Loop Quantum Gravity & Quantum Information

Eugenio Bianchi

Institute for Gravitation & the Cosmos

The Pennsylvania State University

XXXV International Workshop on High Energy Physics

Logunov Institute for High Energy Physics

December 1st, 2023

THE QUANTUM INFORMATION STRUCTURE OF SPACETIME

$|\psi\rangle = |$  $\rangle$

Quantum Gravity – Question:

* What is the quantum nature of spacetime geometry?

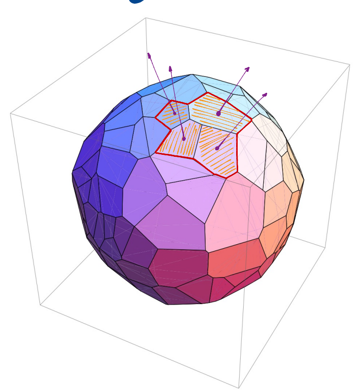
Quantum Information – Tools:

* Entanglement entropy $S_R(|\psi\rangle)$ of a spacetime region.

- d.o.f.? Entanglement of observables vs gauge-dependent quantities
- region? Subsystem but $\mathcal{H} \neq \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$
- locality? Adjacency of two regions encoded in $|\psi\rangle$
- area law? $S_R(|\psi\rangle) = \dfrac{\text{Area}(\partial R)}{4G\hbar} + \ldots$ under what conditions on R and $|\psi\rangle$

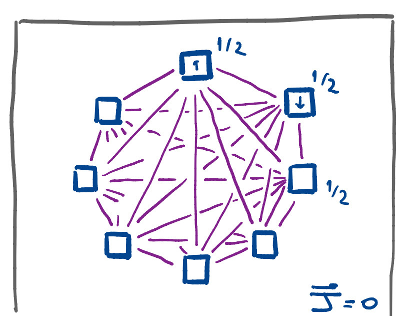
Model: 2d quantum geometry on S^2 (boundary of a node in loop quantum gravity)

Quantum Polyhedron



$\longrightarrow$

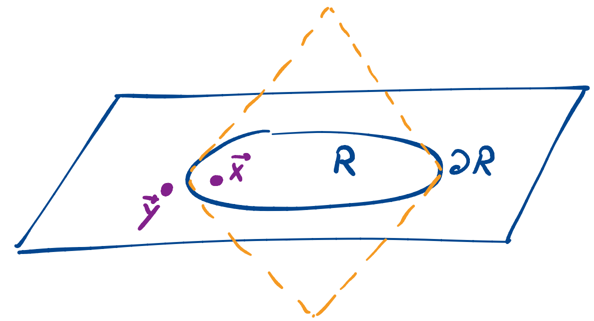
$\longleftarrow$



Spin System (Random Heisenberg)

Area Law & QFT on a Classical Spacetime

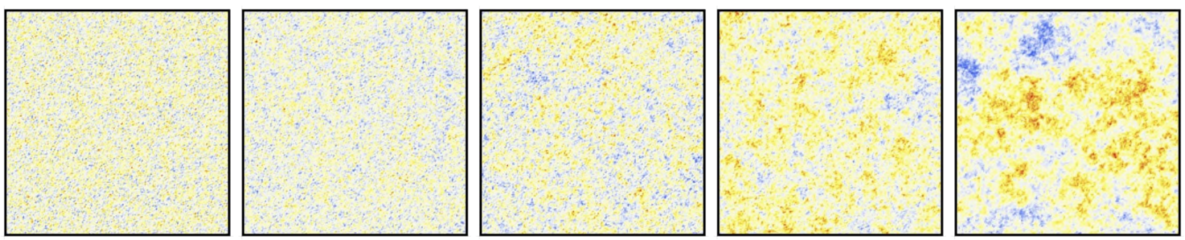
Quantum Field on Minkowski Spacetime



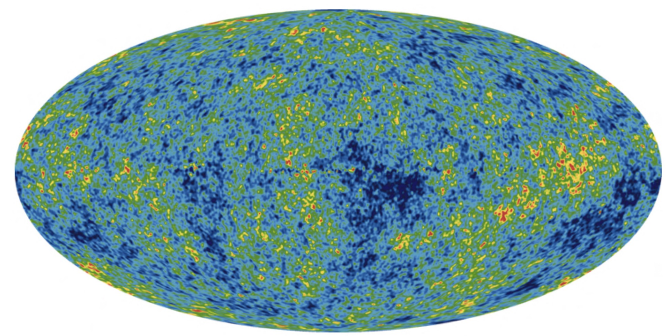
- Vacuum Correlations $\langle 0 | \varphi(\vec{x},t)\, \varphi(\vec{y},t) | 0 \rangle \underset{|\vec{x}-\vec{y}| \ll \ell}{\sim} \dfrac{1}{|\vec{x}-\vec{y}|^2}$
- Robust property (Hadamard Condition)

Cosmological Perturbations

$$\hat{g}_{\mu\nu}(\vec{x},t) = g^{FLRW}_{\mu\nu}(t) + \hat{h}_{\mu\nu}(\vec{x},t)$$



$\sim\!\!\rightarrow$



$\longrightarrow$ t

$\sqrt{P(k)} \sim \sqrt{\frac{1}{k}k^3}$

$\sqrt{P(k_*)} \sim \sqrt{\dfrac{G\hbar H_*^2}{N_*^2}} \sim 10^{-5}$

$\dfrac{\delta T}{T} \sim 10^{-5}$

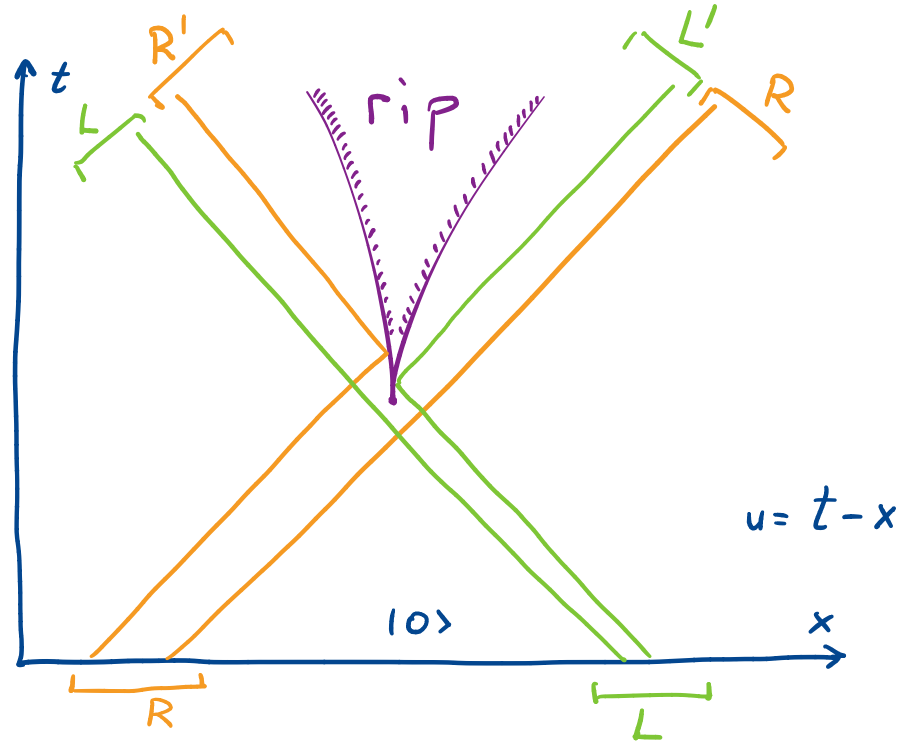
Entanglement Entropy of $|\Psi\rangle$ restricted to R [Sorkin 1983]

$$S_R(|\Psi\rangle) = \frac{\mathrm{Area}(\partial R)}{\varepsilon^2} + \dots$$

Area Law Divergence

Zero Law & Ripping Apart Spacetime

- Quantum Field in Minkowski Spacetime
- Initial State $|0\rangle$, vacuum $(1+1,\ m=0)$



- Rip in space $\Rightarrow$ Large energy flux

  * large gravitational backreaction

$$G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle_r$$

[Anderson & DeWitt, Found. Phys. (1986) "Does the topology of space fluctuate?"]

- Energy-Flux / Entanglement Entropy Relation

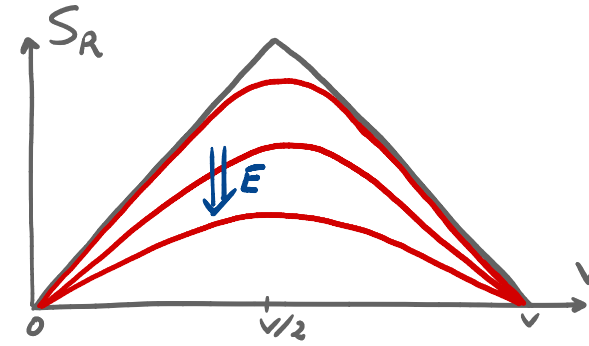
$$F(u) = \frac{\hbar}{2\pi} \left(6\, \dot{S}^2(u) + \ddot{S}(u) \right)$$

[EB & Smerlak, PRD (2014)]

- Ripping apart the entanglement structure of a quantum field cuts out the causal domain to the future of the rip

Area Law & Semiclassicality

- In CM & QFT, as we lower the energy, we transition from volume-law to area-law
- In CM, zero law states are high-energy (not Fock in QFT)

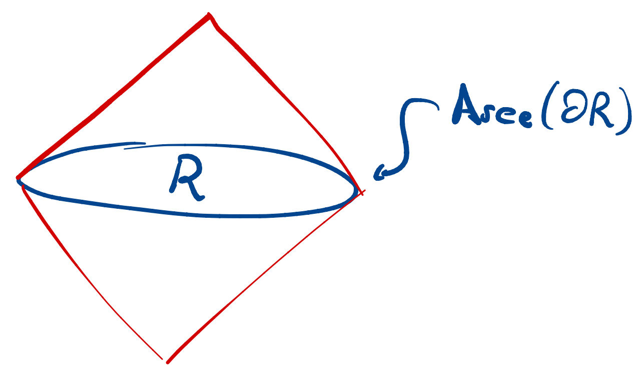


- In QG, we don't have an immediate notion of energy or energy-density

$\Longrightarrow$ Reverse Perspective: Entanglement as a Probe

Architecture Conjecture

> Semiclassical $|\psi\rangle$ in QG belong to the area-law corner of $\mathcal{H}_{phys}$
>
> $$S_R(|\psi\rangle) = 2\pi \frac{\langle Area(\partial R)\rangle}{\ell_P{}^2} + \dots$$

