Lattice study of rotating QCD properties

V. Braguta

JINR

24 July, 2024

In collaboration with

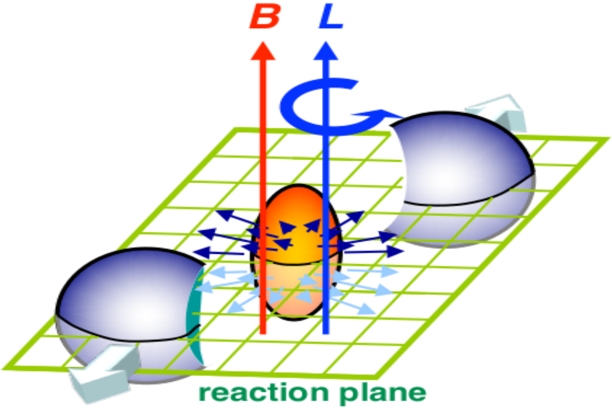
- M. Chernodub
- A. Kotov
- I. Kudrov
- A. Roenko
- D. Sychev

JETP Lett. 112, 9–16 (2020); JETP Lett. 117, 9, 639-644 (2023); Phys.Rev.D 103 (2021) 9, 094515; PoS LATTICE2022 (2023) 190; Phys.Lett.B 855 (2024) 138783; Phys.Lett.B 852 (2024) 138604; e-Print: 2310.16036

Outline:

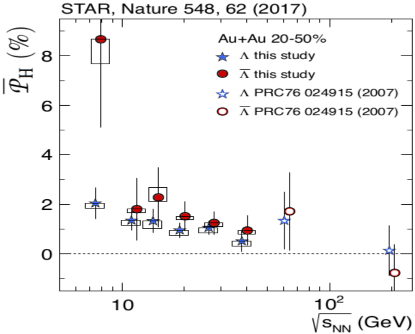
- Introduction
- Moment of inertia of QGP
- Inhomogeneous phase transitions in QGP
- Conclusion

Rotation of QGP in heavy ion collisions



- QGP is created with non-zero angular momentum in non-central collisions

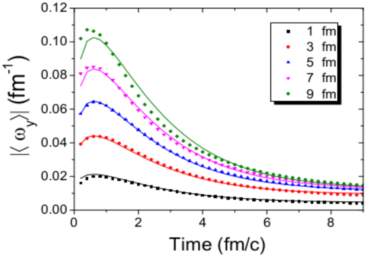
Rotation of QGP in heavy ion collisions

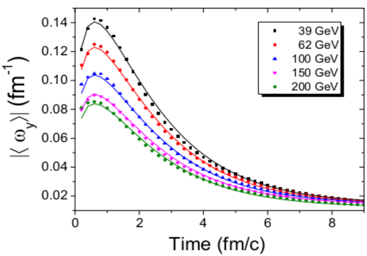


Angular velocity from STAR (Nature 548, 62 (2017))

- $\Omega = (P_\Lambda + P_{\bar\Lambda})\frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- $\Omega \sim 10$ MeV ($v \sim c$ at distances 10-20 fm, $\sim 9 \times 10^{21} s^{-1}$)
- Relativistic rotation of QGP

Rotation of QGP in heavy ion collisions

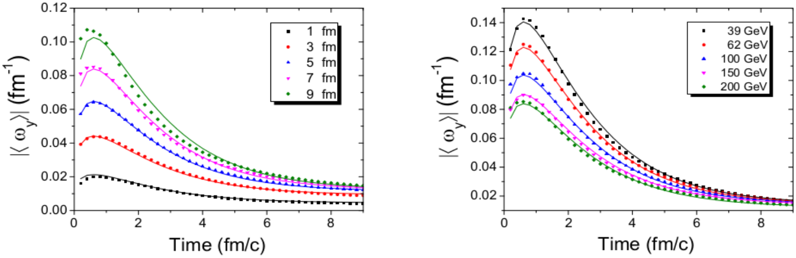




Hydrodynamic simulations (Phys.Rev.C 94, 044910 (2016))

- Au-Au: *left* $\sqrt{s} = 200$ GeV, *right* $b = 7$ fm,
- $\Omega \sim (4 - 28)$ MeV
- Relativistic rotation of QGP

Rotation of QGP in heavy ion collisions



Hydrodynamic simulations (Phys.Rev.C 94, 044910 (2016))

- Au-Au: *left* $\sqrt{s} = 200$ GeV, *right* $b = 7$ fm,
- $\Omega \sim (4 - 28)$ MeV
- Relativistic rotation of QGP

How relativistic rotation influences QCD?

