right], \qquad (\mathbf{n} = \text{unit vector})
\end{aligned}$$

- In the following, we neglect quark-antiquark couplings $\mathcal{K}_{q\bar{q}'} \simeq 0$

$$H_{\rm eff}(X,Y,Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}\mathbf{L}^2 + 2A_Q\,(\mathbf{L}\cdot\mathbf{S}) + 2\mathcal{K}_{qQ}\left[(\mathbf{s}_q\cdot\mathbf{s}_Q) + (\mathbf{s}_{\bar{q}}\cdot\mathbf{s}_{\bar{Q}})\right] + b_Y\,\frac{S_{12}}{4}$$

Low-Lying S-Wave Tetraquark States

- In the $|s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$ and $|s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$ bases, the positive parity S-wave tetraquarks are listed below; $M_{00} = 2m_{\mathcal{Q}}$

Label	J^{PC}	$\lvert s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	$\lvert s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$	Mass
X_0	0^{++}	$\lvert 0, 0; 0, 0\rangle_0$	$(\lvert 0, 0; 0, 0\rangle_0 + \sqrt{3}\lvert 1, 1; 0, 0\rangle_0)/2$	$M_{00} - 3\kappa_{qQ}$
X_0'	0^{++}	$\lvert 1, 1; 0, 0\rangle_0$	$(\sqrt{3}\lvert 0, 0; 0, 0\rangle_0 - \lvert 1, 1; 0, 0\rangle_0)/2$	$M_{00} + \kappa_{qQ}$
X_1	1^{++}	$(\lvert 1, 0; 1, 0\rangle_1 + \lvert 0, 1; 1, 0\rangle_1)/\sqrt{2}$	$\lvert 1, 1; 1, 0\rangle_1$	$M_{00} - \kappa_{qQ}$
Z	1^{+-}	$(\lvert 1, 0; 1, 0\rangle_1 - \lvert 0, 1; 1, 0\rangle_1)/\sqrt{2}$	$(\lvert 1, 0; 1, 0\rangle_1 - \lvert 0, 1; 1, 0\rangle_1)/\sqrt{2}$	$M_{00} - \kappa_{qQ}$
Z'	1^{+-}	$\lvert 1, 1; 1, 0\rangle_1$	$(\lvert 1, 0; 1, 0\rangle_1 + \lvert 0, 1; 1, 0\rangle_1)/\sqrt{2}$	$M_{00} + \kappa_{qQ}$
X_2	2^{++}	$\lvert 1, 1; 2, 0\rangle_2$	$\lvert 1, 1; 2, 0\rangle_2$	$M_{00} + \kappa_{qQ}$

- The spectrum of these states depends on just two parameters, $M_{00}(Q)$ and κ_{qQ}, $Q = c, b$, hence very predictive
- Some of the states, such as X_0, X_0', X_2, still missing and are being searched for at the LHC

Analysis of Tetraquark Y-States in the Diquark Model

- Effective Hamiltonian for the mass spectrum

$$\begin{aligned} H_{\text{eff}} &= 2m_{\mathcal{Q}} + \frac{1}{2} B_{\mathcal{Q}} \mathbf{L}^2 + 2a_Y (\mathbf{L} \cdot \mathbf{S}) + \frac{1}{4} b_Y \langle S_{12} \rangle \\ &+ 2\kappa_{cq} \left[(\mathbf{S}_q \cdot \mathbf{S}_c) + (\mathbf{S}_{\bar{q}} \cdot \mathbf{S}_{\bar{c}}) \right] \end{aligned}$$

- There are four $L = 1$ and one $L = 3$ tetraquark P-wave states with $J^{PC} = 1^{--}$ and two $L = 1$ states with $J^{PC} = 1^{-+}$

Label	J^{PC}	$\lvert s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	Mass
Y_1	1^{--}	$\lvert 0,0;0,1\rangle_1$	$M_{00} - 3\kappa_{qQ} + B_Q \equiv \tilde{M}_{00}$
Y_2	1^{--}	$\left(\lvert 1,0;1,1\rangle_1 + \lvert 0,1;1,1\rangle_1\right)/\sqrt{2}$	$\tilde{M}_{00} + 2\kappa_{qQ} - 2A_Q$
Y_3	1^{--}	$\lvert 1,1;0,1\rangle_1$	
Y_4	1^{--}	$\lvert 1,1;2,1\rangle_1$	
Y_5	1^{--}	$\lvert 1,1;2,3\rangle_1$	$M_{Y_2} + 2\kappa_{qQ} - 14A_Q + 5B_Q - 8b_Y/5$
$Y_2^{(+)}$	1^{-+}	$\left(\lvert 1,0;1,1\rangle_1 - \lvert 0,1;1,1\rangle_1\right)/\sqrt{2}$	$\tilde{M}_{00} + 2\kappa_{qQ} - 2A_Q$
$Y^{(+)}$	1^{-+}	$\lvert 1,1;1,1\rangle_1$	$\tilde{M}_{00} + \kappa_{qQ} - 2A_Q + b_Y$

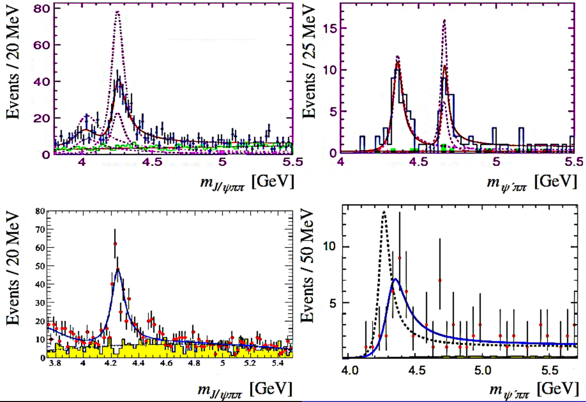
- Tensor couplings non-vanishing only for the states with $S_{\mathcal{Q}} = S_{\bar{\mathcal{Q}}} = 1$
- Y_3 and Y_4 are not the mass eigenstates of the Hamiltonian

Experimental situation with the tetraquark Υ states rather confusing

- Summary of the Y states observed in Initial State Radiation (ISR) processes in e^+e^- annihilation [BaBaR, Belle, CLEO]

$$e^+e^- \to \gamma_{\rm ISR}\ \ J/\psi\pi^+\pi^-;\ \ \gamma_{\rm ISR}\ \ \psi'\pi^+\pi^-$$

$$\Longrightarrow\ \ Y(4008),\ Y(4260),\ Y(4360),\ Y(4660)$$

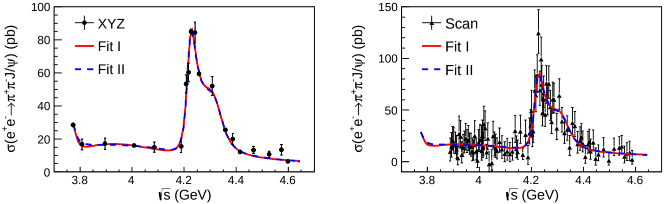


$e^+e^- \to J/\psi\pi^+\pi^-$ cross section at $\sqrt{s} = (3.77 - 4.60)$ GeV

(BESIII, PRL 118, 092001 (2017))