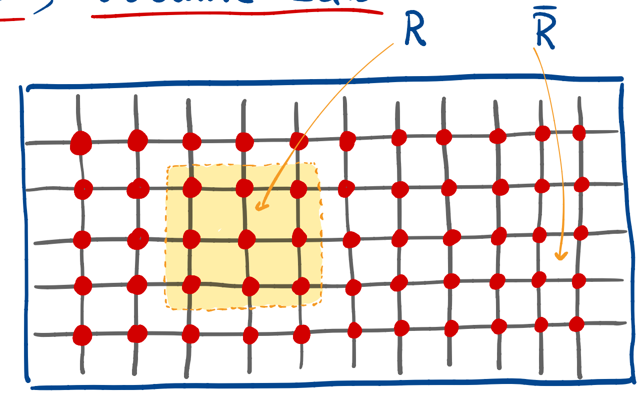


Volume Law and Zero Law States are genuine quantum geometries far from | classical spacetime + quantum perturbation

Bianchi - Myers [1212.5183] (CQG)
Bianchi - Guglielmon - Hackl - Yokomizo [1609.02219] (PRD)
Baytas - Bianchi - Yokomizo [1805.05856] (PRD)
Bianchi - Dona - Vilensky [1812.10996] (PRD)

Hierarchy of States: Zero-Law, Area-Law, Volume Law

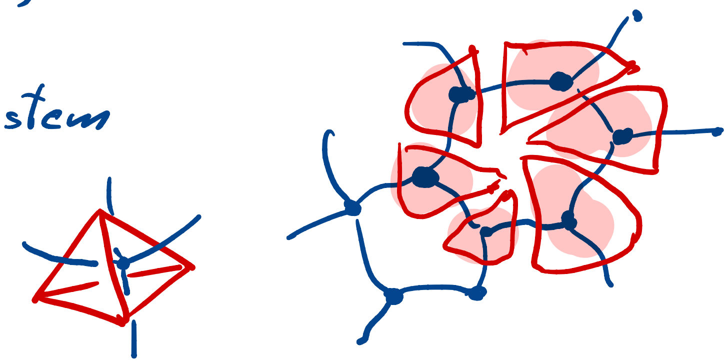
- Model System: qubits in 3d cubic lattice



$$|\psi\rangle \in \mathcal{H} = \bigotimes_{n=1}^{N} \mathcal{H}_n = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$$

$$S_R(|\psi\rangle) = -\mathrm{Tr}_R(\rho_R \log \rho_R) \,, \quad \rho_R = \mathrm{Tr}_{\bar{R}}(|\psi\rangle\langle\psi|)$$

* Cf: Loop Quantum Gravity as a many body system
SU(2) lattice



a Volume-Law States $\quad S_R(|\psi\rangle) = \gamma\, V_R + \dots$

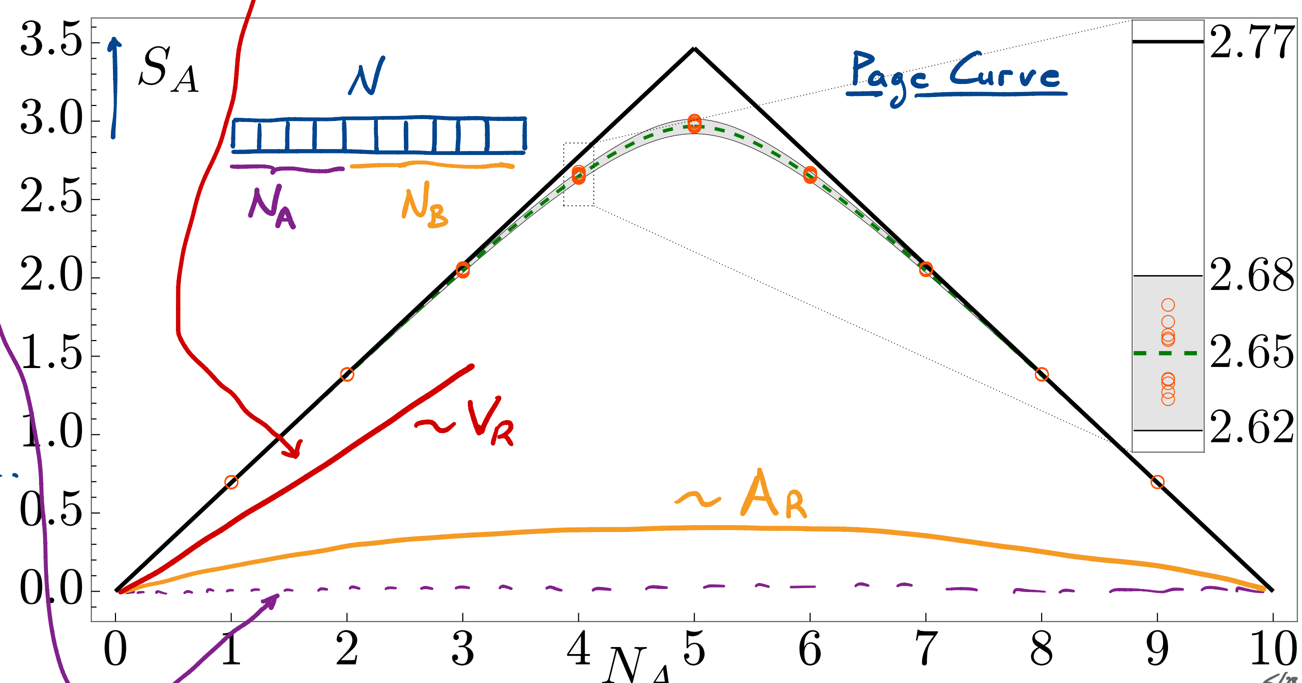
- Random States
- High Temperature

b Zero-Law States $\quad S_R(|\psi\rangle) = c_0 + \dots$

- QG basis states
- High Energy (no T)

c Area-Law States $\quad S_R(|\psi\rangle) = \alpha A_R + \dots$

- Ground State of Local H
- Long-Range Correlations



Bianchi-Livine [2302.05922] (QG Handbook)

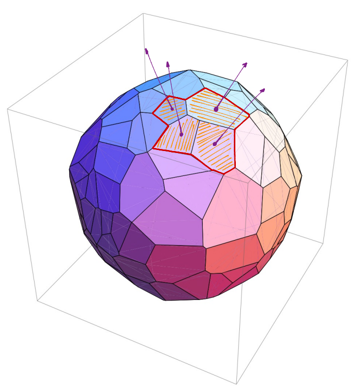
□ Entanglement entropy $S_R(|\psi\rangle)$ of a spacetime region in QG

Questions:

- d.o.f. ? Entanglement of observables vs gauge-dependent quantities
- region ? Subsystem but $\mathcal{H} \neq \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$
- locality ? Adjacency of two regions encoded in $|\psi\rangle$
- area law ? $S_R(|\psi\rangle) = \dfrac{\text{Area}(\partial R)}{4G\hbar} + \dots$ under what conditions on R and $|\psi\rangle$

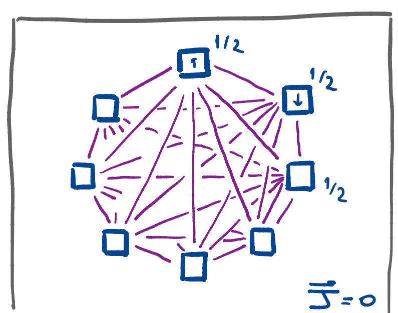
Model: 2d quantum geometry on S^2 (boundary of a node in loop quantum gravity)

Quantum Polyhedron



$\longrightarrow$

$\longleftarrow$



Spin System (Random Heisenberg)

Quantum Geometry : Regions and Symmetry-Resolved Entanglement

$\boxed{1}$ Symmetry-Resolved Entanglement in a Spin System

$\boxed{2}$ Quantum Geometry on S^2 : The Quantum Polyhedron

$\boxed{3}$ Regions & Entanglement Entropy in a Quantum Polyhedron

based on :

- EB - Donà - Kumar (to appear)
- EB - Hackl - Kieburg - Rigol - Vidmar [2112.06959] PRX (2022)
- EB - Donà [1904.08370] PRD (2019)
- EB - Donà - Speziale [1009.3402] PRD (2011)

see also :

- EB - Livine [2302.05922] QG handbook (2023)
- Murthy - Babakani - Iniguez - Srednicki - Yunger Halpern [2206.05310] PRL (2023)

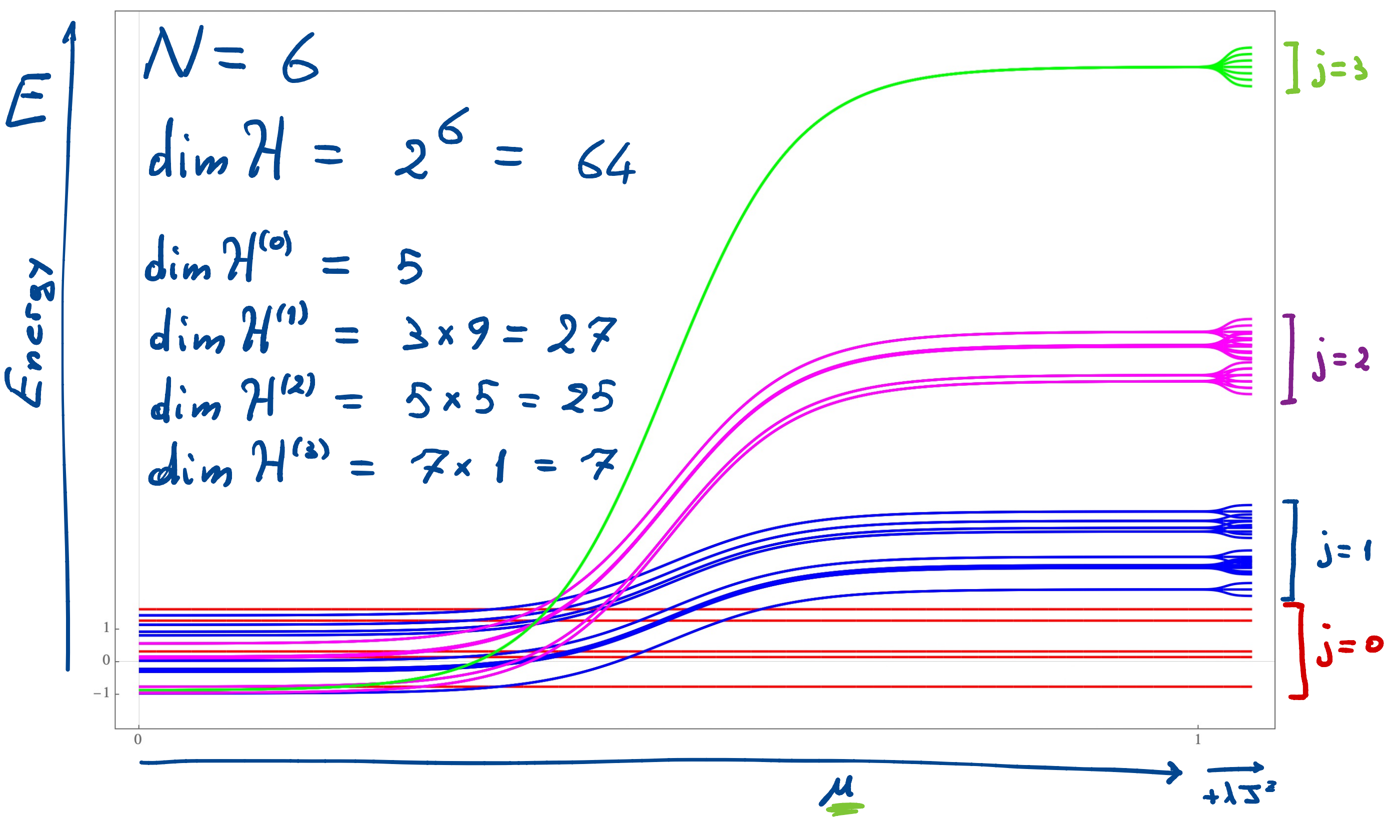
1 Symmetry-Resolved Sectors in a Spin System

