Protvino, HEPFT-2024

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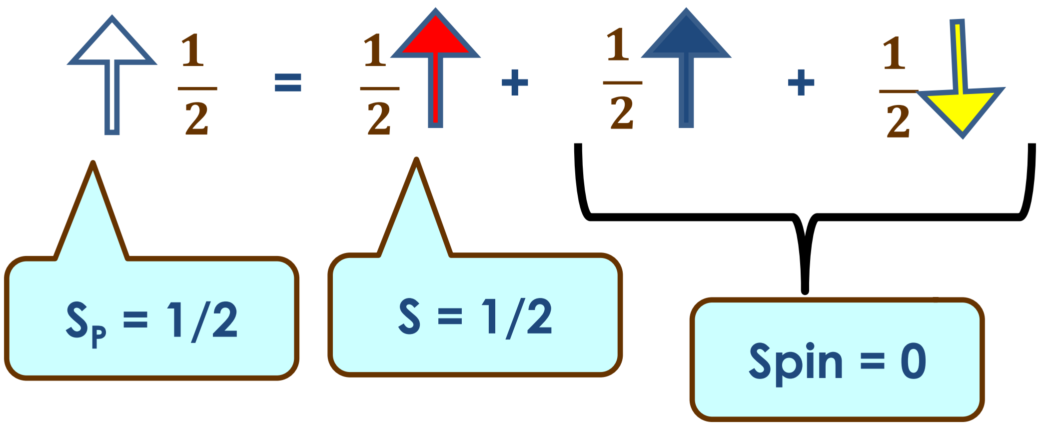
Present State of the Proton Spin Problem

Proton spin puzzle/ Spin crisis

Proton spin $S_p = 1/2$. In the simplest model, proton consists of three quarks of different colours, spin of each quark = 1/2, so



No spin problem with the proton spin description if proton consists of 3 quarks only

However, experiments on Deep-Inelastic Scattering off polarized protons brought a problem. At high energies, nucleons (protons) consist of partons, i.e. quarks and gluons

Spin/Angular Moment conservation relates the hadron spin to the parton (quarks and gluon) spins

Proton spin =1/2. Proton consists of quarks (quark spin = 1/2) and gluons (gluon spin = 1)

Proton spin is made out of the parton spins, so it is expected that

$$\frac{1}{2} = S_q + S_g$$

quarks

gluons

First experimental investigation of the nucleon spin was carried out by European Muon Collaboration (EMC) in 1988

$$S_q = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x)$$

Quark helicity distribution

$$S_g = \int_0^1 dx\, \Delta G(x)$$

Gluon helicity distribution

Angular momentum conservation: $S_q + S_g = 1/2$

However in 1988, EMC reported that $S_q + S_g < 1/2$

This was named **Proton Spin Puzzle/ Spin Crisis**

To explain Puzzle, there were introduced additional contributions: **Angular Orbital Moments** of quarks and gluons, L_q and L_g Nevertheless it did not solve the problem:

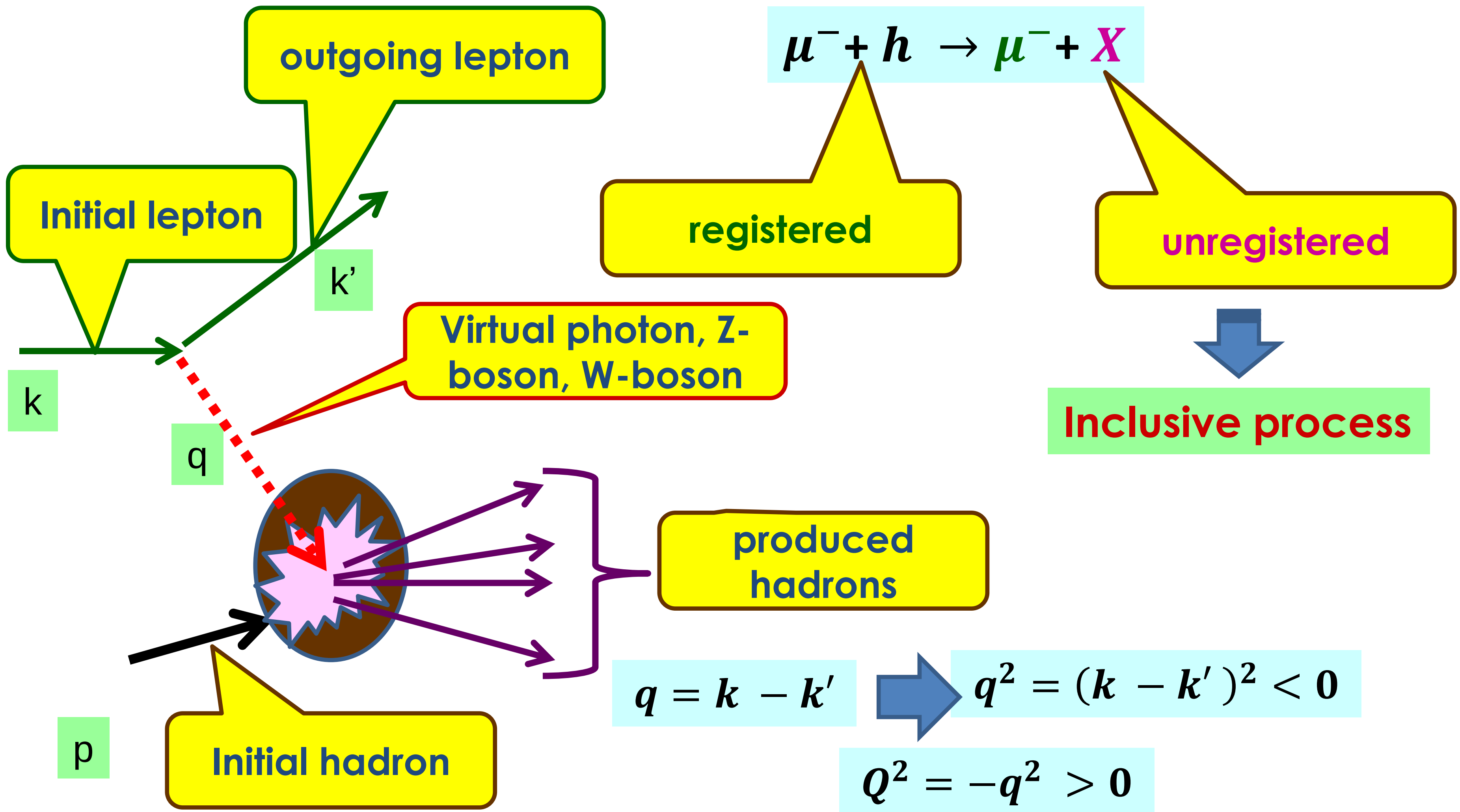
$$S_q + S_g + L_q + L_g < 1/2$$

But it has not helped to solve the puzzle

Experimental data on proton spin at high energies arrive from lepton-hadron Deep-Inelastic Scattering (DIS)

Deep-inelastic lepton-hadron scattering

Aim: probing electromagnetic structure of hadrons



Standard parametrization of $W^{spin}{}_{\mu\nu}$

Hadron mass

Longitudinal component of spin

transverse component of spin

$$W^{spin}{}_{\mu\nu} = i\,\epsilon_{\mu\nu\lambda\rho}\; m_h q_\lambda / pq \left[S^{||}{}_\rho\, g_1(x.\,Q^2) + S^{\perp}{}_\rho\, g_\perp(x.\,Q^2) \right]$$

Spin-dependent structure functions

$$g_\perp = g_1 + g_2$$

$$S^{||}{}_\rho \approx p_\rho / m_h$$

Each structure function depends on the invariant energy *w = 2pq* and virtuality of the photon Q^2

$$x = Q^2/2pq, \qquad 0 < x < 1$$

Spin structure functions are asymmetries:

$$g_1 \sim \sigma_L(\uparrow\uparrow) - \sigma_L(\uparrow\downarrow) \qquad g_\perp = g_1 + g_2 \sim \sigma_T(\uparrow\uparrow) - \sigma_T(\uparrow\downarrow)$$

Spins are longitudinal

Spins are transverse

subscripts:

L –longitudinal
T - transverse

At high energies, when masses are neglected, $S_L \leftrightarrow h$

helicity

Experimental data on S_q and S_g
come from investigation of structure function g_1
of Deep-Inelastic Scattering at COMPASS and RHIC

Taken from wwwcompass.cern.ch

COMPASS is a high-energy physics experiment at the Super Proton Synchrotron (SPS) at CERN in Geneva, Switzerland. The purpose of this experiment is the study of hadron structure and hadron spectroscopy with high intensity muon and hadron beams.

