Pion degrees of freedom in nuclear matter from 1971 till tomorrow

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Plan

- Historical aspects.
- Briefly about possibility of π condensate nuclei-stars.
- General description of pions in *dense* baryon matter.
- In-medium *pion* effects in HIC.
- Pion softening effects in neutrino radiation of NSs.

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- Other problems. What further?

Pion degree of freedom in hadron matter – *history from Migdal group, in brief*:

- **1971-1978 I period.**

1971 A.B. Migdal ZhETF **61** studied **pion vac. instability in strong fields, suggested possibility of p-wave pion condensation in dense nuclear matter and principal possibility of existence of superdense π condensate nuclei.**

1972-74 he suggested that m.b. pion condensate exists already in atomic nuclei and should exist in interiors of massive NSs (summed up in two papers in JETP1974,1976 published with Markin and Mishustin).

1975-1978 Saperstein, Troitsky, Fayans and others analyzed experiments in nuclei and did not find any manifestation pion condensate in atomic nuclei but it was supported the fact that pion mode in nuclei is already soft.

1976 Migdal, Popov, D.V., JETP, worked out relativistic semiclassical approach to Dirac eq. Problem came up from question of Gell-Mann to Migdal why he speaks on π^--cond. whereas e^- are much lighter? Including e+ π^- screening we suggested possibility of **nuclei-stars of any size (up to size of NS)**. (D.V., Chernoutsan,Sorokin, JETP Lett.).

1978 Migdal Rev.Mod.Phys., & book (in Rus.) \`Fermions and bosons in strong fields.\`

Important parallel research was done in west: Scalapino, Sawyer 1972, Baym 1973, Campbell, Dashen, Manassah 1975 (worked out pion cond. in chiral symmetry model).

Pion degree of freedom in hadron matter – *history in Moscow time, in brief***:**

1978-1983 II period.

1978-1983 Saperstein, Troitsky, Fayans, Tolokonnikov, Borzov et al. recalculated Landau-Migdal parameters with separation of soft pion mode in nuclei.

1978 Migdal, Chernoutsan, Mishustin suggested that pion condensate may appear by strong I-order phase transition in cold NSs with a blowing off a part of matter. D.Pines 1980 remarked that NS M_{max} should be $>1.4M_{sol}$, (**Now NSs $M \approx 2M_{sol}$ are found**→π condensation effect on EoS should be weaker than it was thought earlier)

1978 D.V, Mishustin, Pisma JETP, **began to study pion mode in nuclear matter at T≠0.**

1980 D.V, Anisimov JETP, suggested p-wave pion condensate superconductivity in NS with a layer structure in a magnetic field. Since p-wave nature of condensate v**ortices are not threads but slabs** (**$k \approx p_{Fn}$, H**-plane), condensate survives up to very strong magnetic field **$H_{c2} \sim p_{Fn}^2/e \sim 10^{19}$ G .**
In this work first estimate of magnetic field in HIC was done:

$h \sim H_\pi (Ze^6)^{1/3}$, $H_\pi = m_\pi^2 c^3/e\hbar \approx 3.5 \cdot 10^{18}$ ΓG. supported later by work Skokov, Illarionov, Toneev 2009.

D.V, Mishustin 1981-82, Dyugaev 1982-83 **studied pion fluctuation (liquid) phase at T≠0 above critical point of Migdal's solid-like π condensation.**

Migdal 1983: book `Theory of finite Fermi systems. .. (II ed. in Rus.)

Pion degree of freedom in hadron matter - *history in Moscow time, in brief:*

- **1983-1990 III period.**
- 1981 Nagamiya published review in PRC on results of HIC experiments at Bevalac. Only ideal gas models were used with free pions.

  1984 Schulz, D.V. **included in-medium effects in pion production in HIC.**
- 1984-87 Senatorov, D.V. **Pion in-medium effects in problem of cooling of NS, resulted later in `Nuclear medium cooling scenario.'**
- Senatorov, D.V.,Kämpfer 1987 suggested possibility of 2 neutrino bursts due to pi-condensate phase transition possibly occurring during a supernova explosion.
- Senatorov, D.V. 1988, D.V.1989: we applied nonequilibrium diagram technique to description of pion modes in HIC and NSs.
- 1990 Migdal, Saperstein**,** Troitsky, D.V. review appeared in 3-volumes of Phys.Rep. & 1991 book in Rus. `Pion degrees of freedom in nuclear matter.'

Idea of supercharged nuclei and nuclei-stars

Migdal ZhETF1971,1974, Migdal, Popov, D.V. JETP Lett.1976, DV, Sorokin, Chernoutsan **1977.**

Large size nucleus represents the potential well for negatively charged bosons. With increasing Z, boson ground energy level decreases to 0 for $|V| \to m_b$ and reaction n→ p+b^- occurs. Energy is

$$\mathcal{E} = -16\mathrm{MeV}\cdot A - \int d^3r n_p V - \cancel{\int d^3r \frac{(\nabla V)^2}{8\pi e^2}} + \cancel{\int d^3r\,(|\nabla\phi|^2} - (V^2 - m_b^2)|\phi|^2)\,.$$

Coulomb charged boson

$$\cancel{\Delta}\phi + (V^2 - m_b^2)\phi = 0 \text{ yields } V = -m_b \text{ for } r < R.$$

$$\cancel{\Delta V} = 4\pi e^2(n_p + 2V|\phi|^2)$$

$$\mathcal{E} = -16\mathrm{MeV}\cdot A + m_b A/2 \text{ for } Z = A/2 \text{ and } \mathcal{E} < 0 \text{ for } m_b < 32 \text{ MeV}.$$

if there were charged light bosons with mass m_b <32 MeV there would exist nuclei-stars which interior is such as in atomic nuclei, $n \approx n_0$, n_0 is density of atomic nucleus, A≈2Z.

but pion has mass 140 MeV

Rotating supercharged nuclei & nuclearites, cf. D.V. PRD2023

Rotation acts as 0-component of vector electric potential well (!)

V=- $V_0 \simeq m_\pi^* - \Omega\nu$ winding number $\nu = 0, \pm 1, \ldots$ $\phi = \phi_0 \chi(r) e^{i\xi(\theta) - i\mu t + i p_z z}$, $\xi = \nu\theta$,

Ground state has large winding number. $\mathcal{E} - \mathcal{E}_{in} \simeq (m_\pi^* - \Omega\nu - 32\mathrm{MeV})Z$

Rapid rotation may lead to long-living π^- condensate nuclearites, since *for large A surface radiation is weak*.

Ideas of pion condensation in dense matter and its influence on EoS

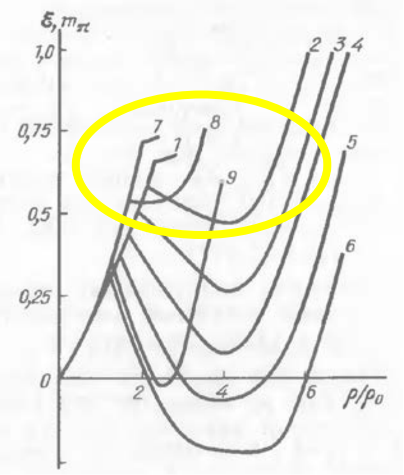
In nuclear matter square of effective pion mass/gap (at k~p_F) decreases with increasing density and becomes negative for *$n > n_c > n_0$, and Migdal's pion condensate appears*

$$\delta E = [m_\pi^2 + k_m^2 + \Pi(\omega_c, k_m)]|\phi|^2 + \frac{\lambda(\omega_c, k_m)|\phi|^4}{2} = -\frac{\omega^{*4}}{4\lambda} \sim -m_\pi^4 \frac{(n-n_c)^2}{n_c^2}$$

Gain owing π-cond.

loss- NN repulsive correlations

In dependence on uncertainties in knowledge of NN interaction (LM parameter g') at large densities energy per baryon has one of forms:



Migdal,Saperstein,Troitsky,D.V. Phys.Rep.192 (1990).

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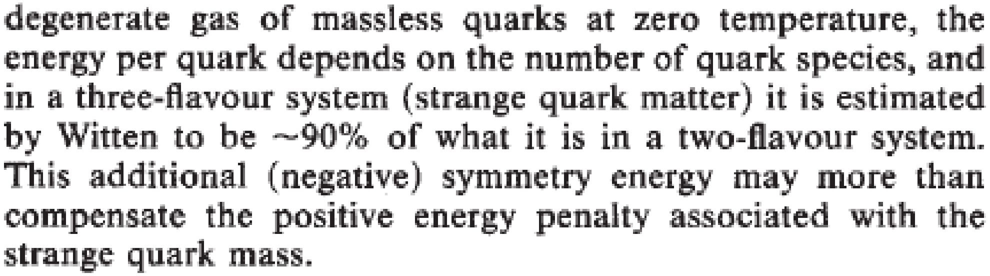
Calculations are done with FP EoS (K=220 MeV, curves 1-6) and Walecka EoS (K=540 MeV, 7-9) , with varying LM parameter g' =0.65,0.6,…0.4, showing effects of weak II-order pion cond. transition (curves 1,7) , and I-order transition (2-4), and with stable superdense matter (5,6,9).

Modern observations measured NSs with mass >2 M_{sol} → stiff EoS, stiffer than FP EoS. Realistic values g' (n_0)=0.6-0.7, so possibility of superdense pion condensate nuclei requiring g' (~$3n_0$)=0.4-0.45 looks now rather exotic and needs decrease of g' (n) with n.

Quark nuggets, quark stars, hybrid stars, etc.

Ideas do not die but reincarnate. **New field: Quark strange nuggets, strange stars**

1984 Witten, then Farhi, Jaffe,De Rujula, Glashow →



degenerate gas of massless quarks at zero temperature, the energy per quark depends on the number of quark species, and in a three-flavour system (strange quark matter) it is estimated by Witten to be ~90% of what it is in a two-flavour system. This additional (negative) symmetry energy may more than compensate the positive energy penalty associated with the strange quark mass.

Alcock, C., Fahri, E., and Olinto, A., **1986**, charge distrib. ыuch as in pion cond. case in Migdal, Popov, D.V. **1976,** D.V, Sorokin,Chernoutsan **1977**.

See review on strange quark matter by Clemente et al, arXiv 2404.12094

1988 Bethe, Brown, Cooperstein, hybrid stars (hadron shell, quark core)

1984 Glashow charged **DM CHAMPs** (charged matter particles)

2005 Glashow, U tera-quarks and O-{UUU} (e=-2) baryons, 2006 Khlopov, m_O ~1TeV, OHe DM

2019 Gani, Khlopov, D.V., DM O-nuclearites, **no kin. energy, but charge screening**

electroneutrality

$$\mathcal{E} = \boxed{-16\ \mathrm{MeV}\cdot A} - \int d^3r(\cancel{n_p - 2n_O})V \qquad n_O = n_p/2$$

Triple role of mesons in matter:

collective excitations, mediators of *baryon-baryon* interaction and condensates of classical fields

In spite of this up to now in many models pions in matter are considered as obeying **vacuum dispersion law** that leads to **various inconsistencies**

(minimal cooling paradigm for description of cooling of NS, ideal pion gas in HIC)

Pion-nucleon- Δ-isobar interaction

N (934 MeV), vertex of πNN p-wave attraction is

$V^{n.rel}_{\pi NN} = -if_{\pi NN}\sigma_\alpha\tau_\beta\partial_\alpha\pi_\beta, \quad \alpha, \beta = 1, 2, 3,$ $\boxed{f_{\pi NN} \simeq 1/m_\pi}$ **degeneracy factor 4**

Δ (1232): $m_\Delta - m_N \approx 2m_\pi$ $\boxed{f_{\pi N\Delta} \simeq 2.15\, f_{\pi NN},}$ **Larger coupling, degeneracy factor 16**

Strong *p*-wave πNN, πNΔ attraction resulting in possibility of p-wave pion condensation for *n*>(1.5-3) n_0

S-wave πN interaction is weaker at least at $n \sim n_0$:. Model dependence. In isospin sym. matter with Manohar-Georgi Lagrangian s-wave pion cond. For ~2 n_0, with sigma model and Gasser-Sainio-Svarc models no pion condensation up to high densities. In neutron matter no s-wave cond., cf. D.V. PRD 2022.

Approximations:

Relativistic π are incorporated explicitly with the help of the full spectral function

$$A_\pi = -2\Im D_\pi^R .$$

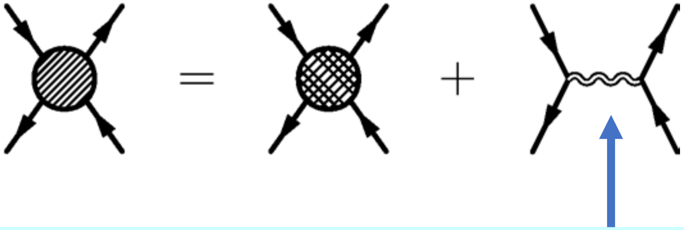
$$A_\pi(t,\vec{r},\omega,\vec{k}) = \frac{\Gamma_\pi(t,\vec{r},\omega,\vec{k})}{[\omega^2 - k^2 - m_\pi^2 - \Re\Sigma^R(t,\vec{r},\omega,\vec{k})]^2 + \Gamma_\pi^2(t,\vec{r},\omega,\vec{k})/4},$$

Heavy *N* (7 times heavier than π) are good quasiparticles, except vicinity of
π condensation transition and except for large *T* (*T*>100 MeV),

Δ (1232) isobars have width ~100 MeV in vacuum which is included by shift of energy.

Fermi liquid approach: separation of soft and hard modes

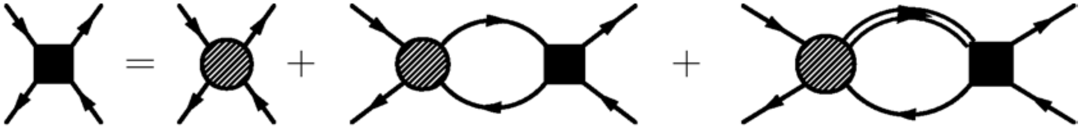
- explicit pionic degrees of freedom



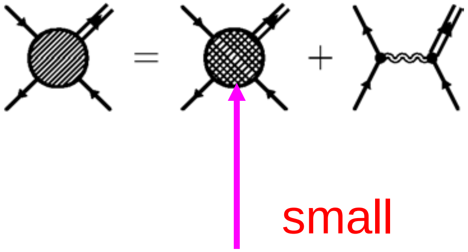
pion with residual (irreducible in NN^{-1} and ΔN^{-1}) s-wave π N interaction and $\pi\pi$ scattering``

- explicit Δ degrees of freedom

explicit nucleon-nucleon hole and Delta-nucleon hole degrees of freedom



Part of the interaction involving Δ isobar is analogously constructed:


small

- Reduction of the more local interaction to the point-like interaction

$$= C_0\,(f_{12} + g_{12}\boldsymbol{\sigma_1}\boldsymbol{\sigma_2})\,,$$

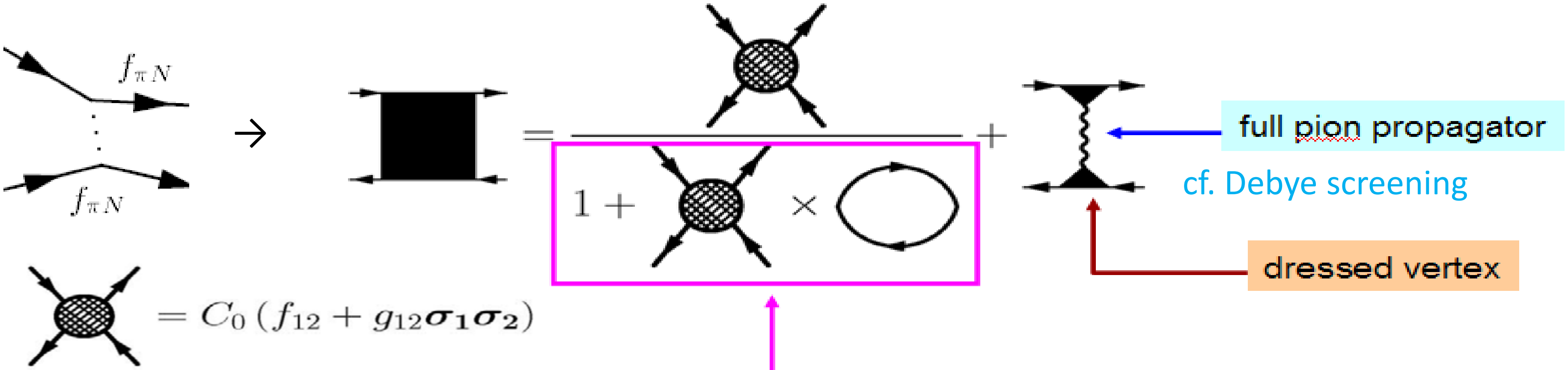
Assumption that the Landau-Migdal parameters, f_{12}, g_{12}, are constants Is a rough approximation.

