History of Lattice Field Theory from a Statistical Perspective

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The conceptual basis for lattice field theory was elaborated in the 1970s and 1980s, in particular by Ken Wilson. We also mention early unpublished work by Jan Smit; independent activities in the Soviet Union are reviewed in A.M. Polyakov, "A view from the island" (1992, Berezinskii).

Since the early 1990s, lattice physicists form an established, intercontinental community, now involving $\mathcal{O}(1000)$ researchers, with a large, annual Lattice Conference. Evolution from $\mathcal{O}(200)$ up to ≈ 500 participants (2020: cancelled, 2021: online, 2023: first time hybrid).

Usually considered branch of Theoretical Physics, but sometimes also interpreted as a third line of research, in addition to theory and experiment ($e.g.$ statistical and systematic errors). Five lattice textbooks so far.

- Functional integral formulation of Quantum Field Theory. Lattice UV regularization in Euclidean space-time, link to Statistical Mechanics.
- Monte Carlo simulations generate configurations $[\Phi]$ with probability $p[\Phi] \propto \exp(-S_{\mathrm{E}}[\Phi]) \;\Rightarrow\;$ non-perturbative results, $\langle \dots \rangle$.
- Gauge invariant, compact link variable $\to$ no gauge fixing needed.
- Bottleneck: fermion determinant $\to$ in 20th century often "quenched" $(\det = 1)$, works for QCD up to $\approx 15\,\%$ systematic error.
- 21st century: Dynamical quarks, $e.g.$ QCDSF Collaboration: (GB, Germany, Japan, Mexico, Russia; 2010), %-level accuracy!
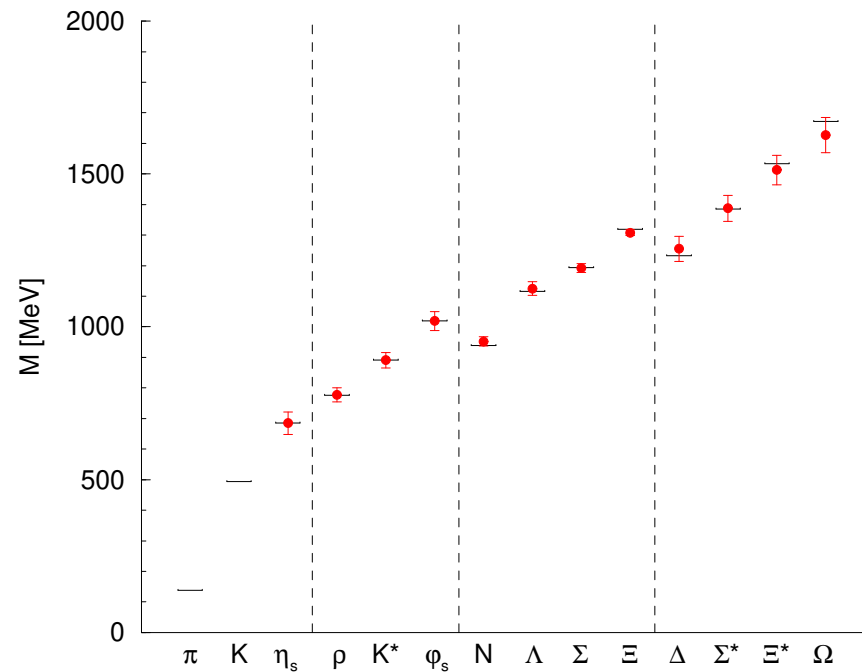
Conclusive results for light hadron spectrum, despite Wilson's pessimism in 1989: non-perturbative QCD from first principles, explains $\simeq 98\,\%$ of the mass of macroscopic objects.

Still working on excited states, $e.g.$ Roper resonance. Lattice spacing $a = 0.05 \ldots 0.1\,\mathrm{fm}$; below conceptual problems with "topological freezing".

Anyhow, and 21st century, from post-dictions to pre-dictions. Example M_{Bc} predicted by HPQCD Collaboration (2005): $6.82(8)$ GeV; measured by CDF (2006): $6.78(7)$ GeV.

However: the subject of this talk is the **historical evolution of lattice field theory** (not only QCD), based on **statistical data**.

In 1991/2, the online preprint repository `arXiv` became operational.

Just around that time the lattice community was formed, thanks to established conceptual basis and widely accessible computing facilities.

Its specific `hep-lat` section provides a unique opportunity to statistically monitor the evolution of lattice activities over 3 decades. Systematically used by lattice people.

For other physics communities, it would be hard to obtain such a statistics.

We used INSPIRE to count:

E : **Entries, all articles with primary arXiv section** `hep-lat`
P : **Papers, subset of** E **published as regular papers**
C : **Citations to all articles in** E
H : **Hirsch Index, based on** E

Comparison to the **sum over 7 related arXiv sections:** `hep-ph`, `hep-th`, `hep-lat`, `gr-qc`, `nucl-th`, `quant-ph`, `cond-mat*` (all subsections).

As a **single parameter**, we also display $\underline{\Sigma := E + P + 0.05C}$ (weights according to statistical trends).