- Y(4008) is not confirmed; Y(4260) is split into 2 resonances: Y(4220) and Y(4320), with Y(4220) probably the same as Y(4260)

Parameters	Fit result
$M(R_1)$	$3812.6^{+61.9}_{-96.6}$ $(\cdots)$
$\Gamma_{\rm tot}(R_1)$	$476.9^{+78.4}_{-64.8}$ $(\cdots)$
$M(R_2)$	4222.0 ± 3.1 (4220.9 ± 2.9)
$\Gamma_{\rm tot}(R_2)$	44.1 ± 4.3 (44.1 ± 3.8)
$M(R_3)$	4320.0 ± 10.4 (4326.8 ± 10.0)
$\Gamma_{\rm tot}(R_3)$	$101.4^{+25.3}_{-19.7}$ $(98.2^{+25.4}_{-19.6})$



Two Experimental Scenarios for the Υ States

- SI (Based on CLEO, BaBaR, Belle): Y(4008), Y(4260), Y(4360), Y(4660)
- SII (BESIII, PRL 118, 092001 (2017): Y(4220), Y(4320), with Y(4390), Y(4660) the same as in SI
- Parameters in SI and SII and $\pm 1\sigma$ errors (all in MeV). Here, $c1$ and $c2$ refer to two solutions of the secular equation

	a_Y	b_Y	κ_{cq}	M_{00}
SI (c1)	-22 ± 32	-89 ± 77	89 ± 11	4275 ± 54
SI (c2)	48 ± 23	11 ± 91	159 ± 20	4484 ± 26
SII (c1)	-3 ± 18	-105 ± 32	54 ± 8	4380 ± 25
SII (c2)	48 ± 8	-32 ± 47	105 ± 4	4535 ± 10

- SII (based on BESIII data) is favored, with a_Y and κ_{cq} values similar to the Ω_c analysis

Energy of Orbital Excitation

- Fixing $\kappa_{cq} = 67$ MeV(from the S states); fitted the two scenarios $\Longrightarrow$ clear preference for SII, with parameters as follows (in MeV)

Scenario	M_{00}	a_Y	b_Y	$\chi^2_{\min}$/n.d.f.
SI	4321 ± 79	2 ± 41	-141 ± 63	12.8/1
SII	4421 ± 6	22 ± 3	-136 ± 6	1.3/1

- SII: $M_{00} \equiv 2m_{\mathcal{Q}} + B_{\mathcal{Q}} \Longrightarrow B_{\mathcal{Q}} = 442$ MeV
- Comparable to the orbital angular momentum excitation energy in charmonia

$$B_{\mathcal{Q}}(c\bar{c}) = M(h_c) - \frac{1}{4}\left[3M(J/\psi) + M(\eta_c)\right] = 457 \text{ MeV}$$

- κ_{cq} and a_{Y} for Y states are similar to the ones in (X, Z) and Ω_c
- Precise data on the Y-states is needed to confirm or refute the diquark picture

Predictions for the $L = 3$ Vector Tetraquark

- Among the five vector states, Y_5 is the heaviest one as its angular momentum $L = 3$
- Note that the tensor term $\langle Q(\mathbf{S}_{cq}, \mathbf{S}_{[\bar{c}\bar{q}]})\rangle_{L=3}$ should be modified
- Mass formula

$$M_5 - M_2 = 5B_{\mathcal{Q}} - 14a_Y + 2\kappa_{cq} - \frac{8}{5}\, b_Y$$

- Prediction from the Diquark Model

$$M_5 = \begin{cases} 6539\ \mathrm{MeV}, & \mathrm{SI(c1)} \\ 6589\ \mathrm{MeV}, & \mathrm{SI(c2)} \\ 6862\ \mathrm{MeV}, & \mathrm{SII(c1)} \\ 6899\ \mathrm{MeV}, & \mathrm{SII(c2)} \end{cases}$$

- Should be taken with cousion: b_Y can differ for $L = 3$ states

$L = 1$ Multiplet Predictions

J^{PC}	$\lvert S_{\mathcal{Q}}, S_{\bar{\mathcal{Q}}}; S, L\rangle_J$	N_1	$2(L \cdot S)$	$S_{12}/4$	Mass (MeV) best fit	EFG
3^{--}	$\lvert 1, 1; 2, 1\rangle_3$	2	4	$-2/5$	4630	4381
2^{--}	$\lvert 1, 1; 2, 1\rangle_2$	2	-2	$+7/5$	4254	4379
2_a^{--}	$\lvert \frac{(1,0)+(0,1)}{\sqrt{2}}; 1, 1\rangle_2$	1	+2	0	4398	4315
2^{-+}	$\lvert 1, 1; 1, 1\rangle_2$	2	+2	$-1/5$	4559	4367
2_b^{-+}	$\lvert \frac{(1,0)-(0,1)}{\sqrt{2}}; 1, 1\rangle_2$	1	+2	0	4398	4315
1^{-+}	$\lvert 1, 1; 1, 1\rangle_1$	2	-2	$+1$	4308	4345
1_b^{-+}	$\lvert \frac{(1,0)-(0,1)}{\sqrt{2}}; 1, 1\rangle_1$	1	-2	0	4310	4284
0^{-+}	$\lvert 1, 1; 1, 1\rangle_0$	2	-4	-2	4672	4304
0_b^{-+}	$\lvert \frac{(1,0)-(0,1)}{\sqrt{2}}; 1, 1\rangle_0$	1	-4	0	4266	4269
0_a^{--}	$\lvert \frac{(1,0)+(0,1)}{\sqrt{2}}; 1, 1\rangle_0$	1	-4	0	4266	4269

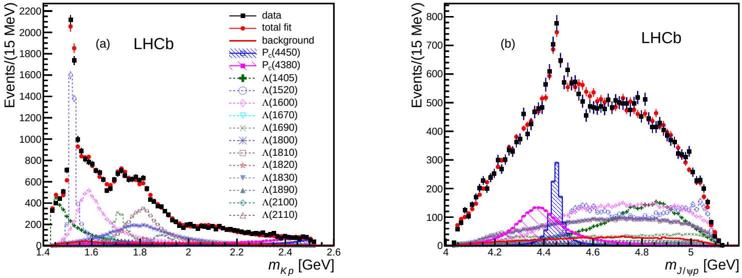
- N_1 is the number of “bad” diquarks and antidiquarks
- EFG data are from the paper by Ebert, Faustov & Galkin [EPJC 58 (2008) 399]

Doubly Heavy Pentaquarks

Doubly Heavy Pentaquarks

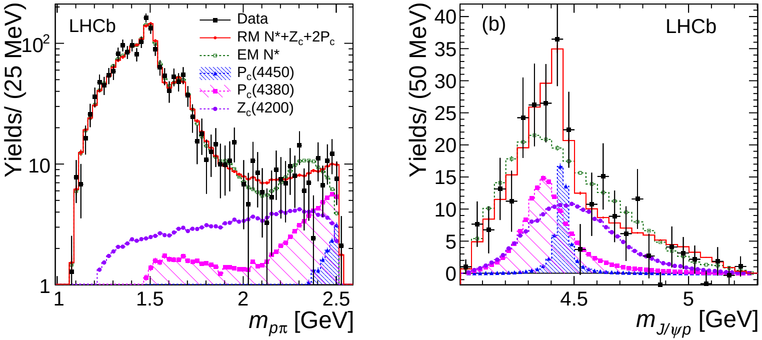
$\Lambda_b \to p + K^- + J/\psi$ Decay: 2015 Results by LHCb

