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Glueballs and the pomeron

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Abstract – Glueballs are considered to be bound states of constituent gluons. The relativistic wave equation for two massive gluons interacting by the funnel-type potential is analyzed. Using two exact asymptotic solutions of the equation, we derive an interpolating mass formula and calculate glueball masses in agreement with the lattice data. We obtain the complex non-linear Pomeron trajectory, $\alpha_P(t)$, in the whole region of t. The real part of the trajectory corresponds to the soft pomeron, parameters of which are found from the fit of recent HERA data.

Introduction. – Quantum Chromodynamics allows for an effective description of hadrons as bound states of constituent particles. The classification of baryons and mesons with the quark hypothesis is a first historical example. There should exist a way to connect the constituent approaches to QCD. This long-standing problem is far from being completely solved.

A good test of our understanding of the non-perturbative (NP) aspects of QCD is to study particles where the gauge field plays a more important dynamical role than in the standard hadrons. QCD allows the existence of purely gluonic bound states, glueballs [1–3], but no firm experimental discovery of such states has been obtained yet. Glueballs are particles whose valence degrees of freedom are gluons. An important theoretical achievement in this field has been the computation of the glueball spectrum in lattice QCD [4].

The glueball spectrum has also been computed by using an effective approach like the potential model [5–7]. The potential model is very successful to describe the bound states of quarks. It is also a possible approach to study glueballs [7,8]. Recent results in the physics of glueballs have been reviewed in [3].

At the present, one of open topics in hadron physics is the relation between glueballs and the pomeron. The pomeron is the Pomeranchuk trajectory (P trajectory) [9]. In the many high-energy reactions with small momentum transfer the soft pomeron exchange, gives the dominant contribution [10].

Recent small-$(-t)$ ZEUS and H1 data for exclusive ρ and ϕ photoproduction [11,12] point out that the P trajectory is rather *non-linear*. The data have been explained by adding in a hard pomeron contribution, whose magnitude is calculated from the data for exclusive J/Ψ photoproduction [13,14]. The ZEUS, H1 as well as CDF data on $p\bar{p}$ elastic scattering data have also been analyzed by using the non-linear P trajectory [15]. But non-linearity of the P trajectory is still an open question. The amount of non-linearity is unknown.

There has been a long-standing speculation that the physical particles on the P trajectory might be glueballs [1,3]. In this work we take a picture where the t-channel pomeron is dual to glueballs, *i.e.*, purely gluonic bound states in the s-channel. We use the potential approach, which is the natural framework for studying the Regge trajectories and their properties [16,17]. The potential model, which is so successful to describe bound states of quarks, is also a possible approach to study glueballs [7,18]. We derive an interpolating mass formula for glueball masses and analytic expression for the P trajectory, $\alpha_P(t)$, in the whole region of variable t. Calculation results are compared with the lattice data.

Glueballs. – Glueballs are purely gluonic bound states allowed by QCD. At present, there is the understanding of the deep relation between the properties of the glueball states and the structure of the QCD vacuum. The basic idea is that the vacuum is filled with $J^{PC} = 0^{++}$ transverse electric glueballs which form a negative-energy condensate [19].

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Two gluons in a color singlet state have always positive charge conjugation ($C = +$). The lightest glueballs, which have $C = +$, can be successfully modeled by a two-gluon system in which the constituent gluons are massless helicity-1 particles [20]. The proper inclusion of the helicity degrees of freedom dramatically improves the compatibility between lattice QCD and potential models [20].

The modern development in glueball spectroscopy from various perspectives has been discussed in [3]. At present, several candidates for the low-mass glueballs with quantum numbers $J^{PC} = 0^{++}$, 2^{++}, 0^{-+} and 1^{--} are under discussion.