We present <u>global</u> and <u>national</u> statistics, with <u>extensive</u> and <u>intensive</u> data ($i.e.$ <u>absolute</u> and <u>relative to the population</u>), along with three socio-economic indices:

- **GPD** (Gross Domestic Product)

- **GDPpp** (GPD per person, or capita)

- **EI** (Education Index) $:= \frac{1}{2}\left(\frac{\text{EYS}}{18} + \frac{\text{MYS}}{15}\right) \in [0, 1]$
  EYS: expected schooling years of children, normalized by duration for a Master degree
  MYS: mean schooling years of adults, normalized by the projected maximum in 2025

[Source: United Nations Development Programme, http://hdr.undp.org/en/]

$\text{EI} \simeq \text{HDI}$, Human Development Index: health, income, education.
(Data for "skilled labour force" are messy).

• National statistics: An article counts for a country if at least one author has a working address there (authors' origin and nationality don't matter).

One article can count for several countries (tough typically small collaborations).

Data were taken in July 2020 from `https://old.inspirehep.net`; this INSPIRE version is dis-activated now, unfortunately.

[The new version gives with "date" much larger numbers, apparently due to multiple counting (date when submitted, revised, published).

The command "de" ("date earliest") is supposed to avoid this, but it returns weird results, which are much too low ($e.g.$ just one `hep-lat` entry in 2019).

Moreover, the new INSPIRE version does not have the option "country code", hence it does not provide national statistics.]
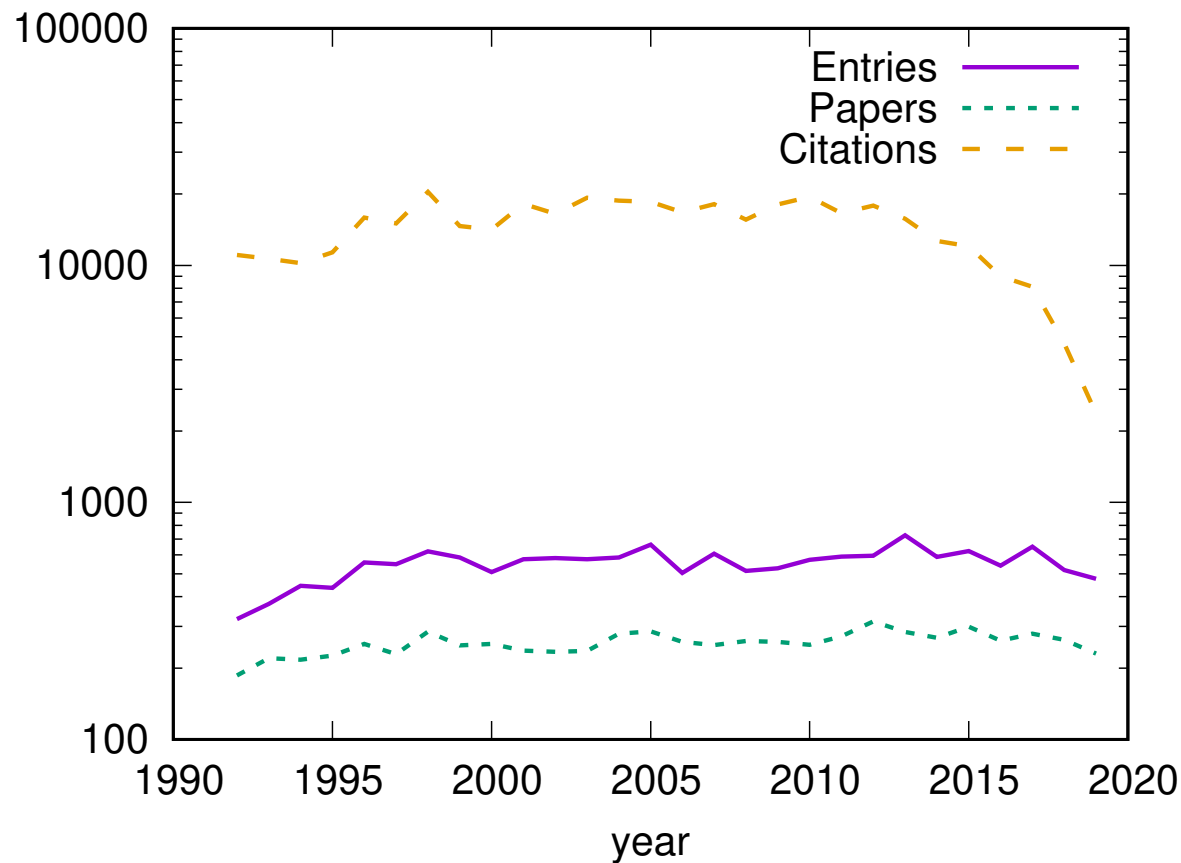
Evolution of `hep-lat` **entries** (E)**, papers** (P) **and citations** (C) **from 1992 to 2019.** Here we exclude 1991 (only few sporadic entries), and 2020 (we could only capture the first half). About 10 entries and 4 papers per week. (Late decrease of C: these articles had less time to be cited.)
E, P : **stagnation since about 1998**
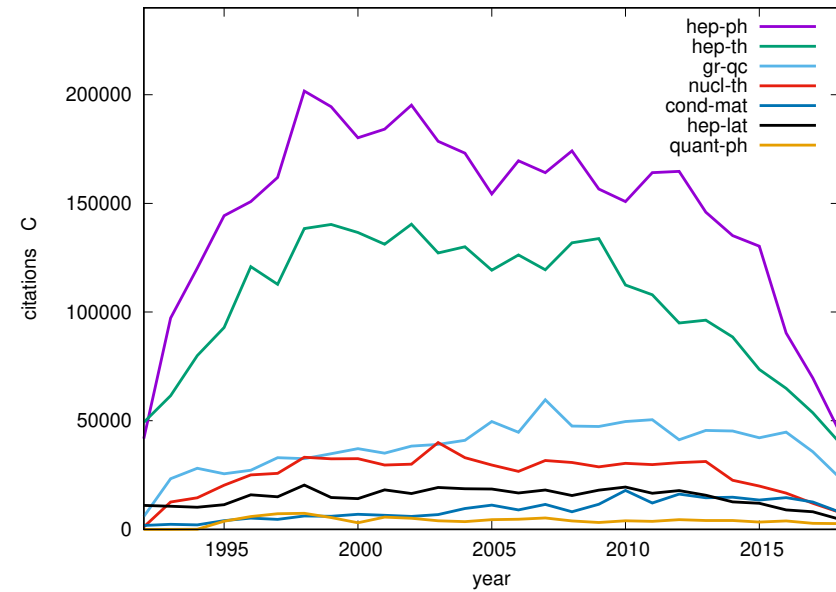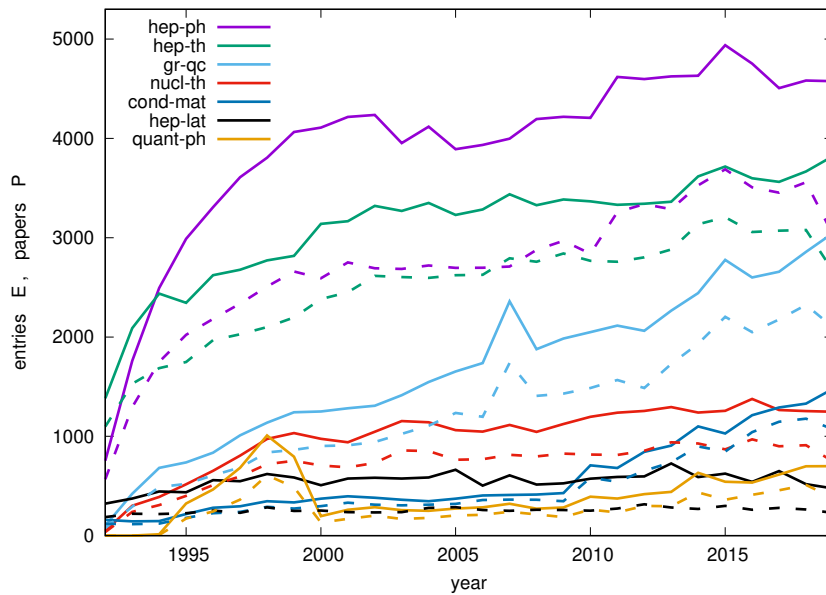
Evolution of E [solid] and P [dashed], 1992 to 2019 (left); and of C, 1992 to 2018, in hep-ph, hep-th, gr-qc, nucl-th, cond-mat, **hep-lat**, quant-ph. **In other sections, E and P have a stronger trend up.**