- Two peaks in invariant-mass, $m_{p\,J/\psi} = \sqrt{(p_P + p_{J/\psi})^2}$, distribution were interpreted as hidden-charm pentaquarks
  1. $P_c^+(4380)$: spin-parity $J^P = 3/2^-$ (preferred)
     $M = (4380 \pm 8 \pm 29)$ MeV, $\Gamma = (205 \pm 18 \pm 86)$ MeV
  2. $P_c^+(4450)$: spin-parity $J^P = 5/2^+$ (preferred)
     $M = (4449.8 \pm 1.7 \pm 2.5)$ MeV, $\Gamma = (39 \pm 5 \pm 19)$ MeV
- Assignments $(3/2^+, 5/2^-)$ and $(5/2^+, 3/2^-)$ are possible



LHCb Results on $\Lambda_b \to p + J/\psi + \pi^-$ Decay

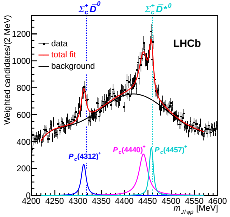
- Evidence of these resonances was also pointed out in the other decay $\Lambda_b \to p + J/\psi + \pi^-$ [LHCb, PRL, 2016]
- Combined significance is calculated to be 3.1σ
- Contributions from pentaquarks are shown as shaded



$\Lambda_b \to p + J/\psi + K^-$ Decay: 2019 Results by LHCb

- Λ_b-baryon decay $\Lambda_b \to p + J/\psi + K^-$ was studied on 9 times more data based on Run 1 and 2 than on Run 1
- Three narrow peaks were observed in $m_{J/\psi\, p}$ distribution

State	Mass [MeV]	Width [MeV]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	(< 27)	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	(< 49)	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	(< 20)	$0.53 \pm 0.16^{+0.15}_{-0.13}$



- $P_c(4312)$ is a new resonance
- $P_c(4450)$ splits into $P_c(4440)$ and $P_c(4457)$
- $P_c(4380)$ under question
- Spin-parities are unknown yet

Results by D0 & ATLAS Collaborations

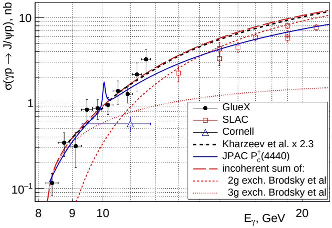
- D0 Collab. [V. M. Abasov *et al.*, arXiv:1910.11767]
  - Analysis is based on 10.4 fb^{-1} of data
  - Enhancement in $J/\psi\, p$ invariant mass distribution originated by decays of b-flavored hadrons
  - Consistent with a sum of $P_c(4440)^+$ and $P_c(4457)^+$
  - Significance is 3.0σ
  - No evidence of $P_c(4312)^+$ state
  - $R = N(4312)/[N(4440) + N(4457)] < 0.6$ at 95% C.L.

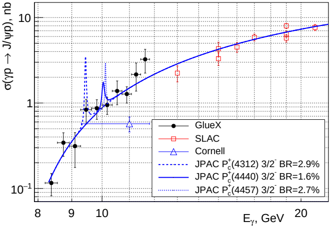
- ATLAS Collab. [I. Eletskikh, ATL-PHYS-PROC-2020-007]
  - Based on 4.9 fb^{-1} at 7 TeV and 20.6 fb^{-1} at 8 TeV
  - $\Lambda_b \to J/\psi\, p\, K^-$ with large $m_{p\,K^-}$ invariant mass
  - Model without pentaquarks is not excluded
  - Data prefer model with two or more pentaquarks
  - Masses and widths of two $P_c(4380)^+$ and $P_c(4450)^+$ pentaquarks are consistent with those from LHCb
  - Data are also compatible with the three narrow LHCb pentaquarks

$\gamma + p \to J/\psi + p$ Scattering: 2019 Results by GlueX

GlueX Collab. [A. Ali *et al.*, PRL 123 (2019) 072001]

- Hall D of Jafferson Lab., data of 2016–2017
- Photon energy $E_\gamma \in [8.2\,\mathrm{GeV}, 11.8\,\mathrm{GeV}]$
- For $J^P = 3/2^-$ $\mathcal{B}(P_c^+ \to J/\psi\, p) < 2.0\%$; consistent with LHCb
- Upper limits on BF do not exclude the molecular model of P_c^+ but are an order of magnitude lower than predictions in hadrocharmonium model





Theoretical Interpretations of Narrow Pentaquarks

- Molecular Picture:
  Open charm-meson and charm-baryon bound states
  - Masses are slightly below meson-baryon thresholds
  - S-wave molecular-like states
  - Negative parity $P = (-1)^{L+1}$
- Hadrocharmonium Picture:
  Compact charmonium state inside the proton interior
- Compact Multiquark Picture:
  - Quarks and antiquarks are tightly bound into colorless state
  - Introduction of point-like diquarks and antidiquarks simplifies consideration drastically

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Diquark Model of Pentaquarks

- Antiquark $\bar{q}_k^\gamma$ and two diquarks $\mathcal{Q}_{i\alpha}$ and $\mathcal{Q}'_{j\beta}$ are the building blocks of pentaquarks
- At least, three approaches are suggested for hidden-charm pentaquarks in the compact diquark model
- Heavy triquark — heavy diquark model within the "Dynamical Diquark Model" [R. Lebed, PLB 749 (2015) 454]
- Heavy tetraquark — heavy antiquark model [A. Ali, I. Ahmed, M. J. Aslam, and A. Rehman, PRD 94 (2016) 054001]
- Doubly-heavy triquark — light diquark model [A. Ali and AP, PLB 793 (2019) 365; A. Ali et al., JHEP 10 (2019) 256]
- For ground-state pentaquarks and their first orbital excitations, basic vectors of the states constructed in $L - S$ scheme can be easily related to each other with using spin recouplings

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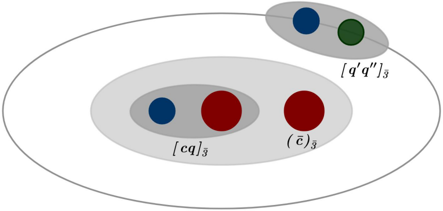
Diquark Model of Pentaquarks