Low energy excitations in nuclear Fermi liquid (Migdal approach)

No free pions in matter! **Strong p-wave pion-nucleon interaction!**

Resummed NN interaction

based on a separation of long and short scales



full pion propagator

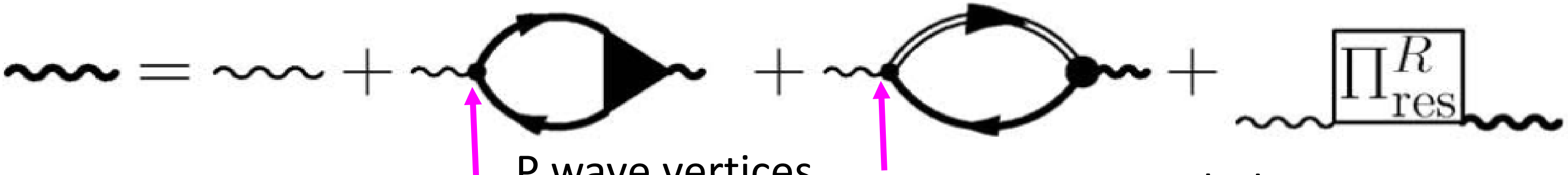
cf. Debye screening

dressed vertex

$$= C_0\,(f_{12} + g_{12}\boldsymbol{\sigma_1}\boldsymbol{\sigma_2})$$

Info. on short-range interact. is extracted from atomic nuclei exp. and model calcul.

Poles yield zero-sound modes in scalar and spin channels



P wave vertices

includes s-wave inter. & pion fl.

$$V^{n.rel}_{\pi NN} = -if_{\pi NN}\sigma_\alpha\tau_\beta\partial_\alpha\pi_\beta, \quad \alpha,\beta = 1, 2, 3,$$

Migdal Rev.Mod.Phys. 50, 1978, Migdal,Saperstein,Troitsky,D.V. Phys.Rep.192 (1990).

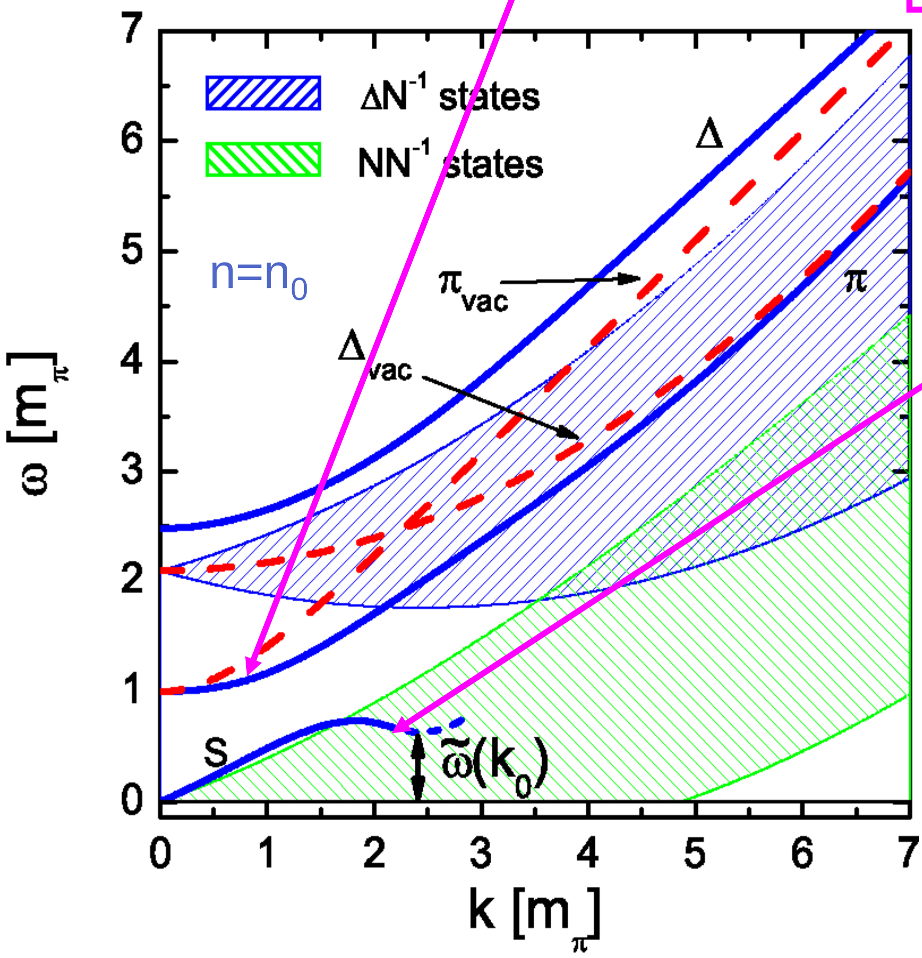
Pion spectrum in iso-symmetric nuclear matter

in simplified quasiparticle-based picture

$$-2\,\mathrm{Im}\,D_\pi^{\mathrm{ret}}(\omega,k) = A_\pi(\omega,\boldsymbol{k}) \approx \sum_{i=\pi,\Delta,S} \frac{2\pi\,\delta(\omega-\omega_i(\boldsymbol{k}))}{\left(2\omega-\frac{\partial\Pi^R}{\partial\omega}\right)\Big|_{\omega=\omega_i(\boldsymbol{k})}} + \frac{2\beta k\omega}{\widetilde{\omega}^4(k)+\beta^2k^2\omega^2}\,\theta(\omega<v_F k)$$

Attraction at π branch is in accord. with π atom data

$\sqrt{m_\pi^2+\alpha_0\vec{k}^2}$, $\alpha_0\simeq 0.4$ for $n\simeq n_0$ Instead of $\boxed{\omega_k=\sqrt{m_\pi^2+\vec{k}^2}}$



$\widetilde{\omega}^2(k) = -D^{-1}(0,k)$

Landau damping

$\beta = m_N^{*2}\,k\,f_{\pi NN}^2/\pi$

$\widetilde{\omega}(k_0)$ <m_π *for n> n $_{c1}$~0.5-0.8 n$_0$*

at k≠ 0 (p-wave) strong **pion fluctuations** for T ≠0, n>n$_{c1}$ DV NPA 1993

$\Pi^{\mathrm{F}}(k,\omega,T)=$

***liquid phase** of quantum pion condensate*

for $\widetilde{\omega}^2(k_{\min}\simeq p_{\mathrm{F}})<0$ (for n>n $_c$~ (1.5-3) n$_0$)

instability $\varphi\sim\exp(-i\omega t)$ $\omega\propto -i\widetilde{\omega}^2(k_{\min})/\beta$

→ P-wave crystal-like phase of pion cond. Always 1-order phase tr.

Pion in matter is very complex collective excitation

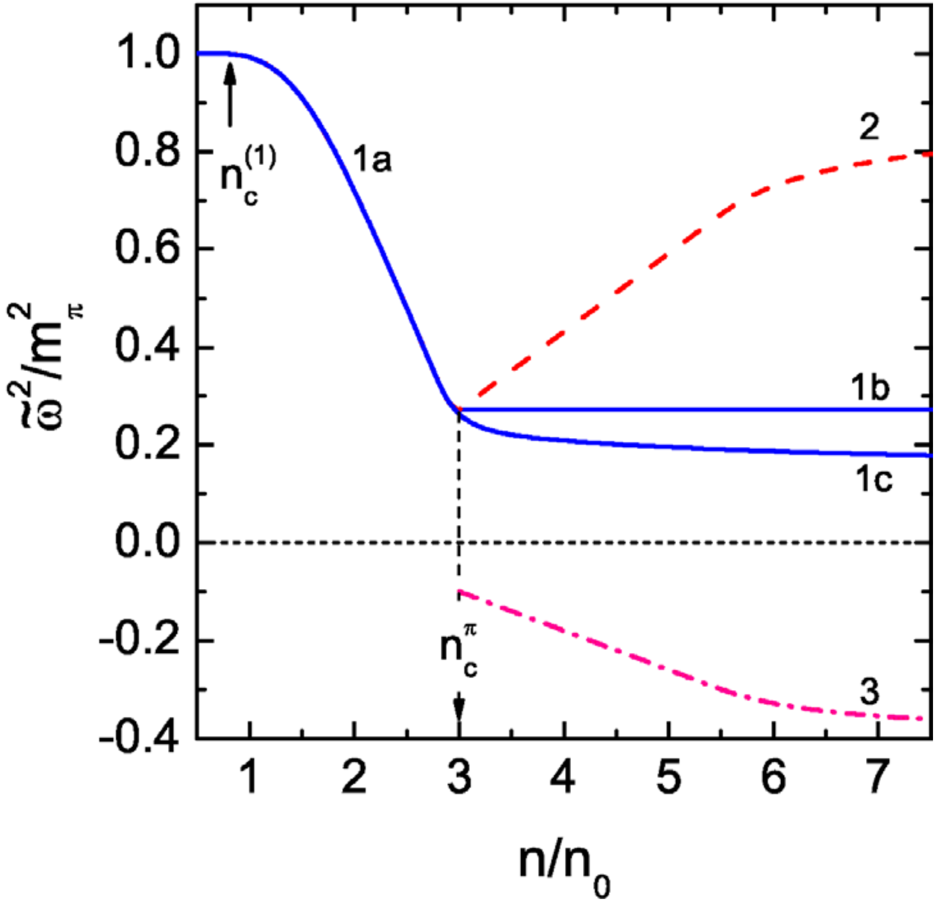
- pion softenning

Neutral pion propagator for **$n_c^{\pi} > n > n_{c1} \sim 0.5\text{-}0.8\, n_0$** $\omega \ll k_0 v_F$ $k \sim k_0 \simeq p_F$

$$D_{\pi}^{R}(\omega, k) \simeq \frac{1}{-\tilde{\omega}^2 - \gamma\,(k-k_0)^2 + i\beta(k)\,\omega}$$

Effective pion gap squared

$n_c^{\pi} > n > n_{c1}$ liquid phase of pion cond. in HIC, cf. D.V. 1989, Nucl. Phys.A555 (1993)



reconstruction of pion spectrum on top of the pion condensate

If LM parameters increase with density -- saturation of pion softening and **no pion condensate**

amplitude of the pion condensate (liquid-crystal or m.b. solid) , e.g.

$$\varphi(\boldsymbol{r}) = (1/\sqrt{2}) a\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \quad \text{(traveling wave)},$$

$$\varphi(\boldsymbol{r}) = a \sin \boldsymbol{k}\cdot\boldsymbol{r} \quad \text{(standing wave)}.$$

A picture for HIC at Bevalac, GSI, AGS energies: Here $N_\pi<N_N$

- After a short strongly nonequilibrium q-g and/or hadron stage, formation of hadron fireball + peripheral region.
- Local quasi-equilibrium T(t,**r**), n(t,**r**) stage of fireball expansion till breakup.
- Long-path-length particles probe dense hot interior: *dilepton and photon are radiated in direct reactions*, *similar to* neutrino radiation from NS (except first minutes-hours after NS formation)
- Pions with $k<m_\pi$ and $K^{+,0}$ (s=1), having larger path lengths, fly away from intermediate fireball expansion stage since $\lambda_{K^+} \gg \lambda_{K^-}$ $\lambda_{K^-} \simeq 3\ \mathrm{fm}$ for $\rho = 0.5\rho_0$.
- Nucleons, pions with $k>1.5\ m_\pi$ and K^- (s=-1) having shorter path lengths probe breakup stage, $n\sim0.6\ n_0$,

Migdal,Saperstein,Troitsky,D.V. Phys.Rep.192 (1990), D.V. NPA555 (1993)

Picture for HIC at SPS, RHIC, LHC energies is different:

Here $N_\pi >> N_N$.→ baryon-less matter, hadron blurring, cf. D.V. NPA2004,2008.

For low energy pions ($k<<m_\pi$) there is stage of elastic collisions, Bose enhancement effects and m.b. BEC, cf. D.V. JETP 1994, Kolomeitsev.D.V. EPJA2018.

Nucleons are blurred die to multiple collisions with pions with momenta $k\sim T\sim m_\pi$

Manifestation of in-medium effects in exp. pion distributions in HIC

``Free`` pion distribution in nonequilibrium matter

$$n_\pi(t,\mathbf{r},\mathbf{k}) = 2\sqrt{m_\pi^2+k^2}\langle\Phi(t)|\pi^+\pi|\Phi(t)\rangle = 2i\sqrt{m_\pi^2+k^2}\,D_\pi^{-+}(t,\mathbf{r},t,\mathbf{k})\,.$$

In thermal equilibrium matter

$$iD_\pi^{-+}(\omega,k) = -\frac{2\,\mathrm{Im}\,D_\pi^{\mathrm{ret}}(\omega,k)}{\exp\left(\frac{\omega}{T}\right)-1}\,.$$

Senatorov, D.V. PhLett B219 (1989)

For a sudden breakup of system

$$n_k^\pi = \frac{d^3N_\pi}{\frac{d^3k}{(2\pi)^3}V(t_0)} = -2\sqrt{m_\pi^2+k^2}\int\frac{2\,\mathrm{Im}\,D_\pi^{\mathrm{ret}}(\omega,k,\rho(t_0),T(t_0))}{\exp\left(\frac{\omega}{T(t_0)}\right)-1}\frac{d\omega}{2\pi}\,,$$

in simplified quasiparticle picture

$$n_k^\pi = \sum_j \Gamma_j\frac{1}{\exp\left(\frac{\omega_j(k)}{T(t_0)}\right)-1} + \delta n$$

in-medium branches
+ dressing factors

$$\Gamma_i = \frac{2\sqrt{m_\pi^2+k^2}}{\left(2\omega - \frac{\partial\,\mathrm{Re}\,\Pi^{\mathrm{ret}}}{\partial\omega}\right)\Big|_{\omega=\omega_i(k)}}\,,\quad i=\pi,\ \Delta$$

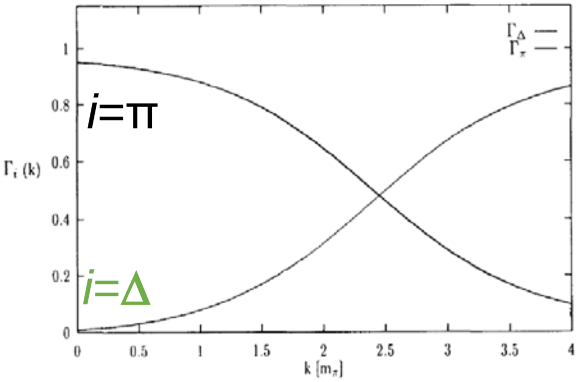


Fig. 2. Admixture factors Γ_π, Γ_Δ.

