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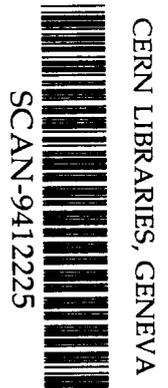
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ABSTRACT

The perturbative asymptotic of the Regge trajectories, $\alpha(t)$, at $-t \to \infty$ is deduced on the basis of a known solution of relativistic wave equation with the Coulomb-like potential. We obtain the perturbative asymptotic at $-t \to \infty$ of the leading $S = 1$ quarkonium Regge trajectories of the form $\alpha_R(t) = 0$, and leading $S = 1/2$ nucleon Regge trajectories of the form $\alpha_N(t) = -1/2$.

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1. In recent experiments at HERA [1] has been observed a sample of events with a large rapidity gaps, which are conveniently described in the language of complex angular momentum. These results have renewed interest to Regge theory, especially in that aspect which synthesizes hard collisions phenomena with the classical physics of large rapidity-gaps (see [2,3] and references therein). As it was argued in [4] conventional Regge approach to high-energy collisions is most probably correct. It describes accurately all pp and $p\bar{p}$ elastic scattering from ISR to Tevatron energies [5], successfully predicted the γp total cross section [6] and, through optical theorem, the total and inclusive cross sections as well.

Fundamental objects of the Regge theory are the t-channel Regge trajectories, $\alpha(t)$, which determine the behaviour of the scattering amplitude in the s channel. The main features of soft hadronic processes can be understood in terms of the exchange of particles, which lie on linear Regge trajectories. Moreover, as it was shown in [7], exclusive processes whose cross sections are determined by Regge pole trajectory exchange, $\alpha(t)$, at small momentum transfers, t, are controlled by these same exchanges at very large spacelike t too. Hereby trajectory must to be nonlinear and one of the most crucial distinction between small $|t|$ behaviour of $\alpha(t)$ and large $|t|$ behaviour of $\alpha(t)$ involves the asymptotic form of Regge trajectories at $-t \to \infty$.

In this letter we dealt with the perturbative asymptotic of Regge trajectories, $\alpha(t)$, at $-t \to \infty$ on the basis of a known solution of relativistic wave equation with Coulomb-like potential. We use a generally agreed fact that in perturbative QCD the essential interaction at small distances is instantaneous Coulomb gluon-exchange (quark-parton Coulomb scattering). We obtain the perturbative asymptotic of the leading $S = 1$ quarkonium Regge trajectories of the form $\alpha_{q\bar{q}}(t) = 0$ at $-t \to \infty$. For the leading nucleon (in general case baryon) Regge trajectory we obtain the asymptotic of the form $\alpha_N(t) = -1/2$, $-t \to \infty$. This results obtained for $\alpha(t)$ at large spacelike t coincide with the predictions of ref. [8] obtained in the Born approximation of perturbative QCD.

2. An important aspect of Regge theory is the mechanism by which Regge

trajectories in t channel determine the behaviour of the scattering amplitude in the t channel. This makes it possible to determine the properties of the trajectories from an analysis of the data on the scattering of the particles. Presented in [9,10] experimental data on inclusive π^0 and η production in 100 and 200 GeV/c $\pi^{\pm}p$ collisions have compared in detail with the prediction of triple Regge model. So far as these reactions are theoretically clean with ρ (π^0 production) or A_2 (η production) exchange, there were extracted the Regge trajectories, $\alpha(t)$, in the t range of -1 to -8 GeV^2. There was shown that the ρ trajectory flattens off at about -0.7 or -0.5. The full inclusive value, $\alpha(-t \to \infty) = -0.92$. At all values of $-t = 0 - 8$ CeV^2, agreement with the triple Regge model is good.

Asymptotic behaviour of the Regge trajectories at $-t \to \infty$ has been discussed by many authors [7,11-14]. Important information on perturbative asymptotic of trajectories one can obtain by comparing predictions for the scattering amplitude, $T(s,t)$, of the "quark counting rule" [12,15] and the Regge pole approach at $s \to \infty$, $-t$ fixed. If one assume a smooth interpolation between these two predictions [14] one obtain the condition

$$\alpha(t) = const, \qquad -t \to \infty, \tag{1}$$

which seems do not contradict experimental data [9,10]. There have been a considerable efforts to extend the constituent interchange model (CIM) (see [12]) from the fixed-angle region into the fixed t-region. These efforts have resulted in prediction for the large $-t$ behaviour of ρ trajectory

$$\alpha_\rho(t) = -1. \tag{2}$$

The same asymptotic behaviour of all leading $S = 1$ quarkonium Regge trajectories has been obtained in our ref. [16] on the basis of the analysis of a relativistic quasipotential equation with the Cornell potential, and in ref. [17] on the basis of solution of Klein-Gordon equation containing Lorentz vector and Lorentz scalar potentials. Main results of these investigations is that the asymptotic behaviour

of the Regge trajectories at large $-t$ is determined by one-gluon exchange at small distances. The Regge trajectory has been determined to be function $l(E^2)$ where E^2 is the square of quarkonium mass and l is the relative orbital angular momentum of the quarks.