- Antiquark $\bar{q}_k^\gamma$ and two diquarks $\mathcal{Q}_{i\alpha}$ and $\mathcal{Q}'_{j\beta}$ are the building blocks of pentaquarks
- At least, three approaches are suggested for hidden-charm pentaquarks in the compact diquark model
- Heavy triquark — heavy diquark model within the "Dynamical Diquark Model" [R. Lebed, PLB 749 (2015) 454]
- Heavy tetraquark — heavy antiquark model [A. Ali, I. Ahmed, M. J. Aslam, and A. Rehman, PRD 94 (2016) 054001]
- Doubly-heavy triquark — light diquark model [A. Ali and AP, PLB 793 (2019) 365; A. Ali et al., JHEP 10 (2019) 256]
- For ground-state pentaquarks and their first orbital excitations, basic vectors of the states constructed in $L - S$ scheme can be easily related to each other with using spin recouplings

Doubly-Heavy Triquark — Light Diquark Model

- Heavy diquark couples with *c*-antiquark in the color-triplet doubly-heavy triquark (DHT)
- Light diquark being a color antitriplet makes pentaquark colorless
- DHT is practically static
- Light diquark is “rotating” around triquark
- Light diquark is easier to excite orbitally than constituents inside the DHT



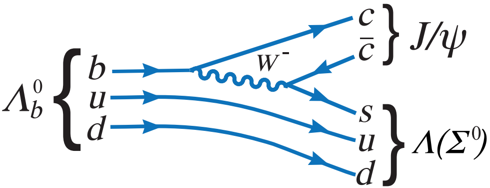
Mass Predictions for Unflavored Pentaquarks

J^P	This work	AAAR	J^P	This work	AAAR
	$S_{ld} = 0$, $L = 0$			$S_{ld} = 1$, $L = 1$	
$1/2^-$	3830 ± 34	4086 ± 42	$1/2^+$	4144 ± 37	3970 ± 50
	4150 ± 29	4162 ± 38		4209 ± 37	4174 ± 44
$3/2^-$	$\boxed{4240 \pm 29}$	4133 ± 55		4465 ± 32	4198 ± 50
	$S_{ld} = 1$, $L = 0$			4530 ± 32	4221 ± 40
$1/2^-$	4026 ± 31	4119 ± 42		4564 ± 33	4240 ± 50
	4346 ± 25	4166 ± 38		4663 ± 32	4319 ± 43
	4436 ± 25	4264 ± 41	$3/2^+$	4187 ± 37	
$3/2^-$	4026 ± 31	4072 ± 40		4250 ± 37	
	4346 ± 25	4300 ± 40		4508 ± 32	
	4436 ± 25	4342 ± 40		4570 ± 32	
$5/2^-$	4436 ± 25	4409 ± 40		4511 ± 33	
	$S_{ld} = 0$, $L = 1$			4566 ± 32	
$1/2^+$	4030 ± 39	4030 ± 62		4656 ± 32	
	4351 ± 35	4141 ± 44	$5/2^+$	4260 ± 37	4450 ± 44
	4430 ± 35	4217 ± 40		4581 ± 32	4524 ± 41
$3/2^+$	4040 ± 39			4601 ± 32	4678 ± 44
	4361 ± 35			4656 ± 32	4720 ± 44
	$\boxed{4440 \pm 35}$		$7/2^+$	4672 ± 32	
$5/2^+$	$\boxed{4457 \pm 35}$	4510 ± 57			

Isospin Violation in Λ_b-Decays

LHCb Collab. [R. Aaij *et al.*, Phys. Rev. Lett. 124 (2020) 111802]

- Data: 1.0 fb^{-1} at 7 TeV , 2.0 fb^{-1} at 8 TeV, and 5.5 fb^{-1} at 13 TeV
- Isospin-0 FS: $\Lambda_b^0 \to J/\psi\,\Lambda^0$
  Isospin-1 FS: $\Lambda_b^0 \to J/\psi\,\Sigma^0$
- Decays through the $\Delta I = 0$ transition $b \to sc\bar{c}$
- Amplitide's ratio: $|A_1/A_0| < 1/20.9$ at 95% C.L.
- Rules out isospin violation at 1% rate



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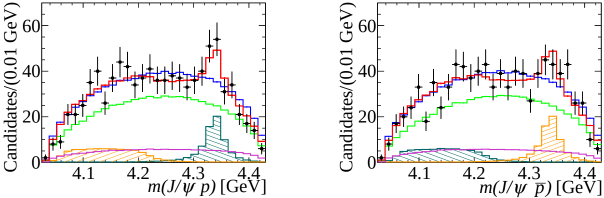
Structures in $J/\psi\, p$ and $J/\psi\, \bar{p}$ Systems in $B_s^0 \to J/\psi\, p\, \bar{p}$ Decay

LHCb Collab. [R. Aaij *et al.*, arXiv:2108.04720]

- Data of 2011 — 2018; correspond int. luminosity of 9 fb^{-1}
- No evidence is seen either for $P_c(4312)^+$ or glueball $f_J(2220)$
- Evidence for a Breit-Wigner shaped resonance is obtained

J^P	p $(\times 10^{-3})$	σ	M_0 (MeV)	Γ_0 (MeV)
$1/2^-$	0.5 ± 0.3	3.5 ± 0.1	$4335^{+3}_{-3} \pm 2$	$23^{+11}_{-8} \pm 14$
$1/2^+$	0.2 ± 0.1	3.7 ± 0.1	$4337^{+7}_{-4} \pm 2$	$29^{+26}_{-12} \pm 14$
$3/2^-$	0.3 ± 0.2	3.6 ± 0.1	$4337^{+5}_{-3} \pm 2$	$23^{+16}_{-9} \pm 14$
$3/2^+$	2 ± 1	3.1 ± 0.1	$4336^{+3}_{-2} \pm 2$	$15^{+9}_{-6} \pm 14$

- Limited sample size; impossible to distinguish among J^P



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Mass Predictions for Strange Pentaquarks

- Inclusion of strange quark(s) into the content makes spectrum of hidden-charm pentaquarks very rich
- They can be classsified according to their strangeness and color connection of four quarks
  - Singly-strange: $(\bar{c}_{\bar{3}}\,[cs]_{\bar{3}}\,[qq']_{\bar{3}})$ and $(\bar{c}_{\bar{3}}\,[cq]_{\bar{3}}\,[sq']_{\bar{3}})$
  - Doubly-strange: $(\bar{c}_{\bar{3}}\,[cs]_{\bar{3}}\,[sq]_{\bar{3}})$ and $(\bar{c}_{\bar{3}}\,[cq]_{\bar{3}}\,\{ss\}_{\bar{3}})$
  - Triple-strange: $(\bar{c}_{\bar{3}}\,[cs]_{\bar{3}}\,\{ss\}_{\bar{3}})$
- Can be produced in weak decays of Ξ_b- and Ω_b-baryons at LHC
  - $\Xi_b^- \to P_\Lambda^0 + K^- \to J/\psi + \Lambda^0 + K^-$
  - $\Xi_b^{-,0} \to P_\Sigma^{0,+} + K^- \to J/\psi + \Sigma^{0,+} + K^-$
  - $\Omega_b^- \to P_{\Xi_{10}}^0 + K^- \to J/\psi + \Xi'^0 + K^-$
  - $\Omega_b^- \to P_{\Omega_{10}}^- + \phi \to J/\psi + \Omega^- + \phi$
- Ω_b-decays gives a new avenue to study pentaquarks with “bad” light diquarks