- System of N spin $\frac{1}{2}$, $\boxed{\mathcal{H}_N = \bigotimes_{n=1}^{N} \mathcal{H}^{(1/2)}}$,
- "Local" Observables $\vec{S}_n$ $\rightsquigarrow$ local basis $|s_1^z\rangle |s_2^z\rangle \cdots |s_N^z\rangle$ $\quad s_n^z = \pm\frac{1}{2}$
- Hamiltonian: chaotic, SU(2) symmetric

$$\boxed{\begin{aligned} &[H, \vec{J}\,] = 0 \\ &\vec{J} = \sum_{n=1}^{N} \vec{S}_n \end{aligned}}$$

e.g.: $H = \frac{1}{\sqrt{N}} \sum_{n,n'} c_{nn'} \vec{S}_n \cdot \vec{S}_{n'} + \mu \vec{J}^2$

- Symmetry-Resolved Sectors:



$N = 6$

$\dim \mathcal{H} = 2^6 = 64$

$\dim \mathcal{H}^{(0)} = 5$

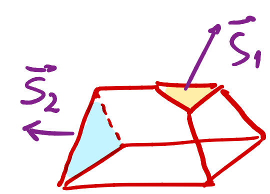
$\dim \mathcal{H}^{(1)} = 3 \times 9 = 27$

$\dim \mathcal{H}^{(2)} = 5 \times 5 = 25$

$\dim \mathcal{H}^{(3)} = 7 \times 1 = 7$

$$\boxed{\mathcal{H}_N = \bigoplus_{j=0}^{N/2} \mathcal{H}_N^{(j)}}$$

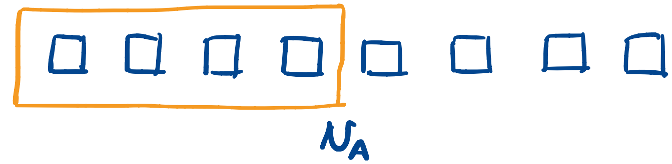
$\mathcal{H}^{(0)}$ = Quantum Polyhedron



K-Local Observables vs G-Local Observables

- Algebra of Observables on $\mathcal{H}_N$, generated by S_n^i, $n = 1, \dots, N$

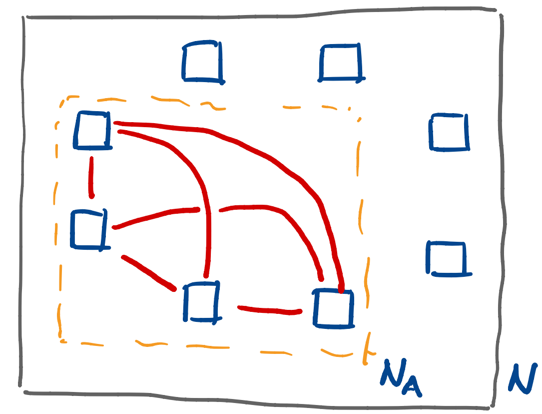
- K-local subalgebra in A



  generated by $\boxed{S_a^i}$, $a = 1, \dots, N_A$

- G-local subalgebra in A, condition $[O_A, \vec{J}\,] = 0$

  generated by $\boxed{\vec{S}_a \cdot \vec{S}_{a'}}$, $a, a' = 1, \dots, N_A$



* Note :

- both subalgebras define local subsystems

- G-local observables preserves energy sectors, while K-local do not

$$\langle E_n | \vec{S}_a \cdot \vec{S}_{a'} | E_{n'} \rangle = \begin{pmatrix} j{=}0 & 0 & & \\ 0 & j{=}1 & 0 & \\ & 0 & j{=}2 & \\ & & & j{=}3 \end{pmatrix}$$

$$\langle E_n | S_a^x S_{a'}^z | E_{n'} \rangle = \begin{pmatrix} j{=}0 & & & \\ & j{=}1 & & \\ & & j{=}2 & \\ & & & j{=}3 \end{pmatrix}$$

G-Local Subsystem in $\mathcal{H}_N^{(j)}$

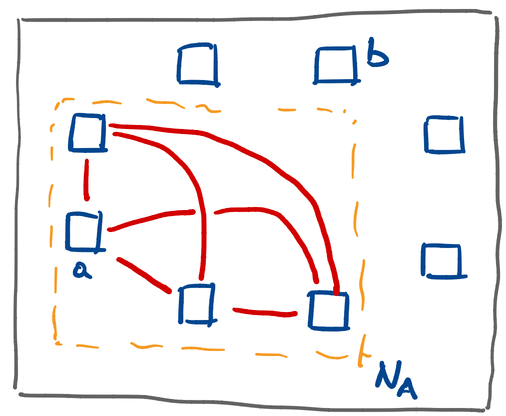
* Note: Sector $\mathcal{H}_N^{(j)}$ does not have a local tensor-product structure $\mathcal{H}_N^{(j)} \neq \mathcal{H}_A \otimes \mathcal{H}_B$

$\longrightarrow$ Construction of local subsystem from G-local observables

- Algebra of observables on $\mathcal{H}_N^{(j)}$, $[O, \vec{J}] = 0$, e.g. $\vec{S}_n \cdot \vec{S}_{n'}$ $\quad n = 1, \dots, N$
- Observables in subsystem A $\longrightarrow$ $O_A = \vec{S}_a \cdot \vec{S}_{a'}$ $\quad a = 1, \dots, N_A$
- Observables in complement $\bar{A}$ $\longrightarrow$ every O that commutes with all O_A

$[O_A, O_{\bar{A}}] = 0$

e.g. $\begin{cases} \vec{S}_b \cdot \vec{S}_{b'} & b = 1, \dots, N_B \\ \vec{J}_A^{\,2} = \left(\sum_{a=1}^{N_A} \vec{S}_a \right)^2 \end{cases}$



* Note: $\vec{S}_a \cdot \vec{S}_b$ does not belong neither to A nor to $\bar{A}$

- Center: $\mathcal{Z} = A \cap \bar{A} = \{ \vec{J}_A^{\,2} \}$

- Decomposition:

$$\boxed{\mathcal{H}_N^{(j)} = \bigoplus_{j_A} \left(\mathcal{H}_A^{(j_A)} \otimes \mathcal{H}_{\bar{A}}^{(j, j_A)} \right)}$$

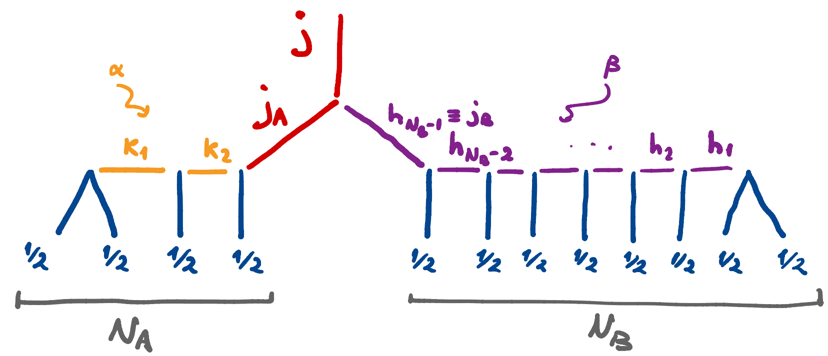
☒ Basis adapted to G-local observables and entropy

- Diagonalize first the observables in the center $\mathcal{Z} = \mathcal{O}_A \cap \mathcal{O}_{\bar{A}} = \{\vec{J}_A^{\,2}\}$, then ON basis of $\mathcal{H}_A^{(j_A)}$ and of $\mathcal{H}_{\bar{A}}^{(j,j_A)}$

$$\boxed{|\psi^{(j)}\rangle = \sum_{j_A=0}^{N_A/2} \sqrt{p_{j_A}} \sum_{\alpha,\beta} \psi^{(j)}_{j_A\alpha\beta}\; |j_A,\underline{\alpha}\rangle\, |(j,j_A),\underline{\beta}\rangle}$$

$*N_A$ even

- Technique: recoupling scheme

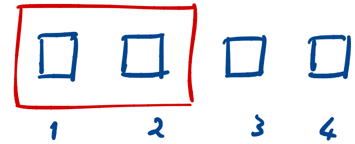


- G-local subsystem $\rho_{AG} = \bigoplus_{j_A} p_{j_A}\, \rho_{j_A}$
- G-local entanglement entropy

$$\boxed{S_{AG}(|\psi^{(j)}\rangle) = -\mathrm{Tr}(\rho_{AG}\log\rho_{AG}) = \sum_{j_A} p_{j_A}\left(-\mathrm{Tr}\,\rho_{j_A}\log\rho_{j_A}\right) - \sum_{j_A} p_{j_A}\log p_{j_A}}$$

Example: Quantum Tetrahedron and Subsystems

- $N = 4$ spin $\frac{1}{2}$, Sector $\mathcal{H}_4^{(j=0)}$



- State $|\psi_0\rangle = \frac{1}{\sqrt{3}} \sum_{m=-1,0,1} \underbrace{|t, m\rangle_{12}}_{} \, |t, -m\rangle_{34}$

triplet $|t, m\rangle = \begin{cases} |\uparrow\uparrow\rangle & m=+1 \\ \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) & m=0 \\ |\downarrow\downarrow\rangle & m=-1 \end{cases}$

- Subsystem A = measurements of $\vec{S}_1$ and $\vec{S}_2$

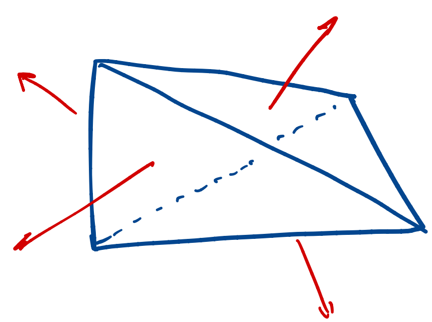
K $\quad S_{AK}(|\psi_0\rangle) = \log 3$

- Subsystem obs A = measurements of $\vec{S}_1 \cdot \vec{S}_2$, G-local obs.