Glueball masses have been calculated by many authors. A new method called the Vacuum Correlator Model (VCM) has been used in [21]. In this model all non-perturbative and perturbative dynamics of quarks and gluons is universally described by lowest cumulants, *i.e.*, gauge invariant correlators of the type $\langle F_{\mu\nu}(x_1)\dots F_{\lambda\sigma}(x_\nu)\rangle$. More discussions on the subject can be found in [3].

Authors of [7] compared different models for glueballs. They concluded that a semi-relativistic Hamiltonian is an essential ingredient to handle glue states. All the analyzed models used an SL-basis to include the spin of gluons. These arguments support the use of an *effective gluon mass* to describe the glueball dynamics of QCD.

Reggeons and the pomeron. – There exists a conviction that the Regge trajectories are linear in the whole region, that is, not only in the bound-state region ($t = E^2 > 0$) but in the scattering region ($t < 0$) too. However, one of the most crucial distinctions between small-($-t$) behavior and large-($-t$) behavior of trajectories $\alpha(t)$ involves the asymptotic form of Regge trajectories at $-t \to \infty$.

The asymptotic behavior of the Regge trajectories at $-t \to \infty$ has been discussed by many authors [22–24]. The constituent interchange model (CIM) [22] results in the prediction for the large-($-t$) behavior of ρ trajectory

$$\alpha_\rho(t) = -1, \quad t \to -\infty, \qquad (1)$$

that means the ρ trajectory is non-linear. General properties of the trajectories have been considered in classical papers [25,26].

There is a renewed interest in the studies of the dynamics of the Regge trajectories [27]. The conception of linear Regge trajectories is not consistent with experimental data and expectations of perturbative QCD (pQCD) at large $-t \gg \Lambda_{QCD}$ [22]. In the experiment far more complicated behavior of the ρ-meson trajectory, $\alpha_\rho(t)$, was discovered; the ρ trajectory flattens off at about -0.6 or below.

Regge trajectories with the same asymptotic behavior for all leading $S = 1$ meson and quarkonium Regge trajectories were obtained in our refs. [28,29]. The calculated effective ρ Regge trajectory matching the experimental data on the spectrum of the ρ trajectory as well as those on the charge-exchange reaction $\pi^- p \to \pi^0 n$ at $t < 0$. The trajectory deviates considerably from a linear in the

space-like region and is asymptotically linear in the time-like region, matching nicely between the two.

The pomeron is the highest-lying Regge trajectory. In the many high-energy reactions with small momentum transfer the pomeron exchange, gives the dominant contribution [10,30]. The classic soft pomeron is constructed from multi-peripheral hadronic exchanges. It is usually believed that the soft P trajectory is a linear function,

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P(0)t, \qquad (2)$$

where the intercept $\alpha_P(0) = 1$ and the slope $\alpha'_P(0) = 0.25\,\mathrm{GeV}^{-2}$. These fundamental parameters are the most important in high-energy hadron physics. Usually, they are determined from experiment in hadron-hadron collisions.

To explain the rising hadronic cross-sections at high energies, the classic soft pomeron was replaced by a soft supercritical pomeron with an intercept $\alpha_P(0) \simeq 1.08$. What is the pomeron by definition?

The pomeron is the vacuum exchange contribution to scattering at high energies at leading order in $1/N_c$ expansion. In gauge theories with string-theoretical dual descriptions, the pomeron emerges unambiguously. In the QCD framework the pomeron can be understood as the exchange of at least two gluons in a color singlet state [31]. The pQCD approach to the pomeron, the Balitskiĭ -Fadin-Kuraev-Lipatov (BFKL) pomeron, has been discussed in [32,33]. The pomeron can also be associated with a reggeized massive graviton [34].

The approximate linearity (2) is true only in a small-($-t$) region. The ZEUS, H1 as well as CDF data on $p\bar{p}$ elastic scattering data have also been analyzed by using the non-linear P trajectory [15]. Important theoretical results have been obtained in [35–37]. The results imply that the effective P trajectory flattens for $-t > 1\,\mathrm{GeV}^2$ that is evidence for the onset of the perturbative 2-gluon pomeron. These results may shed some light on the self-consistency of recent measurements of hard-diffractive jet production cross-sections in the UA8, CDF and HERA experiments.