On February 1997 the experiment was approved conditionally by CERN and the final Memorandum of Understanding was signed in September 1998. The spectrometer was installed in 1999 - 2000 and was commissioned during a technical run in 2001. Data taking started in summer 2002 and continued until fall 2004. After one year shutdown in 2005, COMPASS will resume data taking in 2006.

Nearly 240 physicists from 11 countries and 28 institutions work in COMPASS

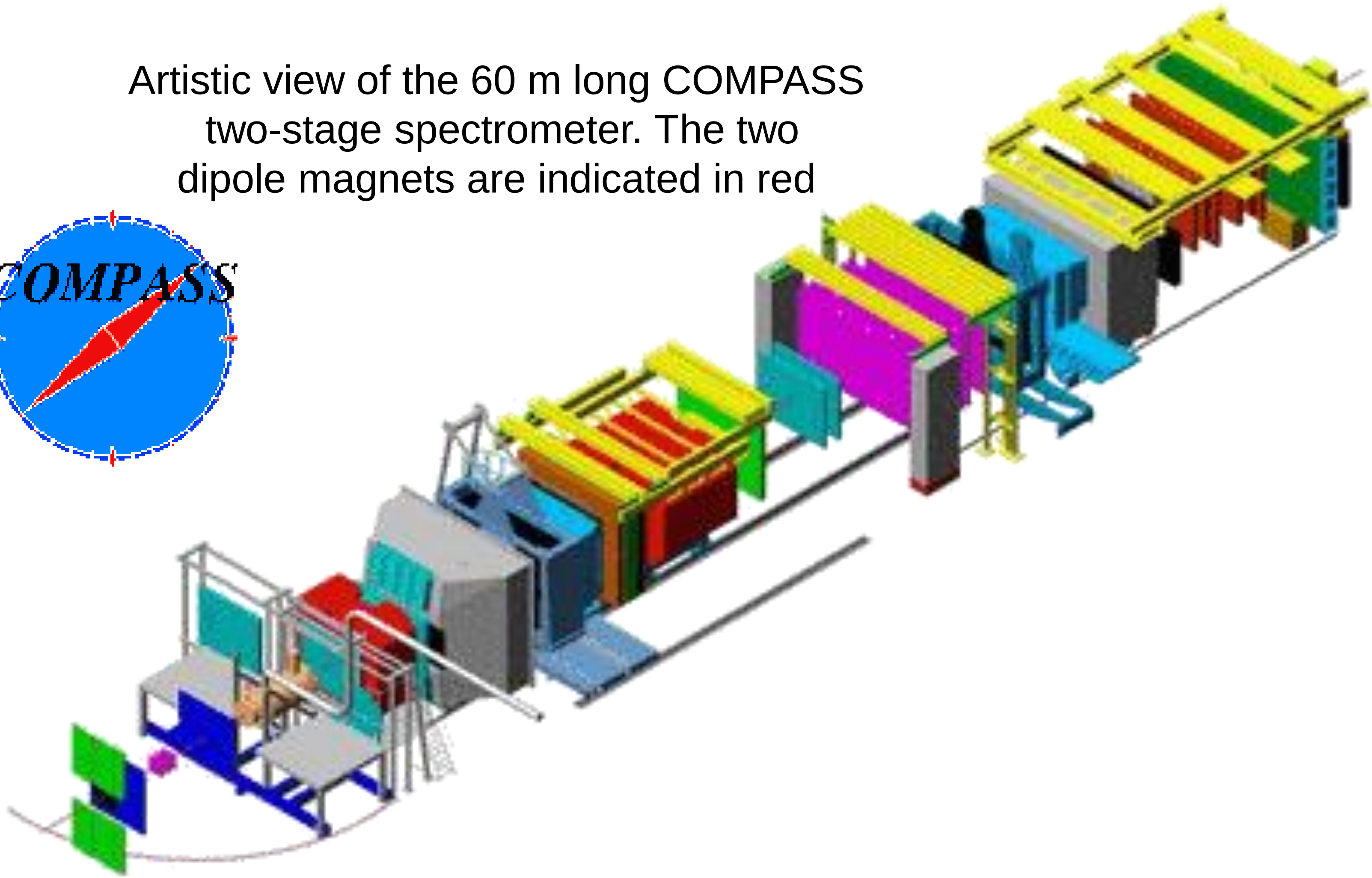
Taken from wwwcompass.cern.ch

COMPASS

COmmon Muon Proton Apparatus for Structure and Spectroscopy

Artistic view of the 60 m long COMPASS two-stage spectrometer. The two dipole magnets are indicated in red

COMPASS



Spin Physics

RHIC is the world's only machine capable of colliding high-en beams of polarized protons, and is a unique tool for exploring puzzle of the proton's 'missing' spin.

In addition to colliding heavy ions, RHIC is able

The Importance of Spin

Aim of the RHIC experiments: to obtain S_q and S_g

$$S_q = \frac{1}{2}\int_0^1 dx\, h_q(x)$$

Quark helicity distribution

$$S_g = \int_0^1 dx\, h_g(x)$$

Gluon helicity distribution

Actually they obtained $\overline{S}_q$ and $\overline{S}_g$

$$\overline{S}_q = \frac{1}{2}\int_{x_1}^1 dx\, h_q(x)$$

$$\overline{S}_g = \frac{1}{2}\int_{x_2}^1 dx\, h_q(x)$$

$x_1 = 0.001$

$x_2 = 0.05$

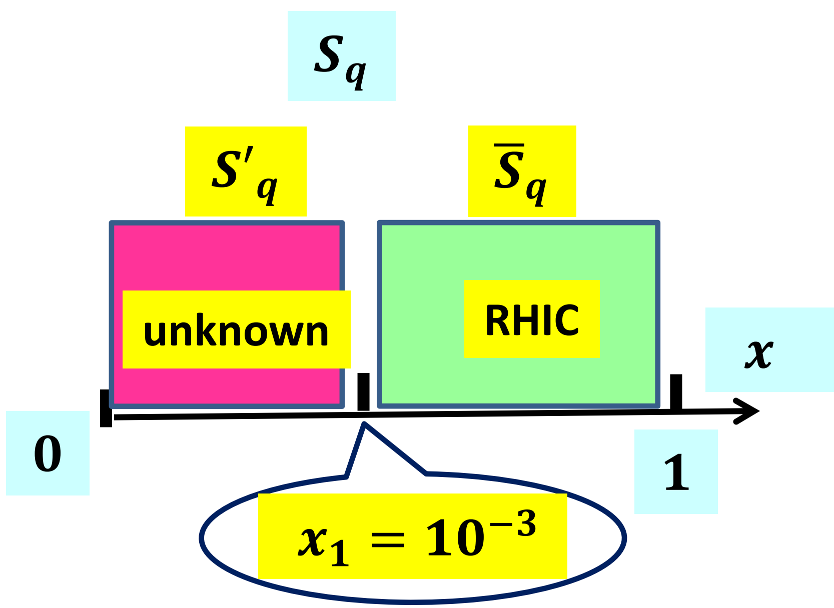
Recent RHIC data (2015) obtained by measuring g_1:

$S_q = 0.15 \div 0.20$ at $0.001 < x < 1$

$S_g = 0.13 \div 0.26$ at $0.05 < x < 1$

$Q^2 = 10\ GeV^2$

knowledge of $h_q(x)$ and $h_g(x)$ at smaller **x** is out of the RHIC reach



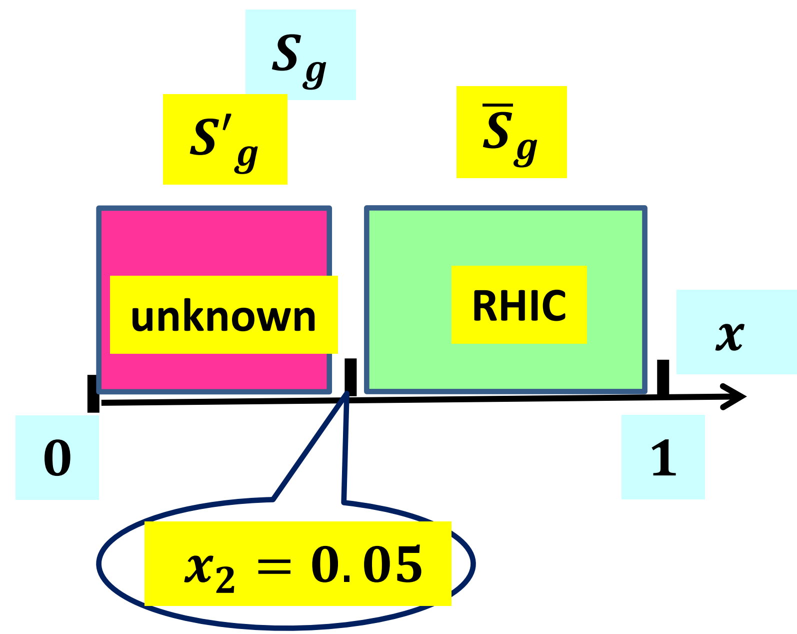
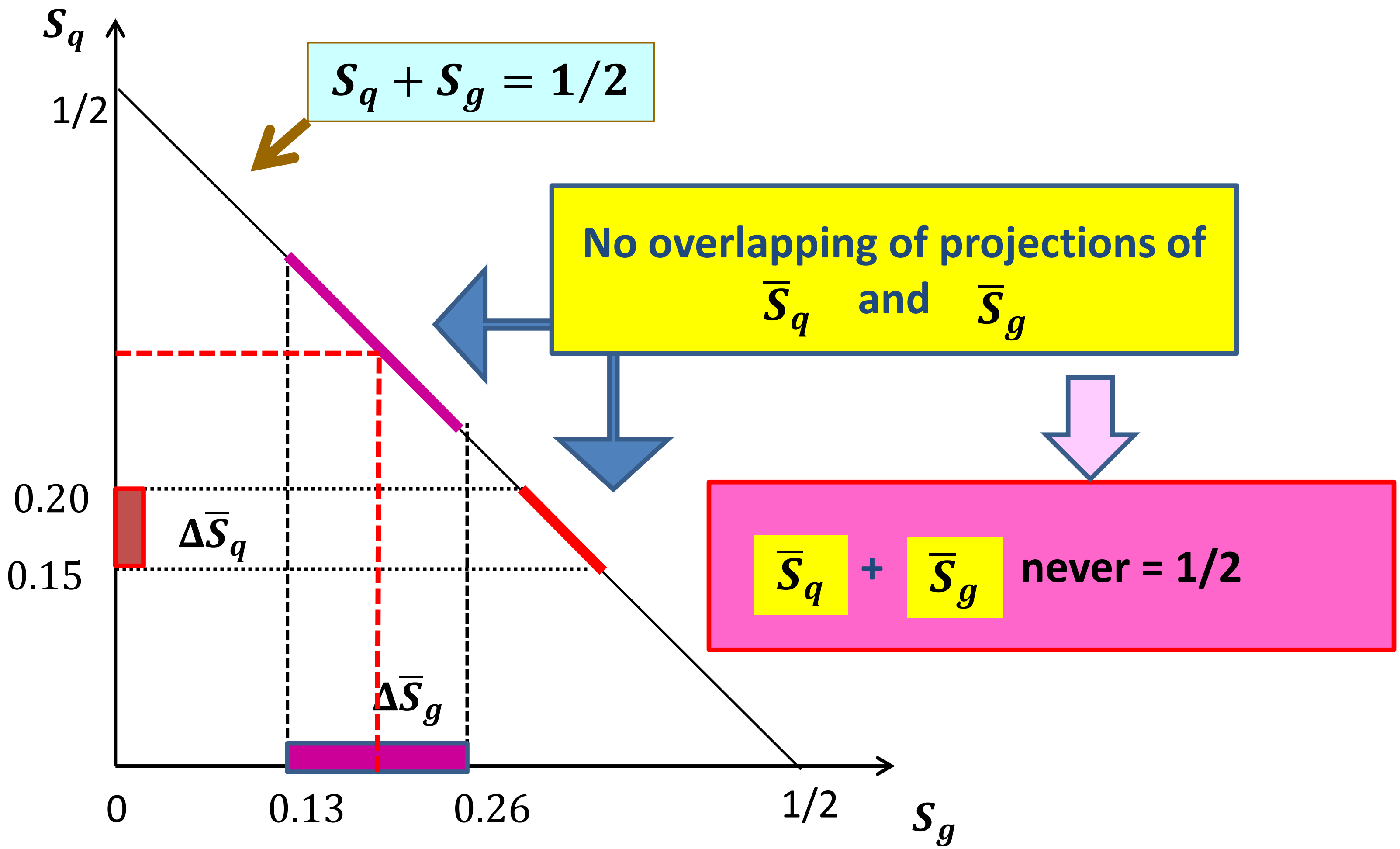


Illustration of the RHIC data:



Missing contributions to the proton spin:

$$S'_q = \frac{1}{2}\int_0^{x_1} dx\, h_q(x)$$

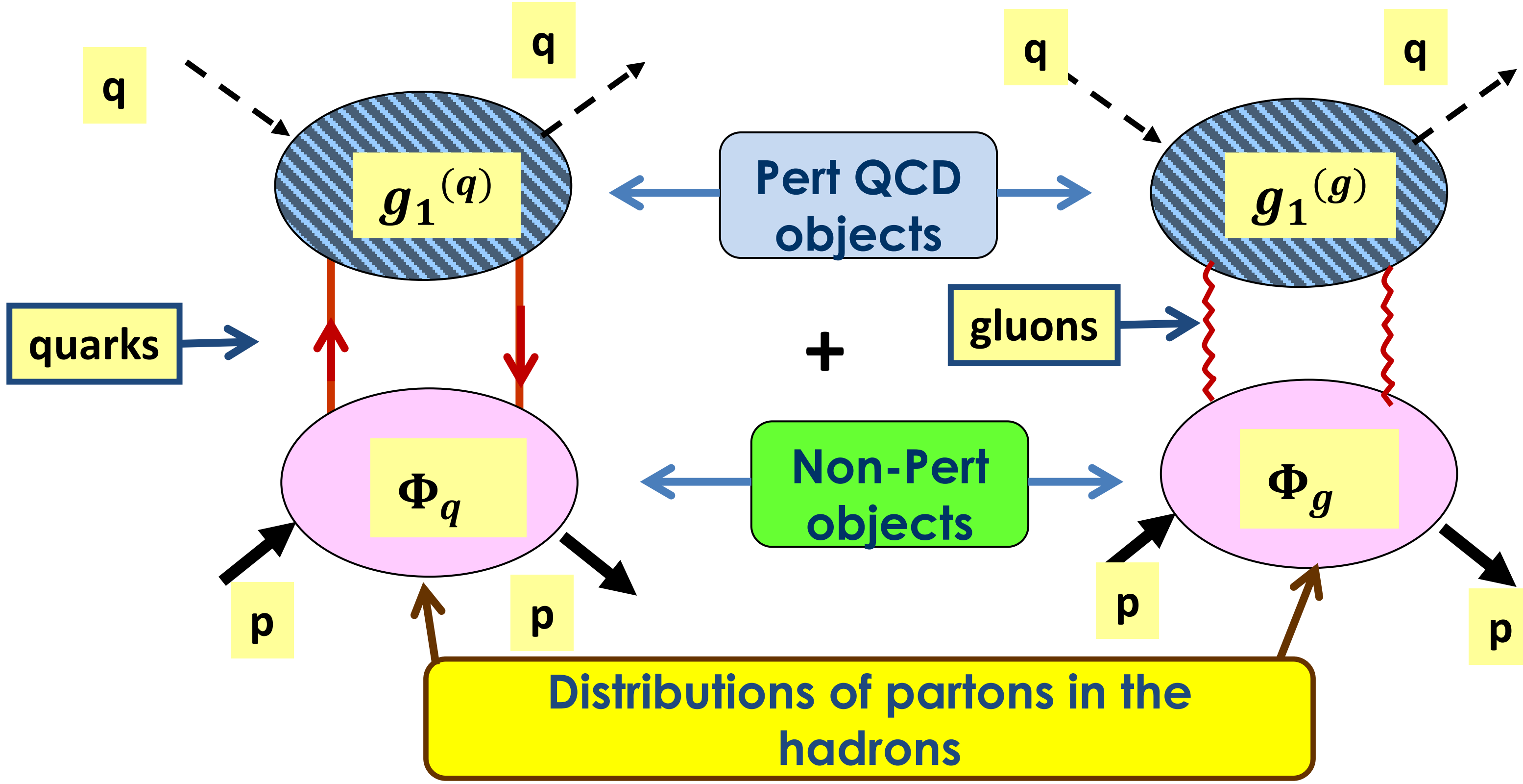
$$S'_g = \int_0^{x_2} dx\, h_q(x)$$

$$x_1 = 0.001$$

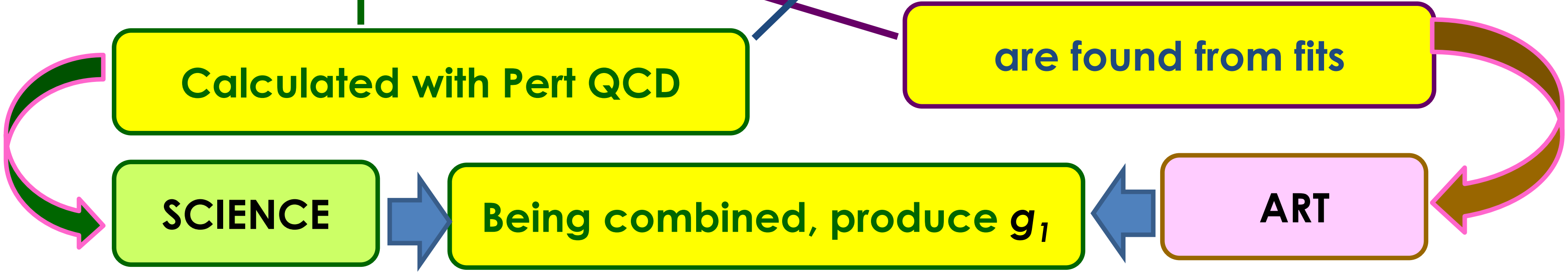
$$x_2 = 0.05$$

They cannot be registered at RHIC, so they should be calculated.
Available theoretical instrument is QCD but it is a regular technical means at large momenta only.
In order to describe an impact of the small momenta region, the QCD Factorization concept is used.

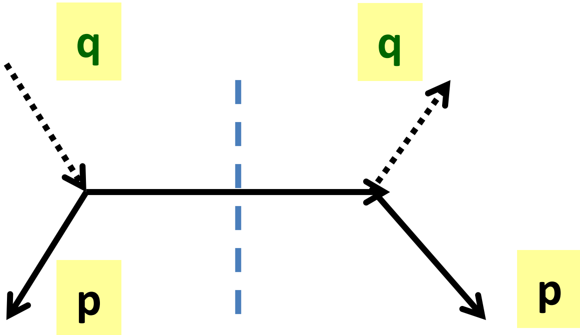
QCD Factorization:



$$g_1 = g_1^{quark} \otimes \Phi_{quark} + g_1^{gluon} \otimes \Phi_{gluon}$$



Perturbative components of g_1
Born approximation



Photon virtuality

Invariant energy

$$x = Q^2/2pq$$

Quark electric charge

$$g_1^{(q)} = e_q^{\,2}\,\delta(x-1)$$

$$g_1^{(g)} = 0$$

We are interested in x <0.05
where Born fails

higher loop calculations are necessary
The contributions most important at small x are
Doubly-Logarithmic (DL)

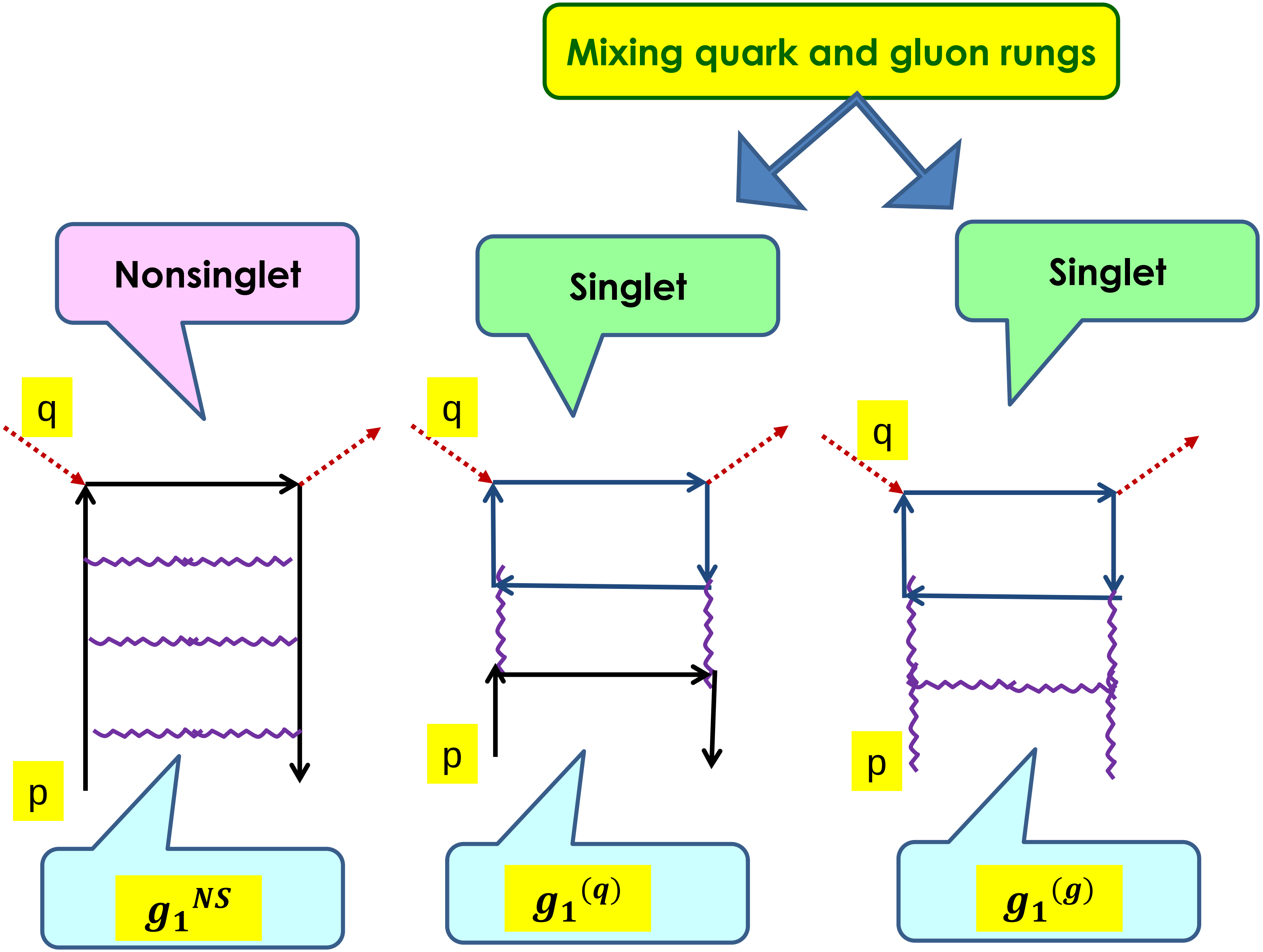
Standard instrument to calculate g_1 or helicities beyond Born is DGLAP Dokshitzer-Gribov-Lipatov-Altarelli-Parisi