Study of rotating QGP

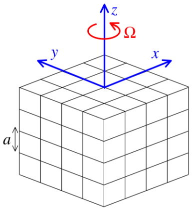
- Our aim: study rotating QCD within lattice simulations
- Rotating QCD at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- Partition function ($\hat{H}$ is conserved)

$$Z = \mathrm{Tr}\,\exp\left[-\beta \hat{H}\right] = \int DA\ \exp\left[-S_G\right]$$

- Euclidean action

$$S_G = -\frac{1}{2g_{YM}^2}\int d^4x\,\sqrt{g_E}\,g_E^{\mu\nu}\,g_E^{\alpha\beta}\,F_{\mu\alpha}^{(a)}\,F_{\nu\beta(a)}$$

$$S_G = \frac{1}{2g_{YM}^2}\int d^4x \mathrm{Tr}\big[(1-r^2\Omega^2)F_{xy}^a F_{xy}^a + (1-y^2\Omega^2)F_{xz}^a F_{xz}^a +$$

$$+(1-x^2\Omega^2)F_{yz}^a F_{yz}^a + +F_{x\tau}^a F_{x\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a -$$

$$-2iy\Omega(F_{xy}^a F_{y\tau}^a + F_{xz}^a F_{z\tau}^a) + 2ix\Omega(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) - 2xy\Omega^2 F_{xz}F_{zy}\big]$$

Details of the simulations

- *Ehrenfest–Tolman effect:* **In gravitational field the temperature is not constant in space at thermal equilibrium**

$$T(r)\sqrt{g_{00}} = const = 1/\beta$$

$$T(r)\sqrt{1 - r^2\Omega^2} = 1/\beta$$

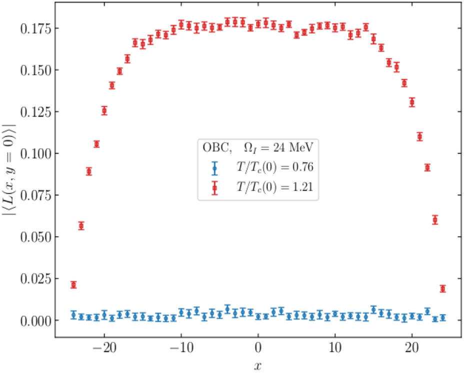
- Rotation effectively heats the system from the rotation axis to the boundaries $T(r) > T(r = 0)$
- **One could expect that rotation decreases the critical temperature**
- We use the designation $T = T(r = 0) = 1/\beta$

Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
- **Neumann b.c.:**
  - Outside the volume $U_P = 1, \quad F_{\mu\nu} = 0$
- The dependence on boundary conditions is the property of all approaches
- One can expect that boundary conditions influence our results considerably, but their influence is restricted due to the screening

Screening of boundary conditions



Details of the simulations

Sign problem

$$S_G = \frac{1}{2g_{YM}^2}\int d^4x \mathrm{Tr}\big[(1-r^2\Omega^2)F^a_{xy}F^a_{xy} + (1-y^2\Omega^2)F^a_{xz}F^a_{xz}+$$

$$+(1-x^2\Omega^2)F^a_{yz}F^a_{yz} + +F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau}-$$

$$-2iy\Omega(F^a_{xy}F^a_{y\tau} + F^a_{xz}F^a_{z\tau}) + 2ix\Omega(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}) - 2xy\Omega^2 F_{xz}F_{zy}\big]$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω

EoS of rotating gluodynamics

- Free energy of rotating QGP

$$F(T,R,\Omega) = F_0(T,R) + C_2\Omega^2 + ...$$

- The moment of inertia

$$C_2 = -\frac{1}{2} I_0(T,R), \quad I_0(T,\Omega) = -\frac{1}{\Omega}\left(\frac{\partial F}{\partial \Omega}\right)_{T,\Omega\to 0}$$

- Instead of $I_0(T,R)$ we calculate $K_2 = -\frac{I_0(T,R)}{F_0(T,R)R^2}$
- Sign of K_2 coincides with the sign of $I_0(T,R)$
- Sometimes instead of Ω^2 we use $v^2 = (\Omega r)^2$ and $v_I^2 = (\Omega_I r)^2$

EoS of rotating gluodynamics

- Classical moment of inertia

$$I_0(R) = \int_V d^3x x_\perp^2 \rho_0(x_\perp)$$

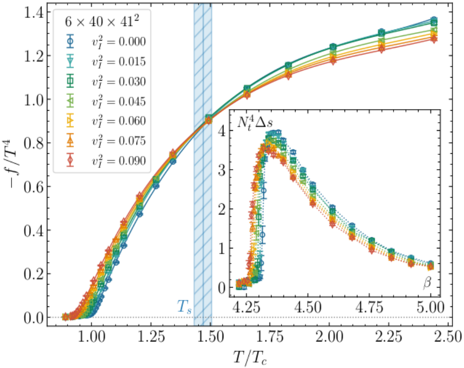
- Related to the trace of EMT $\quad T^\mu_\mu = \rho_0(x_\perp)c^2$
- Generation of mass scale in QCD and scale anomaly

$$T^\mu_\mu \sim \langle G^2 \rangle \sim \langle H^2 + E^2 \rangle$$

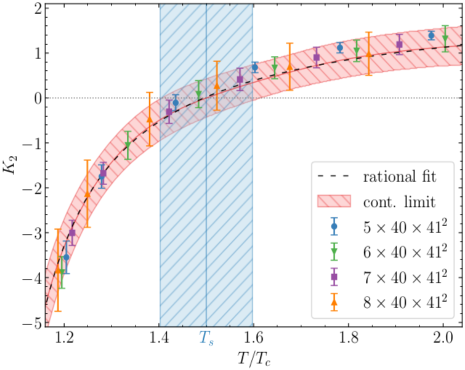
- In QCD the gluon condensate $\langle G^2 \rangle \neq 0$
- *One could anticipate:* $\rho_0 \sim \langle H^2 + E^2 \rangle$?
- $I_0 = I_{fluct} + I_{cond}$ *valid for QCD!*
  $I_{fluct} = \langle J_3^2 \rangle - (\langle J_3 \rangle)^2$
  $I_{cond} = \frac{1}{3} \int d^3x r^2 \langle H^2 \rangle$

Calculation of free energy on the lattice

- $F = -T \log Z$ impossible to calculate on the lattice
- $\frac{\partial F}{\partial \beta} \sim \langle \Delta s(\beta) \rangle = s(\beta)_T - s(\beta)_{T=0}, \quad \beta = \frac{6}{g^2}$
- $\frac{F(T)}{T^4} \sim \int_{\beta_0}^{\beta_1} d\beta' \langle \Delta s(\beta') \rangle$

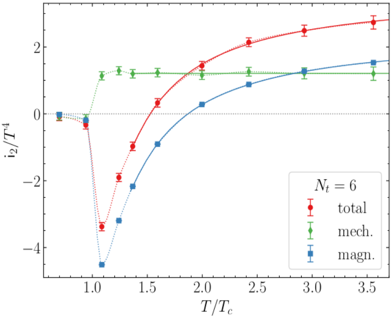


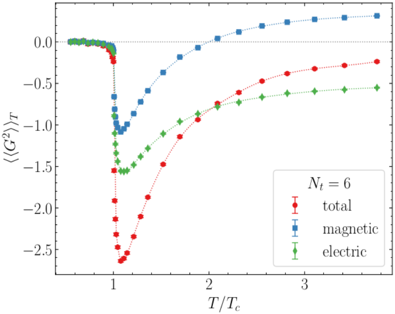
Moment of inertia of gluon plasma



- $I(T, R) = -F_0(T, R) K_2 R^2$
- $I < 0$ for $T < 1.5T_c$ and $I > 0$ for $T > 1.5T_c$
- Negative moment of inertia indicates a thermodynamic instability of rigid rotation
- The region of $I < 0$ is related to magnetic condensate and the scale anomaly
- We believe that the same is true for QCD