A further analysis [36] of inelastic diffraction data at the ISR and SPS-Collider confirms the relatively flat s-independent P trajectory in the high-($-t$) domain, $1 < -t < 2\,\mathrm{GeV2}$, reported by the UA8 Collaboration. This suggests a universal fixed P trajectory at high $-t$. It was shown that a triple-Regge pomeron-exchange parametrization fit to the data requires an s-dependent (effective) P trajectory intercept, $\alpha_P(0)$, which decreases with increasing s, as expected from unitarization (multi-pomeron-exchange) calculations, $\alpha_P(0) = 1.10$ at the lowest ISR energy, 1.03 at the SPS-Collider and perhaps smaller at the Tevatron. In [37] a new $\gamma^* p / p\bar{p}$ factorization test in diffraction, valid below Q^2 about $6\,\mathrm{GeV}^2$ has been investigated. The apparent factorization breakdowns are likely due to the different effective P trajectories in ep and pp interactions.

The issue of soft and hard pomerons has been discussed extensively in the literature [32,33,38]. Both the IR (soft)

pomeron and the UV (BFKL) pomeron are dealt in a unified single step. On the basis of gauge/string duality, the authors describe simultaneously both the BFKL regime and the classic Regge regime. The problem was reduced to finding the spectrum of a single j-plane Schrödinger operator. The results agreed with expectations for the BFKL pomeron at negative t, and with the expected glueball spectrum at positive t, but provide a framework in which they are unified.

A model for the pomeron has been put forward by Landshoff and Nachtmann where it is evidenced the importance of the QCD NP vacuum [9]. The current data is compatible with a smooth transition from a soft to a hard pomeron contribution which can account for the rise of σ_{tot} with s. If soft and BFKL pomeron have a common origin, the discontinuity across the cut in the $\alpha_P(t)$ plane must have a strong t-dependence that points out on non-linearity of the P trajectory [39].

There are currently no any realistic theoretical estimations of the P trajectory. The behavior of the trajectory $\alpha_P(t)$ in the whole region is unknown. Below, in this work we reproduce the P trajectory in the whole region and calculate its parameters $\alpha_P(0)$ and $\alpha'_P(0)$.

The pomeron trajectory. – Let us consider the picture in which the physical particles on the P trajectory are glueballs, *i.e.*, purely gluonic bound states of massive gluons [1,3]. The P trajectory can be obtained by similar way as the reggeon ones [28,29]. We consider clueballs as relativistic two-gluon bound systems. The question arises: what is the potential of gg interaction?

The potential is a non-relativistic concept. Nevertheless, the potential is successfully used in many relativistic models. We do not know the QCD potential in the whole region. It is generally agreed that, in pQCD, as in QED the essential interaction at small distances is instantaneous Coulomb exchange; in QCD, it is qq, qg, or gg Coulomb scattering [39]. For large distances, from lattice-gauge-theory computations [40] follows that the potential is an approximately linear, $V_L(r) \simeq \sigma r$, at $r \to \infty$, where $\sigma \simeq 0.15 \, \text{GeV}^2$ is the string tension.

In the model of [21] all dependence on gluonic fields $\bar{A}_\mu$ is contained in the adjoined Wilson loop $\langle W_{adj}(C) \rangle$, where the closed contour C runs over trajectories $z_\mu(\sigma)$ and $\bar{z}_\mu(\sigma)$ of both gluons. Gluons are linked by an adjoint string. The adjoint string tension $\sigma_a = 9\sigma/4$ is expressed in terms of the well-known fundamental string tension σ for mesons through the Casimir scaling hypothesis. Using typical values for the parameters, $\sigma = 0.15 \, \text{GeV}^2$ and $\alpha_s = 0.4$ for the strong coupling, this model encodes the essential features of glueballs. More discussions on the subject can be found in [3].