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  - Doubly-strange: $(\bar{c}_3\,[cs]_{\bar{3}}\,[sq]_{\bar{3}})$ and $(\bar{c}_3\,[cq]_{\bar{3}}\,\{ss\}_{\bar{3}})$
  - Triple-strange: $(\bar{c}_3\,[cs]_{\bar{3}}\,\{ss\}_{\bar{3}})$
- Can be produced in weak decays of Ξ_b- and Ω_b-baryons at LHC
  - $\Xi_b^- \to P_{cs}(4459)^0 + K^- \to J/\psi + \Lambda^0 + K^-$
  - $\Xi_b^{-,0} \to P_\Sigma^{0,+} + K^- \to J/\psi + \Sigma^{0,+} + K^-$
  - $\Omega_b^- \to P^0_{\Xi_{10}} + K^- \to J/\psi + \Xi'^0 + K^-$
  - $\Omega_b^- \to P^-_{\Omega_{10}} + \phi \to J/\psi + \Omega^- + \phi$
- Ω_b-decays gives a new avenue to study pentaquarks with “bad” light diquarks

Masses of Singly-Strange ($\bar{c}_{\bar 3}\,[cq]_{\bar 3}\,[sq']_{\bar 3}$) Pentaquarks

J^P	This work	AAAR	J^P	This work	AAAR
	$S_{ld} = 0,\ L = 0$			$S_{ld} = 1,\ L = 1$	
$1/2^-$	4112 ± 32	4094 ± 44	$1/2^+$	4348 ± 36	3929 ± 53
	4433 ± 26	4132 ± 43		4414 ± 36	4183 ± 45
$3/2^-$	$\boxed{4523 \pm 26}$	4172 ± 47		4669 ± 32	4159 ± 53
	$S_{ld} = 1,\ L = 0$			4735 ± 32	4189 ± 44
$1/2^-$	4230 ± 30	4128 ± 44		4768 ± 32	4201 ± 53
	4551 ± 25	4134 ± 42		4867 ± 32	4275 ± 45
	4641 ± 25	4220 ± 43	$3/2^+$	4392 ± 36	
$3/2^-$	4230 ± 30	4031 ± 43		4454 ± 36	
	4551 ± 25	4262 ± 43		4713 ± 32	
	4641 ± 25	4303 ± 43		4775 ± 32	
$5/2^-$	4641 ± 25	4370 ± 43		4716 ± 32	
	$S_{ld} = 0,\ L = 1$			4770 ± 32	
$1/2^+$	4312 ± 37	4069 ± 56		4861 ± 32	
	4633 ± 33	4149 ± 45	$5/2^+$	4465 ± 36	4409 ± 47
	4713 ± 33	4187 ± 44		4786 ± 32	4486 ± 45
$3/2^+$	4323 ± 37			4806 ± 32	4639 ± 47
	4643 ± 33			4860 ± 32	4681 ± 47
	$\boxed{4723 \pm 33}$		$7/2^+$	4877 ± 32	
$5/2^+$	$\boxed{4740 \pm 33}$	4549 ± 51			

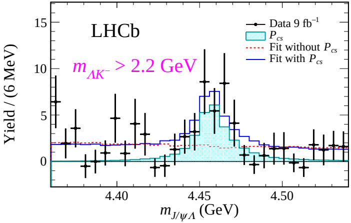
$P_{cs}(4459)^0$-Resonance in $\Xi_b^- \to J/\psi + \Lambda + K^-$ Decay

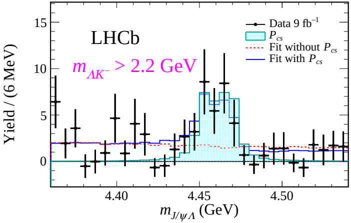
LHCb Collab. [R. Aaij *et al.*, Sci. Bull. 66 (2021) 1278]

- Amplitude analysis of $\Xi_b^- \to \Lambda\, J/\psi\, K^-$ decay is performed using approximately 1750 events
- Narrow structure $P_{cs}(4459)^0$ is seen in $m_{\Lambda\, J/\psi}$ distribution; significance is 3.1σ including systematic uncertainties

$$M_{P_{cs}} = \left(4458.8 \pm 2.9^{+4.7}_{-1.1}\right)\ \text{MeV}, \qquad \Gamma_{P_{cs}} = \left(17.3 \pm 6.5^{+8.0}_{-5.7}\right)\ \text{MeV}$$

- Data cannot confirm or refute the two-peak hypothesis
- Spin-parity remains undetermine due to limited statistics





Masses of Singly-Strange ($\bar{c}_3\,[cq]_{\bar{3}}\,[sq']_{\bar{3}}$) Pentaquarks

J^P	This work	AAAR	J^P	This work	AAAR
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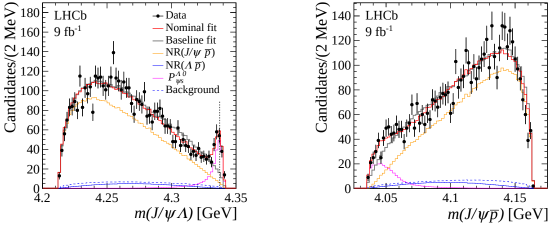
$1/2^-$	4112 ± 32	4094 ± 44	$1/2^+$	4348 ± 36	3929 ± 53
	4433 ± 26	4132 ± 43		4414 ± 36	4183 ± 45
$3/2^-$	$\boxed{4523 \pm 26}$	4172 ± 47		4669 ± 32	4159 ± 53
	$S_{ld} = 1,\ L = 0$			4735 ± 32	4189 ± 44
$1/2^-$	4230 ± 30	4128 ± 44		4768 ± 32	4201 ± 53
	4551 ± 25	4134 ± 42		4867 ± 32	4275 ± 45
	4641 ± 25	4220 ± 43	$3/2^+$	4392 ± 36	
$3/2^-$	4230 ± 30	4031 ± 43		4454 ± 36	
	4551 ± 25	4262 ± 43		4713 ± 32	
	4641 ± 25	4303 ± 43		4775 ± 32	
$5/2^-$	4641 ± 25	4370 ± 43		4716 ± 32	
	$S_{ld} = 0,\ L = 1$			4770 ± 32	
$1/2^+$	4312 ± 37	4069 ± 56		4861 ± 32	
	4633 ± 33	4149 ± 45	$5/2^+$	4465 ± 36	4409 ± 47
	4713 ± 33	4187 ± 44		4786 ± 32	4486 ± 45
$3/2^+$	4323 ± 37			4806 ± 32	4639 ± 47
	4643 ± 33			4860 ± 32	4681 ± 47
	$\boxed{4723 \pm 33}$		$7/2^+$	4877 ± 32	
$5/2^+$	$\boxed{4740 \pm 33}$	4549 ± 51			

Structure in $J/\psi\,\Lambda$ System in $B^- \to J/\psi\,\Lambda\,\bar{p}$ Decay

LHCb Collab. [R. Aaij *et al.*, arXiv:2210.10346]