3. In particle collisions at high energies in processes with extremely large momentum transfers $(-t \to \infty)$ the asymptotic behaviour of the scattering amplitude is determined by the dynamics of interactions at small distances and small time interval, i.e. inner structure of hadrons. It has been well tested that hard processes are governed by short range part of the strong interaction. The differential cross section at large $-t = Q^2$ depend essentially on the number of constituents. Two quarks in meson (or three quarks in nucleon) represent the basic quark configuration, because it is need consider the hadron state vector in the Fock space. From the point of view of "quark counting rule" an account of the contribution of the higher corrections to the differential cross section gives the corrections of the order $s^{-4n_{q\bar{q}}}$ [15], where $n_{q\bar{q}}$ is a number of additional $q\bar{q}$ pairs.

It is generally agreed that in perturbative QCD, as in QED, the essential interaction at small distances will be instantaneous Coulomb gluon exchange, that is quark-quark Coulomb scattering. The dynamics is the Coulomb interaction,

$$V(r) = -\frac{\tilde{\alpha}}{r}, \qquad r \to 0, \tag{3}$$

where $\tilde{\alpha}$ being the effective strong coupling constant, $\tilde{\alpha} = \frac{4}{3}\alpha_s$ for mesons. The potential (3) corresponds (at large $Q^2 = |t|$) to the scattering amplitude in the Born approximation with gluon propagator

$$D(q^2) = \frac{1}{q^2} \tag{4}$$

at large $|q^2| = |t|$. Recall that the Coulomb potential (3) in momentum space is

$$V(t) = \frac{4\pi\tilde{\alpha}}{t}, \tag{5}$$

that is the Fourier transform of (5) leads to the potential (3).

Now let us show that the potential (3) leads to the asymptotic behaviour of Regge trajectories of the type (1). Basic process under dealt with is the elementary process of qq interaction. If at small Q^2 we consider a hadron to be consisted from constituent (or valent) quarks, at large Q^2 the hadron is considered to be consisted from a large number of partons (quarks and gluons). Two interacting quarks one may consider both in the bound state region (s channel) and in the scattering region (t channel). Wave equation, say, Klein- Gordin describes two interacting quarks both at $t > 0$ and at $t < 0$. At $t > 0$ we have matter with bound state problem, and at $t < 0$ - with the scattering problem.

Short-range interquark interaction is of the vectoral type, and Klein-Gordon equation with potential (3) for the equal mass case $m_1 = m_2 = m$ has a form

$$\left[\vec{\nabla}^2 + \frac{1}{4} \left(E + \frac{\tilde{\alpha}}{r} \right)^2 - m^2 \right] \psi(\vec{r}) = 0. \tag{6}$$

At large negative $E^2 = t$ in accordance with asymptotic freedom principle quarks behave itself as quasi-free particles and eq. (6) describes the scattering of the quarks-partons. But at $E^2 > 0$ this equation represents eigenvalue problem for discrete spectrum. In this (bound state) region solution of (6) is well known and correspond eigenvalues are given by [18]

$$E_n^2 = \frac{4m^2}{1 + \tilde{\alpha}^2 \left[n' + 1/2 + \sqrt{(l + 1/2)^2 - \tilde{\alpha}^2} \right]^{-2}}, \tag{7}$$

where n', l are the radial and orbital quantum numbers respectively. In the region under consideration, i.e. $Q^2 = -t \to \infty$, effective coupling constant $\tilde{\alpha}$ is very small value and formula (7) (after expansion with respect to $\tilde{\alpha}^2$) one can rewrite in more simple form

$$E_n^2 = 4m^2 \left[1 - \frac{\tilde{\alpha}^2}{(n' + l + 1)^2} \right]. \tag{8}$$

Note that the expression (8) for the square of invariant mass of two particles has a correct relativistic form, $E^2 = 4(\vec{p}^2 + m^2)$, with $\vec{p}^2 = -\tilde{\alpha}^2 m^2 / (n' + l + 1)^2$ or $p = i\tilde{\alpha}m/(n' + l + 1)$, because the expansion (8) have proceeded by expansion

with respect to parameter $\tilde{\alpha}^2$, describing the quark interaction, and not a kinematic variable.