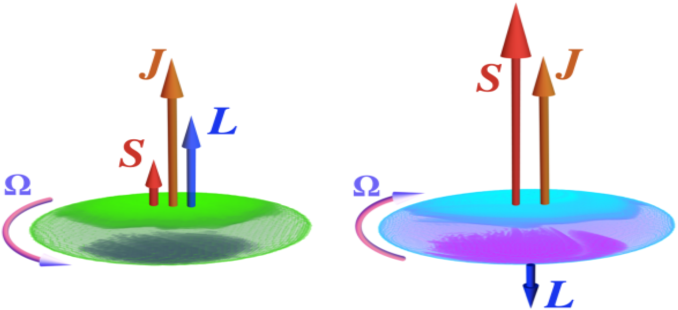
Moment of inertia of gluon plasma





- $i_2 = \frac{I_2}{V R_\perp^2}$, $\quad I_2 = I_{mech} + I_{magn}$
  $I_{mech} = \langle J_z^2 \rangle - (\langle J_z \rangle)^2$
  $I_{magn} = \frac{1}{3} \int d^3x r^2 \langle H^2 \rangle$
- $\langle G^2 \rangle_T = \langle E^2 \rangle_T + \langle H^2 \rangle_T$

Negative Barnett effect(?)



- $J = I_2 \Omega = -\left(\frac{\partial F}{\partial \Omega}\right)_T$
- $\mathbf{J} = \mathbf{L} + \mathbf{S}, \quad \mathbf{L} \parallel \boldsymbol{\Omega}$

Confinement/deconfinement phase transition

- Polyakov loop

$$L(r) = \left\langle \mathrm{Tr}\mathcal{T}\exp\left[ig\int_{[0,\beta]} A_4(r)\,dx^4\right]\right\rangle = e^{-F_q/T}$$

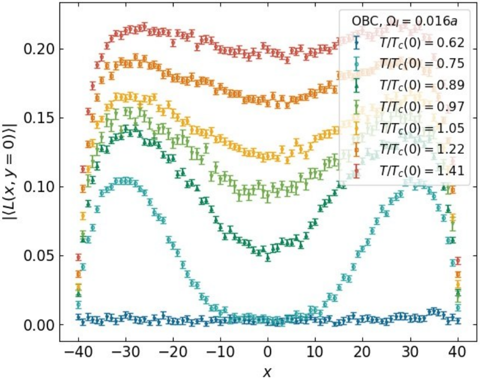
  - Confinement: $F_q = \infty \Rightarrow L = 0$
  - Deconfinement $F_q = \text{finite} \Rightarrow L \neq 0$

- Susceptibility of the Polyakov loop

$$\chi = N_s^2 N_z \left(\langle |L|^2\rangle - \langle |L|\rangle^2\right)$$

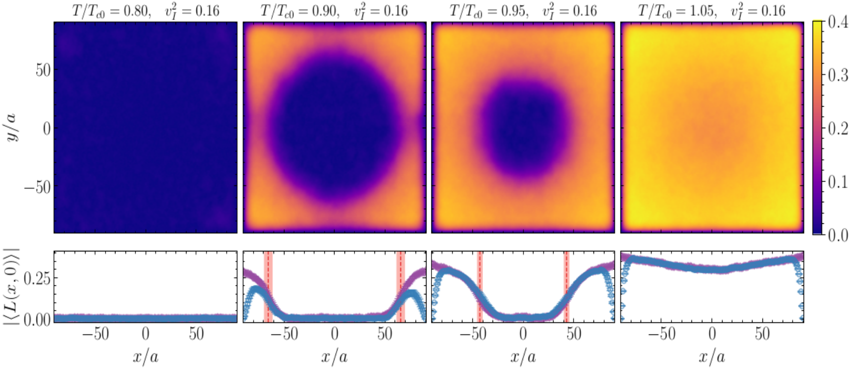
- T_c is determined from Gaussian fit of the $\chi(T)$