Total from 1991 to July 2020 (note for `hep-lat`: $P < E/2$) :

	hep-ph	hep-th	gr-qc	nucl-th	cond-mat	hep-lat	quant-ph
E	111515	89279	48927	28522	16969	**15610**	11602
P	76520	70561	35703	20215	13677	**7165**	7484
C	3960720	2857462	1043823	682874	247734	**402121**	106901
P/E	0.69	0.79	0.71	0.71	0.81	**0.46**	0.65
C/E	35.5	32.0	21.3	23.9	14.6	**25.8**	9.21
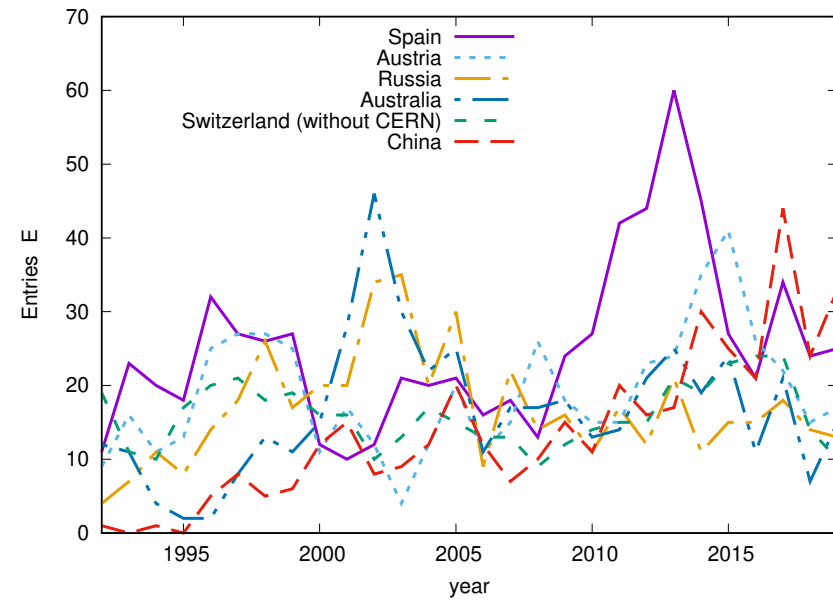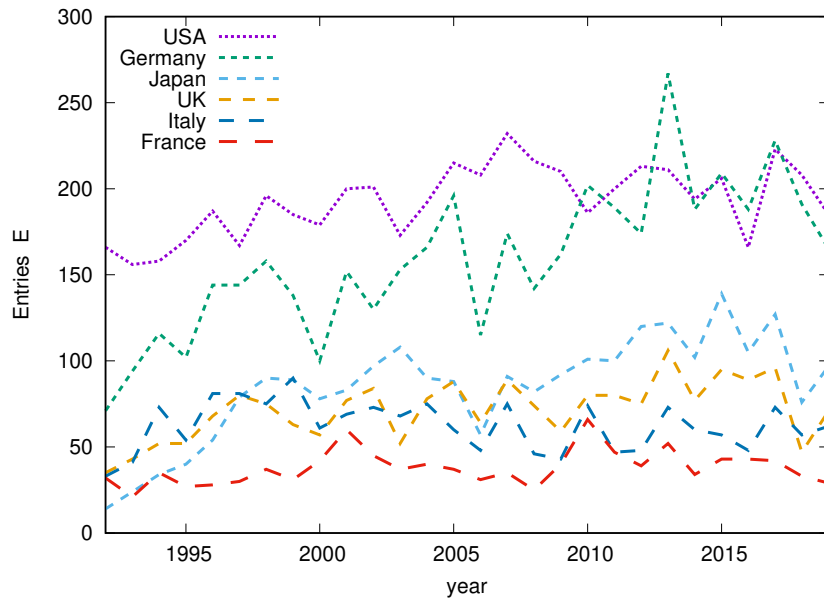
Evolution of the number of `hep-lat` entries (E) of the 6 leading nations (left) and of the nations ranking $7 \ldots 12$ (right).

Since 2010, Germany caught up with the USA as the most productive countries, followed by Japan, UK, Italy, France, . . . cf. Table 1.

Strong peaks for Spain and Austria. Lately China is moving up.

Table 1: Extensive `hep-lat statistics`: ranking according to the Hirsch Index H for all nations with $H > 8$, which is very similar to the Σ-rank based on $\Sigma = E + P + 0.05C$. The ranks refer to nations only (in case of an identical H, Σ decides). Population in millions of inhabitants, Gross Domestic Product (GDP) in 10^9 US \$ (purchasing parity to its value in 2011), both averaged over the years $1992 - 2019$.
The European Union is shown for comparison, with 28 nations, still including the UK. The data for Switzerland exclude CERN.

	Σ-rank	population	GDP	E	P	C	Σ	H
European Union		495.0	15900.4	9020	4265	249236	**25746.8**	**182**
1. USA	1	294.3	14050.1	5472	2565	175639	**16818.9**	**167**
2. Germany	2	81.3	3167.4	4523	2107	143667	**13813.3**	**155**
3. UK	3	61.4	2136.2	2026	950	67386	**6345.3**	**121**
4. Japan	4	127.6	4458.1	2419	1078	55729	**6283.5**	**103**
5. France	6	61.1	2232.9	1070	540	42060	**3713.0**	**101**
6. Italy	5	58.5	2066.6	1765	877	41252	**4704.6**	**89**
CERN				615	306	27553	**2298.7**	**79**
7. Switzerland	7	7.5	400.7	614	295	24073	**2112.7**	**75**
8. Spain	8	43.6	1326.8	708	370	19310	**2043.5**	**71**
9. Australia	9	18.9	801.9	471	250	14208	**1431.4**	**64**
10. Hungary	10	10.1	208.5	292	151	18824	**1384.2**	**61**
11. Cyprus	14	1.0	23.1	389	193	8794	**1021.7**	**54**
12. Austria	11	8.3	333.8	534	254	10300	**1303.0**	**51**

	Σ-rank	population	GDP	E	P	C	Σ	H
13. China	13	1326.5	9638.8	415	223	9708	**1123.4**	**50**
14. Denmark	15	5.4	233.1	322	174	10188	**1005.4**	**50**
15. Russia	12	145.5	2822.8	475	252	8608	**1157.4**	**48**
16. Canada	17	32.5	1269.4	325	164	8387	**908.3**	**46**
17. India	16	1142.0	4509.3	316	183	8198	**908.9**	**45**
18. Ireland	18	4.2	183.7	292	117	8565	**837.3**	**45**
19. Taiwan	20	22.5	702.2	196	100	5491	**570.6**	**40**
20. Finland	23	5.3	191.6	165	81	4460	**469.0**	**36**
21. Netherlands	19	16.3	696.5	203	98	4544	**528.2**	**35**
22. Israel	22	6.6	197.8	149	96	4622	**476.1**	**35**
23. South Korea	21	48.3	1242.6	238	82	4448	**542.4**	**34**
24. Poland	24	38.3	696.8	181	82	3192	**422.6**	**30**
25. Brazil	25	184.4	2397.1	117	58	3939	**372.0**	**29**
26. Sweden	27	9.2	365.9	99	50	2162	**257.1**	**27**
27. Slovenia	30	2.0	51.1	67	33	2174	**208.7**	**26**
28. Portugal	26	10.4	264.0	138	55	2252	**305.6**	**25**
29. Slovakia	28	5.4	111.5	82	42	2354	**241.7**	**25**
30. Greece	29	10.9	273.9	96	49	1756	**232.8**	**22**
31. Belgium	31	10.7	409.5	54	28	1792	**171.6**	**20**
32. Mexico	33	106.6	1710.8	58	26	760	**122.0**	**15**
33. Ukraine	32	47.5	319.9	85	34	417	**139.9**	**11**
34. Turkey	34	68.2	1192.0	25	19	347	**61.3**	**10**