4. If we invert the E^2 given by (8), and express the orbital angular momentum as the function of the E^2, $l(E^2)$, we will be have

$$l(E^2) = -n' - 1 + \frac{\tilde{\alpha}(|E^2|)}{\sqrt{1 - \frac{E^2}{4m^2}}}, \qquad E^2 < 4m^2, \tag{9}$$

where we emphasize that the effective coupling, $\tilde{\alpha}$, is the function of $Q^2 = |E^2|$. Because l is the orbital angular momentum we define the Regge trajectory as $\alpha(E^2) = l(E^2) + S$ where $l(E^2)$ given by (9) and S is the total spin of interacting quarks

$$\alpha(t) = S - n' - 1 + \frac{\tilde{\alpha}(|t|)}{\sqrt{1 - \frac{t}{4m^2}}}, \qquad -t \to \infty, \tag{10}$$

For leading $S = 1$ ($n' = 0$) meson trajectories expression (10) gives ($\tilde{\alpha}(|t|) = \frac{4}{3}\alpha_s(|t|)$)

$$\alpha_R(t) = \frac{\frac{4}{3}\alpha_s(|t|)}{\sqrt{1 - \frac{t}{4m^2}}} \simeq 0, \qquad -t \to \infty. \tag{11}$$

Similarly, in the case of a leading $S = 1/2$ ($n' = 0$) baryon exchange trajectory with the help of general formula (10) we get at large spacelike t

$$\alpha_N(t) \simeq -\frac{1}{2}, \qquad -t \to \infty. \tag{12}$$

Leading $S = 1/2$ baryon trajectory interpolate a sequence of baryon states with total spin $S = 1/2$ that reflects the quark-diquark structure of baryon when two quarks consisting diquark have the total spin $s = 0$; these two quarks interact each other by means of additional color-magnetic spin force.

These results on perturbative asymptotic of Regge trajectories are in agreement with prediction of ref. [8] which have been obtained in the Born approximation of perturbative QCQ. In [8] has been note that a mesonic charge-exchange Reggeon at large spacelike t can be simply identified with $q_a \bar{q}_b$ exchange in t channel. Thus to lowest order one expects in QCD

$$\alpha_R(t) = \frac{1}{2} + \frac{1}{2} - 1 = 0, \qquad -t \to \infty; \tag{13}$$

This means that Regge trajectory asymptotically decreases to $\alpha_R(t) = 0$ at large $-t$. Similarly, in the case of baryon trajectory one expects

$$\alpha_N(t) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} - 1 - 1 = -\frac{1}{2}, \qquad -t \to \infty. \tag{14}$$

One can lead analogous arguments in the case of the Pomeron trajectory too. Really, if one assumes the two-gluon Pomeron model [19] with the total spin S of two interacting gluons to be equal 2, $S = 2$, then the formula (10) gives the perturbative asymptotic for the Pomeron trajectory

$$\alpha_P(t) = 1 + \frac{\tilde{\alpha}(|t|)}{\sqrt{1 - \frac{t}{4m^2}}} \simeq 1, \qquad -t \to \infty. \tag{15}$$

This prediction for the Pomeron trajectory very similar to those obtained in the framework of the leading and subleading $log's$ summation in QCD [13]

$$\alpha_P(t) \simeq 1 + O(\tilde{g}^2(t)), \qquad -t \to \infty. \tag{16}$$

where $\tilde{g}^2(t))$ stands for the running coupling of QCD.

5. Considering the properties of the Regge trajectories it is need to stress an important aspect, namely that these functions give a smooth transition from the s channel to the t channel, i.e. its connect the scattering region with bound state region, as well as the scattering problem with bound state problem. We have obtained the perturbative asymptotic of Regge trajectories from the point of view of the potential approach which assumes a unify consideration both the scattering problem and bound state problem. Our results are in agreement with predictions of ref. [8] obtained in the Born approximation of perturbative QCD.

But these results, as in [8], on the asymptotic behaviour of Regge trajectories contradict to the experimental data [9,10]. This contradiction between the theoretical estimations and the experimental data have resolved in [8] by showing that the hard QCD part of the trajectory is weakly coupled and that its contribution will be hidden until much high energy.

As for experimental data [9,10], it is need to note the following. Extracted at these experiments Regge trajectories have obtained by model-dependent way (with

the use of the triple Regge approximation). For invariant variable t, for example, there has been used approximate relation $t \simeq -\vec{p}_\perp^2/x$, which is valid only at small t, $p_\perp$ and $x \to 1$. There are other uncertainties conjugated with the definition of the fractional variable x. Namely where have used x as the lab energy divided by the maximum possible energy at the given t value. Changing the definition so that the denominator is just the beam energy would change the fitted $\alpha(t)$. Nevertheless the main result of these experiments seems to be valid: Regge trajectories flatten off at some constant value.

Acknowledgments

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