In the adjoined and fundamental representations, the final form of interaction of two gluons is [21]

$$V_{gg}(r) = -\frac{\alpha_a}{r} + \sigma_a r - C_0, \qquad (3)$$

where $\alpha_a \equiv \alpha^{adj} = 3\alpha_s^{fund}$, $\sigma_a \equiv \sigma^{adj} = \frac{9}{4}\sigma^{fund}$; α_s^{fund} is the strong coupling, $\sigma^{fund} \equiv \sigma \simeq 0.15 \, \text{GeV}^2$ is the string tension, and C_0 is the arbitrary parameter.

In hadron physics, the nature of the potential is very important. There are normalizable solutions for scalar-like potentials, but not for vectorlike [41]. The effective interaction has to be scalar in order to confine particles (quarks and gluons) [41].

To reproduce the P trajectory we need to obtain an analytic expression for the squared gluonium mass, E^2. For this, we solve the relativistic semi-classical wave equation, which for two interacting gluons of equal masses $\mu_1 = \mu_2 = \mu_0$ is [42,43]

$$-\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2 \theta}\frac{\partial^2}{\partial \varphi^2} \right) \tilde{\psi}(\vec{r}) = p^2(E, r)\tilde{\psi}(\vec{r}), \qquad (4)$$

where $p^2(E, r) = E^2/4 - [\mu_0 + V_{gg}(r)]^2$.

The correlation of the function $\tilde{\psi}(\vec{r})$ with the wave function $\psi(\vec{r})$ in case of the spherical coordinates is given by the relation $\tilde{\psi}(\vec{r}) = \sqrt{\det g_{ij}}\,\psi(\vec{r})$, which follows from the identity: $\int |\psi(\vec{r})|^2 \, \mathrm{d}^3\vec{r} \equiv \int |\psi(\vec{r})|^2 \det g_{ij}\mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}\varphi = 1$, where g_{ij} is the metric tensor ($\det g_{ij} = r^2 \sin \theta$ for the spherical coordinates).

Relativistic wave equations are usually solved in terms of special functions, with the help of specially developed methods or numerically. However, almost together with quantum mechanics, the appropriate method to solve the wave equation has been developed; it is general simple for all the problems, and its correct application results in the exact energy eigenvalues for all solvable potentials. This is the phase-integral method which is also known as the WKB method [44,45].

It is hard to find the exact analytic solution of eq. (4) for the potential (3). But we can find exact analytic solutions for two asymptotic limits of the potential (3), *i.e.* for the Coulomb and linear potentials, separately [42]. The most general form of the WKB solution and the quantization condition can be written in the complex plane [43].

The WKB quantization condition appropriate to (4) with the Coulomb potential is

$$I = \oint_C \sqrt{\frac{E^2}{4} - \mu_0^2 + \frac{2\alpha_a\mu_0}{r} - \frac{\Lambda^2}{r^2}} = 2\pi\left(n_r + \frac{1}{2}\right), \quad (5)$$

where $\Lambda^2 = (l + 1/2)^2 + \alpha_a^2$ and a contour C encloses the classical turning points r_1 and r_2. Using the method of stereographic projection, we should exclude the singularities outside the contour C, *i.e.* at $r = 0$ and ∞. Excluding these infinities we have, for the integral (5),

$$I = 2\pi(\alpha_a\mu_0/\sqrt{-E^2/4 + \mu_0^2} - \Lambda), \qquad (6)$$

and for the energy eigenvalues this gives [46],

$$E_n^2 = 4\mu_0^2\left[1 - \frac{\alpha_a^2}{(n_r + 1/2 + \Lambda)^2}\right]. \qquad (7)$$