Virtual soft pions contribute for n>> $n_{c1}=(0.5\text{-}0.8)n_0$

$$\delta n_k^\pi \approx k_0\sqrt{m_\pi^2+k_0^2}\,T^2\beta/[2(\sqrt{\gamma\tilde\omega^3(k_0)})] \qquad \beta T \ll \tilde\omega^2(k_0)$$

Criterion of sudden breakup in reality:

$$\tau_{\mathrm{breakup}} < |\omega_\pi(k) - \sqrt{m_\pi^2+k^2}|^{-1} \qquad \tau_{\mathrm{breakup}} < |\omega_\Delta(k) - \sqrt{m_\pi^2+k^2}|^{-1}$$

at $n_b \approx n_{c1} \approx 0.6\,n_0$ and T_b breakup is always sudden for ω_i (k, n_b, T_b.) close to ω_{free} (k) but most of soft pions are absorbed during breakup stage, they contribute to EoS at n>> n_{c1} but not to pion yield at $n_b \approx n_{c1} \approx 0.6\,n_0$

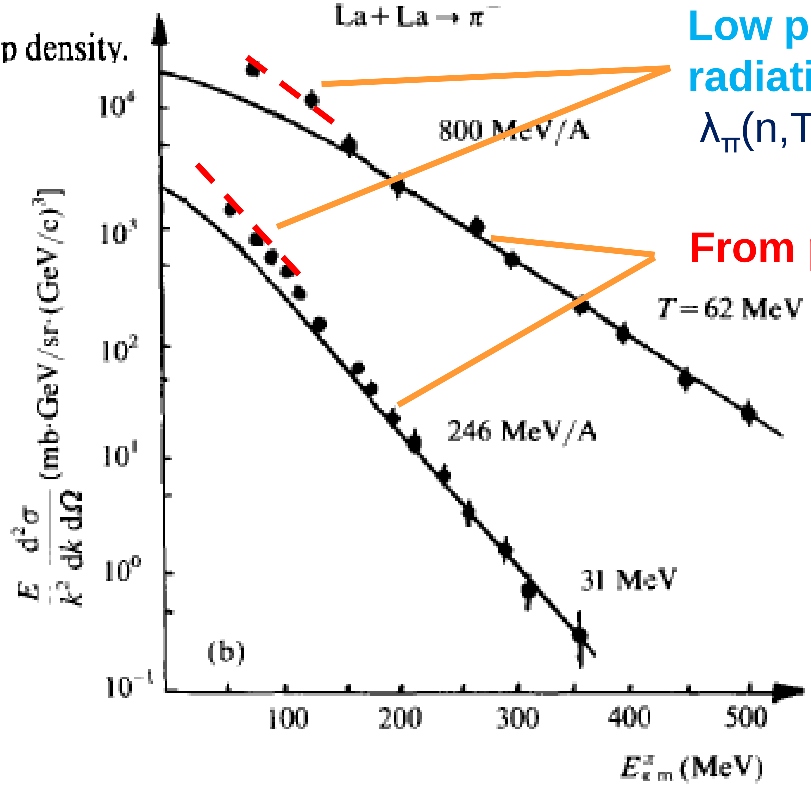
Pion production in La+La collisions

$$E\frac{d\sigma}{dk}=\sum_i \frac{A^{5/3}\omega_k\Gamma_i}{\rho_b[\exp(\omega_i/T_b)-1]} \qquad \Gamma_i=\frac{2\omega_k}{2\omega_i(k)-\partial\,\mathrm{Re}\,\Pi^A/\partial\omega}\bigg|_{\omega=\omega_i(k)}.$$

$$\times 15.64\,\mathrm{mb/GeV^2}\,c^3\,\mathrm{st}\,,$$

where ρ_b is the assumed break up density.

from D.V. Nucl. Phys. A555 (1993)



Low p-enhancement is described by radiation of pions in direct reactions

$\lambda_\pi(n,T)>R(n)$ at $n>n_b$

From prompt freeze out

With in-medium effects included, pions with k>(150-200) MeV have short mean-free path and radiate from thermal freeze-out, $n\approx 0.6\ n_0$

EoS for HIC with inclusion of in-medium pions

DV Phys.At.Nucl 1989, NPA 1993

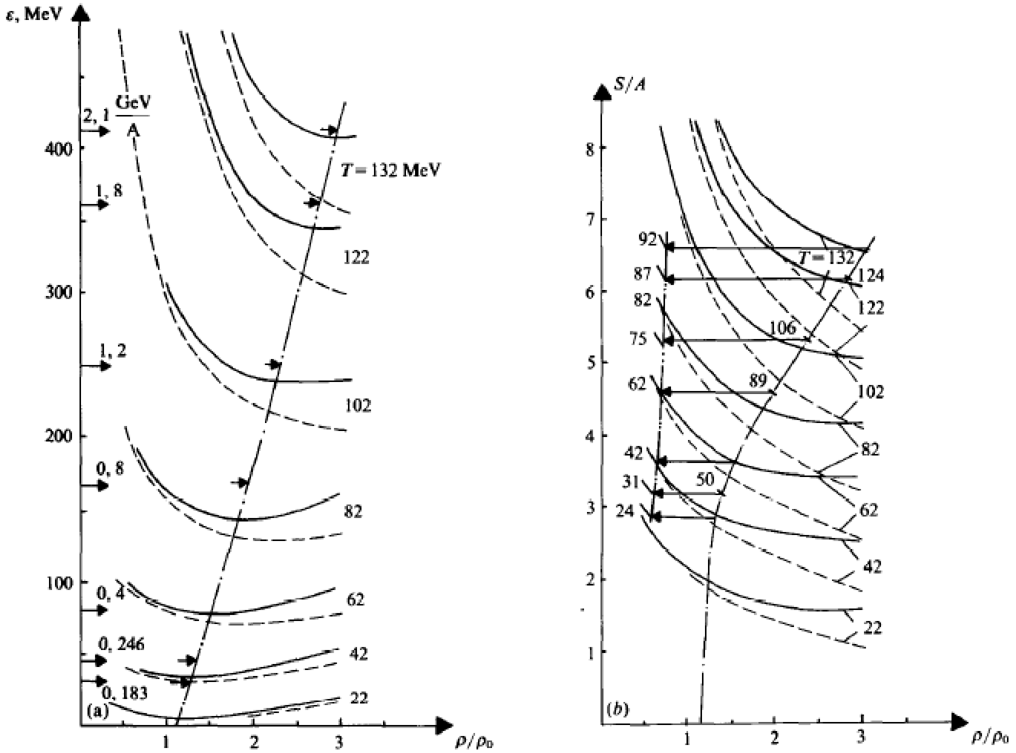
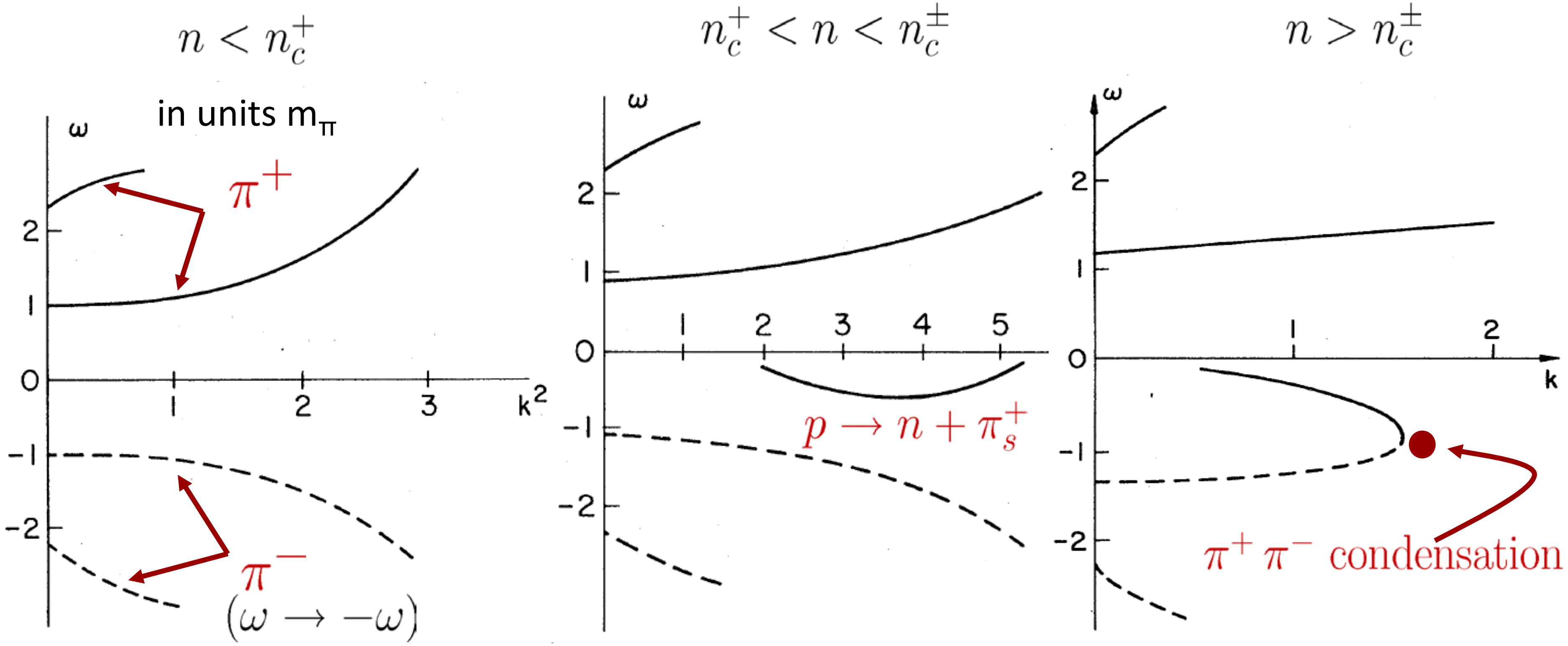


Fig. 3. Energy and entropy per nucleon as functions of the density at different temperatures. Solid curves: nucleons are calculated in the MW model, pions, with account of nuclear-matter polarization. Dashed curves: without taking into account the π-condensate liquid phase. The dash-dotted line is the initial configuration, determined by minimizing $\mathscr{E}(\rho)$. The dash-double-dotted line (b) is $\rho_b(T)$. Horizontal arrows: different collisional energies. It is assumed that $\mathscr{E}_{\rm bind}(\rho_0, T=0) = -16$ MeV.

Solid curves with soft virtual p-wave pions (**not included in modern HIC codes**), dashed –quasiparticle branches

Charged pion spectra in neutron matter,& π+- condensation in NS



[Migdal,Markin,Mishustin, JETP (1974)]

$$n_c^+ \lesssim n_0\,, \quad n_c^\pm \sim (1-3)\,n_0\,, \quad n_c^{\pi^0} \sim (1-3)\,n_0$$

In variational EoS [Akmal, Pandharipande, Ravenhall, PRC58 (1998)]: Takatsuka,Tamagaki 1997

π± condensate: $n_c \simeq 2\,n_0 \quad N = Z$ $\quad$ $n_c \approx (1.5\text{-}2)n_0$ N>> Z

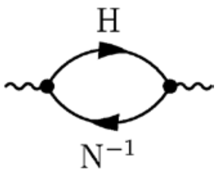
neutral pion condensate: $n_c \simeq 2\,n_0 \quad N = Z\,, \qquad n_c = 1.3\,n_0 \quad N >> Z$

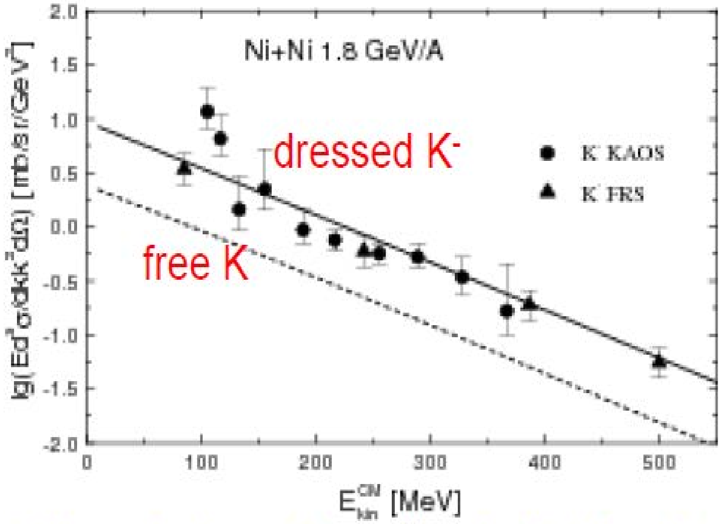
Pion in matter is not free particle! Looks trivial but many authors still work with free pions.

Strangeness kaon and hyperon-nucleon particle-hole modes

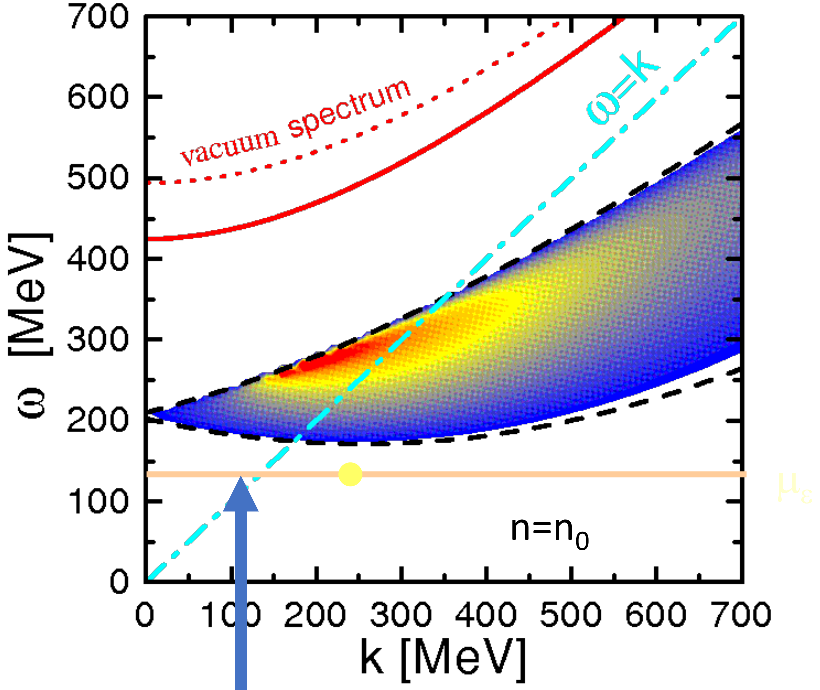
Attractive s/p-wave potential manifested in K^- atoms via

regular part of Π (mainly s-wave)+ p-wave Lambda proton modes





K^- have short mean free path and radiate from freeze out



Hyperonization of NS matter $p + e^- \longrightarrow \Lambda$ for n>(2-4) n_0

and m.b. **antikaon s or p-wave condensation** in dense NS matter, $e \rightarrow K^-$ for $\omega_K<\mu_e$

Kolomeitsev,D.V.,Kampfer, HIC-NPA1995,IntJModPh 1996, NS-PRC2003

Cooling of neutron stars

After passing a minute-hour after formation, during 10^5 years a neutron star cools down by neutrino emission, then by photon black body emission from the surface

$$\lambda_\nu \gg R \simeq 10\mathrm{km}$$

White-body radiation problem (at low $T<T_{\mathrm{opac}}$~1-few MeV) -- direct reactions
D.V., Senatorov, JETP 1986, Yad. Fiz. 1987

Neutrinos bring information straight from the dense interior

similar to di-leptons from HIC

Cooling: crust is light, interior is massive -most important are reactions in dense interior

Main reactions in minimal cooling scenario (MCS)-most popular till now

1965 Tsuruta, Cameron and Bahcall, Wolf-standard scenario (SC), D. Page **2004** MCS, Similar scenario by Yakovlev et al.

Phase-space separation

one-nucleon reactions: $n \to p + e + \bar{\nu}$ direct Urca (DU) needs n_p/n >11 % , $2p_{Fp} > p_{Fn}$

Some EoSs permit such n_p/n , other do not.