G $\quad S_{AG}(|\psi_0\rangle) = \underline{\underline{0}}$ because eigenstate

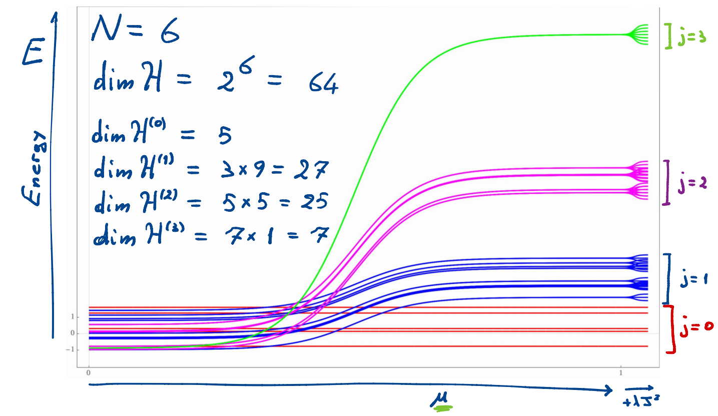
$(\vec{S}_1 + \vec{S}_2)^2 |\psi_0\rangle = 1\,(1+1)\,|\psi_0\rangle$

* Note: $|\psi_0\rangle \in \mathcal{H}^{(0)}$



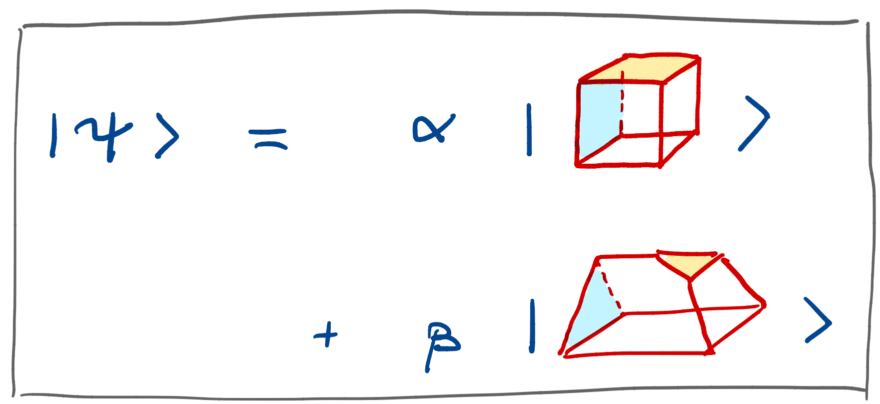
$S_{AK}(|\psi_0\rangle) = \log 3 > \log \underline{\dim \mathcal{H}^{(0)}} = \log 2$

Quantum Geometry : Regions and Symmetry-Resolved Entanglement



$\boxed{1}$ Symmetry-Resolved Entanglement in a Spin System

$\boxed{2}$ Quantum Geometry on S^2 : The Quantum Polyhedron



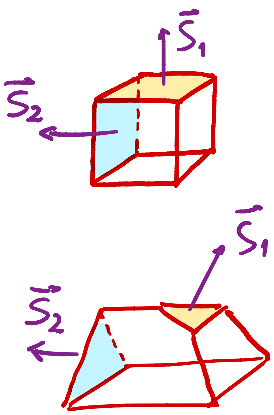
$\boxed{3}$ Regions & Entanglement Entropy in a Quantum Polyhedron

$\boxed{2}$ The Classical Polyhedron : a 2d model of discrete geometry on S^2

Polyhedron in $\mathbb{R}^3$ with N faces of fixed area

N vectors with constraint $\sum_{n=1}^{N} \vec{S}_n = 0$

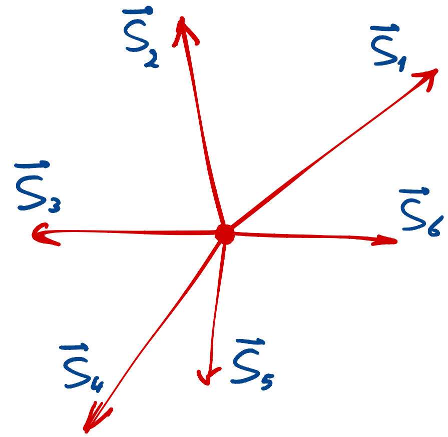
Phase space (Kapovich-Millson) and adjacency basins



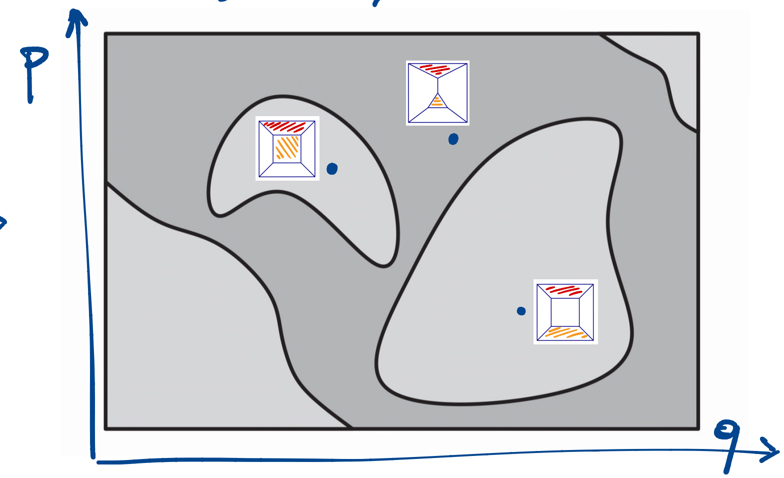
$\Longrightarrow$

$\Longleftarrow$

(Minkowski 1897)



$\Longleftrightarrow$



- Intrinsic Geometry on S^2, e.g. adiacency relations and deficit angles ε_n induced from normals $\vec{S}_n$

  Classical, Discrete

- Quantization $\rightarrow$ SU(2) intertwiner space $\mathcal{H}_N^{(0)}$ and quantum polyhedron

  (d.o.f. at nodes in loop quantum gravity)

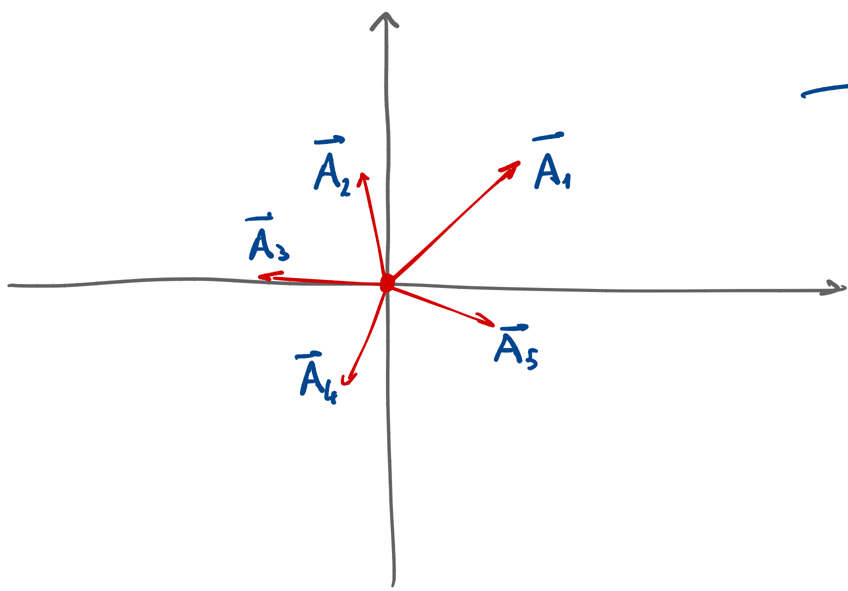
Bianchi-Donà-Speziale [1009.3402] PRD (2011)

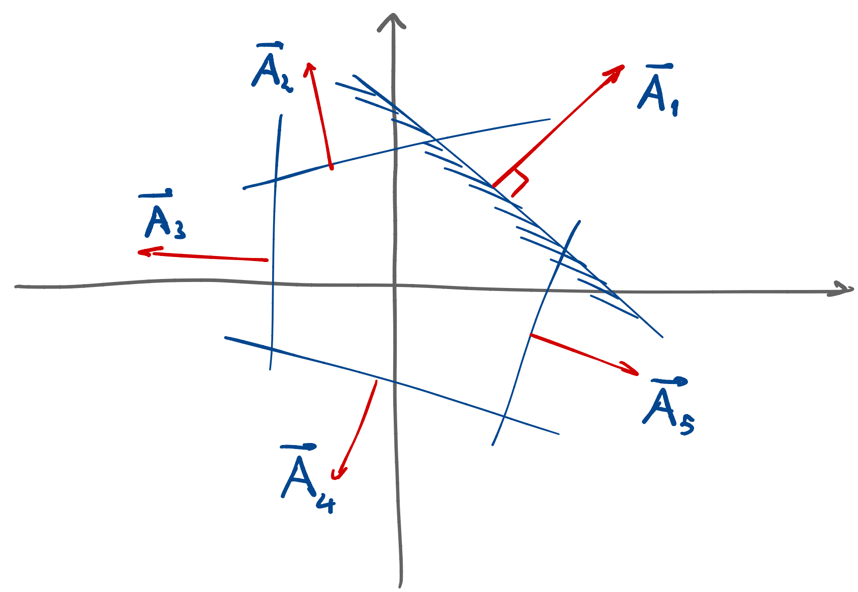
▨ Classical Polyhedron : Convex Hull

- N vectors $\vec{A}_n$ that sum to zero , $\sum_{n=1}^{N} \vec{A}_n = 0$

$\Longrightarrow$ Polyhedron in 3d Euclidean Space [Minkowski: 1897]

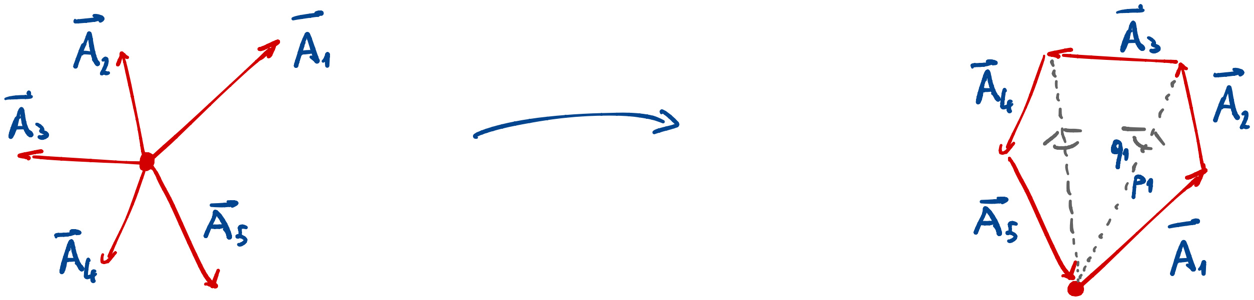
- N faces of area $|\vec{A}_n|$
- unit-normals to faces $\frac{\vec{A}_n}{|\vec{A}_n|}$





▨ Classical Polyhedron : Phase Space

- Phase space of a polyhedron with N faces of fixed area



q_1 = Angle between planes $\vec{A}_1, \vec{A}_2$ and $\vec{A}_1+\vec{A}_2, \vec{A}_3$