Table 2: Like Table 1, but joint statistics for the 7 arXiv sections under consideration, for all nations with H > 62.

	Σ-rank	E	P	C	Σ	H
1. USA	1	85601	63422	4006414	**349343.7**	**583**
European Union		147464	106804	4961069	**502321.4**	**548**
2. France	3	28598	21090	1368170	**118096.5**	**394**
3. Germany	2	45242	35533	1713406	**166445.3**	**390**
4. UK	4	26174	19927	1089933	**100597.7**	**339**
CERN		9836	7253	682730	**51225.5**	**333**
5. Italy	5	27034	19875	949801	**94399.1**	**309**
6. Spain	9	16466	12223	601111	**58744.6**	**267**
7. Russia	7	22429	15260	586236	**67000.8**	**257**
8. Canada	10	12364	9696	469849	**45552.5**	**250**
9. Japan	6	24611	18620	690235	**77742.8**	**249**
10. Switzerland	14	5734	4142	290056	**24378.8**	**227**
11. Netherlands	16	5634	4307	262786	**23080.3**	**207**
12. Poland	13	8205	5660	245108	**26120.4**	**191**
13. China	8	20475	16372	444642	**59079.1**	**186**
14. India	11	14374	10925	332859	**41942.0**	**186**
15. Sweden	18	4973	3753	198321	**18642.1**	**178**
16. Belgium	17	4955	3798	201122	**18809.1**	**171**
17. Israel	19	5085	3943	176081	**17832.1**	**166**
18. Brazil	12	11460	8961	222670	**31554.5**	**146**
19. South Korea	15	7627	6096	196954	**23570.7**	**142**

	Σ-rank	E	P	C	Σ	H
20. Portugal	22	4166	3087	128890	**13697.5**	**139**
21. Austria	23	3885	2563	112541	**12075.1**	**137**
22. Greece	24	3505	2748	107266	**11616.3**	**134**
23. Australia	20	4185	3219	136125	**14210.3**	**133**
24. Denmark	27	3006	2248	103328	**10420.4**	**133**
25. Taiwan	21	4110	3186	134243	**14008.2**	**129**
26. Finland	29	2460	1818	86101	**8583.1**	**128**
27. Hungary	30	2274	1586	88347	**8277.4**	**121**
28. Chile	28	3089	2567	77183	**9515.2**	**108**
29. Argentina	31	2576	2179	63899	**7950.0**	**104**
30. Mexico	25	4450	3232	75687	**11466.4**	**101**
31. Ireland	36	1476	1078	43298	**4718.9**	**97**
32. Iran	26	4054	3398	64526	**10678.3**	**89**
33. South Africa	34	1903	1483	44556	**5613.8**	**85**
34. Slovenia	41	718	484	28219	**2613.0**	**85**
35. Norway	37	1242	923	35198	**3924.9**	**84**
36. Ukraine	33	2508	1674	42411	**6302.6**	**82**
37. Czech Rep.	35	2106	1501	37754	**5494.7**	**78**
38. Bulgaria	40	1040	684	24849	**2966.5**	**73**
39. Croatia	38	1035	778	25216	**3073.8**	**71**
40. Estonia	47	425	336	20056	**1763.8**	**71**