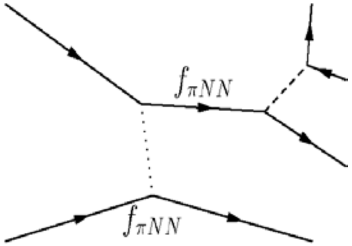
m.b. not allowed in NSs, except most massive ones, yielding too rapid cooling

two-nucleon reactions: $n + n \to n + p + e + \bar{\nu}$ modified Urca (MU)

Main process in SC and MCS, computed with FOPE model of *NN* interaction

Pion is considered as free in SC and MCS

$$\omega^2 \simeq m_\pi^2 + k^2$$



$$\epsilon_\nu^{\rm MU} = \frac{11513}{60480\,\pi} G^2\, g_A^2\, f_{\pi NN}^4\, m_n^3\, m_p\, p_{{\rm F},e}\, T^8\, 1.3 \simeq 8 \cdot 10^{21} (n_p/n_0)^{1/3}\, T_9^8 \times \frac{\rm erg}{{\rm cm}^3 \cdot {\rm s}}$$

(~10^6 smaller emissivity than for DU, if latter was allowed, very weak density dependence)

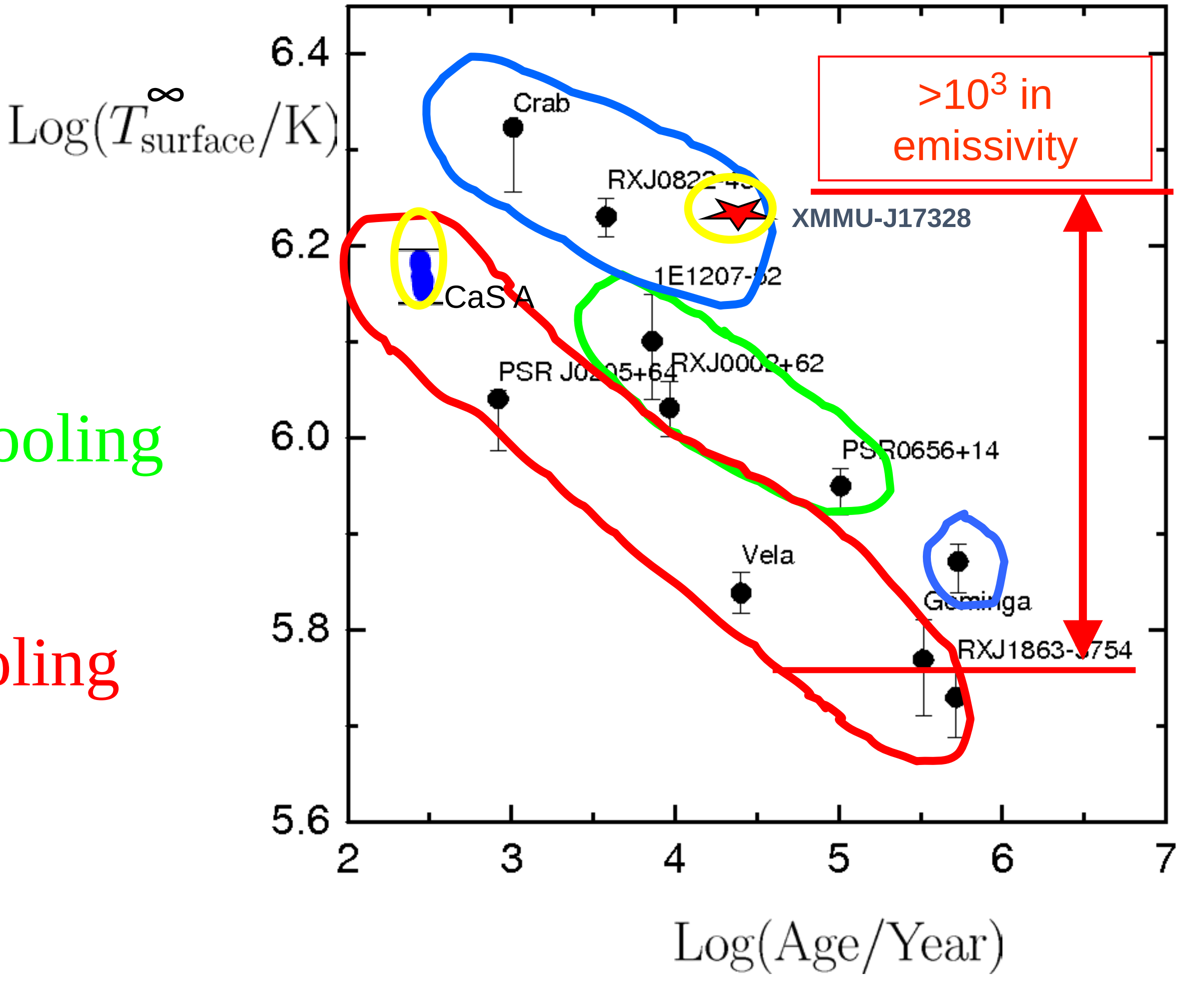
Neutron star cooling

3 groups+Cas A:

slow cooling

intermediate cooling

rapid cooling



Difficult to describe all data in MCS.

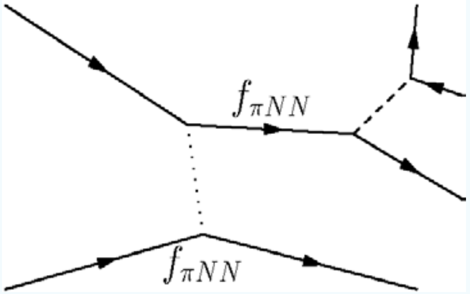
With pion softening effect included data are described within one cooling scenario.

Inconsistencies of FOPE model

The only diagram in FOPE model which contributes to the MU is

Born approximation

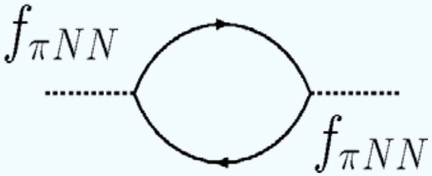
FOPE →



$f_{\pi NN} \approx 1/m_\pi$

For consistency necessary to include corrections of the second-order in $f_{\pi NN}$ in all quantities !!! Otherwise -- problems with unitarity *(summed probability does not yield 1).*

at order $f_{\pi NN}^2$: Π_0 =



no fitting parameters!

Re Π_0 is so attractive that *p*-wave pion condensate in N=Z matter would occur already at $n>0.3\,n_0$ Exp. data show that there is no pion condensation in atomic nuclei but to describe levels in pion atoms one needs $\omega(k) \simeq \sqrt{m_\pi^2 + \boxed{\alpha_0}\vec{k}^2}$

$\alpha_0 \simeq 0.4$ *for ω near m_π at $n = n_0$* $k \lesssim m_\pi,$ rather than $\omega^2 \simeq m_\pi^2 + k^2$

Migdal,Saperstein,Troitsky,D.V. Phys.Rep.192 (1990), Friedman, Gal *Acta Phys.Polon.B* 51 (2020)

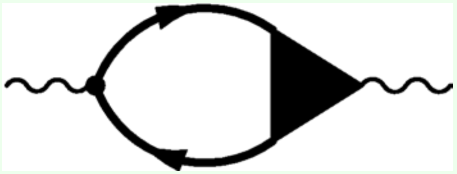
Solution of the puzzle

One should replace FOPE by the full *NN* interaction,

essential part of which is due to MOPE

with vertices corrected by *NN* correlations.

NN^{-1} part of the pion self-energy



$$\approx \Pi_0^R(\omega, k, n)\,\gamma(g', \omega, k, n)$$

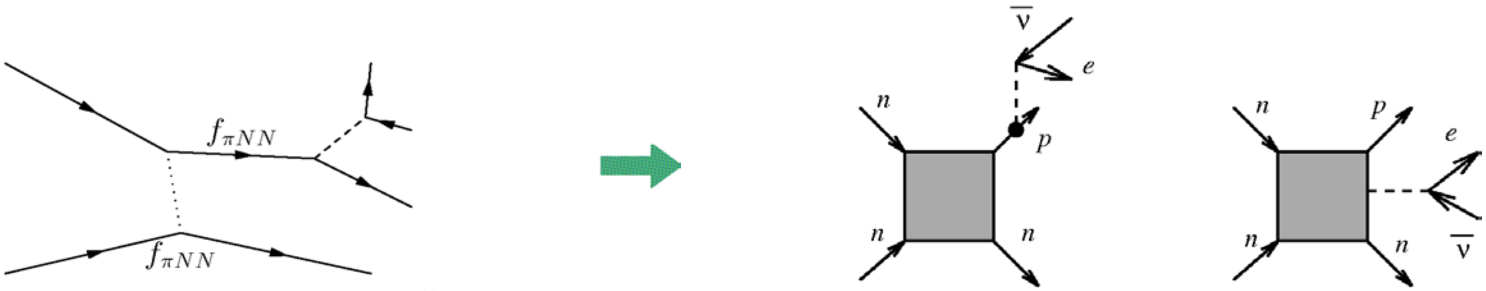
suppressed by the factor $\gamma(g', \omega = 0, k \simeq p_F, n \simeq n_0) \simeq 0.35 \div 0.45.$

→ In isospin-symmetric matter no pion condensation for $n<n_0$

but pion softening with increase of *n* and m.b. p-wave pion condensation for $n>n_c>n_0$

Medium effects in two-nucleon processes

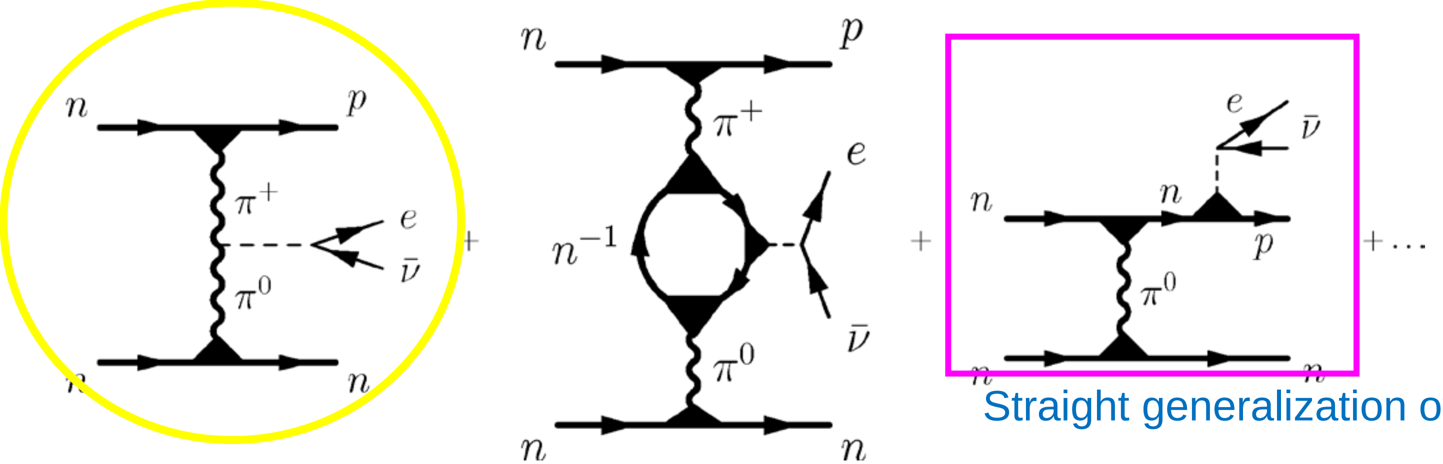
MU(FOPE) :



Key proc. in “Minimal cooling scenario” Page,Yakovlev et al.

D.V., Senatorov JETP (1986), A.B.Migdal,E.Saperstein,M.Troitsky,D.V. Phys.Rep.190 (1990)

MMU



Straight generalization of MU

emissivity: larger → smaller

Very important !

$$\frac{\epsilon_\nu[\mathrm{MMU}]}{\epsilon_\nu[\mathrm{MU}]} \sim 3\left(\frac{n}{n_0}\right)^{10/3} \frac{[\Gamma(n)/\Gamma(n_0)]^6}{(\widetilde{\omega}/m_\pi)^8}$$

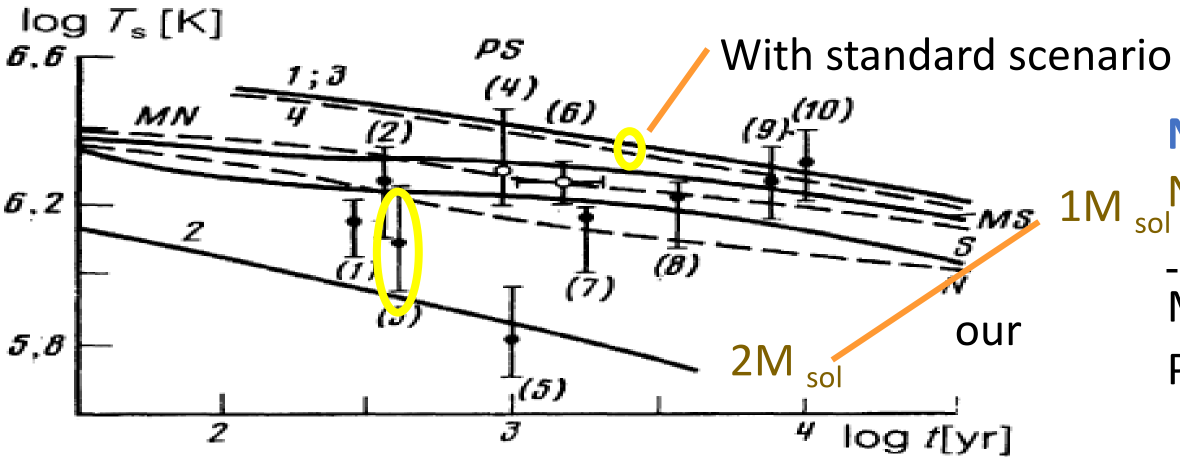
Very strong density dependence

enhancement factor ~10^3 -- 10^5 for n~(1.5-4) n_0

MMU-key proc. in “Nuclear medium cooling scenario” Blaschke, Grigorian, D.V. 2004

D.V., Senatorov JETP Lett. 1984, JETP 63,1986 all the data (**only upper limits on T_s!**) were explained by medium-modified Urca (MMU) process **assuming different masses of NS** (different mean densities)

whereas following Brown, Bethe till 2000-th most of authors thought that all NSs have masses M≈1.4 M_{sol} ,

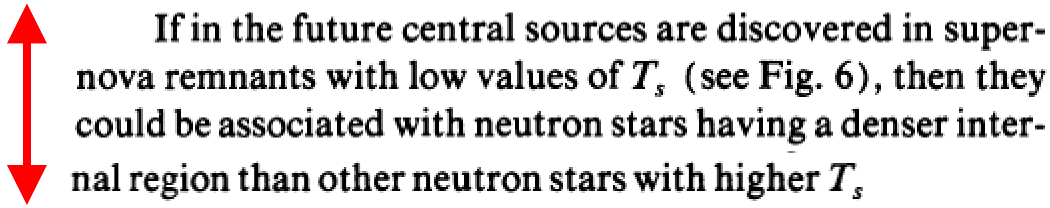


Now it is exp. known that
NS masses are different
Pulsar J0348-04232
M = 2.01(4) M_{sol}
PSRJ0453+1559: M=1.174(4)M_{sol}

Max. M>(2-2.1) M_{sol}

FIG. 6. Comparison of theoretical calculations with data of the Einstein observatory. The abscissa is the logarithm of the time, the ordinate the logarithm of the surface temperature. The equation of state is that of Pandharipande and Smith ($M \approx 1.3\, M_\odot$, $R \approx 8$ km, $R^\infty = 11$ km). The experimental data (see Ref. 7): 1) Cas A, 2) Kepler, 3) Tycho, 4) Crab,[5] 5) SN1006, 6) RCW103, 7) RCW86, 8) W28, 9) G350, 0-18, 10) G22, 7-02. The open circles represent observed sources, the black circles are upper limits. The curves are as follows: *S* for superfluid neutron stars, *N* for normal stars, *MS* for magnetic superfluid stars, and *MN* for magnetic normal stars.[7] Curves 1 and 2 represent our calculations with mean density $\bar{\rho} = \rho_0$, $\tilde{\omega}_\pi = m_\pi$, $\gamma = 0.47$ and $\bar{\rho} = 2\rho_0$, $\tilde{\omega}_\pi = 0.5 m_\pi$, $\gamma = 0.47$, respectively. Curves 3 and 4 are our calculations for the same values of $\bar{\rho}$ but with luminosity from Ref. 27.

It was thought that no source in Cas A , now 20 years of measurements of T_s in Cas A,+ ~20 other data points.