$p_1 = |\vec{A}_1+\vec{A}_2|$

Canonical Variables $\{q_i, p_j\} = \delta_{ij}$ [Kapovich-Millson 1996]

- Quantization: Intertwiner Space $\mathcal{H}^{(0)}$

$\vec{A}_n \longleftrightarrow \vec{S}_n$ spin operators, LQG area $A_n = 8\pi G\hbar\gamma\sqrt{j_n(j_n+1)}$

- Coherent States peaked on a point in phase space

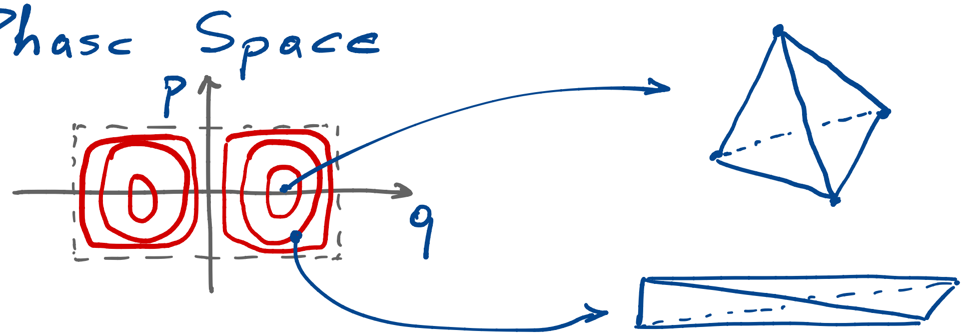
$\longrightarrow$ coherent polyhedron

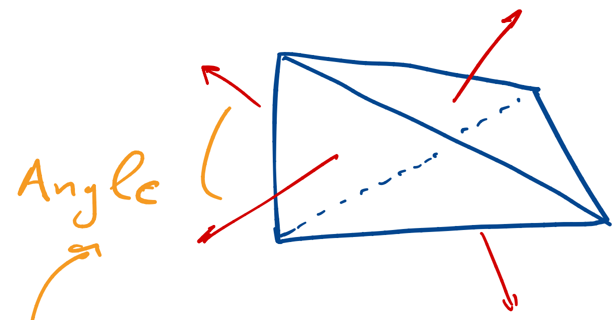
Bianchi-Donà-Speziale [1009.3402] PRD (2011)

[Barbieri, NPB 1998]
[Bianchi-Haggard, PRL 2011]

* Example : $N=4$, Tetrahedron

- Phase Space

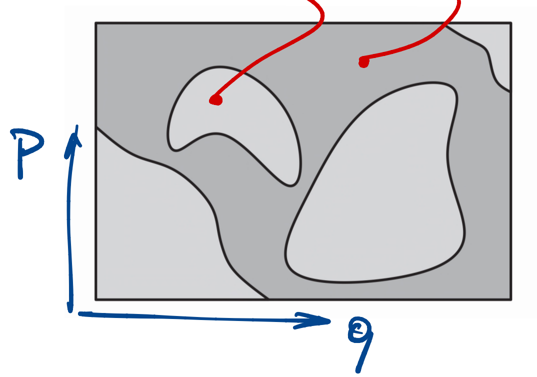


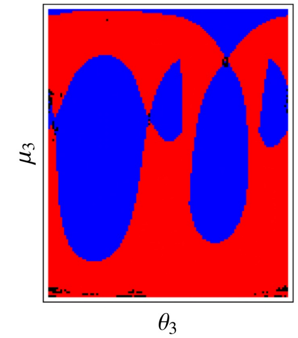


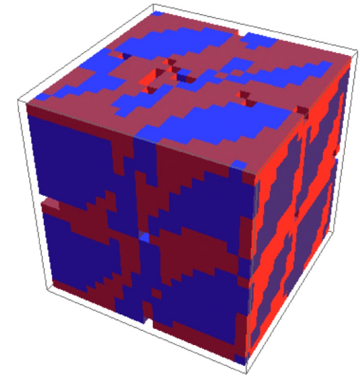
- Classical Observables
  - Area of faces, A_n fixed
  - Angle between faces $\vec{A}_n \cdot \vec{A}_{n'} = A_n A_{n'} \cos \theta_{nn'}$
  - Volume
  - ...

* Example: $N=6$, Cuboid, Pentagonal Wedge, ...

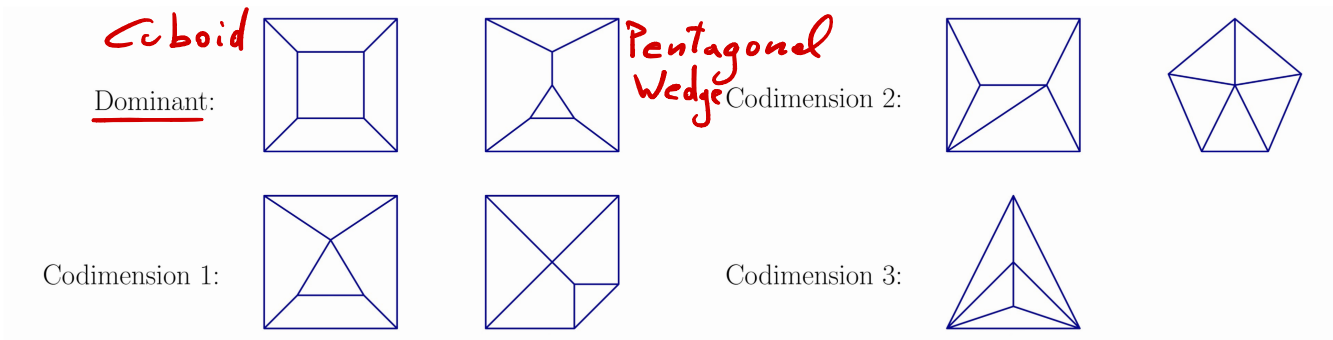
- Phase Space



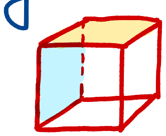
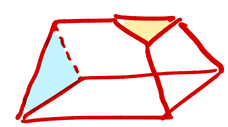




* Adjacency of faces depends on the point in phase space

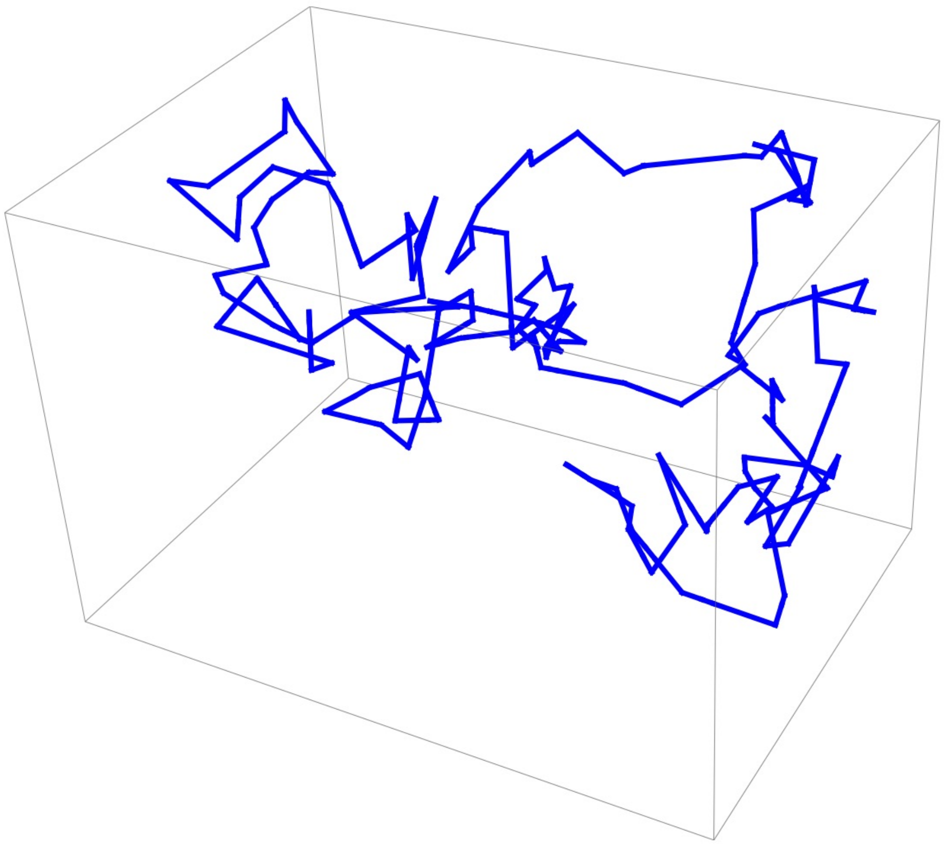


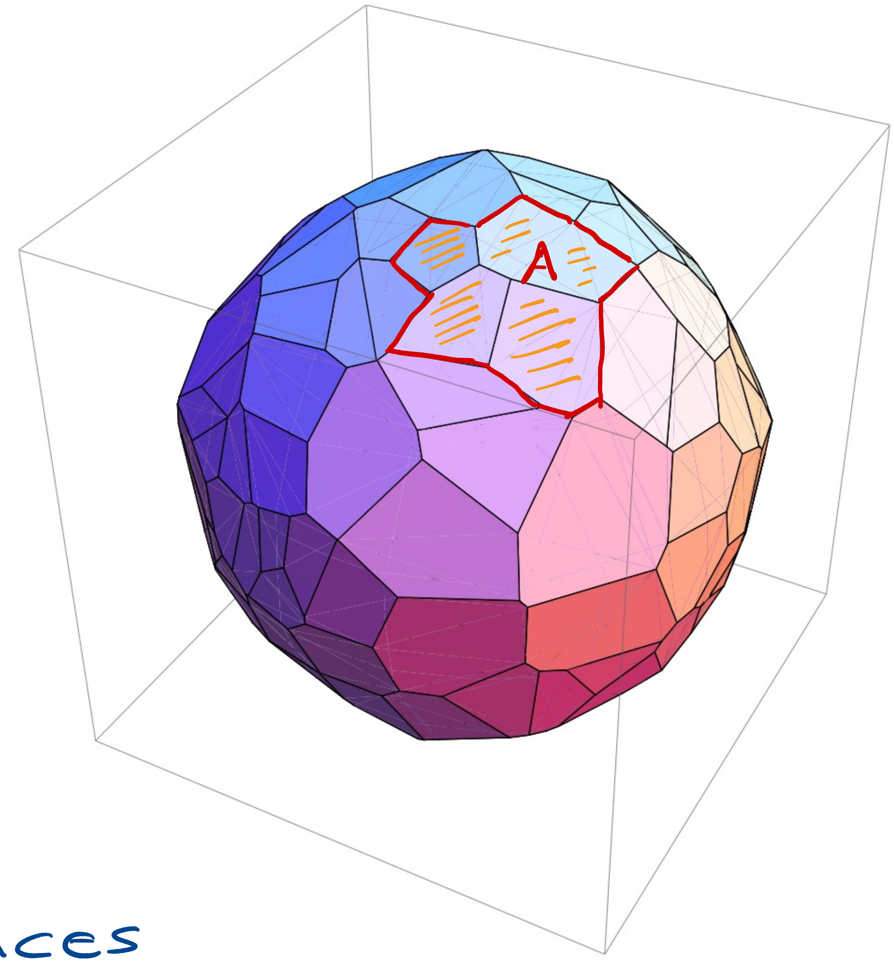
- Superposition of adjacencies

$|\psi\rangle = \alpha$ |  $\rangle$ $+ \ \beta$ |  $\rangle$

- Local Subsystem of adjacent faces $\longrightarrow$ $\Big\{$
  - subalgebra $\vec{S}_a \cdot \vec{S}_{a'}$ $\quad a = 1, \dots, N_A$
  - semiclass. state $|\psi\rangle$ peaked on config.