Based on the Σ-rank, Turkey (32) and Romania (39) enter the top 40.
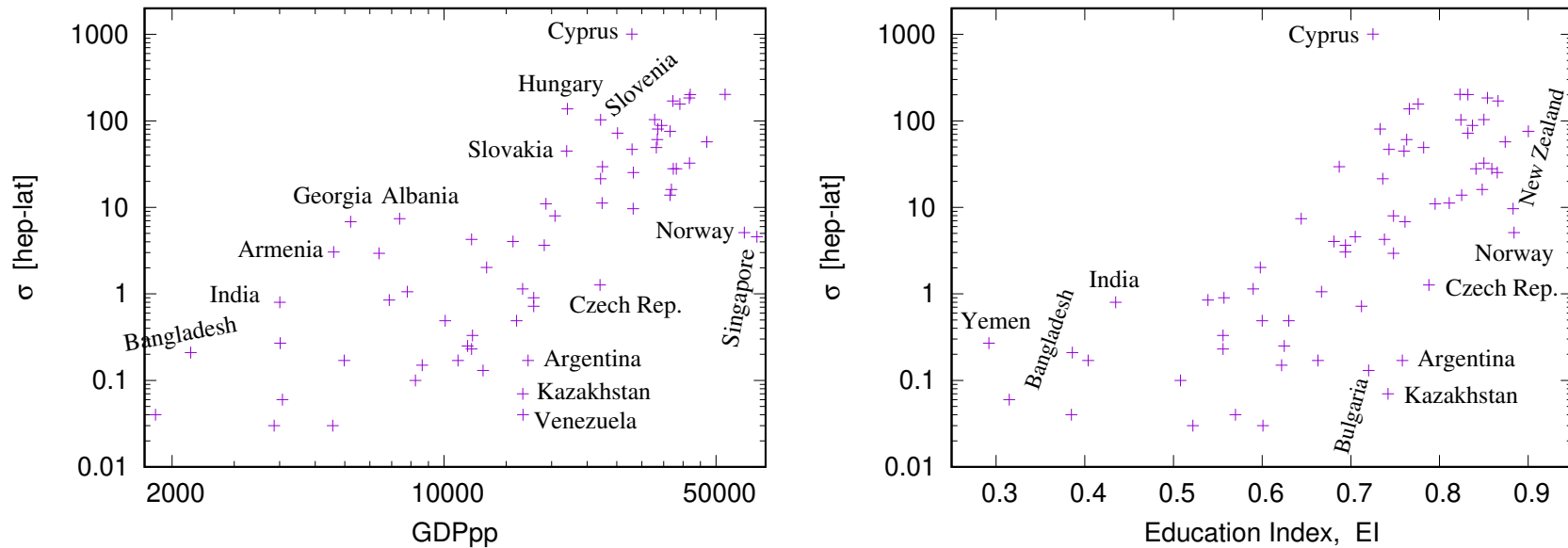
Intensive Statistics

Figure 1: Scatter plot for the 66 nations which contributed hep-lat entries.
We show $\sigma := \Sigma/\text{pop}$ (pop = population in millions) vs. GDPpp (left),
and vs. EI (right). A monotonic tend is visible, but not as clearly as one
might expect. The top 36 nations are included in Table 3.
Nations with σ-values higher than economically expected: Cyprus, Hungary,
Georgia (very high); Slovenia, Slovakia, Albania, Armenia (high).
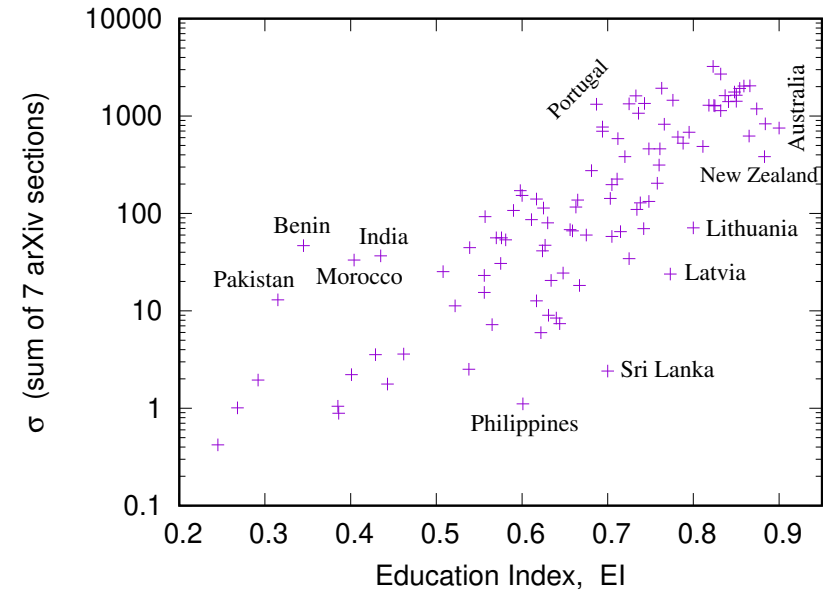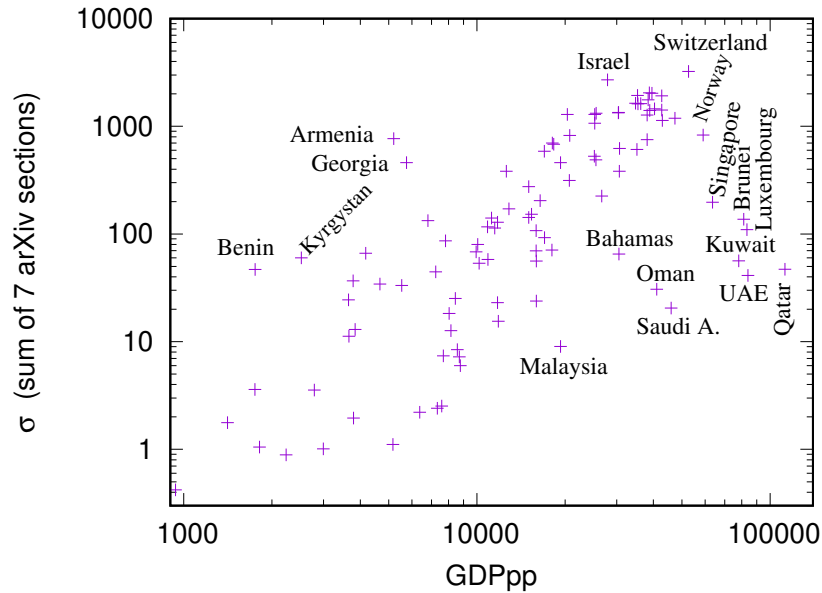
Figure 2: Like Fig. 1, but for the <u>7 arXivs</u>. The plot captures the 100 nations with $H \geq 5$, the leading 40 appear Table 4 below. The monotonic trend is somewhat clearer than in Fig. 1, which is more specific.