Inhomogeneity of Polyakov loop in rotating QGP



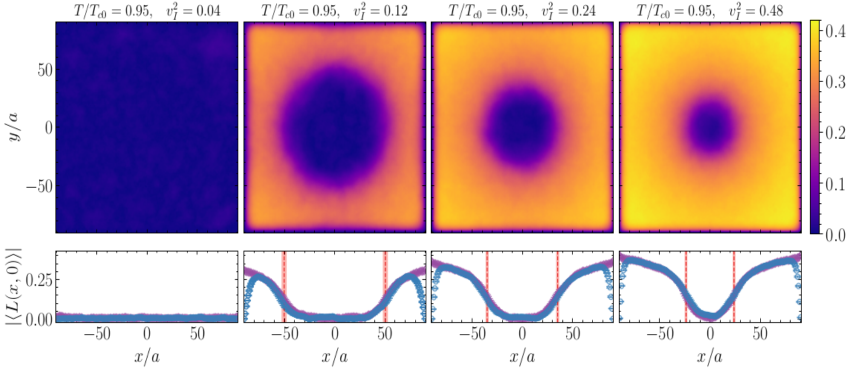
- Weak dependence at large temperatures
- No dependence at low temperatures
- Strong inhomogeneity of Polyakov loop close to $\sim T_c$

Inhomogeneous phase transitions in QGP



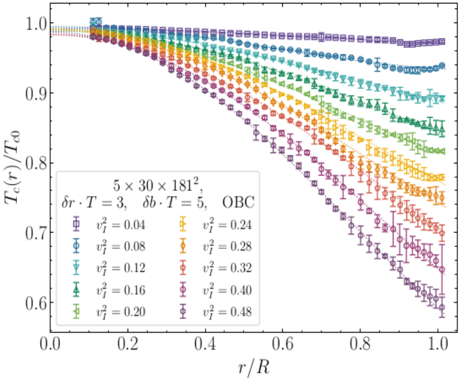
- Cylindrical Symmetry is restored
- The results for PBC and OBC coincides
- Confinement in the center and deconfinement in the periphery
- Inhomogeneous phase takes place below T_c

Inhomogeneous phase transitions in QGP



- The phase transition due to the rotation

Local critical temperature $T_c(r, \Omega_I)$

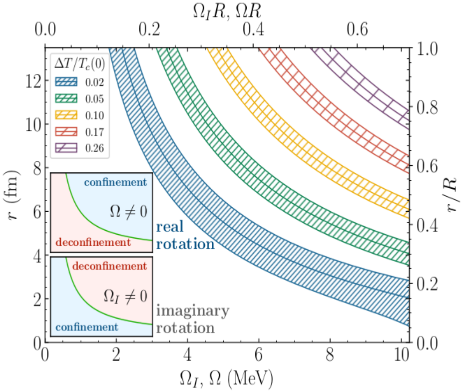


- Our results can be well described by the formula

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - \kappa_2(\Omega_I r)^2$$

- Weak dependence on the simulation parameters

Analytical continuation to real rotation



- Analytical continuation $\Omega_I^2 \to -\Omega^2$:

$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa_2(\Omega r)^2$$

- Inhomogeneous phase can be realised for $T > T_{c0}$
- Deconfinement in the center and confinement in the periphery