- Data of 2011 — 2018; correspond int. luminosity of 9 fb^{-1}
- New resonant structure called $P^{\Lambda}_{\psi s}(4338)^0$ in the $J/\psi\,\Lambda$ system is found with high statistical significance ($> 15\sigma$)
- $P^{\Lambda}_{\psi s}(4338)^0$ with preferred spin-parity $J^P = 1/2^-$ has the mass $M = 4338.2 \pm 0.7 \pm 0.4$ MeV and width $\Gamma = 7.0 \pm 1.2 \pm 1.3$ MeV
- $P^{\Lambda}_{\psi s}(4338)^0$ state is found at the $\Xi_c^+ D^-$ threshold
- No evidence is seen either for unflavored pentaquark or lower mass strange pentaquark $P^{\Lambda}_{\psi s}(4255)^0$



Masses of Triple-Strange ($\bar{c}_3\,[cs]_{\bar{3}}\,\{ss\}_{\bar{3}}$) Pentaquarks

J^P	Mass	J^P	Mass
$S_{ld} = 1,\ L = 0$		$S_{ld} = 1,\ L = 1$	
$1/2^-$	4642 ± 31	$3/2^+$	4804 ± 37
	4974 ± 25		4866 ± 37
	5043 ± 25		5136 ± 32
$3/2^-$	4642 ± 31		5198 ± 32
	4974 ± 25		5118 ± 32
	5043 ± 25		5173 ± 32
$5/2^-$	5043 ± 25		5263 ± 32
$S_{ld} = 1,\ L = 1$		$5/2^+$	4877 ± 37
$1/2^+$	4761 ± 37		5209 ± 32
	4826 ± 37		5208 ± 32
	5092 ± 32		5263 ± 32
	5158 ± 32	$7/2^+$	5279 ± 32
	5171 ± 32		
	5270 ± 32		

- All of them are decaying strongly

Strong Decays of Tetra- and Pentaquakrs

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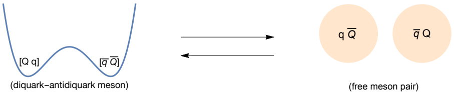
Double Well Potential in Tetraquarks

- Hypothesis: tetraquark can plausibly be represented by two diquarks in double well potential separated by a barrier [L. Maiani, A.D. Polosa & V. Riquer, Phys. Lett. B778 (2018) 247]
- Arguments in favor:
  1. At large distances, diquarks interact like QCD point charges
  2. Confining forces are the same as for quark and antiquark
  3. At shorter distances, forces among constituents in diquarks (e. g. attraction between quarks and antiquarks) reduce the diquark binding energies
  4. These effects increase at decreasing distance and produce repulsion among diquark and antidiquark, i. e. increasing component in potential at decreasing distance
  5. If this effect wins against the decrease due to the color attraction, the barrier is produced



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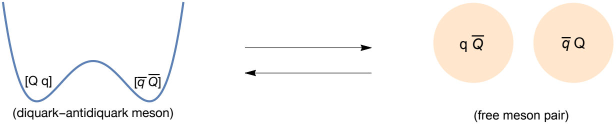
- There are two length scales: diquark radius R_{Qq} & tetraquark radius R_{4q}
- Assumed to be well separated $\lambda = R_{4q}/R_{Qq} \geq 3$
- Tunneling transitions of quarks result into strong decays
- Diquark radius R_{Qq} in tetraquark can be different from diquark radius R_{Qq}^{baryon} in baryon
- Increase of experimental resolution and statistics is crucial to support or disprove this hypothesis



Hidden-Charm Tetraquark Decays to *D*-Mesons

- Diquark-antidiquark system can rearrange itself into a pair of color singlets by exchanging quarks through tunneling transition
- Small overlap between constituent quarks in different wells suppresses quark-antiquark direct annihilation
- Two stage process:
  1. switch of quark and antiquark among two wells
  2. evolution of quark-antiquark pairs into mesons
- Including diquark spins (subscripts), consider the states:

$$\Psi_{\mathcal{D}}^{(1)} = [cu]_0(x)\,[\bar{c}\bar{u}]_1(y), \qquad \Psi_{\mathcal{D}}^{(2)} = \mathcal{C}\Psi_{\mathcal{D}}^{(1)} = [cu]_1(y)\,[\bar{c}\bar{u}]_0(x)$$

- After Fierz rearrangements of color and spin indices, in evident meson notations

$$\Psi_{\mathcal{D}}^{(1)} = A\, D^0 \bar{D}^{*0} - B\, D^{*0} \bar{D}^0 + iC\, D^{*0} \times \bar{D}^{*0}$$
$$\Psi_{\mathcal{D}}^{(2)} = B\, D^0 \bar{D}^{*0} - A\, D^{*0} \bar{D}^0 - iC\, D^{*0} \times \bar{D}^{*0}$$

- *A*, *B*, and *C* are non-perturbative coefficients associated to barrier penetration amplitudes for different total spins of u and $\bar{u}$

Hidden-Charm Tetraquark Decays to Charmonia

- Tunneling transition of light quarks

$$X_u \sim \frac{1}{\sqrt{2}}\left[\Psi_{\mathcal{D}}^{(1)} + \Psi_{\mathcal{D}}^{(2)}\right] = \frac{A+B}{\sqrt{2}}\left[D^0\bar{D}^{*0} - D^{*0}\bar{D}^0\right]$$

- Tunneling transition of heavy quarks

$$X_u \sim a\, i J/\psi \times \left(\omega + \rho^0\right)$$

- Tunneling amplitude in leading semiclassical approximation, $\mathcal{A}_M \sim e^{-\sqrt{2ME}\ell}$, where E and ℓ are barrier height and extension
- For constituent quark masses, m_q and m_c, $E = 100$ MeV and $\ell = 2$ fm, the ratio of amplitides squared

$$R = [a/(A+B)]^2 \sim \left(\mathcal{A}_{m_c}/\mathcal{A}_{m_q}\right)^2 \sim 10^{-3}$$

- With decay momenta $p_\rho \simeq 124$ MeV and $p_{DD^*} \simeq 2$ MeV

$$\frac{\Gamma(X(3872) \to J/\psi\, \rho)}{\Gamma(X(3872) \to D\bar{D}^*)} = \frac{p_\rho}{p_{DD^*}}\, R \sim 0.1$$

- Experiment [PDG]: $\mathcal{B}_{\rm exp}(X(3872) \to J/\psi\, \rho) = (3.8 \pm 1.2)\%$
  $\mathcal{B}_{\rm exp}(X(3872) \to D\bar{D}^*) = (37 \pm 9)\%$