High (low) σ-values w.r.t. GDPpp: Armenia, Georgia, Kyrgyzstan, Benin (Brunei, Luxembourg, Kuwait, UAE, Qatar, Saudi Arabia).

High (low) σ-values w.r.t. EI: Portugal, Benin, Morocco, India, Pakistan (New Zealand, Lithuania, Latvia, Sri Lanka, Philippines).

[descending order in σ]

16

Table 3: Intensive `hep-lat` statistics, ordered according to $\sigma = \Sigma/\text{pop}$. We include further intensive quantities: GDPpp, EI, $(e,p,c) := (E,P,C)/\text{pop}$. Ranking according to σ, for all nations with $\sigma > 3.7$.
We also display the economic rank, e-rank, based on $\Sigma/\text{GDP} \propto \sigma/\text{GPDpp}$. In that regard, Armenia (28) and Ukraine (29) would enter the top 36.

	e-rank	GDPpp	EI	e	p	c	Σ/GDP	σ
1. Cyprus	1	30386	0.725	383.32	190.18	8665.6	**44.17**	**1006.78**
2. Switzerland	3	52698	0.823	81.46	39.14	3194.0	**5.27**	**280.30**
3. Ireland	4	42903	0.832	70.27	28.16	2061.1	**4.56**	**201.48**
4. Denmark	6	42671	0.854	59.22	32.00	1873.8	**4.31**	**184.02**
5. Germany	5	38719	0.866	55.60	25.90	1766.1	**4.36**	**169.81**
6. Austria	8	40348	0.776	64.31	30.59	1240.4	**3.90**	**156.92**
7. Hungary	2	20738	0.766	29.04	15.02	1872.0	**6.64**	**137.66**
8. UK	9	34770	0.850	32.99	15.47	1097.4	**2.97**	**103.33**
9. Slovenia	7	25235	0.824	33.01	16.26	1071.1	**4.08**	**102.83**
10. Finland	10	36205	0.837	31.18	15.30	842.7	**2.45**	**88.61**
11. Italy	12	35411	0.733	30.19	15.00	705.6	**2.28**	**80.47**
12. Australia	14	38067	0.900	24.95	13.25	752.8	**1.79**	**75.84**
13. Israel	11	27906	0.832	22.58	14.55	700.3	**2.41**	**72.14**
14. France	15	35303	0.763	17.52	8.42	688.8	**1.66**	**60.80**
15. USA	19	47324	0.874	18.59	8.72	596.9	**1.20**	**57.15**
European Union		31966	0.792	18.22	8.62	503.5	**1.62**	**52.01**
16. Japan	17	35125	0.782	18.96	8.45	436.8	**1.41**	**49.25**

	e-rank	GDPpp	EI	e	p	c	Σ/GDP	σ
17. Spain	16	30406	0.743	16.24	8.48	442.8	**1.54**	**46.86**
18. Slovakia	13	20657	0.760	15.19	7.78	435.9	**2.17**	**44.76**
19. Netherlands	24	42689	0.850	12.48	6.02	279.3	**0.76**	**32.46**
20. Portugal	20	25515	0.687	13.33	5.31	217.5	**1.16**	**29.52**
21. Canada	25	38828	0.841	10.01	5.05	258.4	**0.72**	**27.98**
22. Sweden	26	39526	0.859	10.77	5.44	235.3	**0.70**	**27.98**
23. Taiwan	23	30654	0.865	8.70	4.44	243.7	**0.81**	**25.33**
24. Greece	22	25240	0.736	8.84	4.51	161.7	**0.85**	**21.43**
25. Belgium	31	38398	0.848	5.07	2.63	168.1	**0.42**	**16.10**
26. Iceland	34	38087	0.825	3.33	3.33	143.3	**0.36**	**13.83**
27. South Korea	30	25473	0.811	4.93	1.70	92.1	**0.44**	**11.24**
28. Poland	27	18267	0.795	4.72	2.14	83.3	**0.61**	**11.02**
29. New Zealand	35	30601	0.883	2.66	1.93	101.4	**0.31**	**9.66**
30. Russia	32	19274	0.748	3.27	1.73	59.2	**0.41**	**7.96**
31. Albania	21	7683	0.644	4.96	0.66	35.4	**0.98**	**7.39**
32. Georgia	18	5752	0.761	2.52	1.60	54.0	**1.32**	**6.82**
33. Norway	43	59066	0.884	2.96	1.27	17.6	**0.09**	**5.11**
34. Singapore	46	63641	0.705	2.22	1.11	25.0	**0.07**	**4.58**
35. Belarus	33	11763	0.738	0.93	0.62	54.4	**0.36**	**4.27**
36. Uruguay	36	15032	0.681	2.11	1.51	8.1	**0.27**	**4.03**

Table 4: Like Table 3, but joint statistics for the 7 arXiv sections.