Eigenvalues (7) are exact, for the Coulomb potential. Analogously, we obtain, for the linear potential [43],

$$E_n^2 = 8\sigma_a \left(2n_r + l - \alpha_a + \frac{3}{2} \right). \qquad (8)$$

For small distances, where the Coulomb-type contribution dominates, the effective strong coupling, α_a, is a small value and (7) can be rewritten in the simpler form

$$E_n^2 \simeq 4\mu_0^2 \left[1 - \frac{\alpha_a^2}{(n_r + l + 1)^2} \right]. \qquad (9)$$

To find gluonium energy eigenvalues (glueball masses) we use the same approach as in [28,29], $i.e.$, we derive an interpolating mass formula for E_n^2, which satisfies both of the above constraints (8) and (9). To derive such a formula, the two-point Padé approximant can be used [46],

$$[K/N]_f(z) = \frac{\sum_{i=0}^{K} a_i z^i}{\sum_{j=0}^{N} b_j z^j}, \qquad (10)$$

with $K = 3$ and $N = 2$. We use $K = 3$ and $N = 2$ because this is a simplest choice to satisfy the two asymptotic limits above.

The result is the interpolating mass formula [28,29],

$$E_n^2 = 8\sigma_a \left(2n_r + l + \frac{3}{2} - \alpha_a \right) - \frac{4\alpha_a^2 \mu_0^2}{(n_r + l + 1)^2} + 4\mu_0^2. \qquad (11)$$

Expression (11) is an Ansatz (as the potential (3)), which is based on two asymptotic expressions (8) and (9). Formula (11) and its derivation are rather simple; this allows us to get an $analytic$ expression for the P trajectory, $\alpha_P(t)$, in the whole region.

Transform (11) into the cubic equation for the angular momentum l,

$$l^3 + c_1(t)l^2 + c_2(t)l + c_3(t) = 0, \qquad (12)$$

where $c_1(t) = 2\tilde{n} + \lambda(t)$, $c_2(t) = \tilde{n}^2 + 2\tilde{n}\lambda(t)$, $c_3(t) = \tilde{n}^2\lambda(t) - \alpha_a^2\mu_0^2/2\sigma_a$, $\tilde{n} = n_r + 1$, $\lambda(t) = 2\tilde{n} - 1/2 - \alpha_a + (4\mu_0^2 - t)/8\sigma_a$. Equation (12) has three (complex in the general case) roots: $l_1(t)$, $l_2(t)$, and $l_3(t)$. The real part of the first root, $\text{Re}\, l_1(t)$, gives the P trajectory,

$$\alpha_P(t) = \begin{cases} f_1(t) + f_2(t) - c_1(t)/3, & Q(t) \geqslant 0, \\ 2\sqrt{-p(t)} \cos\left[\beta(t)/3\right] - c_1(t)/3, & Q(t) < 0, \end{cases} \qquad (13)$$

where

$$f_1(t) = \sqrt[3]{-q(t) + \sqrt{Q(t)}}, \quad f_2(t) = \sqrt[3]{-q(t) - \sqrt{Q(t)}},$$

$$Q(t) = p^3(t) + q^2(t), \quad p(t) = -c_1^2(t)/9 + c_2(t)/3,$$

$$q(t) = c_1^3(t)/27 - c_1(t)c_2(t)/16 + c_3(t)/12,$$

$$\beta(t) = \arccos\left[-q(t)/\sqrt{-p^3(t)} \right].$$

Table 1: Glueball masses and parameters of the gg potential (3).