If in the future central sources are discovered in supernova remnants with low values of T_s (see Fig. 6), then they could be associated with neutron stars having a denser internal region than other neutron stars with higher T_s

Nuclear medium cooling scenario

with stiff EoSs without/with hyperons included

Blaschke, Grigorian, D.V. PRC 2013, EPJA 2016, HDD EoS without hyperons

Grigorian, Maslov, D.V. 2018: Example of cooling calcul. with stiff MKVOR EoS with hyperons

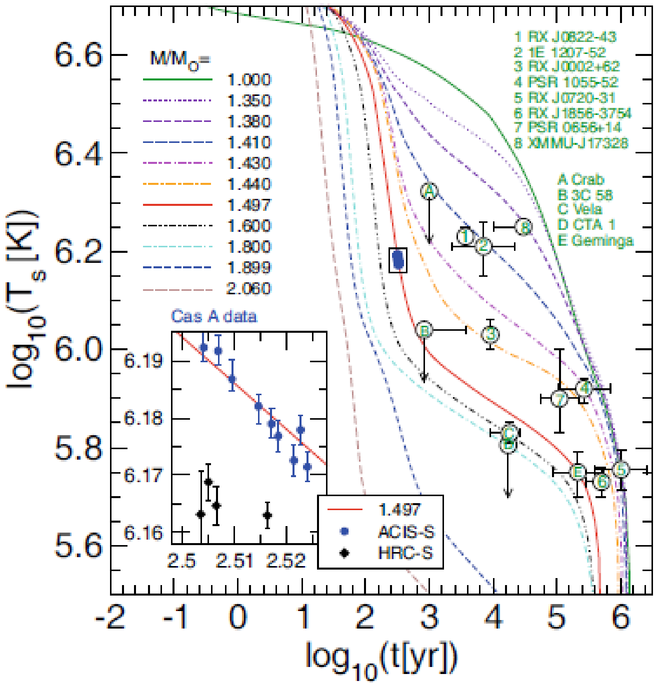
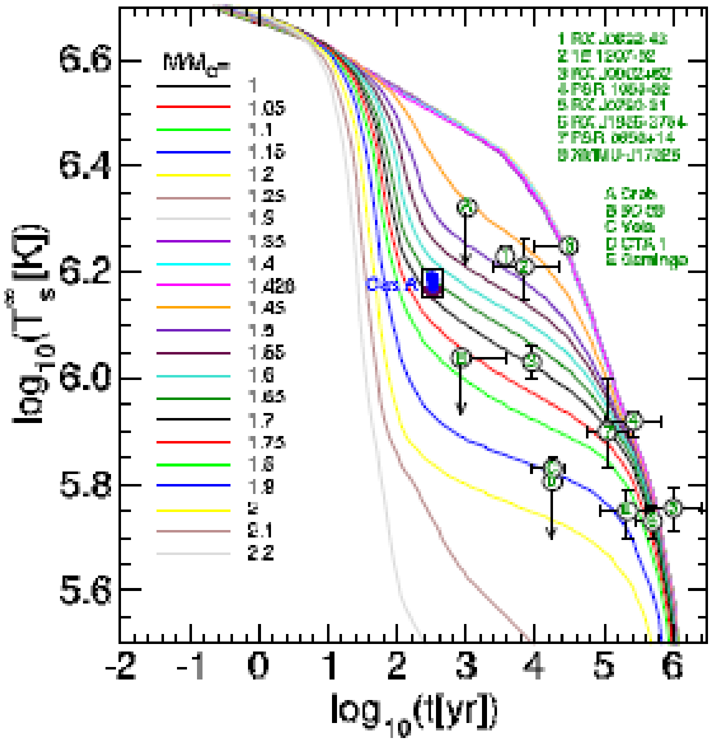


Fig. 4. Cooling curves for a NS sequence according to the hadronic HDD EoS; T_s is the redshifted surface temperature, t is the NS age. The effective pion gap is given by the solid curve 1a+1b in fig. 2, $n_c^\pi = 3n_0$. The $1S_0$ pp pairing gap corresponds to model I. The mass range is shown in the legend. Comparison with Cas A ACIS-S and HRC-S data is shown in the inset. Cooling ACIS-S data for Cas A are explained with a NS mass of $M = 1.497 M_\odot$.



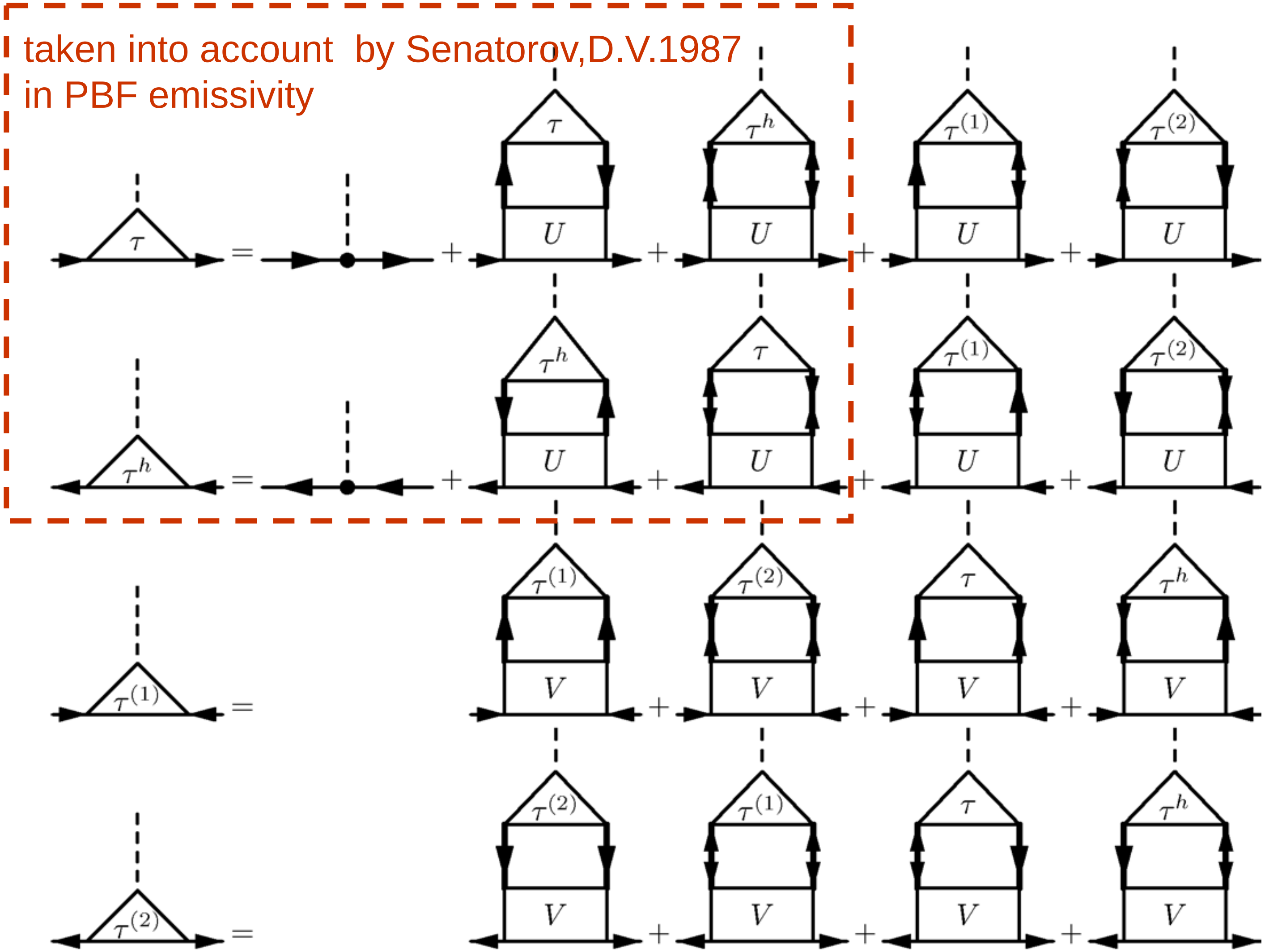
DU is allowed in our EoSs only for most massive NSs

MMU-main processes: low mass objects-slow coolers, more massive objects –more rapid coolers. Data are explained with EoSs, as with hyperons, as without

In presence of pairing:Larkin-Migdal equations

Larkin, Migdal 1963

Neutrino processes in presence of Cooper pairing of nucleons: Kolomeitsev, D.V. 2008



taken into account by Senatorov,D.V.1987
in PBF emissivity

Cannot be written in matrix form in Nambu-Gor'kov space since U ≠ V in p-h and p-p channels. In non-eq. diagr. tech.: Kolomeitsev, D.V. 2011

White body radiation problem in non-eq. closed diagram tech.

In QPA, optic theorem formalism in non-eq. diagram technique

D.V.,Senatorov, Sov.J.Nucl.Phys. 45 (1987)

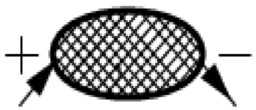
Key idea-in matter there are no asymptotically free states

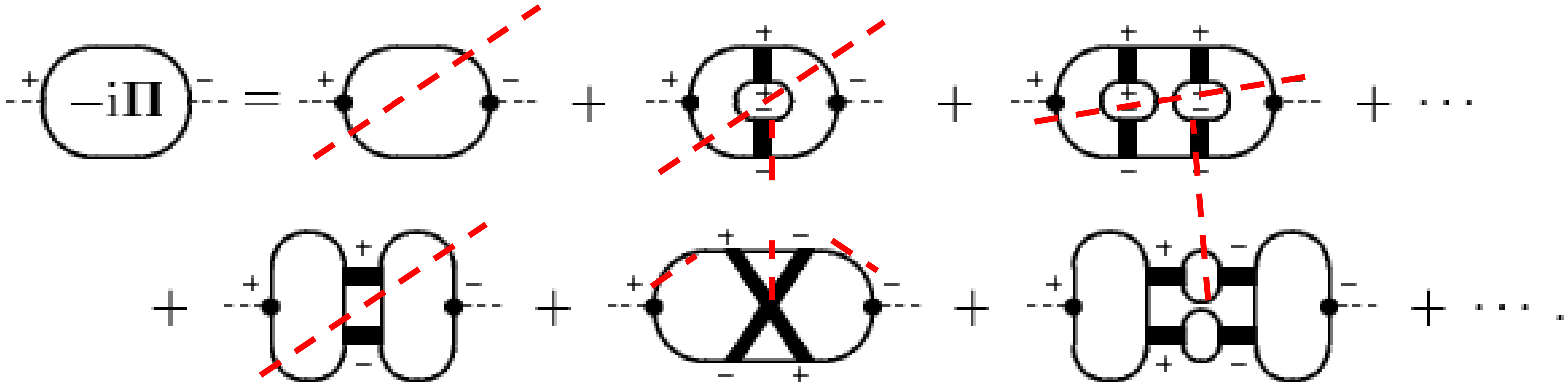
Most general consideration**:** Knoll, D.V. Ann. Phys. 249 (1996)

Direct reactions from piece of matter (*ν* in NS, e+e-, γ, K⁺ in HIC)

expansion in full non-equilibrium G^{-+}

$$\frac{dW}{d^3q/[(2\pi)^3 2\omega_q]} = -i\Pi^{-+} =$$





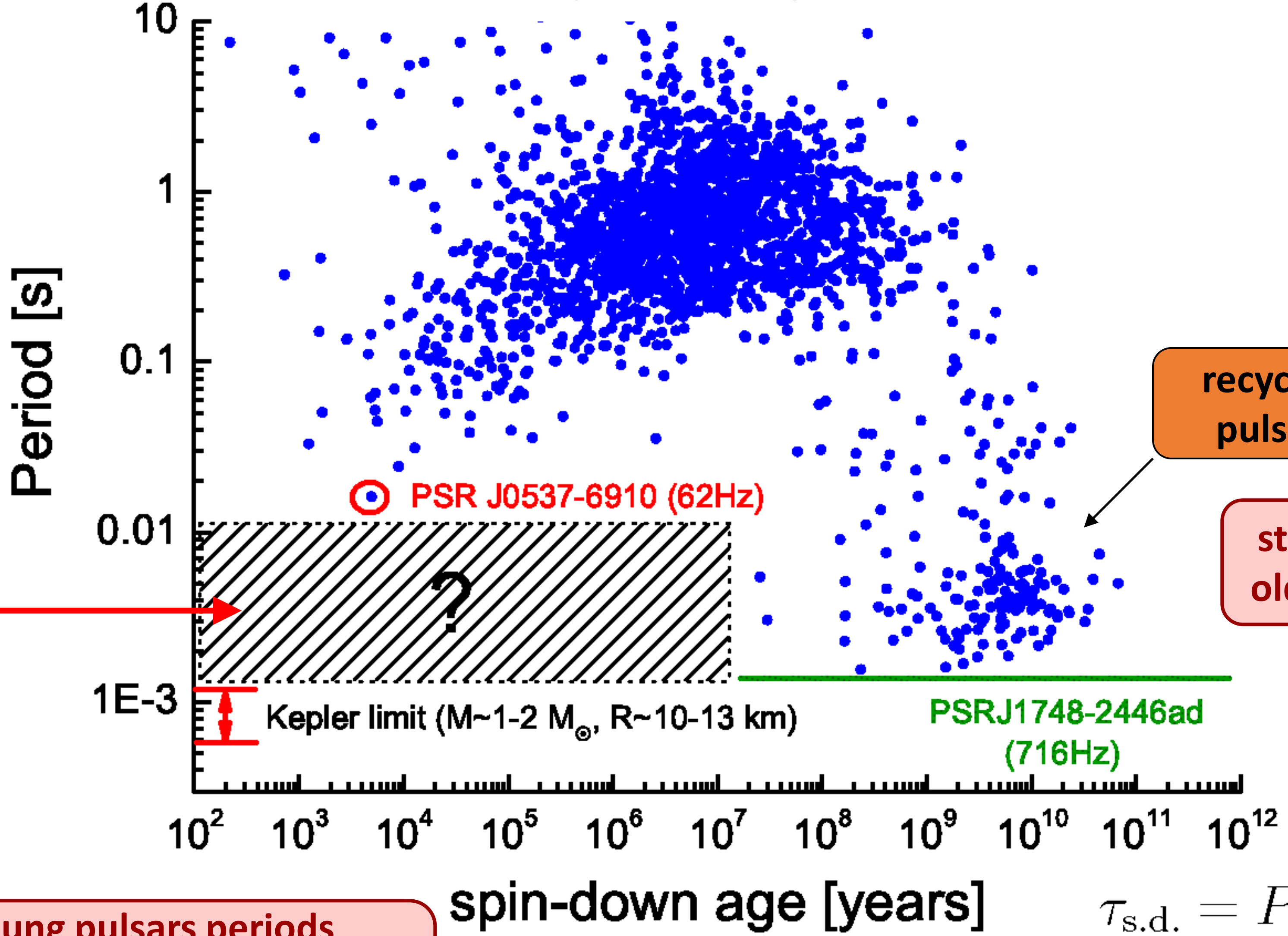
Only for low T<<ε_F, quasiparticle approximation is valid

(each G^{-+} yields T^2, allows to cut diagrams over G^{-+})

For soft radiation: **semiiclassics** (all graphs in first line are of same order):LPM effect

Age-period diagram for pulsars

- The ATNF pulsar catalogue



$$\tau_{\rm s.d.} = P/(2\dot{P})$$

recycled pulsars

stability of old pulsars?

Young pulsars periods (single stars) are much larger than the Kepler limit (why ?)