▪ $N \to \infty$, Tessellation of the Sphere S^2



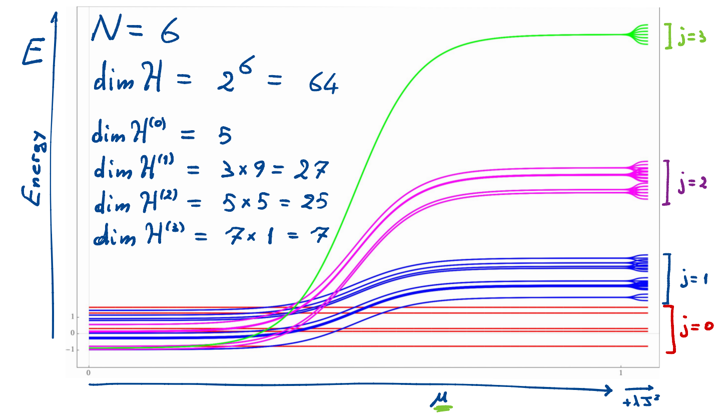


*Note: Adjacency of faces determined by point in phase space

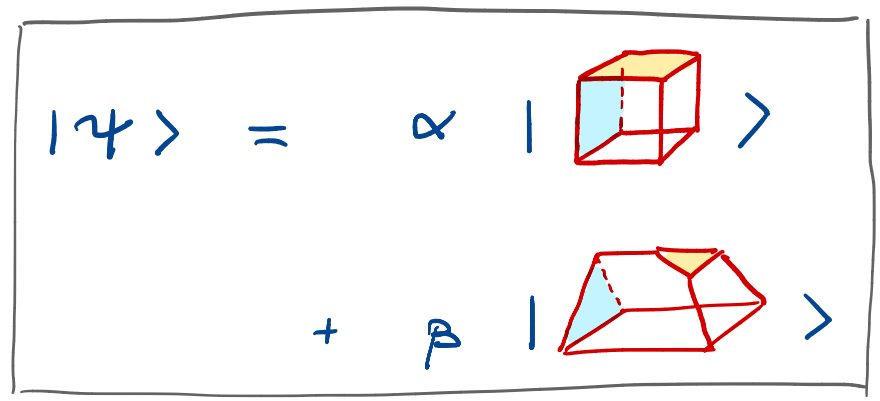
$\rightsquigarrow$ local subsystem A from adjacency matrix

Quantum Geometry : Regions and Symmetry-Resolved Entanglement

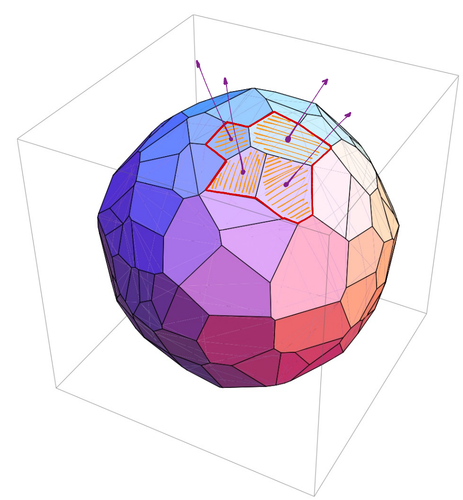
$\boxed{1}$ Symmetry-Resolved Entanglement in a Spin System



$\boxed{2}$ Quantum Geometry on S^2 : The Quantum Polyhedron

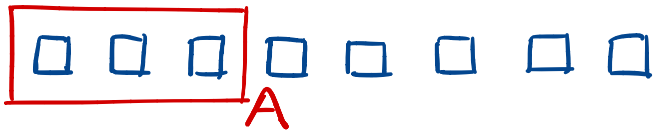


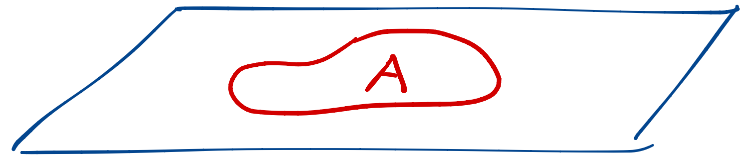
$\boxed{3}$ Regions & Entanglement Entropy in a Quantum Polyhedron



$\boxed{3}$ What is a region in quantum gravity?

- In Many-Body Quantum Systems and in Quantum Field Theory



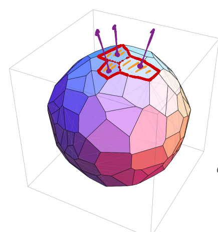
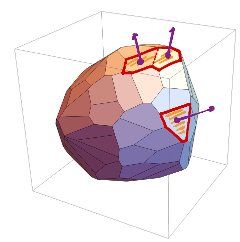


· local subsystem $\rightsquigarrow$ · local subalgebra of observables

$$\mathcal{O}_A \otimes \mathbb{1}_B \quad \text{on} \quad \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$\rightsquigarrow$ · same subalgebra for all states

- In Quantum Gravity: superposition of geometries

$|\psi\rangle = \alpha \,|$  $\rangle + \beta \,|$  $\rangle$

· subsystem $\rightsquigarrow$ sub-algebra of observables

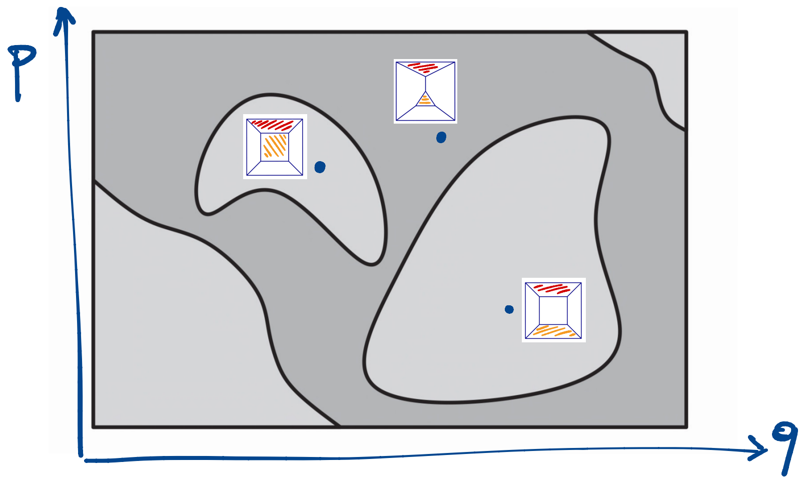
$$\vec{S}_a \cdot \vec{S}_{a'} \qquad \mathcal{H}^{(0)} = \bigoplus_{j_A} \left(\mathcal{H}_A^{(j_A)} \otimes \mathcal{H}_{\bar{A}}^{(0, j_A)} \right)$$

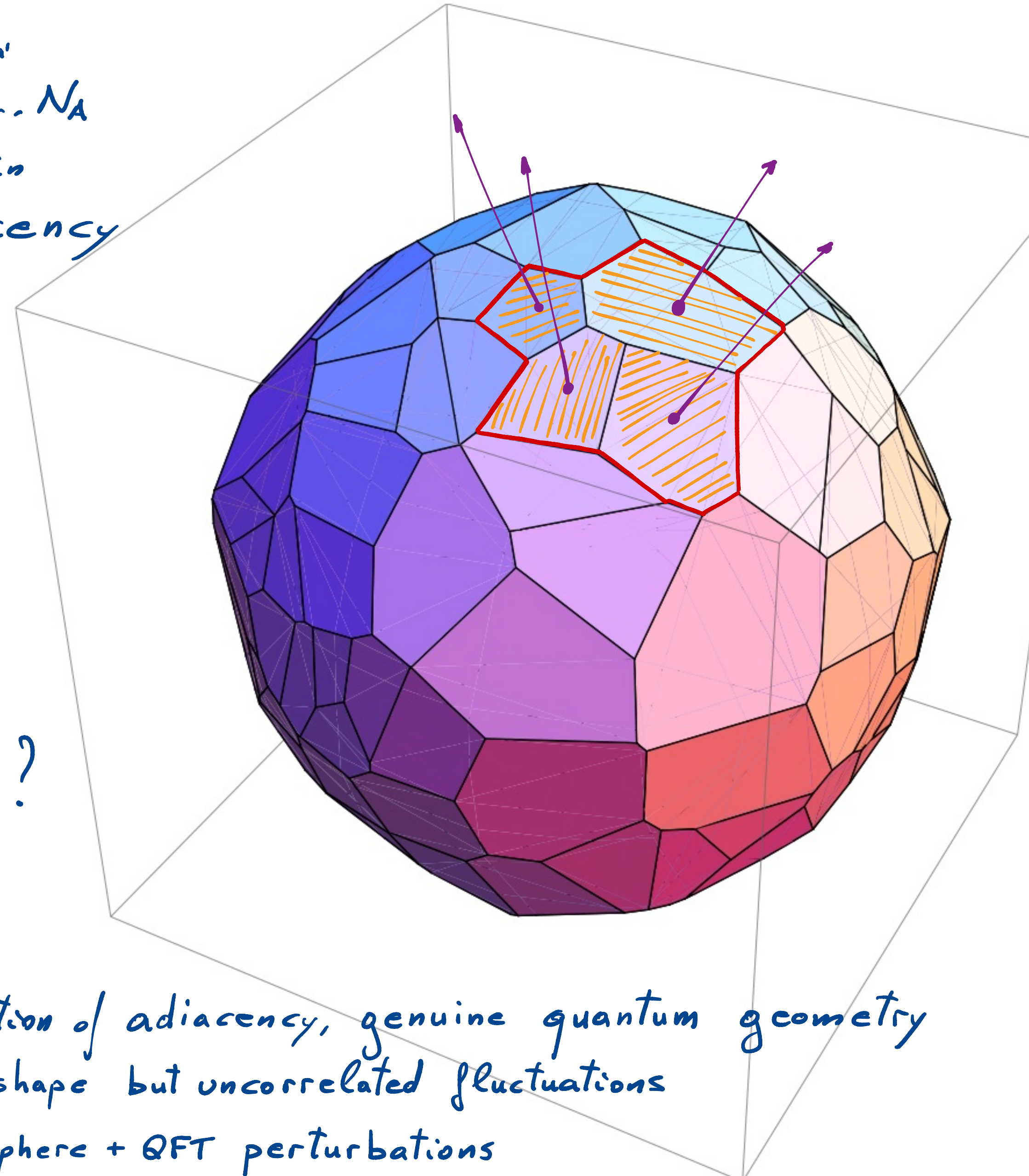
· local subsystem $\rightsquigarrow$ { - sub-algebra of observables
"Geometric Region" { - semiclassical state

$\boxed{3}$ How do we define regions as local subsystems of a quantum polyhedron?