Method	J^{PC}	E_n^{Gl}	Parameters
I	2^{++}	1.740	$\alpha_a = 2.448$ - fixed
	3^{--}	2.452	$\sigma_a = 0.338\,\text{GeV}^2$ - fixed
	4^{++}	2.974	$\mu_0 = 0.495\,\text{GeV}$ - fixed
	5^{--}	3.408	
	6^{++}	3.789	
II	2^{++}	1.984	$\alpha_a = 2.276 \pm 0.041$
	3^{--}	2.689	$\sigma_a = 0.294 \pm 0.003\,\text{GeV}^2$
	4^{++}	3.164	$\mu_0 = 0.968 \pm 0.147\,\text{GeV}$
	5^{--}	3.549	
	6^{++}	3.884	
III	2^{++}	1.695	$\alpha_a = 2.442 \pm 0.044$
	3^{--}	2.393	$\sigma_a = 0.323 \pm 0.071\,\text{GeV}^2$
	4^{++}	3.904	$\mu_0 = 0.478 \pm 0.084\,\text{GeV}$
	5^{--}	3.330	
	6^{++}	3.703	

Expression (13) supports existing experimental data and reproduces the soft P trajectory in the whole region of t (see below). The corresponding parameters α_a, σ_a and μ_0 are listed in table 1.

We calculate glueball masses and the P trajectory for three different sets of parameters (methods): I) the parameters are fixed as in [21]: $\alpha_a = 3\alpha_s = 2.448$, $\sigma_a = 9\sigma/4 = 0.338\,\text{GeV}^2$, and gluon mass $\mu_0 = 1.5\,m_q = 0.495\,\text{GeV}$ (see also [47]) for the typical values $\alpha_s = 0.816$, string tension $\sigma = 0.15\,\text{GeV}^2$, quark mass $m_q = 0.330\,\text{GeV}$ of light mesons; II) the parameters α_a, σ_a, and μ_0 are found from the fit of HERA data for the P trajectory [12]; III) include into the fit the $J^{PC} = 2^{++}$ glueball candidate for $m_G = 1.710\,\text{GeV}$ [48] by supposing that the glueball trajectory is the soft P trajectory. Masses of gluonium leading states, E_n^{Gl}, have been calculated with the use of the interpolating mass formula (11). The methods I) and III) reproduce the trajectory with the properties of the supercritical soft pomeron. The effective intercept and slope estimated by these two methods are:

$$\alpha_P(0) = 1.09 \pm 0.02, \quad \alpha_P'(0) = 0.26 \pm 0.03\,\text{GeV}^{-2}. \qquad (14)$$

The corresponding effective mass of the 2^{++} glueball candidate is $m_G \simeq 1.74\,\text{GeV}$.

Several pomerons are shown in fig. 1. Solid line is the effective P trajectory (13). The trajectory is asymptotically linear at $t \to \infty$ with the slope $\alpha_P' = 1/(8\sigma_a) \simeq 0.38\,\text{GeV}^{-2}$. In the scattering region, the trajectory flattens off at -1 for $t \to -\infty$. We see that the experimental data and simple calculations in the framework of the potential approach support the conception of the soft supercritical pomeron as observed at the presently available energies.
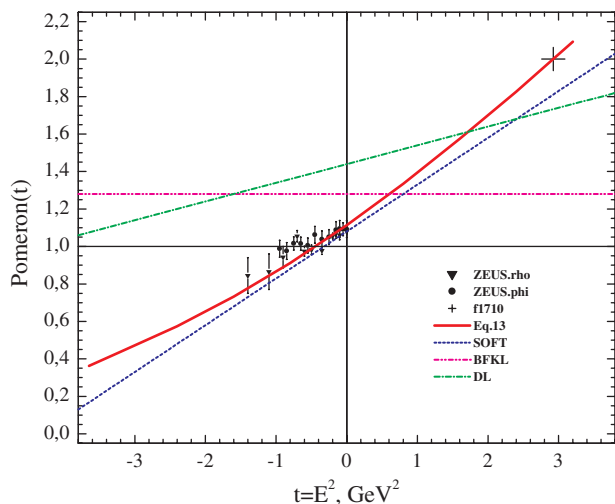
Fig. 1: (Colour on-line) The effective pomeron trajectory. The solid curve is the trajectory (13) with the parameters found from the fit of combined HERA ρ (triangles) and ϕ (circles) data [12], and 2^{++} glueball candidate $f_0(1710)$ (cross) [48]. Other lines show the classic soft, BFKL, and Donnachie-Landshoff hard pomerons.