Viscosity effects: r-mode instability of rotating neutron star and viscosity

In a dense system like a NS, the Rossby waves are sources of r-mode instability

1998 Andersson, Friedman and Morsnik

r-modes can either destroy the star
or the star stops rotating

r-mode is stable if

$$\frac{1}{\tau} = -\frac{1}{\tau_G} + \frac{1}{\tau_\eta} + \frac{1}{\tau_\zeta} > 0$$

In presence of "soft-mode" bulk viscosity increases

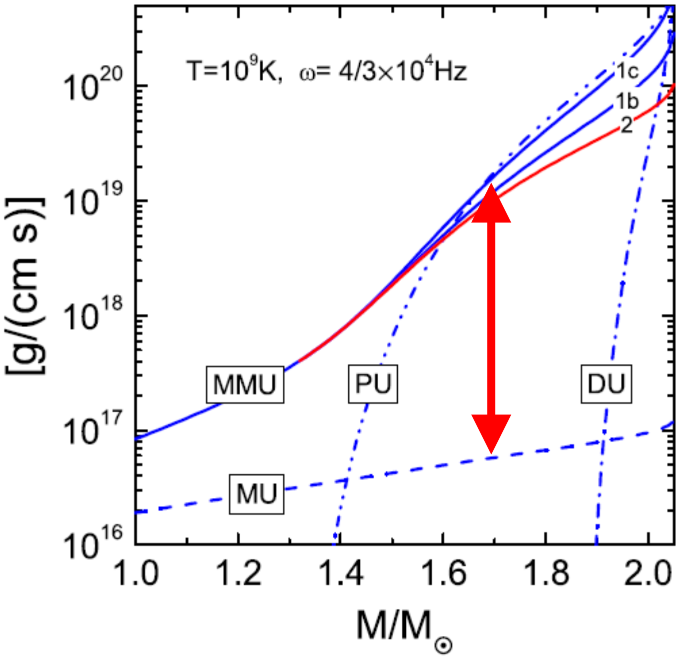


FIG. 21. (Color online) Partial contributions to the profile averaged bulk viscosity $\langle \sum_l \zeta^{(r)}_{\mathrm{s.m.},l}(1+0.86r^2/R^2)\rangle_8$ from DU, MU, MMU, and PU processes as functions of the neutron star mass. Three MMU lines (labeled 1b, 1c, and 2) correspond to different choices of the density dependence of the pion gap for $n > 3n_0$, as shown in Fig. 6; the PU process is calculated using curve 3 in Fig. 6, $T_9 = 1$, $\omega = \frac{4}{3} \times 10^4$ Hz.

Viscosity of the dense nuclear matter can damp r-modes and save the rotating star

Kolomeitsev,D.V. EPJA (2014), PRC (2015)

Key message – pion is essentially softened in dense nuclear medium – **simultaneous analysis is needed of the relevant phenomena in different domains of nuclear physics: low-density domain, atomic nuclei, neutron stars, supernovas, heavy-ion collisions**

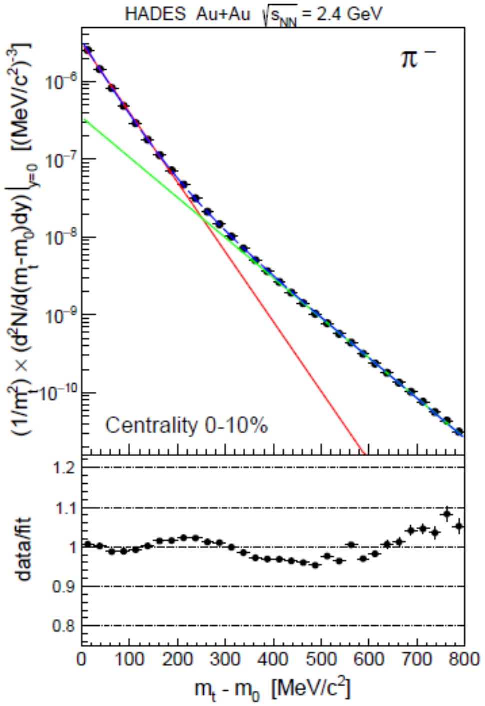
What next?

To marry descriptions of atomic nuclei, HIC and NS with inclusion of dressed pions.

- Careful study of low-energy HIC (where πN interaction effects should look most pronounced)
- Description of pions in baryon matter beyond assumption that LM parameters are constants, applying Phi-derivable models, etc.
- Systematic study effects of pion spectral function at $\omega, k \neq 0$ on EoS.
- Include pions and kaons dressed by p-wave interaction in HIC codes.
- Seek for pion instabilities in peripheral HICs
- Improve description of NS cooling with inclusion of in-medium pions.
- Include in-medium pions in supernova simulation codes.
- Further study precursors of pion BEC in ultrarelativistic HIC, improve description of s-wave πN interaction.
- Further study of hadron blurring at low baryon chem. pot. and $T \sim m_\pi$
- Description of pions in quark matter (p-wave effects, Fermi liquid approach and beyond, etc).
- Hadron blurring and feasible quark-hadron duality in crossover region.
- Rotation and acceleration/deceleration effects on pion distributions.
- Work out analogy with description of cold atoms

Supplementary slides

Bose enhancement and BEC of pions at low momenta << m_π



HADES 2020

For k **< 1.5** m_π pion path length λ~R

Pion number is approximately dynamically conserved, only elastic collisions

⇨ increase of n_π (k **<<** m_π) and BEC at sufficiently rapid cooling of the system

D.V. JETP 78 (1994), Kolomeitsev, D.V. EPJA2018.

Hadron liquid with a small baryon chemical potential at finite temperature

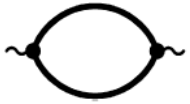
LHC conditions: $N_\pi >> N_N$

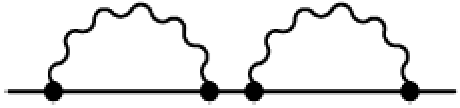
$i\Phi = \frac{1}{2}$  $+ \frac{1}{4}$  $+ \cdots .$

retain

$i\Phi = \frac{1}{2}$ 

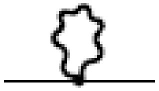
the fermion self-energy 

the boson self-energy 

 $+$  $+ \cdots$

Consider hot pion-nucleon vacuum ($\mu_N \simeq 0$). For temperature $m_\pi \lesssim T \ll m_N$, typical nucleon momenta $\sqrt{2m_N T}$ and typical pion momenta are $\sim T \ll \sqrt{2m_N T}$.

soft thermal loop (STL) approx.: drop pion momentum in nucleon Green func.

$J =$  $\simeq g^2_{p.v.} \frac{T^4}{m^2_\pi},$

Thus Dyson equation for the nucleon gets clear diagrammatic interpretation:

describing the fermion propagation in an external field $\phi_c^{\rm ext}$, $\phi_c^{\rm ext} \cdot (\phi_c^{\rm ext})^* = J/g^2_{p.v.}$. Nucleon undergoes multiple quasi-elastic rescatterings on pairs of quasi-static pion impurities. The value J is proportional to the density of impurities. Thus, it demonstrates the intensity of the multiple elastic scattering.

D.N. Voskresensky, Nucl. Phys. A 744 (2004) 378. Nucl. Phys. A **812**, 158 (2008). ProgPNPh130, 2023

Full blurring of baryon vacuum (baryon blurs-unparticles)

To suppress spin structure assume for simplicity that nucleons are non-rel. part. Then nucleon Dyson eq. yields

$$G_N^R = G_N^{0R} + G_N^{0R} J (G_N^R)^2$$

with $G_N^{0R} = 1/(\epsilon - \epsilon_p)$, $\epsilon_p = \frac{p^2}{2m_N}$. This algebraic eq. is easily resolved:

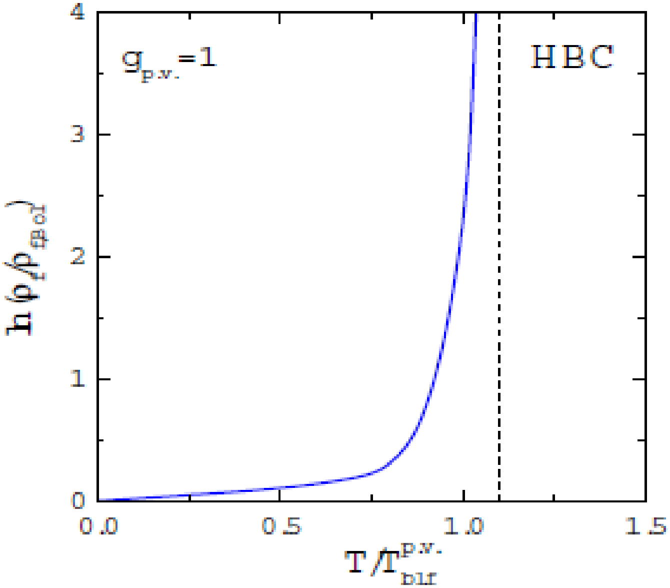
$$G^R = \frac{(\epsilon - \epsilon_p)}{2J} \pm \sqrt{\left(\frac{(\epsilon - \epsilon_p)}{2J}\right)^2 - \frac{1}{J}}$$

where we should take $''-''$ getting appropriate qp. limit for $|\epsilon - \epsilon_p| \sim T \gg J$. In the temperature regime $m_\pi \lesssim T \ll m_N$ we have opposite limit and the nucleon Green function losses its qp. pole structure (!) acquiring a broad width ($\mathrm{Im} G^R \neq 0$) ➡ a blurring of the nucleon continuum. Nucleon momentum distrib.:

$$n_p = -4 \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \frac{2 \mathrm{Im} G^R}{e^{(m_N + \epsilon)/T} + 1} \sim n_p^{Boltz.} \, e^{2\sqrt{J}/T} \left(\frac{T}{\sqrt{J}}\right)^{3/2} \propto T^3 e^{-m^*(T)/T}$$

$$m_{\rm f}^*(T) = m_{\rm f} - 2\sqrt{J},$$

For $T \simeq T^{p.v}_{\rm bl.f}$, $2\sqrt{J}(T^{p.v}_{\rm bl.f}) \simeq m_N$, the effective nucleon mass $m^*(T) \to 0$. $T^{p.v}_{\rm bl.f} \sim 140$MeV



Hot Bose condensation

$$m_{\mathrm{b}}^{*2} \simeq m_{\mathrm{b}}^2 - \frac{4g^2 n_{\mathrm{f}}^{(\pm)}}{\sqrt{J}}.$$

squared of the effective boson mass reaches zero

Logarithm of the ratio of the fermion density to the corresponding Boltzmann quantity for $g_{p.v} = 1$ as function of the temperature in units of $T^{p.v}_{\mathrm{bl.f}}$,
cf. Urbah law; multiple electron-phonon interact. in doped semiconductors.
The vertical dash-line indicates the area of a possible hot boson condensation (HBC),

One could expect an anomalous enhancement of the boson (e.g., pion and kaon) production at low momenta ($p_{\mathrm{b}} \lesssim T$) and an anomalous behavior of fluctuations, e.g., at LHC conditions, as a signature of the hot Bose condensation for $T > T_{\mathrm{cB}}$, if a similar phenomenon occurred in a realistic problem including all relevant particle species.

D.N. Voskresensky, Nucl. Phys. A 744 (2004) 378.

Is π-condensation possible in peripheral HIC?

H.J. Pirner, D.N. Voskresensky / Physics Letters B 343 (1995) 25–30

interpenetrating nuclei

condition $n(p)n(p+p_{\rm lab}+k)=0.$

is safely satisfied

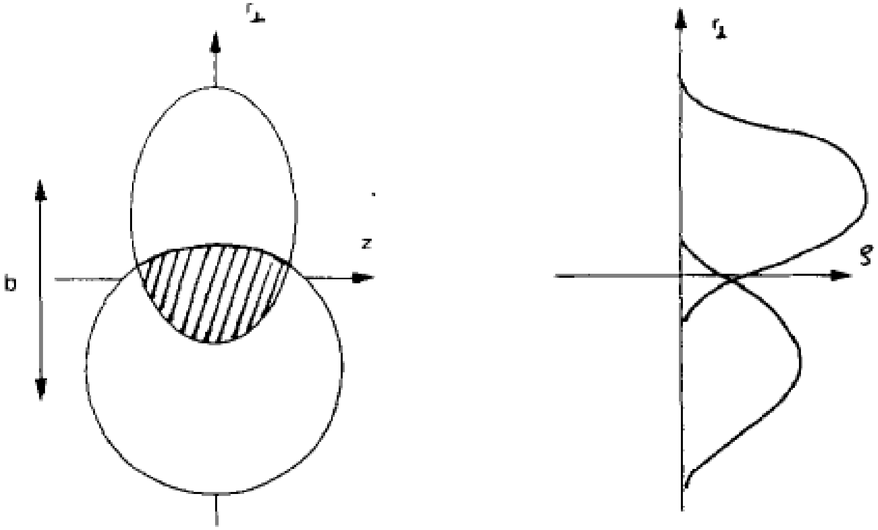


Fig. 1. A typical overlap region and density profiles. Atomic weight $A \simeq 200$, $b \simeq 1.5R$. Lorenz contraction is taken into account for 1 GeV/A collision energy. Hatched area is composite system under consideration.

Pion self-energy for 2 Fermi spheres

~ - 2p $_F$ (n)

2p $_F$ ~ (8**n) $^{1/3}$**

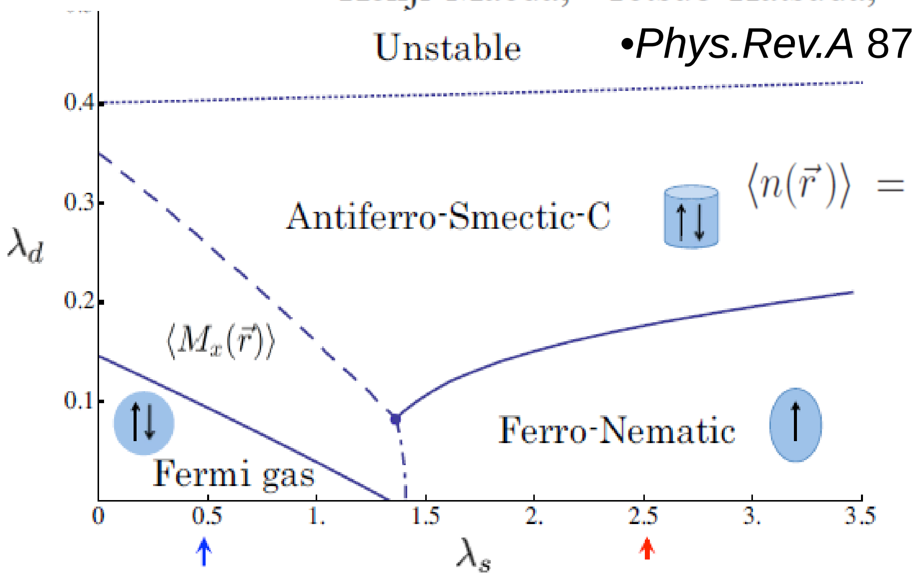
→ effective attraction as at 8 n
(for n=n_0/2 in each nucleus → ef. Density is 4n_0)

→ pion condensation might be observed via pronounced peaks in pion production in a narrow region of momenta $k_\perp \sim (2\text{–}3)m_\pi$, corresponding to rather large rapidity values ($y_{\rm lab} \sim 2$) and also in proton (at $k_0/2$) and neutron ($-k_0/2$) distributions.