- Classical Polyhedron

region R $\longrightarrow$
- subalgebra $\vec{S}_a \cdot \vec{S}_{a'}$ $\quad a, a' = 1, \dots N_A$
- phase-space basin with fixed adiacency





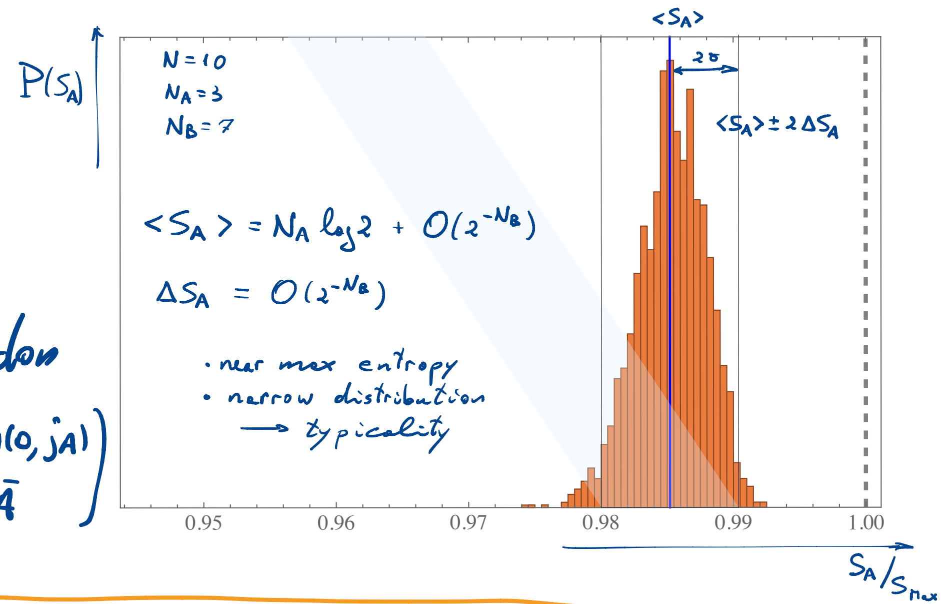
- Quantum Polyhedron $\rightarrow$ Region R ?

- subalgebra $\vec{S}_a \cdot \vec{S}_{a'}$ and
- $\boxed{a}$ Random State $\rightsquigarrow$ Superposition of adiacency, genuine quantum geometry
  $\boxed{b}$ Coherent State $\rightsquigarrow$ Peak over shape but uncorrelated fluctuations
  $\boxed{c}$ Squeezed State $\rightsquigarrow$ $N \to \infty$ Sphere + QFT perturbations

$$\hat{h}_{ab} \sim h^{cl}_{ab} + \delta \hat{h}_{ab}$$

3 a Random Polyhedron: Typical Entanglement Entropy

- Random State of N spins
  → Typical Entanglement Entropy
  [Page, PRL 1993] $|\psi\rangle \in \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$



- Random State of a Quantum Polyhedron

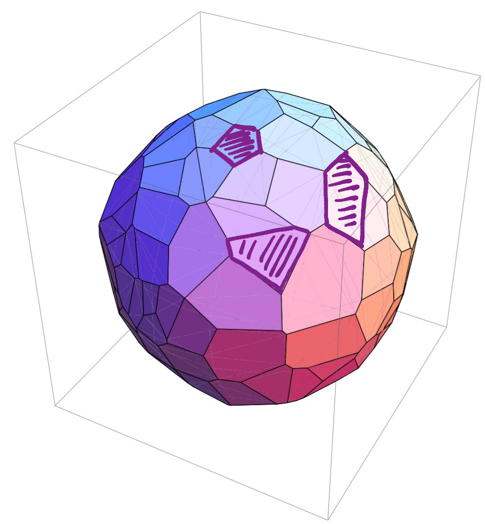
$$|\psi_0\rangle \in \mathcal{H}^{(0)} = \bigoplus_{j_A} \left(\mathcal{H}_A^{(j_A)} \otimes \mathcal{H}_{\bar{A}}^{(0,j_A)}\right)$$

[Bianchi-Donà, PRD 2019]

$$\boxed{\left\langle S_{AG}(|\psi_0\rangle)\right\rangle = \sum_{j_A} \frac{d_A d_{\bar{A}}}{D}\left(\Psi(D+1) - \Psi(\max(d_A, d_{\bar{A}})+1) - \min\left(\frac{d_A-1}{2d_{\bar{A}}}, \frac{d_{\bar{A}}-1}{2d_A}\right)\right)}$$

where $d_A = \dim \mathcal{H}_A^{(j_A)}$, $d_{\bar{A}} = \dim \mathcal{H}_{\bar{A}}^{(0,j_A)}$, $D = \dim \mathcal{H}^{(0)}$

- Random State, $N \to \infty$, $f = \frac{N_A}{N}$ fixed



$$\boxed{\langle S_{AG}(|\psi_0\rangle)\rangle \approx \underline{N_A \log 2} - \frac{1}{2}\log N_A + \frac{3}{2}\frac{N_A}{N} + \log\left(1-\frac{N_A}{N}\right) + \frac{1}{2}(\log 2 - 2 + \gamma_E)}$$

Random State ⇒ <u>Volume Law</u>

<u>Random Superposition of Adjacencies</u>

Genuine Quantum Geometry

[Bianchi-Donà-Kumar (to appear)]

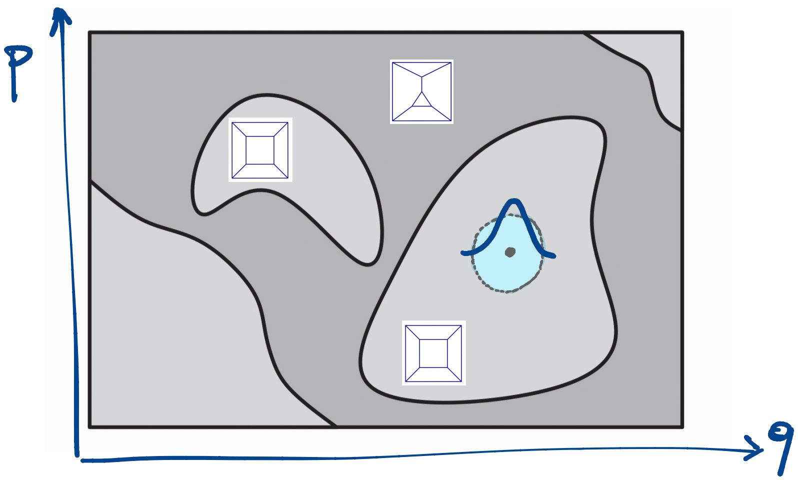
3 b Coherent Polyhedron

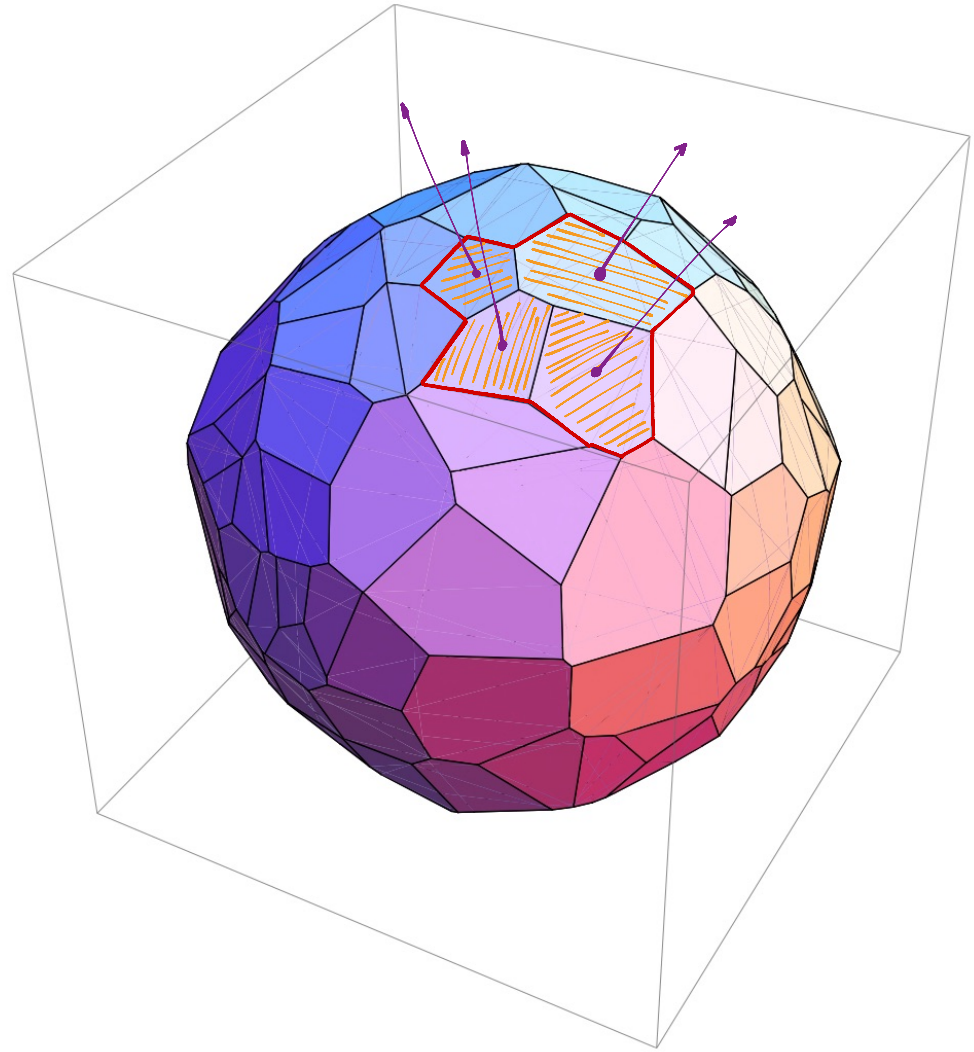
[Livine-Speziale, PRD 2007]

$$|\{\vec{n}_1,\dots,\vec{n}_N\}\rangle = \int dg\,\Big(U(g)|\tfrac{1}{2},\vec{n}_1\rangle\otimes\cdots\otimes U(g)|\tfrac{1}{2},\vec{n}_N\rangle\Big)$$