DGLAP operates with the coefficient functions calculated in first and second orders in the coupling and does not account for total summation of logarithms of x to all orders in the coupling

We account for total resummation of DL contributions and in addition account for the running coupling effects

$$g_1{}^{(q)} = \delta(x-1) + c_1\left(\alpha_s ln\ \ (1/x)\right) + c_2\left(\alpha_s{}^2 ln^3(1/x)\right)\ \ + \cdots$$

$$g_1{}^{(g)} = c'_1\left(\alpha_s ln\ \ (1/x)\right) + c'_2\left(\alpha_s{}^2 ln^3(1/x)\right)\ \ + \cdots$$


Mixing quark and gluon rungs
Nonsinglet
Singlet
Singlet
q
q
q
p
p
p
g_1^{NS}
$g_1^{(q)}$
$g_1^{(g)}$

Both singlet and non-singlet g_1 were calculated in DLA + accounting for running coupling effects. The instrument to calculate them were Infra-Red Evolution Equations

This method was suggested by L.N. Lipatov .
It stems from the observation that the bremsstrahlung photon with minimal transverse momentum (the softest photon) can be factorized out of the radiative amplitudes with DL accuracy V.N. Gribov

Similarly, DL contributions of softest virtual quarks/gluons can be factorized

DL contributions of virtual gluons are infrared (IR)-divergent. When quark masses are neglected, DL contributions from soft quarks also become IR-divergent. In order to regulate them, one can introduce an IR cut-off μ

It is convenient to introduce μ in the transverse momentum space, which makes it possible to use the factorization.

After factorizing the softest quarks and gluons, their transverse momenta act as a new IR cut-off, instead of μ , for integrating over momenta of other virtual partons.

Value of μ obeys the restriction $\mu \ll \Lambda_{QCD}$ in order to allow applying Perturbative QCD, otherwise it is arbitrary. This makes possible to evolve the objects under consideration with respect to μ

It is the reason why the method was named IREE. (M.Krawczyk)
The method proved to be effective and simple instrument for calculations in Double-Logarithmic Approximation (DLA), i.e. when contributions

$$\sim {\alpha_s}^n ln^{2n}(1/x) \qquad (n = 1, 2, \dots)$$

are accounted to all orders in α_s

At the beginning, the IREE method operated with fixed α_s
but later the running coupling effects were incorporated (Ermolaev-Greco-Troyan)

Non-singlet

$$g_1(x, Q^2) = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\,\pi\,i}\, x^{-\omega}\, C(\omega)\, e^{h(\omega)\ln\left(\frac{Q^2}{\mu^2}\right)}\, \phi(\omega)$$

Anomalous dimension

Coefficient function

Initial quark distribution

Expression for the singlet is more involved. It includes mixing of quark and gluon rungs, and initial quark and gluon distributions

Both coefficient functions and anomalous dimensions are calculated in DLA, i.e. each of them sums DL contributions to all orders in the coupling

First, there was calculation of g_1^{NS} in DLA under the ladder approximation

Ermolaev-Manaenkov-Ryskin (1995)

Then contributions of non-ladder graphs were added

Bartels-Ermolaev-Ryskin (1996)

Then g_1^{S} was calculated

Bartels-Ermolaev-Ryskin (1996)

The small-x asymptotics of g_1 was found by purely mathematical means, with Saddle-Point method. All of them proved to be of the Regge type

$$g_1 \sim x^{-\Delta}\left(Q^2/\mu^2\right)^{\Delta/2}$$

intercept

Asymptotic scaling

Any of g_1^{NS}, g_1^{S}, F_1^{NS}, F_1^{S} calculated in DLA asymptotically behaves as

$$f \sim x^{-\Delta}\left(Q^2/\mu^2\right)^{\Delta/2} = \left({Q^2}/{x^2}\right)^{\Delta/2}$$

Albeit their intercepts are different

$$\Delta_{NS}^{(ladder)} = \left(\frac{2\alpha_s C_F}{\pi}\right)^{1/2}$$

$$C_F = \frac{N^2 - 1}{2N} = \frac{4}{3}$$

$$N = 3$$

$$\Delta_{NS} \approx \left(\frac{2\alpha_s C_F}{\pi}\right)^{1/2} \left[\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4/(N^2 - 1)}\right]^{1/2}$$

$$\approx \left(\frac{2\alpha_s C_F}{\pi}\right)^{1/2} \left[1 + 2/N^2\right]$$

Found with numerical calculation

$$\Delta_S = z_h \left(\frac{\alpha_s N}{2\pi}\right)^{1/2}$$

$$z_h = 3.66$$

Later the running coupling effects were accounted for, so the the intercepts became just numbers, without α_s

$$\Delta_{NS} = 0.42 \qquad \Delta_S = 0.86$$

CRITICISM and ALTERNATIVE CALCULATIONS of INTERCEPTS of g_1

Interest to theoretical investigation of g_1 increased in 2015 when
Kovchegov-Pitonyak-Sievert 2015 (KPS)
investigated small-x asymptotics of helicity in DLA with fixed
α_s in the ladder approximation. Their approach differ from ours
First they confirmed our previous
result on Intercept of g_1 in the ladder approximation
Ermolaev-Manaenkov-Ryskin, 1995

Next year KPS considered asymptotics of the singlet g_1
and arrived at a huge disagreement with the result of
Bartels- Ermolaev –Ryskin (BER), 1996

NAMELY, They considered purely gluon DL contributions and represented their result on the intercept as follows:

KPS $$\tilde{\Delta}_{gluon} = \tilde{z}_h \, (\alpha_s N/2\pi)^{1/2}$$

BER $$\Delta_{gluon} = z_h \, (\alpha_s N/2\pi)^{1/2}$$

KPS $\tilde{z}_h = 2.45$ vs $z_h = 3.66$ BER

Strong discrepancy

Publishing such huge discrepancy provoked an extensive interest in the matter, so many authors contributed to this issue

Kovchegov, Pitonyak, Sievert, Borden, Adamiak, Yossathom, Tawabutr, Santiago, Tarasov, Venugoplan, Chirilli, Gougoulic, Nayan Mani Nath, Jayanta Kumar Sarma, Zhou, Boussarie, Hatta, Yuan ..