The perturbative (Coulomb) part of the potential (3) gives the asymptotic expression (9), inverting which we have ($t = E^2$)

$$l(t) = -1 + \frac{\alpha_a}{\sqrt{1 - t/(4\mu_0^2)}}, \qquad t \to -\infty. \qquad (15)$$

If we take into account spins of gluons with the total spin of two interacting gluons $S = 2$ (leading states), then the formula

$$\alpha(t) = l(t) + S \qquad (16)$$

gives the asymptote for the BFKL pomeron predicted by pQCD [33].

The pomeron with such properties has been also used to describe the HERA data on the charm structure function F_2^c [12,49]. It was shown that the two-pomeron picture (soft-plus-hard-pomeron) gives a very good fit to the total cross-section for elastic J/Ψ photoproduction and the charm structure function F_2^c over the whole range of $Q^2 = -t$ [50]. However, the results of these experiments and the found higher-order corrections make it quite unclear what the hard pomeron is.

Conclusion. – Glueballs are a good test of our understanding of the non-perturbative aspects of QCD. Their existence is allowed by QCD and the glueball spectrum has been computed in lattice QCD.

We have considered glueballs as bound states of constituent massive gluons and investigated their properties in the framework of the potential approach. For two-gluon system, we have analyzed exact asymptotic solutions of relativistic wave equation with the funnel-type potential. Using the asymptotes, we have derived the interpolating mass formula (11) and calculated the glueball masses, which are in agreement with the lattice data.

We have considered glueballs as the physical particles on the P trajectory. To reproduce the P trajectory, we have derived the interpolating mass formula (11) for the squared energy eigenvalues, $E_n^2 = E^2(l, n_r)$, of the gg system. We have calculated gluonium masses and obtained the analytic expression (13) for the P trajectory, $\alpha_P(t)$ in the whole region. In the scattering region, at $-t \gg \Lambda_{QCD}$, the trajectory flattens off at -1 and has asymptote $\alpha_P(t \to -\infty) = -1$. In the bound-state region, the P trajectory is approximately linear in accordance with the string model.

However, the non-linearity of trajectories is still an open question. The curvature of the trajectory may come from several linear trajectories, as Donnachie and Landshoff showed. Here we have considered one of possible scenario. We do not have enough experimental data to make final conclusion. It is well known, that the fixed-number of particles within the potential approach cannot be used for a strict relativistic description. A strict description of the pomeron presupposes a multiparticle system. For the perturbative regime with the pomeron scattering, the dominant contribution comes from the BFKL pomeron. However, experimental data and our rather simple calculations support the conception of the soft supercritical pomeron as observed at the presently available energies. The hard BFKL pomeron has intercept $\alpha_P(0) = 1.44$. Next-to-leading-order estimates give for the BFKL intercept 1.26 to 1.3, which is closer to the soft supercritical pomeron.

In this paper we have not considered helicities and spin properties of gluons. This topic has been discussed in details somewhere else [20]. The existing data and the simple analysis performed in this work confirm the existence of the pomeron whose trajectory is non-linear and corresponds to the supercritical soft pomeron at small spacelike t.

In many Regge models [51,52], one-pomeron exchange gives only the dominant contribution into the cross-section. With energy growth, multiple-pomeron exchanges (MPE) and sea quark contributions become important. The MPE contributions are important just at small-x and can give explanation of the small-x charm production data at HERA.

Combined with the eikonal model the MPE contributions give the correct energy dependence of total and total inelastic cross-sections [52] and allow to describe hard distributions of secondary hadrons [51]. From this point of view, the required hard pomeron discussed in [13] effectively accounts for the MPE contributions.

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