$$S_A\big(|\{\vec{n}_1,\dots,\vec{n}_N\}\rangle\big) \sim \frac{1}{2}\log N_A$$

sub extensive





- Peaked on a classical geometry
- Uncorrelated fluctuations

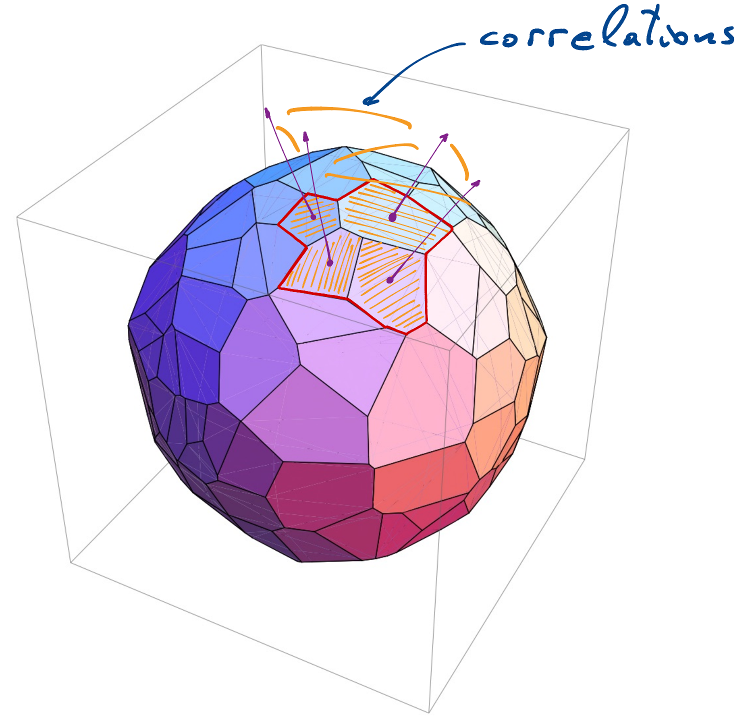
Bianchi - Guglielmon - Hackl - Yokomizo [1609.02219] (PRD)
Freidel - Livine [0911.3553] (JMP)

3 C Squeezed Polyhedron

$$|\gamma\rangle = \frac{1}{Z}\, e^{\sum_{nm} \gamma^{nm} \epsilon_{AB}\, \hat{a}_n^{\dagger A} \hat{a}_m^{\dagger B}} |0\rangle$$

with γ^{nm} squeezing matrix

→ Adjacency matrix for correlations



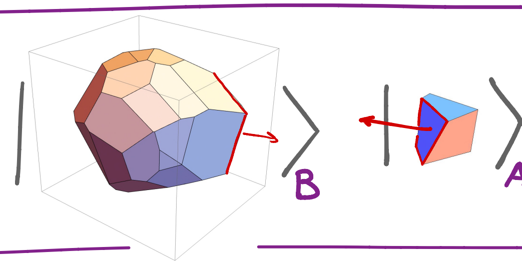
$S_A(|\gamma\rangle) \sim \sqrt{N_A}$ "Area Law" (perimeter)

$N \to \infty$ Sphere + QFT perturbation

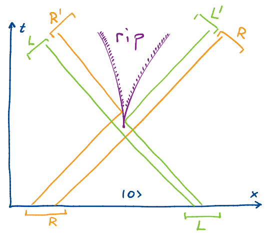
$$\hat{h}_{ab} \sim h^{cl}_{ab} + \delta \hat{h}_{ab}$$

b' Zero-Law Polyhedron → Splits to two polyhedra

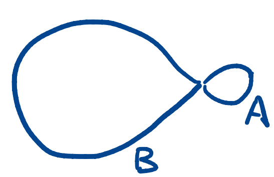
$|\gamma^{nm}\rangle = |\gamma^{bb'}\rangle_B\, |\gamma^{aa'}\rangle_A =$



cf: Rip in Geometry



and



$$\gamma^{nm} = \left(\begin{array}{c|c} \gamma^{aa'} & 0 \\ \hline 0 & \gamma^{bb'} \end{array}\right)$$

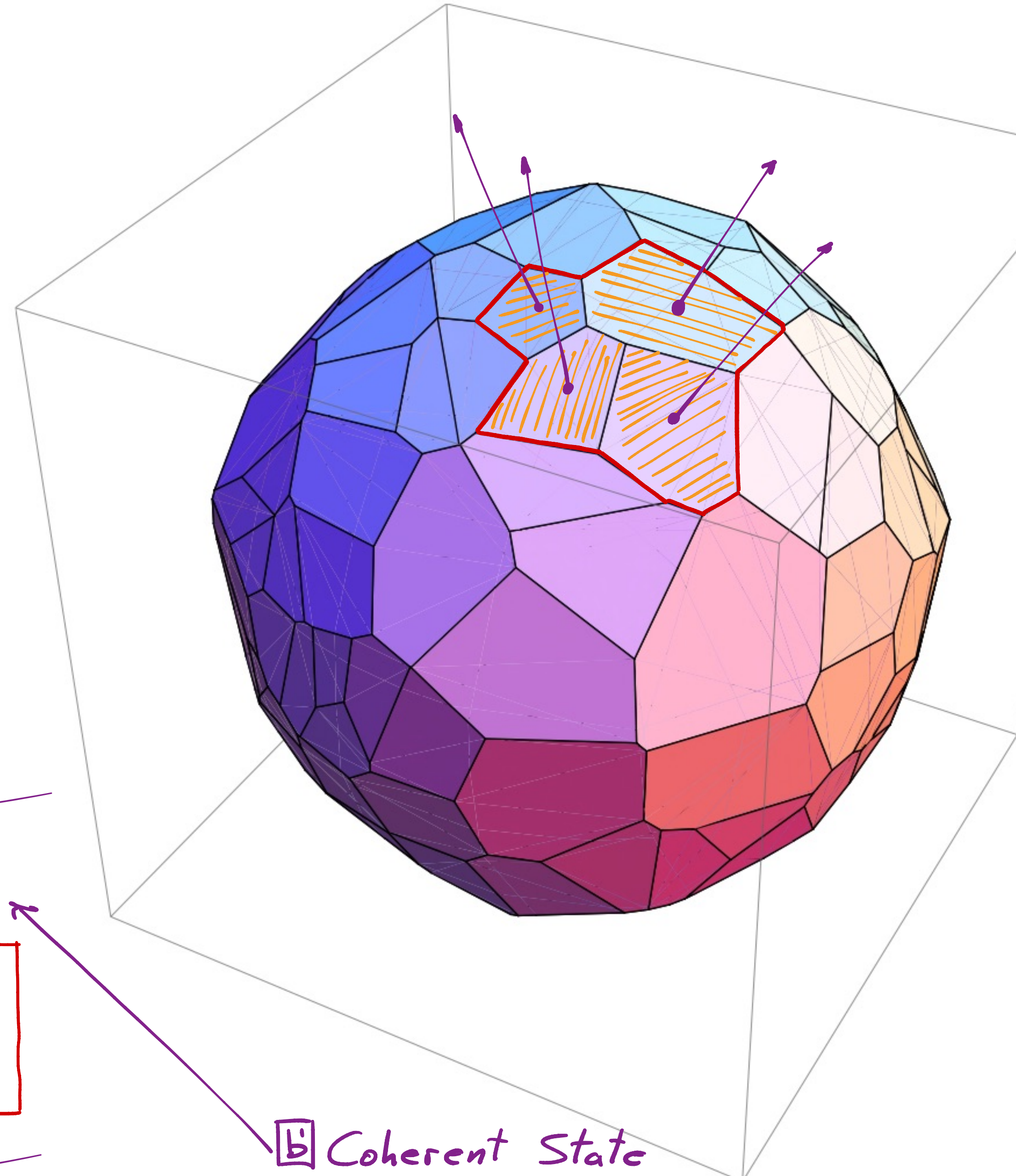
☒ How do we define regions as local subsystems of a quantum polyhedron?

✱ Entanglement as a probe of semiclassicality & locality

[EB-Myers CQG (2014)]

Two ingredients:

- State $|\psi\rangle \in \mathcal{H}^{(0)}$
- G-local subalgebra $A = \{ \vec{S}_a \cdot \vec{S}_{a'} \mid a, a' = 1, .., N_A \}$



· Requirement on $|\psi\rangle$:

$$S_A(|\psi\rangle) = \begin{cases} \cancel{\sim 1 \quad \text{zero law}} \\ \boxed{\sim (N_A)^{\frac{d-1}{d}} \quad \text{"Area" law} \quad \to \text{perimeter for } d=2} \\ \cancel{\sim N_A \quad \text{"Volume" law}} \end{cases}$$

[b] Coherent State

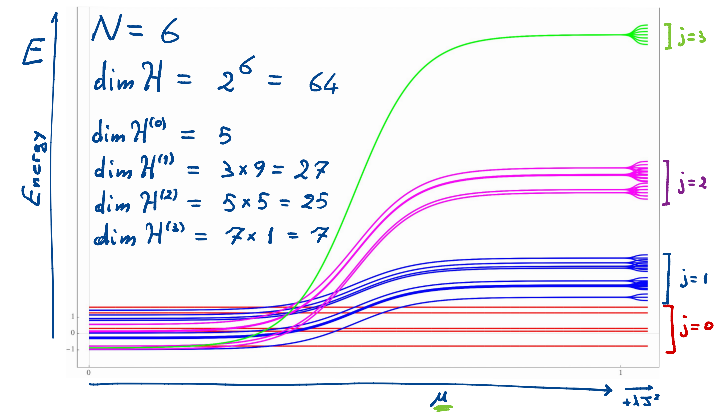
[a] Random State

[c]

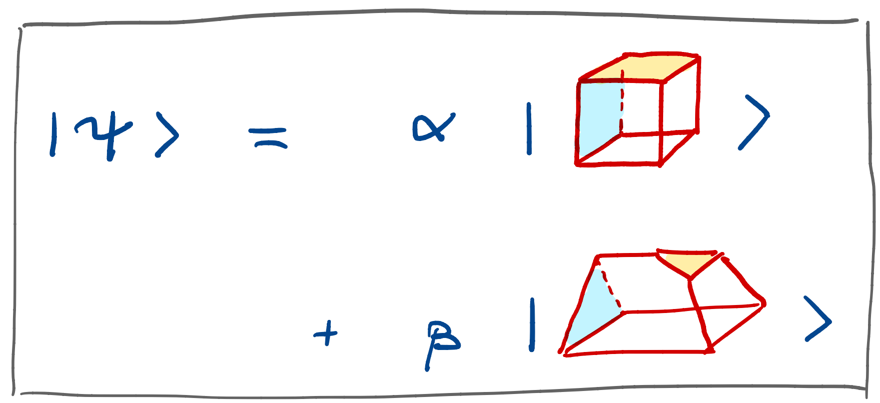
✱ ⇒ Parametrization of the semiclassical corner of the Hilbert space

Quantum Geometry: Regions and Symmetry-Resolved Entanglement

1 Symmetry-Resolved Entanglement in a Spin System



2 Quantum Geometry on S^2: The Quantum Polyhedron



3 Regions & Entanglement Entropy in a Quantum Polyhedron