These authors also studied small- x evolution of helicity, using the JIMWLK -approach

Jalilian-Marian, Iancu, McLerran, Weigert,,Leonidov, Kovner

However, JIMWLK originally was designed for evolution of unpolarized objects , so

Kovchegov- Pitonyak - Sievert

generalized it to study the helicity evolution

and other authors also developed various modifications of JIMWLK trying to obtain most accurate estimates of z_h

This polemics continued till 2023

As a results of this polemics of 2016- 2023, the first estimate of 2016 (called KPS-evolution)
Kovchegov- Pitonyak - Sievert

z_h = 2.45 — KPS 2016

was drastically corrected by
Kovchegov- Pitonyak - Sievert – Cougoulic- Tarasov- Tawabutr

when they constructed KSPTT evolution equation instead of KPS. Their estimate of 2023 is
z_h = 3.6 — KPSCTT 2023

coincides with BER 1996

However, recently accuracy of calculations in the framework of KPSCTT – evolution was increased, so same authors (e.g. Tawabutr) have concluded that there still remains a small disagreement

The newest estimate :

$\tilde{z}_h$ = 3.661 vs z_h = 3.664

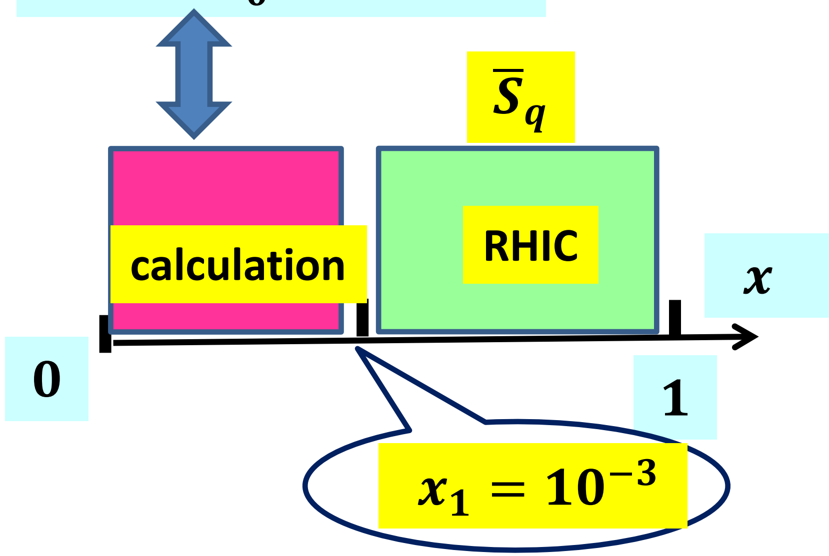
KPSCTT 2023

BER 1996

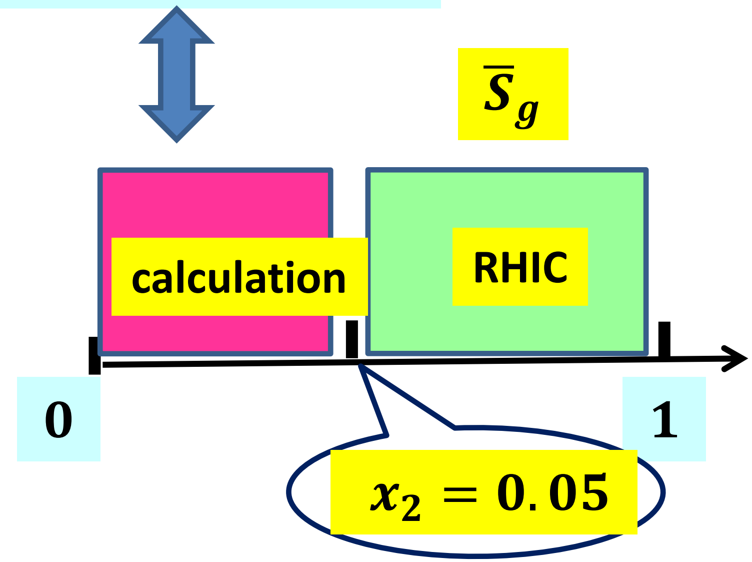
NB it is important to remember that KPSCTT provides asymptotics only whereas our approach first provides explicit expressions for g_1 in DLA and its asymptotics are obtained with Saddle-Point Method from such expressions

Supposedly: $S_q + S_g = (S'_q + \overline{S}_q) + (S'_g + \overline{S}_g) = 1/2$

$$S'_q = \frac{1}{2}\int_0^{x_1} dx\, h_q(x)$$



$$S'_g = \int_0^{x_2} dx\, h_q(x)$$



asymptotic expressions for g_1 were used to calculate S'_q and S'_g

Cougoulic-Kovchegov-Manley-Tarasov-Tawabutr 2023;
Boussarie- Hatta – Yuan, 2019; Kovchegov- Manley, 2023

In more detail: The asymptotic expressions for g_1 were applied to calculate S'_q and S'_q

Cougoulic-Kovchegov-Manley-Tarasov-Tawabutr, 2023

Adamiak-Kovchegov-Tawabutr 2023

It turned out that $S_q + S_g < 1/2$

In order to explain the spin crisis, Angular Orbital Momentum contribution was added to S_q, S_g

Boussarie- Hatta – Yuan, 2019; Kovchegov- Manley, 2023

in hope to obtain

$$S_q + S_g + (L_q + L_g) = \frac{1}{2}$$

All the articles describe L_q , L_g by the same asymptotic formulae as S_q , S_g however the derivation is not clearly presented and the explicit estimates of S_q , S_g are absent

Moreover, any asymptotic expressions should not have been used in these regions

Applicability region of Regge asymptotics

Ermolaev-Greco-Troyan

Regge asymptotics are given by simple and elegant expressions.
However the applicability regions of the asymptotics are poorly known

Asymptotics

We introduce $R_{as}(x) = As(g_{1,})/g_1$

and numerically study its **x**-dependence at fixed Q^2

Asymptotics reliably represent g_1 when R_{as} is close to 1.
Numerical analysis yields

$x = 10^{-3}$ $R_{AS} \approx 0.5$

$x = 10^{-4}$ $R_{AS} \approx 0.7$

$x = 10^{-6}$ $R_{AS} \approx 0.9$ → $x < x_0 = 10^{-6}$

Appicability region for asymptotics

In contrast, we do not use the asymptotics and calculate h_q and h_g in Double-Logarithmic Approximation (DLA)

QCD Factorization:

Perturbative components

$$h_q = f_{qq} \otimes \Phi_q + f_{qg} \otimes \Phi_g \qquad h_g = f_{gq} \otimes \Phi_q + f_{gg} \otimes \Phi_g$$