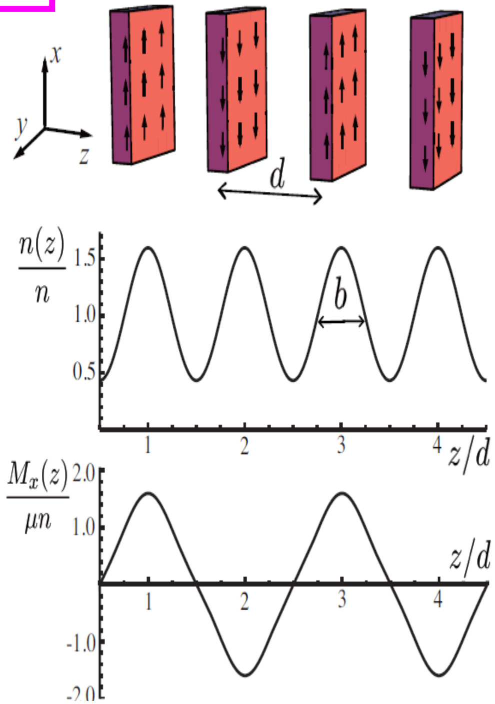
Antiferrosmectic ground state of two-component dipolar Fermi gases
– an analog of meson condensation in nuclear matter

Kenji Maeda,[1] Tetsuo Hatsuda,[2,3] and Gordon Baym[4]

•*Phys.Rev.A* 87 (2013) 2, 021604

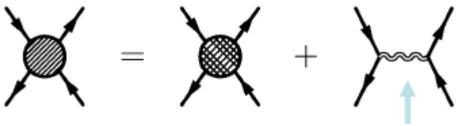


$$\langle n(\vec r)\rangle = \frac{nd}{b\sqrt{\pi}} \sum_{\ell=-\infty}^{\infty} e^{-(z-d\ell)^2/b^2},$$



$$\lambda_d = n\mu^2/\epsilon_F, \quad \lambda_s = ng/\epsilon_F,$$

FIG. 2: Schematic phase structure of dipolar fermions as a function of λ_s and λ_d, showing the Fermi gas phase, the ferronematic phase, the onset of spatially varying magnetization and antiferrosmectic-C phase. The dashed line shows where the AFSC phase becomes favorable compared with the uniform unmagnetized interacting Fermi gas, and the dash-dot line the transition between the uniform Fermi gas and the FN phase. Beyond the upper dotted line the system becomes unstable against collapse.



analog of tensor forces, p-wave πNN inter.

$$V(\vec r_1, \vec r_2)^{ij}_{\alpha\alpha',\beta\beta'} = \boxed{\frac{\mu^2}{r^3}\{\sigma^i_{\alpha\alpha'}(\delta_{ij} - 3\hat r_i \hat r_j)\sigma^j_{\beta\beta'}\}} + g\delta_{\alpha\alpha'}\frac{\delta_{ij}}{3}\delta(\vec r_1 - \vec r_2)\delta_{\beta\beta'},$$

analog of NN correlations

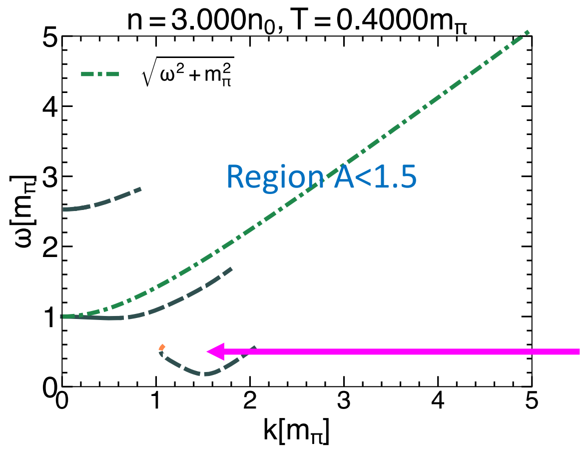
^{161}Dy and ^{163}Dy are the most magnetic fermionic atoms
novel nearly quantum degenerate dipolar Bose-Fermi gas

Similarly, for fermionic polar molecules, e.g., ^{40}K-Rb with large electric dipole moments

Extra supplementary slides

Pion spectrum at nonzero temperature, *N=Z*

in this model pion cond. appears at T=0 for n =2.5 n $_0$



$$\frac{1}{\pi}\int_0^\infty d\omega\,\left[\omega A\left(\omega,k\right)\right] = 1$$

Broader spectral function for T≠0

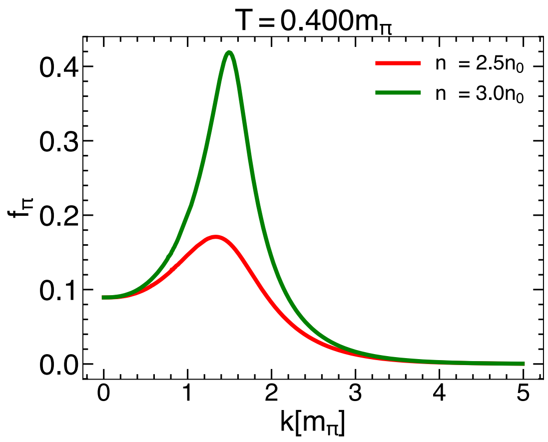
Liquid phase of π condensate

Spin-sound is smeared but with increasing *n* for n> n$_{c1}$ appears Lorenzian peak at *k* ~*p$_F$*

$$f_\pi\left(k\right) = \frac{1}{\pi}\int_0^\infty d\omega\,\frac{\omega}{e^{\frac{\omega}{T}}-1}A\left(\omega,k\right)$$

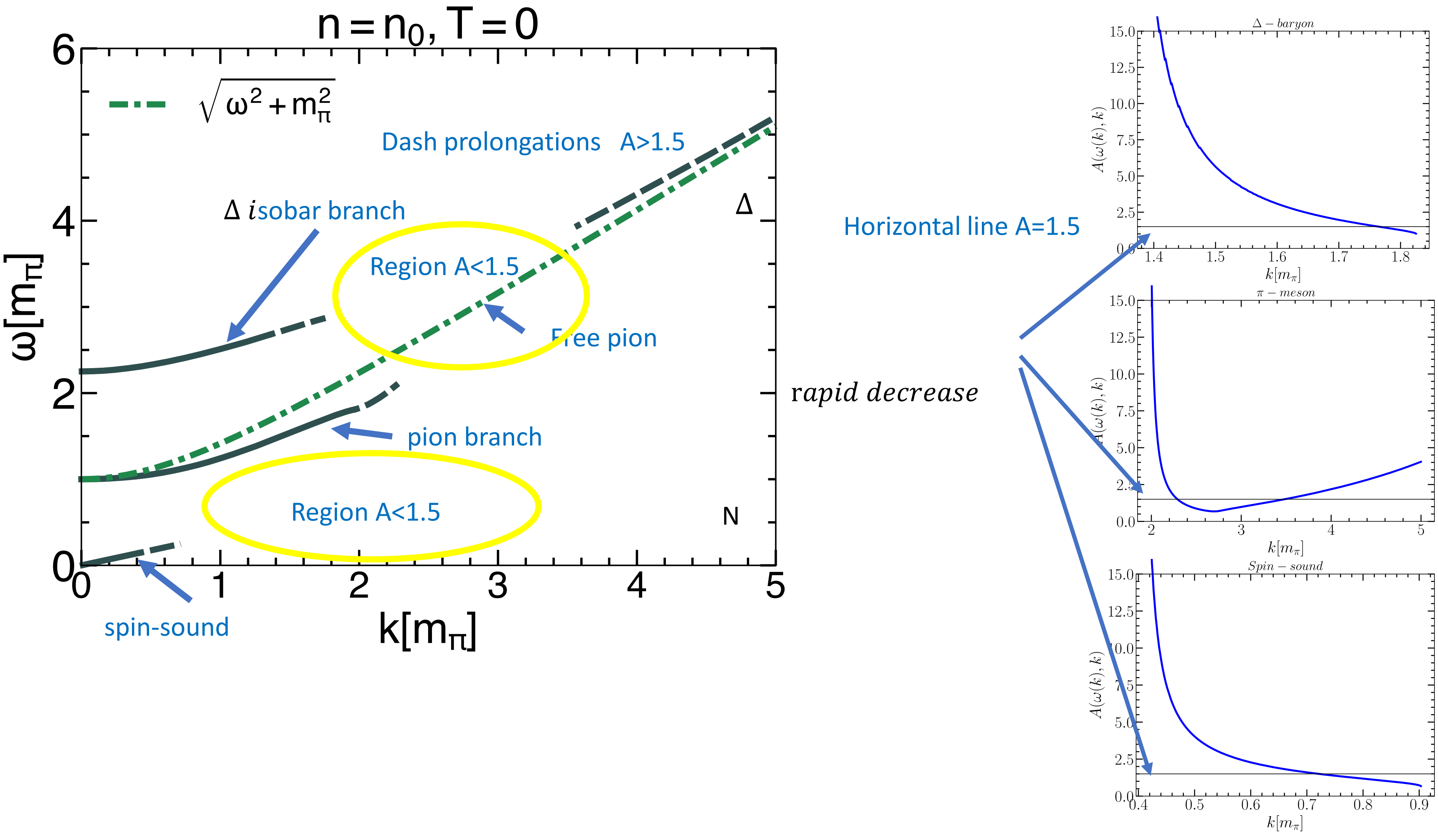
$$n\left(T\right) = 3\int_0^\infty \frac{dk k^2}{2\pi^2}f_\pi\left(k\right)$$

Instead of free pion distribution



With taking into account of the full spectral function A(ω,k) in isosymmetric matter

Sum-rule $$\frac{1}{\pi}\int_0^\infty d\omega\,\left[\omega A\left(\omega,k\right)\right]=1$$

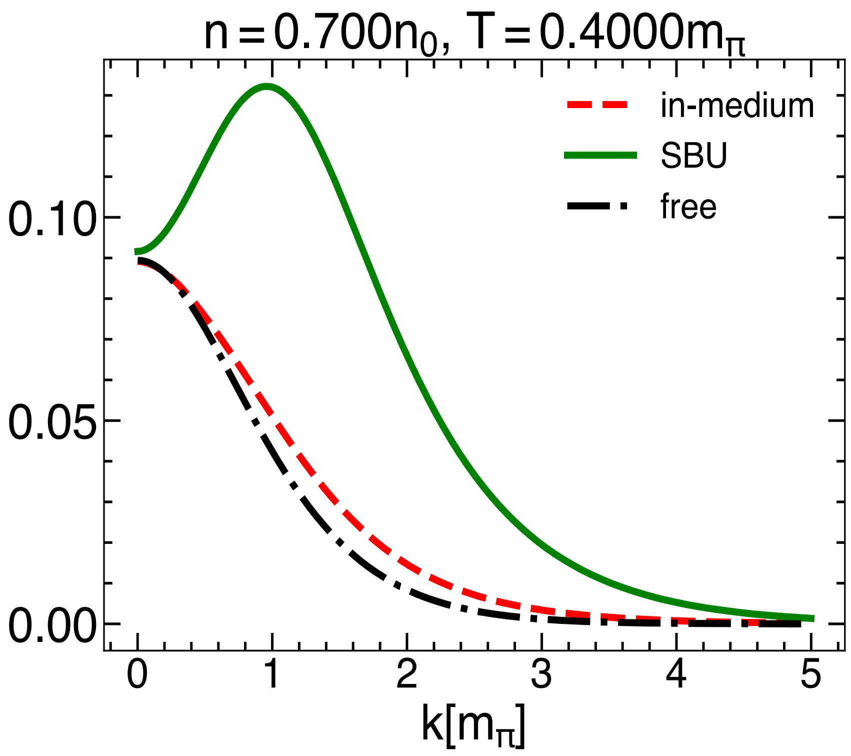


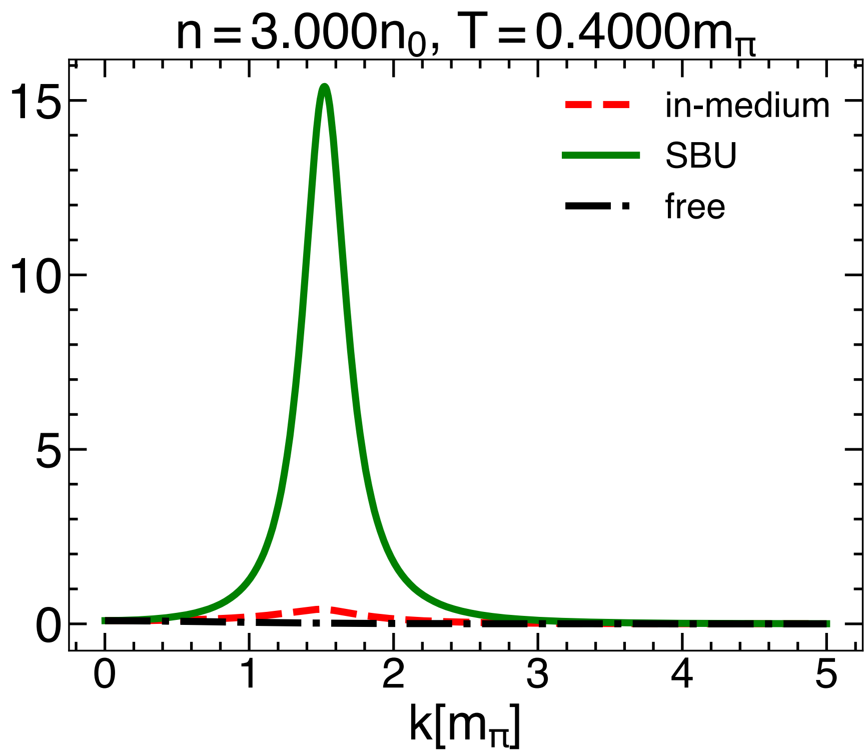
Pion distribution in medium

$$f_\pi(k) = \frac{1}{\pi}\int_0^\infty d\omega \, \frac{\omega}{e^{\frac{\omega}{T}}-1} A(\omega, k)$$

Pion distribution in detector at sudden breakup (SBU)

$$f_\pi^{SBU}(k) = \frac{1}{\pi}\int_0^\infty d\omega \, \frac{\omega_k}{e^{\frac{\omega}{T}}-1} A(\omega, k) \qquad \omega_k = \sqrt{m_\pi^2 + \vec{k}^2}.$$



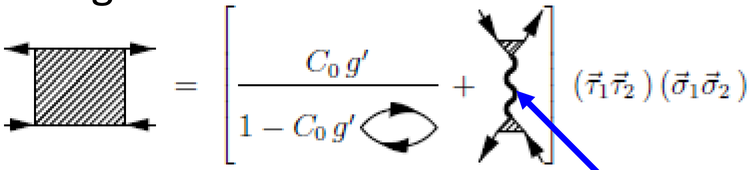


Where are virtual pions in nuclei (at $n \sim n_0$) ?

G. Brown, Buballa, Li, Wambach NPA 1995: no excess of pions in nuclei

Our arguments:

NN correlations are strong $\Gamma \approx 0.4$ for $n \sim n_0$



$$R = \frac{\sigma[\mathrm{FOPE}]}{\sigma[\mathrm{MOPE}]} \simeq \frac{\Gamma^4(\omega \simeq 0, k \simeq p_{\mathrm{F},N})(m_\pi^2 + p_{\mathrm{F},N}^2)^2}{\tilde\omega^4(p_{\mathrm{F},N})} < 1$$

Soft pions yield only small contribution to the sum-rule $\delta S \simeq 0.1 \ll 1$ for $\omega < k v_F$

But from description of π-atoms follows for $\omega \sim m_\pi$, $k \ll m_\pi$ →

$\omega(k) \simeq \sqrt{m_\pi^2 + \alpha_0 \vec k^2}$, $\alpha_0 \simeq 0.4$ not unity for $n \approx n_0$

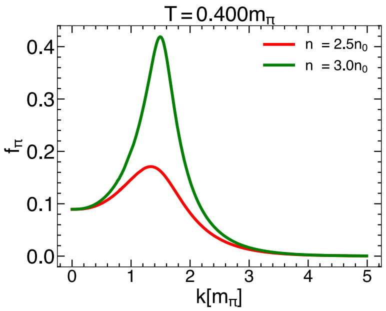
Pions at $\omega \ll m_\pi$ are soft in sense that $|D(\omega = 0, p_F)(m_\pi^2 + p_F^2)|^2 \sim 30 - 50.$ at $n \approx n_0$

Pions are there but their contribution strongly depends on the frequency and momentum.