Decomposition of the action

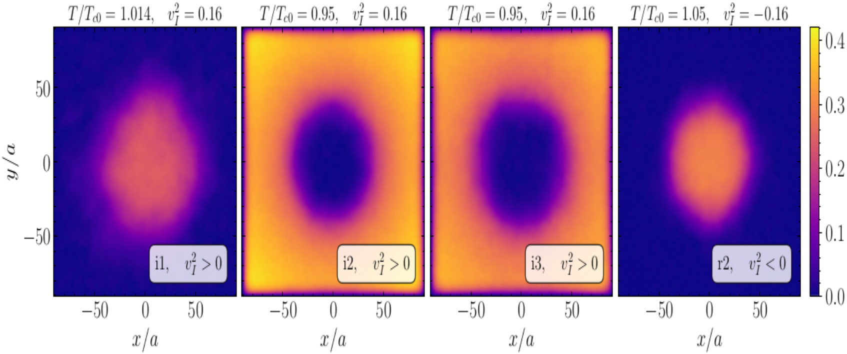
- Rotating action in the cylindrical coordinates

$$S = S_0 + S_1\Omega_I + S_2\Omega_I^2$$

  - $S_1 = -\frac{1}{g^2}\int d^4x\ r\left[F^a_{r\hat{\varphi}}F^a_{\tau r} - F^a_{\hat{\varphi}z}F^a_{\tau z}\right]$
  - $S_2 = \frac{1}{2g^2}\int d^4x\ r^2\left[(F^a_{\hat{\varphi}z})^2 + (F^a_{r\hat{\varphi}})^2\right]$
- S_1 is the total angular momentum and gives $I > 0$
- S_2 is the centrifugal force and gives $I < 0$

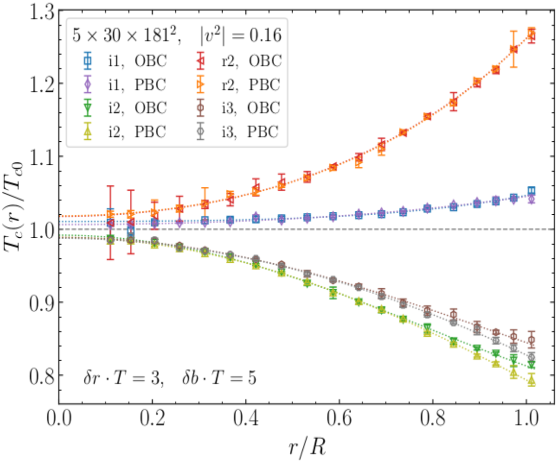
How S_1 and S_2 influence on the inhomogeneous phase transition?

Decomposition of the action



- S_2 is similar to the total acton and gives the dominant contribution
- S_1 effect is opposite to the the total acton

Decomposition of the action



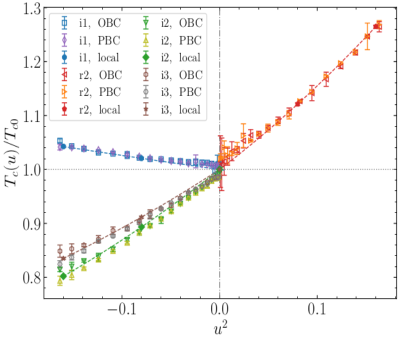
- S_1 increases the local critical temperature
- S_2 decreases the local critical temperature
- The contribution of S_2 is dominant

Local thermalization hypothesis

$$S = \frac{1}{2g^2} \int d^4x \Big[(F^a_{\tau r})^2 + (F^a_{\tau\hat\varphi})^2 + (F^a_{\tau z})^2 + (F^a_{rz})^2 + \\ + \left(1 - (\Omega r)^2\right) (F^a_{\hat\varphi z})^2 + \left(1 - (\Omega r)^2\right) (F^a_{r\hat\varphi})^2 + \\ + 2ir\Omega (F^a_{r\hat\varphi} F^a_{\tau r} - F^a_{\hat\varphi z} F^a_{\tau z}) \Big]$$

- For slow rotation $\Omega\zeta \ll 1$ the coefficients vary slowly
- Local thermalization approximation: study the action with the coefficients freezed at $r = r_0$