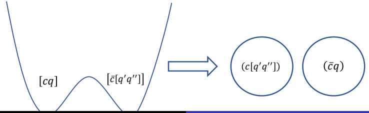
Double Well Potential in Pentaquarks

- Hypothesis: pentaquark can be represented by heavy diquark and heavy triquark in double well potential separated by barrier [A. Ali et. al., JHEP 10 (2019) 256]
- There are two triquark-diquark representations

$$\Psi_1^D = \frac{1}{\sqrt{3}}\left[\frac{1}{\sqrt{2}}\,\epsilon_{ijk}\bar{c}^i\left[\frac{1}{\sqrt{2}}\,\epsilon^{jlm}c_l q_m\right]\right]\left[\frac{1}{\sqrt{2}}\,\epsilon^{knp}q'_n q''_p\right] \equiv [\bar{c}\,[cq]]\,[q'\,q'']$$

$$\Psi_2^D = \frac{1}{\sqrt{3}}\left[\frac{1}{\sqrt{2}}\,\epsilon_{ikj}\bar{c}^i\left[\frac{1}{\sqrt{2}}\,\epsilon^{knp}q'_n q''_p\right]\right]\left[\frac{1}{\sqrt{2}}\,\epsilon^{jlm}c_l q_m\right] \equiv [\bar{c}\,[q'\,q'']]\,[cq]$$

- From color algebra, these states are related, $\Psi_2^D = -\Psi_1^D$, but other internal dynamical properties can be different
- Color connection of quarks in Ψ_1^D is used for mass spectrum
- Ψ_2^D color structure is suitable for study strong decays

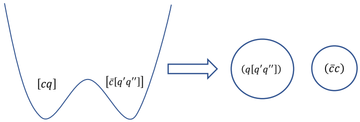


Double Well Potential in Pentaquarks

- Color-singlet combinations are meson-baryon alternatives

$$\Psi_1^H = \left(\frac{1}{\sqrt{3}}\,\bar{c}^i c_i\right)\left[\frac{1}{\sqrt{6}}\,\epsilon^{jkl} q_j q'_k q''_l\right] \equiv (\bar{c}c)\,[qq'q'']$$
$$\Psi_2^H = \left(\frac{1}{\sqrt{3}}\,\bar{c}^i q_i\right)\left[\frac{1}{\sqrt{6}}\,\epsilon^{jkl} c_j q'_k q''_l\right] \equiv (\bar{c}q)\,[cq'q'']$$
$$\Psi_3^H = \left(\frac{1}{\sqrt{3}}\,\bar{c}^i q'_i\right)\left[\frac{1}{\sqrt{6}}\,\epsilon^{jkl} c_j q_k q''_l\right] \equiv (\bar{c}q')\,[cqq'']$$
$$\Psi_4^H = \left(\frac{1}{\sqrt{3}}\,\bar{c}^i q''_i\right)\left[\frac{1}{\sqrt{6}}\,\epsilon^{jkl} c_j q_k q'_l\right] \equiv (\bar{c}q'')\,[cqq']$$

- Ψ_1^H and Ψ_2^H only satisfy HQS condition
- Light $[q'q'']$-diquark is transmitted intact, retaining its spin quantum number, from *b*-baryon to pentaquark



Double Well Potential in Pentaquarks

- Keeping the color of the light diquark unchanged, convolution of two Levi-Civita tensors entering the triquark gives

$$\Psi_1^D = -\frac{\sqrt{3}}{2}\left[\Psi_1^H + \Psi_2^H\right],$$

- Color reconnection is not enough to reexpress pentaquark operator as direct product of the meson and baryon operators
- Spins of quarks and diquarks should be projected onto definite hadronic spin states
- One needs to know Dirac structure of pentaquark operators to undertake the Fierz transformations in Dirac space
- Exemplify this by considering $P_c(4312)$ pentaquark

Mass Predictions for Unflavored Pentaquarks

J^P	This work	AAAR	J^P	This work	AAAR
	$S_{ld} = 0,\ L = 0$			$S_{ld} = 1,\ L = 1$	
$1/2^-$	3830 ± 34	4086 ± 42	$1/2^+$	4144 ± 37	3970 ± 50
	4150 ± 29	4162 ± 38		4209 ± 37	4174 ± 44
$3/2^-$	$\boxed{4240 \pm 29}$	4133 ± 55		4465 ± 32	4198 ± 50
	$S_{ld} = 1,\ L = 0$			4530 ± 32	4221 ± 40
$1/2^-$	4026 ± 31	4119 ± 42		4564 ± 33	4240 ± 50
	4346 ± 25	4166 ± 38		4663 ± 32	4319 ± 43
	4436 ± 25	4264 ± 41	$3/2^+$	4187 ± 37	
$3/2^-$	4026 ± 31	4072 ± 40		4250 ± 37	
	4346 ± 25	4300 ± 40		4508 ± 32	
	4436 ± 25	4342 ± 40		4570 ± 32	
$5/2^-$	4436 ± 25	4409 ± 40		4511 ± 33	
	$S_{ld} = 0,\ L = 1$			4566 ± 32	
$1/2^+$	4030 ± 39	4030 ± 62		4656 ± 32	
	4351 ± 35	4141 ± 44	$5/2^+$	4260 ± 37	4450 ± 44
	4430 ± 35	4217 ± 40		4581 ± 32	4524 ± 41
$3/2^+$	4040 ± 39			4601 ± 32	4678 ± 44
	4361 ± 35			4656 ± 32	4720 ± 44
	$\boxed{4440 \pm 35}$		$7/2^+$	4672 ± 32	
$5/2^+$	$\boxed{4457 \pm 35}$	4510 ± 57			

Double Well Potential in Pentaquarks

- Diquark-diquark-antiquark operators with spinless heavy and light diquarks

$$\Psi_1^{H(1)}(x,y) = \frac{1}{3}\left(\tilde{c}^i(x)\,\sigma_2\right)\left(c_i(y)\,\sigma_2\,q_k(y)\right)d_0^k(x)$$

$$\Psi_2^{H(1)}(x,y) = \frac{1}{3}\left(\tilde{c}^i(x)\,\sigma_2\right)\left(c_k(y)\,\sigma_2\,q_i(y)\right)d_0^k(x)$$

- For the lowest lying pentaquark, $q = u$ and $d_0 = [u\,C\,\gamma_5\,d]$, being scalar diquark
- Quarks are considered in the non-relativistic limit
- After Fierz transformation of Pauli matrices and suppressing position dependence, they can be rewritten in terms of hadrons

$$\Psi_1^{H(1)} = -\frac{i}{\sqrt{2}}\left[a\,\eta_c + b\,(\boldsymbol{\sigma}\,\boldsymbol{J/\psi})\right]p, \quad \Psi_2^{H(1)} = -\frac{i}{\sqrt{2}}\left[A\,\bar{D}^0 + B\left(\boldsymbol{\sigma}\,\bar{\boldsymbol{D}}^{*0}\right)\right]\Lambda_c^+$$

- A and B (a and b) are non-perturbative coefficients associated with barrier penetration amplitudes for light (heavy) quark
- They are equal in the limit of naive Fierz coupling

Double Well Potential in Pentaquarks

- Similarly, diquark-diquark-antiquark operators containing heavy diquark with $S_{hd} = 1$ and light diquark $S_{ld} = 0$

$$\boldsymbol{\Psi}_1^{H(2)}(x,y) = \frac{1}{3}\left(\tilde{c}^i(x)\,\sigma_2\right)\left(c_i(y)\,\sigma_2\,\boldsymbol{\sigma}\,q_k(y)\right)d_0^k(x)$$

$$\boldsymbol{\Psi}_2^{H(2)}(x,y) = \frac{1}{3}\left(\tilde{c}^i(x)\,\sigma_2\right)\left(c_k(y)\,\sigma_2\,\boldsymbol{\sigma}\,q_i(y)\right)d_0^k(x)$$