		e-rank	e	p	c	Σ/GDP	σ
1.	Switzerland	5	760.78	549.55	38484.1	**60.84**	**3234.53**
2.	Israel	2	770.45	597.42	26678.9	**90.16**	**2701.83**
3.	Germany	8	556.15	436.80	21062.6	**52.55**	**2046.09**
4.	Sweden	11	541.20	408.43	21582.7	**50.95**	**2028.76**
5.	France	7	468.31	345.36	22404.5	**52.89**	**1933.89**
6.	Denmark	16	552.87	413.46	19004.5	**44.70**	**1916.56**
7.	Belgium	13	464.85	356.31	18868.3	**45.93**	**1764.58**
8.	United Kingdom	12	426.24	324.50	17749.2	**47.09**	**1638.20**
9.	Finland	15	464.80	343.50	16268.2	**44.79**	**1621.71**
10.	Italy	14	462.38	399.94	16245.2	**45.68**	**1614.58**
11.	Austria	27	467.86	308.66	13553.1	**30.09**	**1454.18**
12.	Netherlands	24	346.27	264.71	16151.2	**33.14**	**1418.55**
13.	Canada	22	380.91	298.71	14475.0	**35.89**	**1403.37**
14.	Spain	17	377.60	280.30	13784.6	**44.28**	**1347.12**
15.	Cyprus	6	489.74	267.04	11504.6	**58.43**	**1332.02**
16.	Portugal	9	402.44	298.21	12450.9	**51.88**	**1323.19**
17.	Estonia	4	310.98	245.85	14675.1	**64.03**	**1290.59**
18.	Slovenia	10	353.76	238.47	13903.5	**51.14**	**1287.40**
19.	Iceland	25	476.67	380.00	8336.7	**33.02**	**1273.50**
20.	United States	30	290.89	215.52	13614.6	**24.86**	**1187.14**
21.	Ireland	29	355.19	259.41	10419.3	**25.69**	**1135.56**
22.	Greece	18	322.66	252.97	9874.5	**42.41**	**1069.35**

	e-rank	e	p	c	Σ/GDP	σ
European Union		297.91	215.77	10022.5	**31.59**	**1014.81**
23. Norway	42	262.81	195.31	7447.9	**14.00**	**830.50**
24. Hungary	19	226.14	157.72	8785.9	**39.70**	**823.16**
25. Armenia	1	274.94	209.09	5706.5	**150.46**	**769.35**
26. Australia	39	221.73	170.55	7212.3	**17.72**	**752.90**
27. Croatia	20	235.43	176.97	5735.7	**39.12**	**699.18**
28. Poland	21	214.02	147.64	6393.5	**37.49**	**681.34**
29. Taiwan	34	182.44	141.43	5959.0	**19.95**	**621.82**
30. Japan	40	192.89	145.94	5409.8	**17.44**	**609.32**
31. Chile	23	190.64	158.42	4763.3	**33.86**	**587.22**
32. Czech Rep.	33	201.85	143.87	3618.6	**21.02**	**526.65**
33. South Korea	36	158.01	126.29	4080.2	**18.97**	**488.31**
34. Russia	31	154.18	104.90	4029.9	**23.74**	**460.58**
35. Georgia	3	158.70	117.25	3671.7	**88.78**	**459.54**

(We exclude tiny nations with $< 50\,000$ inhabitants, in particular Monaco and the Vatican).

Strong deviations between the rankings w.r.t economy or population, $i.e.$ e-rank or σ, for Georgia, Armenia, Estonia (up); Norway, Austria, Australia, Netherlands (down). According to the e-rank, Bulgaria (26), Benin (28), Kyrgyzstan (32), Ukraine (35) are among the top 35.

Summary and Highlights

We analyzed the `hep-lat` articles (primary section) in the period from 1992 - 2020.

On average: 10.4 entries per week (stagnation since ≈ 1998), 25.8 times cited, 46 % get published as regular papers (below other arXiv sections, peculiar role of contributions annual Lattice Conference proceedings).

66 countries contributed (out of 195)

Extensive national statistics

- Top ten in `hep-lat` (based on Hirsch Index)
  USA, Germany, UK, Japan, France, Italy, Switzerland (without CERN),
  Spain, Australia, Hungary (China: place 13, Russia: 15).
  Since 2010, Germany caught up with the USA.

- For comparison: summed over 7 related arXiv sections
  USA, France, Germany, UK, Italy, Spain, Russia, Canada, Japan, Switzerland
  (China: place 13)

Intensive statistics (relative to population)

- Top ten in `hep-lat`
  Cyprus, Switzerland, Ireland, Denmark, Germany, Austria, Hungary, UK, Slovenia, Finland (USA: place 15, Russia: place 30)

- Summed over 7 arXiv sections
  Switzerland, Israel, Germany, Sweden, France, Denmark, Belgium, UK, Finland, Italy (USA: place 20, Russia: place 34)

★ Best performance in `hep-lat`
  with respect to GDPpp: Cyprus, Hungary, Georgia
  and Education Index: Cyprus, India, Yemen

  7 arXiv sections, regarding GDPpp: Armenia, Georgia, Kyrgyzstan, Benin
  regarding Education Index: Portugal, Benin, Morocco, India, Pakistan