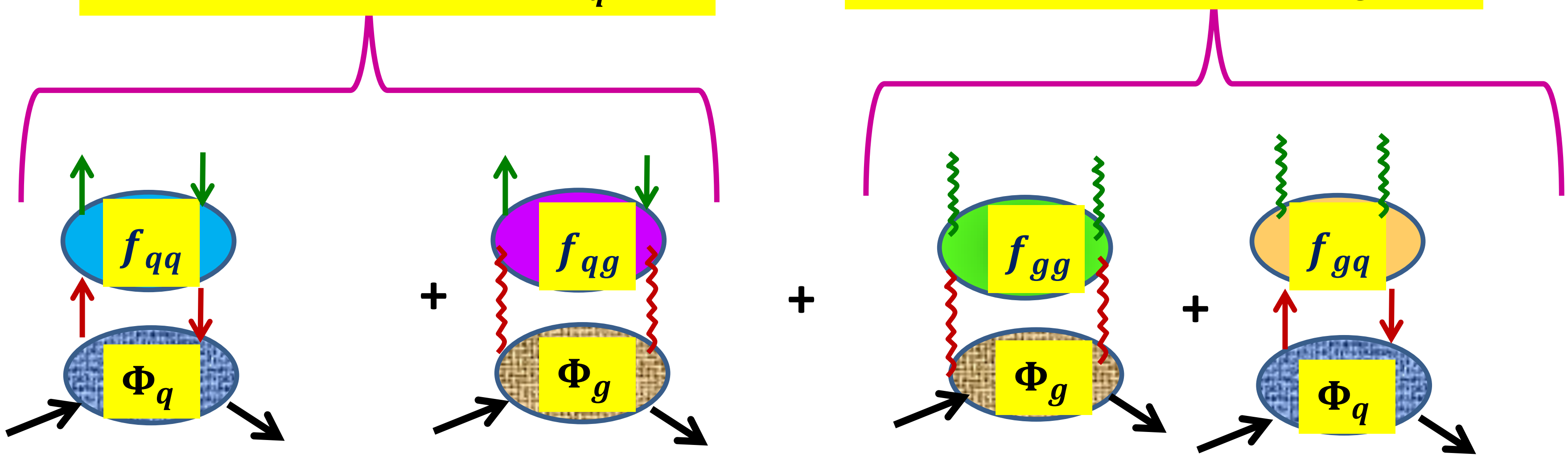
parton distributions

Each intermediate state consists of 2 partons :

Single Parton Collision Approximation

QCD Factorization for $h_q(x)$

QCD Factorization for $h_g(x)$



Perturbative components are calculated in DLA.
Non-Perturbative components are phenomenological objects. They are different for different forms of QCD Factorization

We choose Collinear Factorization. The standard fits for parton densities are:

Can be dropped at small x

$N, a, b, c, d > 0$

$$\Phi_{q,g} = N\, x^{-a}\, (1 - x)^{b} (1 - c x^{d})$$

Mimics resummation of DL and should be dropped when the resummation is taken into account

As a result, at small x

$$\Phi_q \approx N_q \qquad \Phi_g \approx N_g$$

Unknown and cannot be fixed from theoretical grounds

Fix N_q and N_g from the RHIC data on $\bar{S}_q$ and $\bar{S}_g$ respectively

$$\bar{S}_q = \frac{1}{2} N_q \int_{x_1}^{1} dx\, f_{qq}(x) + N_g \frac{1}{2} \int_{x_1}^{1} dx\, f_{qg}(x)$$

$$\bar{S}_g = N_q \int_{x_2}^{1} dx\, f_{gq}(x) + N_g \int_{x_2}^{1} dx\, f_{gg}(x)$$

algebraic equations for $N_{q,g}$

Solving this system, express $N_{q,g}$ through $\bar{S}_{q,g}$

$$S'_q = \frac{1}{2} N_q \int_0^{x_1} dx\, f_{qq}(x) + N_g \frac{1}{2} \int_0^{x_1} dx\, f_{qg}(x)$$

$$S'_g = N_q \int_0^{x_2} dx\, f_{gq}(x) + N_g \int_0^{x_2} dx\, f_{gg}(x)$$

All terms in the r.h.s., are known, so it is possible to perform the integrations

This is program of straightforward calculation of parton contributions to the nucleon spin. However, its implementation is technically difficult because exact expressions for $f_{ik}(x)$ are quite complicated

Instead, we use an approximation for them to obtain a tentative solution to the proton spin puzzle

STEP 1

Main contribution comes from the purely gluon amplitude f_{gg}, so consider it only and neglect contributions of virtual quarks

Then obtain

$$f_{gg}(x) = Im \int_{-i\,\infty}^{i\,\infty} \frac{d\,\omega}{2\,\pi\,i}\, x^{-\omega} F(\omega)$$ where $$F(\omega) = 4\pi^2\sqrt{\omega^2 - a}$$

and $a = 4\alpha_s N/\pi$

Expression for helicity when only gluons accounted for

The integral is expressed through the Modified Bessel Function I_1:

$$M_{gg} = -4\,\pi \frac{\sqrt{a}}{\xi} I_1\left(\xi\sqrt{a}\right)$$ with $$\xi = ln(1/x)$$

And the Imaginary part:

$$Im\, M_{gg} = 4\,\pi^2\, \frac{d}{d\,\xi}\left(\frac{\sqrt{a}}{\xi} I_1\left(\xi\sqrt{a}\right)\right)$$

Mellin transform

Small-x asymptotics is of the Regge type:

$$Im\, M_{gg} \sim 4\,\pi^2\, \frac{\sqrt{a}}{\xi^{3/2}}\, e^{\xi\sqrt{a}} \sim \frac{\sqrt{a}}{\xi^{3/2}}\, x^{-\sqrt{a}}$$

intercept

The genuine intercepts of the helicities and g_1 are known in DLA. They include both gluon and quark contributions

Ermolaev-Greco-Troyan

$$\omega_0 = 0.86$$

Both virtual quarks and gluons contribute

Perfectly agrees with the estimate

$$\omega_0 = 0.88 \pm 0.14$$

obtained by fitting HERA results

Kochelev-Lipka-Nowak-Vento-Vinnikov

STEP 2

Replace the purely gluonic intercept a by the genuine intercept ω_0

It corresponds to accounting for contributions of both virtual quarks and gluons. Therefore, we get a simple interpolation formula

So, we obtain approximate expressions for the quark and gluon helicities

$$h_q = C_q \frac{I_2(z)}{z} \qquad h_g = C_g \frac{I_2(z)}{z}$$

with $z = \omega_0 ln(1/x)$

Unknown, include non-perturbative contributions

Fix them, using the RHIC data

RHIC data

$\bar{S}_q = [0.15 \div 0.20]$

$\bar{S}_g = [0.13 \div 0.26]$

$$\bar{S}_q = \frac{1}{2} C_q A_q \qquad \bar{S}_g = C_g A_g$$

$$A_q = \int_{x_1}^{1} dx \frac{I_2(z)}{z} = 0.138$$

$$A_g = \int_{x_2}^{1} dx \frac{I_2(z)}{z} = 0.874$$

$C_{q,g}$ are known, so we can calculate S'_q and S'_g

$$S'_q = \frac{1}{2} C_q B_q \qquad S'_g = C_g B_g$$

where

$$B_q = \int_0^{x_1} dx \frac{I_2(z)}{z} = 0.0243 \qquad B_g = \int_0^{x_2} dx \frac{I_2(z)}{z} = 0.0747$$