- Being direct product of spinor and vector, they need to be devided into two states with spins $J = 1/2$ and $J = 3/2$
- For $P_c(4312)$ interpreted as $J^P = 3/2^-$ pentaquark, decompositions in term of hadrons are as follows

$$\boldsymbol{\Psi}_1^{H(3/2)} = \frac{i\sqrt{2}}{3}\left\{b'\,\boldsymbol{J/\psi} - 2ic'\left[\boldsymbol{\sigma}\times\boldsymbol{J/\psi}\right]\right\}p$$

$$\boldsymbol{\Psi}_2^{H(3/2)} = -\frac{i\sqrt{2}}{3}\left\{B'\,\bar{\boldsymbol{D}}^{*0} - 2iC'\left[\boldsymbol{\sigma}\times\bar{\boldsymbol{D}}^{*0}\right]\right\}\Lambda_c^+$$

- $P_c(4312)$ is mainly decaying either to $J/\psi\,p$ final state, in which it was observed, or to $\Lambda_c^+\,\bar{D}^{*0}$

Hidden-Charm Pentaquark Decays

- Tunneling amplitude in leading semiclassical approximation, $\mathcal{A}_M \sim e^{-\sqrt{2ME}\ell}$, where E and ℓ are barrier height and extension
- For constituent quark masses, m_u and m_c, $E = 100$ MeV and $\ell = 2$ fm, the ratio of amplitides squared

$$R_{\rm penta} = \frac{|b'|^2 + 4|c'|^2}{|B'|^2 + 4|C'|^2} \sim \left(\frac{\mathcal{A}_{m_c}}{\mathcal{A}_{m_u}} \right)^2 \sim 10^{-3} \sim R$$

- With decay momenta $p_p \simeq 660$ MeV and $p_{\Lambda_c} \simeq 200$ MeV

$$\frac{\Gamma(P_c(4312) \to J/\psi \, p)}{\Gamma(P_c(4312) \to \Lambda_c^+ \bar{D}^{*0})} = \frac{p_p}{p_{\Lambda_c}} R_{\rm penta} \sim 10^{-3}$$

- If this approach is correct, $P_c(4312)$ should be searched in $\Lambda_b^0 \to \Lambda_c^+ \bar{D}^{*0} K^-$ decay
- This can also be applied to decays of $P_{cs}(4459)$ pentaquark

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Evidence of $J/\psi\, K_s^0$ Structure in $B^0 \to J/\psi\, \phi\, K_s^0$ Decay

LHCb Collab. [R. Aaij *et al.*, PRL 131 (2023) 131901]

1. Search for an isospin partner of $Z_{sc}(4000)^+$ (aka $T^\theta_{\psi s1}(4000)^+$) in the isospin-conjugate decay channel $B^+ \to J/\psi\, \phi\, K^+$
2. Evidence for a new $T^\theta_{\psi s1}(4000)^0$ state at 4σ
3. Mass difference 12^{+11+6}_{-10-4} MeV is consistent with isospin partners

Fully Charm Tetraquarks

Xin Chen (ATLAS Collab.)

1. ATLAS searched for potential fully charm tetraquarks decaying into a pair of J/ψ-mesons, or into $J/\psi + \psi(2S)$, in the four muon final state
2. Significant excess in these channels can be explained by $X(6900)^0 = T_{\psi\psi}(6900)^0$ which is consistent with LHCb and CMS results

Alexis Pompili (CMS Collab.)

1. Explored di-J/ψ mass spectrum 3 structures pattern was found, confirming the $X(6900)^0$ observed by LHCb, observing the $X(6600)^0$ and having an evidence for $X(7100)^0$

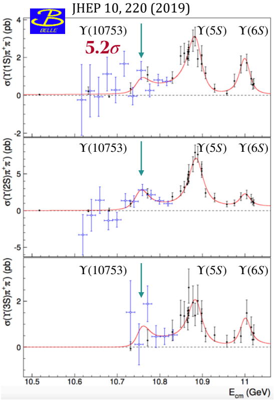
$\Upsilon(10753)$ Studies at Belle and Belle II

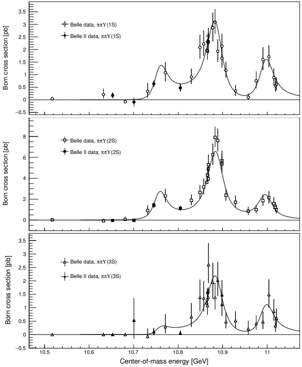
R. Mizuk et al., JHEP 1910 (2019) 220 [arXiv:1905.05521]
I. Adachi et al., arXiv:2401.12021

$\Upsilon(1S)\pi^+\pi^-$

$\Upsilon(2S)\pi^+\pi^-$

$\Upsilon(3S)\pi^+\pi^-$


JHEP 10, 220 (2019)



Significance (Belle + Belle II): 4.1σ in $\Upsilon(1S)\pi^+\pi^-$ & 7.5σ in $\Upsilon(2S)\pi^+\pi^-$

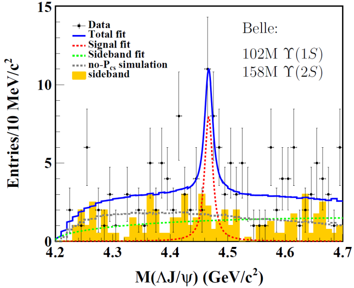
$M = (10756.6 \pm 2.7 \pm 0.9)$ MeV & $\Gamma = (29.0 \pm 8.8 \pm 1.2)$ MeV

Pentaquarks in $\Upsilon(1S)$ and $\Upsilon(2S)$ Inclusive Decays

R. Mizuk, talk at ICHEP-2024
X. Dong et al., arXiv:2403.04340

- Search for $\Upsilon(1S,2S) \to P_\psi^N X \to (J/\psi\, p)\, X$ ⇒ no pentaquark signals
- Search for $\Upsilon(1S,2S) \to P_{\psi s}^\Lambda X \to (J/\psi\, \Lambda)\, X$ ⇒ local significance 4.0σ

Combined $\Upsilon(1S)$ and $\Upsilon(2S)$ Data



- $M = 4469.5 \pm 4.1 \pm 4.1$ MeV
  $\Gamma = 14.3 \pm 9.2 \pm 6.3$ MeV
- LHCb measurements:

  $M = 4458.8 \pm 2.9^{+4.7}_{-1.1}$ MeV

  $\Gamma = 17.3 \pm 6.5^{+8.0}_{-5.7}$ MeV
- 3.3σ significance with systematics

Summary

- During 20 years after the famous $X(3872)$ discovery, a lot of interesing and unexpected experimental results on multiquark systems were obtained
- This area of research is highly motivated by these results which require deeper theoretical understanding
- Several theoretical approaches are developing, being rather successful in expanations, but still they remain compatitive and experiments do not favor anyone yet
- A lot of theoretical predictions for multiquark states are waiting their experimental tests and, I hope, this will be possible in a near future