Local thermalization hypothesis



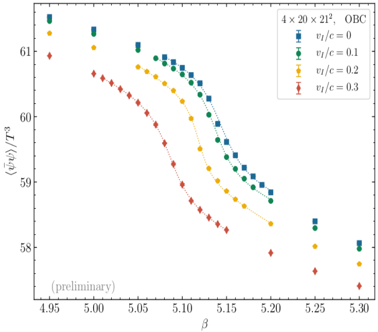
- Good agreement with the full action for sufficiently small Ω
- A lot of advantages
  - The higher order coefficients can be found $T_c(r,\Omega)/T_{c0} = 1 + \sum_n c_n(\Omega r)^{2n}$
  - Weak dependence on the BC
  - One can study small lattices
  - Allows to understand inhomogeneous phase transitions

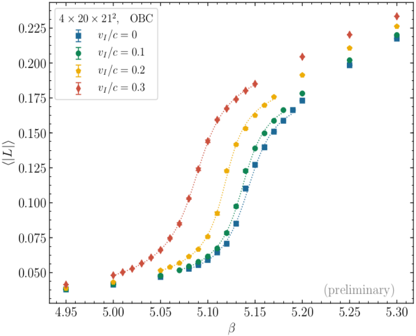
Origin of the inhomogeneous phase transitions

$$S_G = \int d^4x \left[\beta\left((F^a_{x\tau})^2 + (F^a_{y\tau})^2 + (F^a_{z\tau})^2 + (F^a_{xz})^2\right) + \right.$$
$$\left. + \tilde{\beta}\left((F^a_{yz})^2 + (F^a_{xy})^2\right)\right]$$

- Linear in Ω term can be neglected
- External gravitational field leads to the asymmetric action
  $\beta = \frac{1}{2g^2}, \quad \tilde{\beta} = \frac{1}{2\tilde{g}^2}, \quad \frac{\tilde{\beta}}{\beta} = 1 - (\Omega r)^2$
- The asymmetry $\tilde{\beta}/\beta$ is larger in the periphery region leading to the shift of the critical temperature

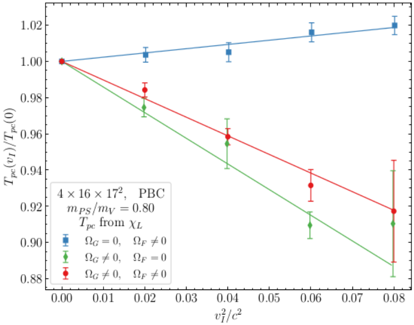
Simulation with fermions

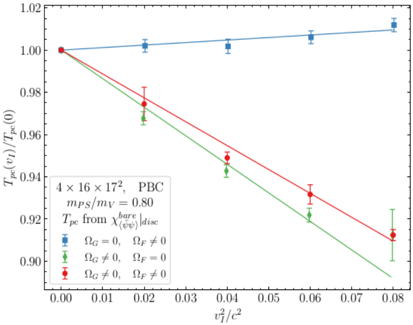




- Lattice simulation with Wilson fermions
- Critical couplings of both transitions coincide
- Critical temperatures are increased

Simulation with fermions





- QCD action: $S = S_f(\Omega_F) + S_g(\Omega_G)$
- One can introduce velocities for gluons Ω_G and fermions Ω_F
- $\Omega_F \neq 0, \Omega_G = 0$ decreases critical temperatures
- $\Omega_F = 0, \Omega_G \neq 0$ increases critical temperatures
- The gluon sector gives the dominant contribution

Conclusion

- Lattice study of rotating gluodynamics and QCD have been carried out
- We calculated the moment of inertia of GP. It is negative at temperatures $T < 1.5T_c$ and positive at larger temperatures
- We observed inhomogeneous phase transitions in GP: deconfinement in the central and confinement in the periphery regions
- External gravitational field leads to asymmetryc action and shift of the critical temperature in the periphery regions
- We believe that all observed effects remain in QCD

Conclusion

- Lattice study of rotating gluodynamics and QCD have been carried out
- We calculated the moment of inertia of GP. It is negative at temperatures $T < 1.5T_c$ and positive at larger temperatures
- We observed inhomogeneous phase transitions in GP: deconfinement in the central and confinement in the periphery regions
- External gravitational field leads to asymmetryc action and shift of the critical temperature in the periphery regions
- We believe that all observed effects remain in QCD

THANK YOU!