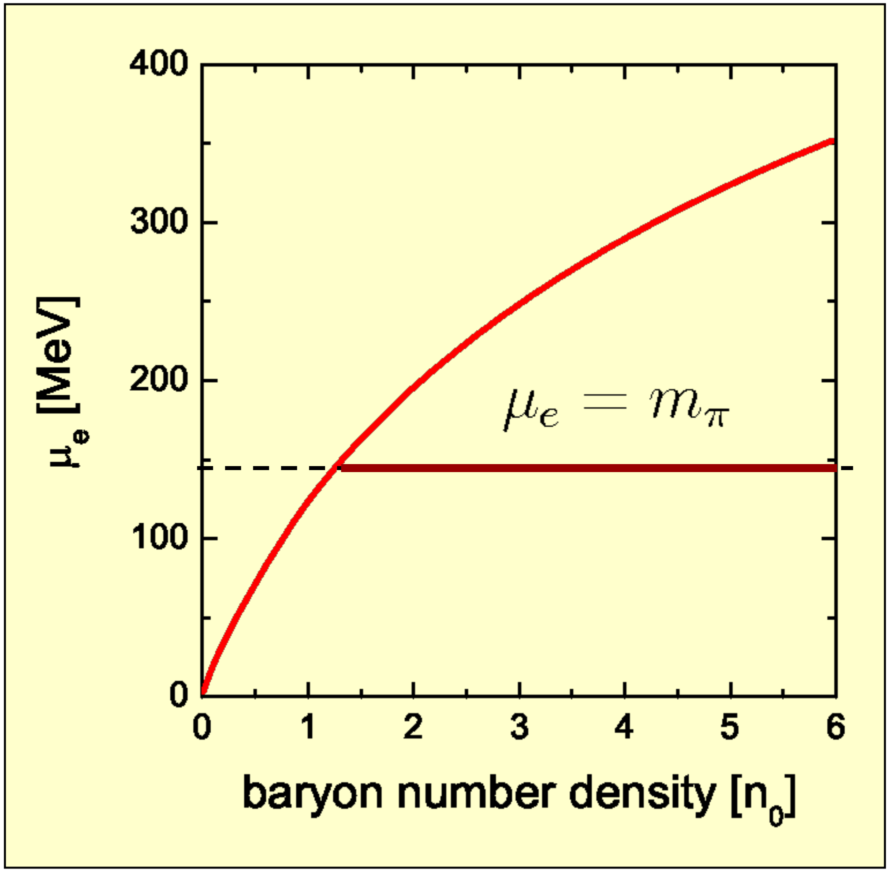
Virtual soft π significantly contribute for T≠ 0 . **π distribution in matter**

$$<\hat\phi^\dagger_{\pi^-}(\vec k)\hat\phi_{\pi^-}(\vec k)>_T \simeq VT/[2\tilde\omega^2(k)].$$

$$f_\pi(k) = \frac{1}{\pi}\int_0^\infty d\omega \frac{\omega}{e^{\frac{\omega}{T}} - 1} A(\omega, k)$$



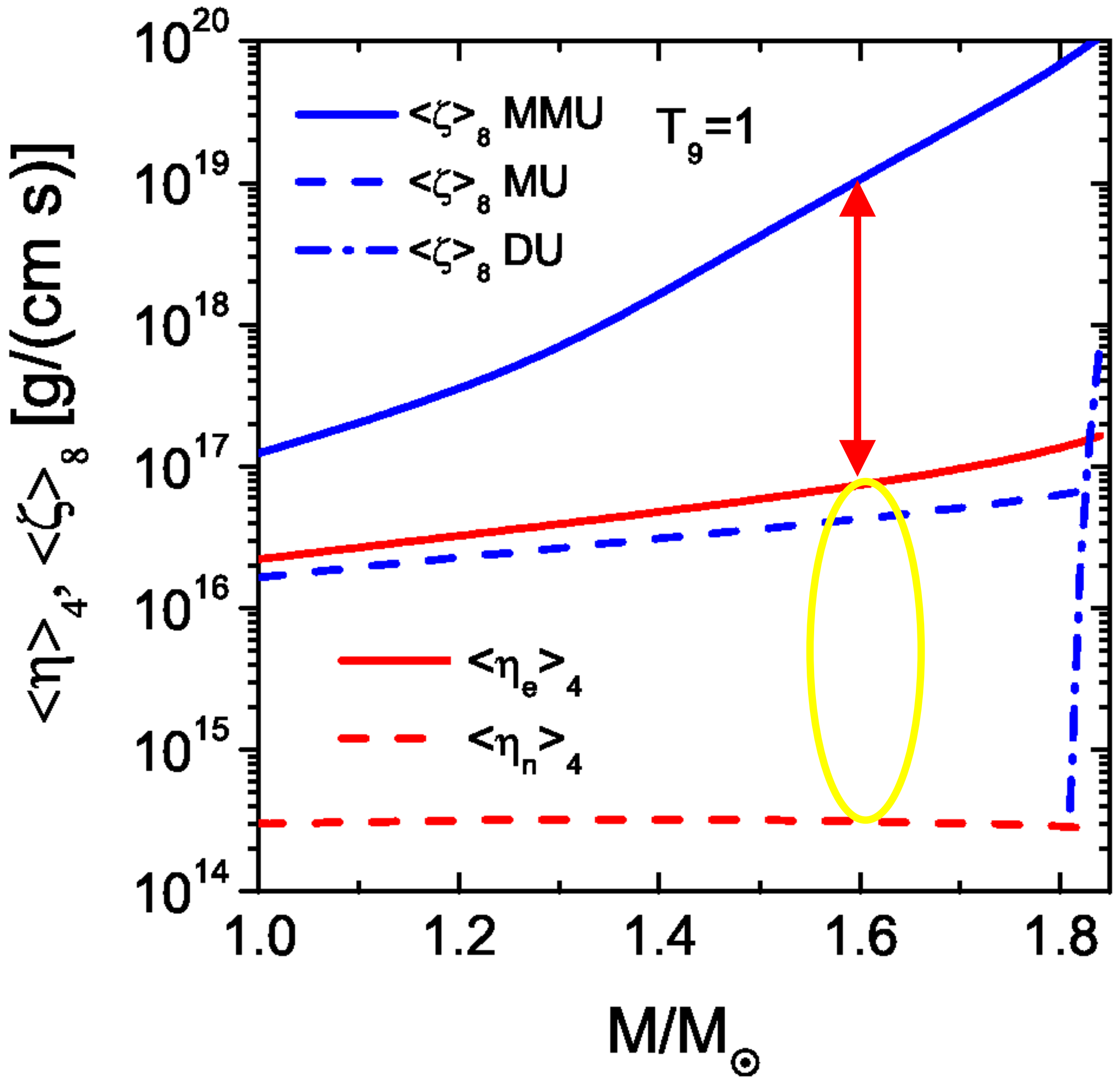
Effect of pion fl. with $k_0 \sim p_{FN}$ on EoS for $n > n_{c1}$ → **liquid phase**, large bulk-viscosity-pion glass (?)



Weak reactions start

$$e^- \longrightarrow \pi^- + \nu_e$$

→ **Bose-Einstein π-cond. at k=0** In MCS one silently ignores BEC of pions! But within their concept it must be included! If included for *n*< (1.5-2) n_0, it would result in a too rapid cooling for all NS with M> M_{sol} .



M_{DU} =1.90 M_{sol}

$$\langle \dots \rangle_l = \frac{1}{R^{l+1}} \int_0^R (\dots)\, r^l \, dr$$

Main term is from MMU

again pion softening

$$\zeta^{[\mathrm{MMU}]}_{\mathrm{s.m.}} \propto F_{\mathrm{MMU}}(n) T^7$$

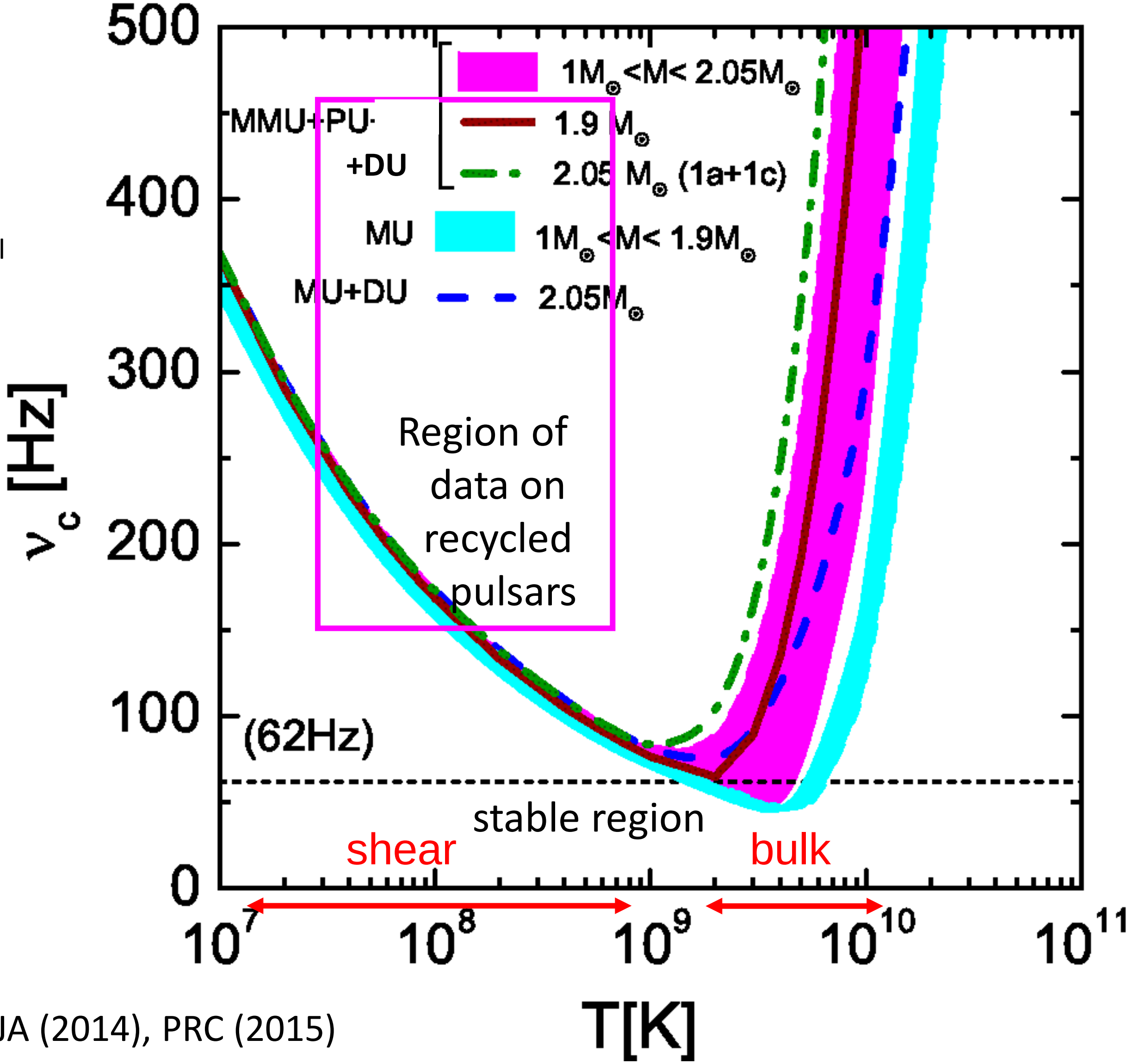
Kolomeitsev,D.V. EPJA (2014), PRC (2015)

R-mode stability window

$$\tau_G^{-1}(\nu_c) = \tau_\eta^{-1}(\nu_c) + \tau_\zeta^{-1}(\nu_c) \longrightarrow \nu_c = \nu_c(T)$$

$$\nu = \Omega/2\pi$$

M $_{DU}$ =1.90 M $_{sol}$



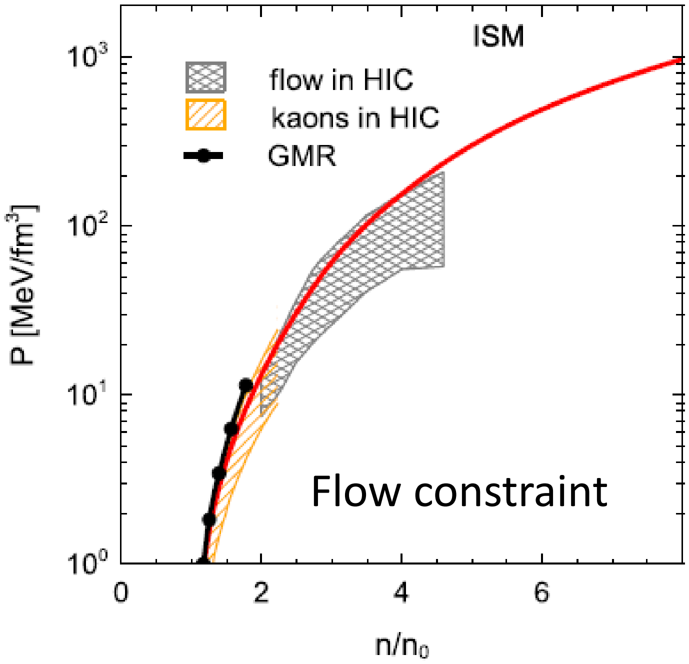
Kolomeitsev,D.V. EPJA (2014), PRC (2015)

62 Hz threshold is reached with MMU for M>M $_c^{MMU}$=1.84 M $_{sol}$

EoS with scaled masses-couplings with included hyperons and Deltas satisfies main known constraints

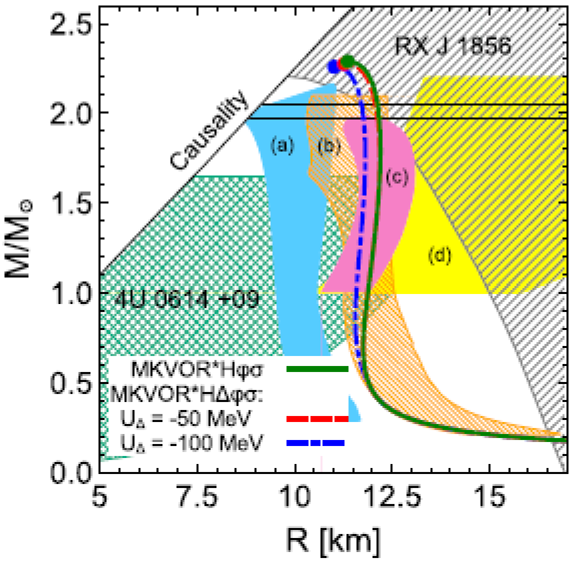
Our EoS with a cut in ρ sector:

Kolomeitsev, Maslov, D.V. NPA 2017



Flow constraint

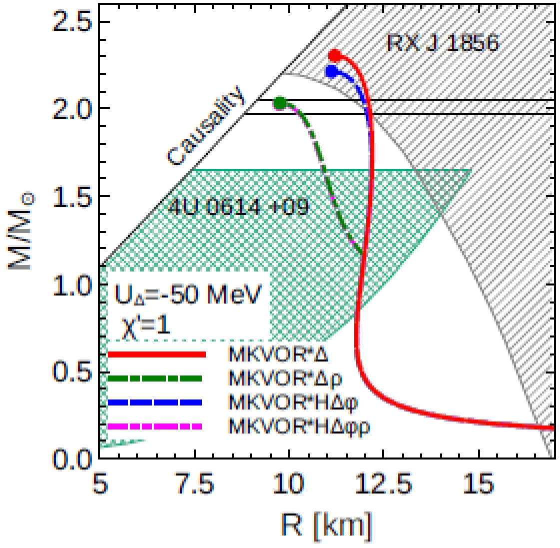
M $_{max}$ >1.97 M $_{sol}$ constraint



Charged ρ condensation: D.V. Phys.Lett. (1997)

EoS with included charged ρ$^-$ condensate also fulfills constraints

Kolomeitsev, Maslov,D.V. NPA 2018



Constraints on hadron EoS

EoS of the cold hadronic matter should:
(cf. Klähn et al. 2006, Kolomeitsev, Maslov, DV 2015-2018)**:**

satisfy empirical constraints on global characteristics of atomic nuclei

constraints on the pressure of the nuclear mater from the description of particle transverse and elliptic flows and K^+ production in HIC (**flow constraint**);

allow for the **heaviest known compact stars with mass** $M > 2.01 \pm 0.04, M_{\odot}$

allow for adequate description of the NS cooling, most probably without DU neutrino processes $n \rightarrow p + e + \bar{\nu}$ ıe majority of the known pulsars with M<(1.4-1.5) M_{sol}.
(**DU constraint**)

yield a mass-radius relation comparable with the empirical constraints (gravitational wave constraint)

solve **hyperon and Delta puzzles**

being extended to T≠0, appropriately describe supernova explosions and proto-neutron stars, and heavy-ion collision data till T~T_{CEP} , etc