Obtain

$$S_q = \bar{S}_q + S'_q = \bar{S}_q[1 + B_q/A_q] = \bar{S}_q[1 + 0.18]$$

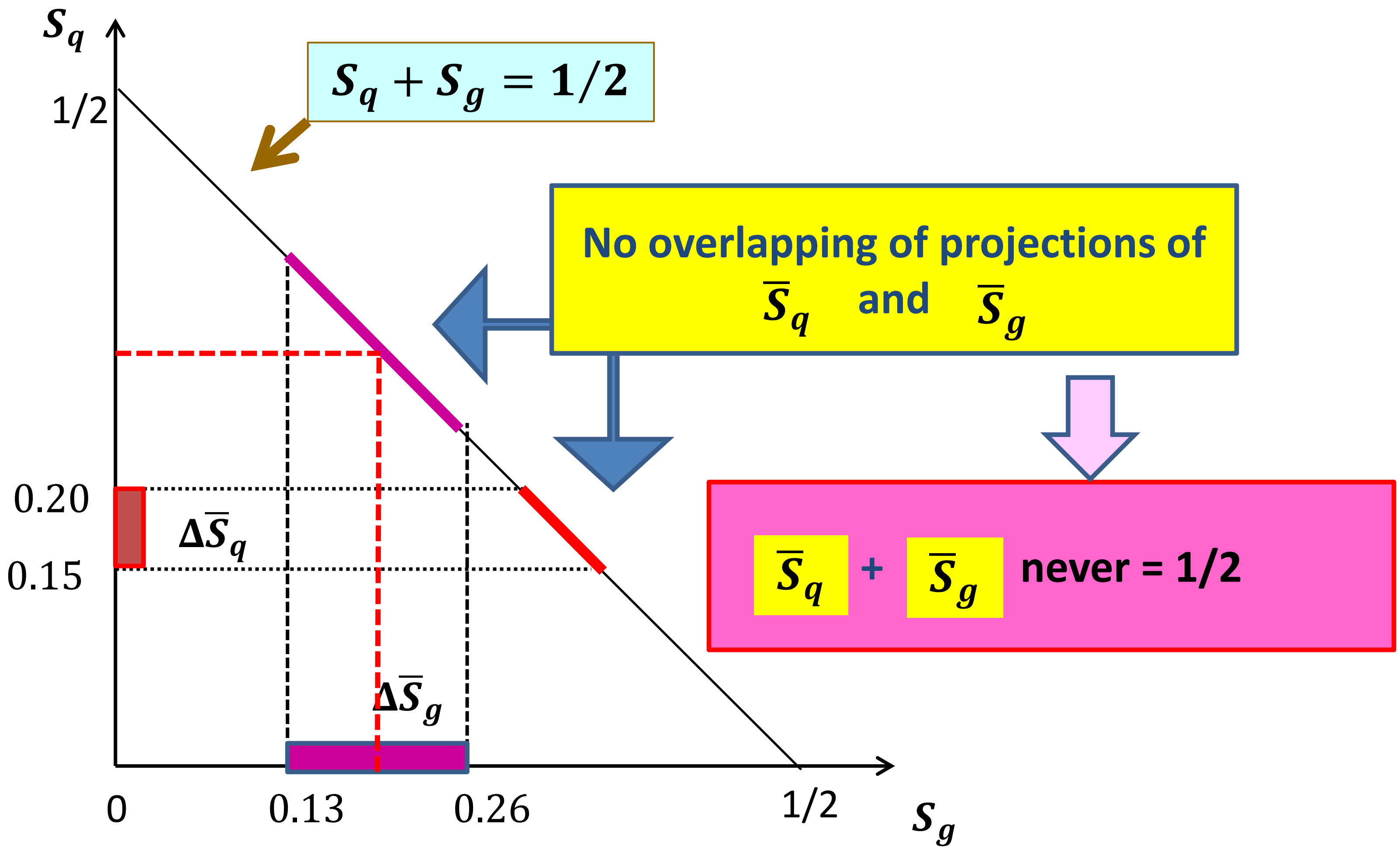
$$S_g = \bar{S}_g + S'_g = \bar{S}_g[1 + B_g/A_g] = \bar{S}_g[1 + 0.85]$$

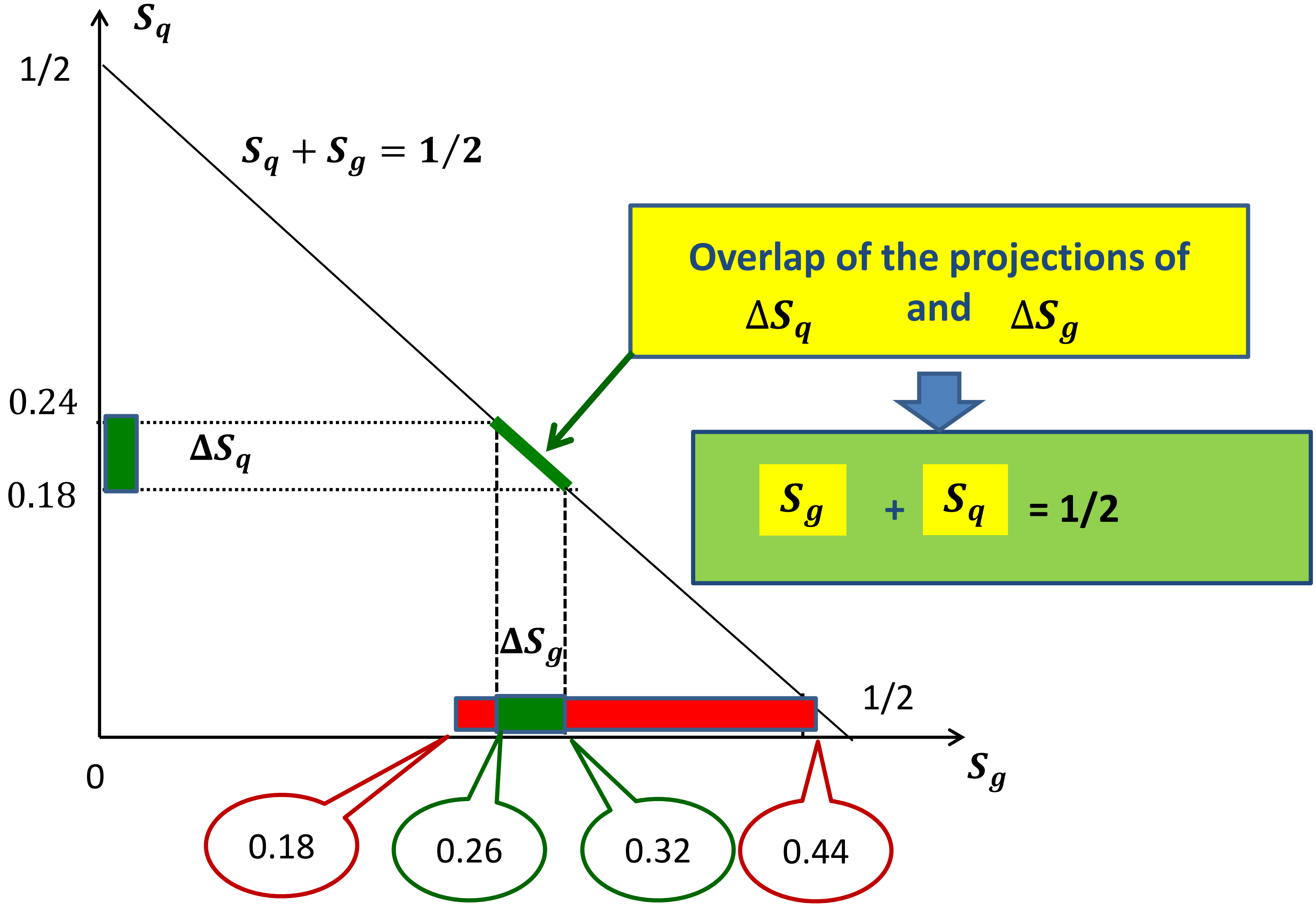
$0.18 \leq S_q \leq 0.24$

$0.24 \leq S_g \leq 0.72$

$0.42 \leq S_P \leq 0.72$

Illustration of the RHIC data:





RHIC: Q^2 = 10 GeV^2

Impact of Q^2 – dependence on the spin problem is very weak

CONCLUSIONS

Using DLA for calculation of the parton contributions S_q and S_g leads to perfect agreement with the value 1/2 of the proton spin.

In contrast to the preceding studies, we do not use asymptotics for the parton contributions because the asymptotics should not have been used outside their applicability region, otherwise it may lead to wrong conclusions

On the contrary, calculations in DLA make it possible to solve this problem because DL contributions are leading ones at small x

In order to simplify calculations, we start with accounting for the gluon contribution to the parton helicities and then implicitly add quark contributions through the intercept value. Non-perturbative contributions to the helicities cannot be calculated with QCD methods, so we fix them with using the RHIC data. As a result, the sum of the parton helicities in DLA proved to be in agreement with the value

Including into consideration Orbital Angular Momenta of quarks and gluons is not crucial for solving the Proton Spin Puzzle but we find it interesting and plan to do it in the future