Theory of gravity in the limit of Newton's constant tending to zero

V.I. Zakharov

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Preliminaries

The terminology used in the title
is not common (yet) and explained later

(vaguely: $G_N \to 0$ limit incorporates equilibrium theory)

Or, the talk is a brief review
of some aspects of field theory - thermodynamics relation

Original part is based on common work with
G. Yu. Prokhorov and O.V. Teryaev (Dubna)

Motivation

Discovery of QGP (quark-gluon plasma) in the year 2005 remains ‘driving force‘’ of theory development .

Challenges to theory (low viscosity, fast termalization),

Also, plasma is produced in extreme environment:
STAR collaboration (2017): QGP is rotated and accelerated

Rotation and acceleration are two basic non-inertial motions considered by Einstein in his first papers on GR.

In general, accelerated frame through the equivalence principle is related to gravity. We will try to make this relation more precise.

D Kharzeev and K Tuchin's paper (2005)

Noted that the temperature of plasma is "close' to the so called Unruh temperature

$$T_{Unruh} = \frac{\hbar a}{2\pi c k_B} ,$$

where $\hbar$ is the reduced Planck constant, c is the speed of light, and k_B is the Boltzmann constant.

Phenomenologically T is temperature of plasma, acceleration a in terms of tension of strings stretched between color charges.

$T_{Kharzeev-Tuchin} \sim T_{hadroniic}$ (with no mention of G_N)

Unruh temperature

One of key notions :

observer moving with acceleration a with respect to Minkowskian vacuum sees thermal distribution of particles with temperature T_{Unruh}
Observer at rest does not see particles

Also, for Hawking radiation

$$(2\pi)T_{horizon}/a_{horizon} = 1$$

Kharzeev+Tuchin: in accelerated frame horizon is formed and we observe radiation from horizon as QGP

Question which we try to answer, what is systematic approach to this kind of physics

Some milestones in FT/Thermodynamics interplay

Equivalence principle
motion in accelerated frame is the same as in external gravitational field:

$$\vec{a}_{kin} \;=\; \vec{a}_{grav}$$

Tolman temperature (1930)
In equilibrium, local temperature depends on grav. field:

$$T_0 = T(z)\left(1 + \frac{gz}{c^2}\right) \quad \text{and} \quad T(r) = \frac{T_0}{\sqrt{-g_{00}(r)}}$$

for Earth field and static-symmetric field, respectively

Physics of equilibrium vs physics of gravity

JM Luttinger (1964)

Transport induced by gradient of temperature $\vec{\nabla} T$ identical to that induced by acceleration $\vec{a}_{grav}$

$$\frac{\vec{\nabla} T}{T} \rightarrow -\frac{\vec{\nabla} \phi_{grav}}{c^2}$$

as a reflection of universality of temperature and gravity

E. Verlinde (2011)
hypothesis: gravity is not fundamental and can be replaced by macroscpic entropic force :

$$\vec{F}(X_0)_{entropic} = T \vec{\nabla}_X S(X)|_{X_0}$$

where S is entropy, X is a characteristic of macrostate

Thermodynamics in non-inertial frames

One averages matrix elements with density operator

$$\hat{\rho} \; = \; \exp(-\hat{H}_{eff})$$

examples: $\hat{H}_{eff} \; = \; \hat{H}_0 \; - \vec{\omega} \cdot \hat{\vec{M}}$ (rotation)

where $\vec{\omega}$ is angular velocity, $\hat{\vec{M}}$ is angular momentum

$$\hat{H}_{eff} \; = \; \hat{H}_0 \; - \vec{\omega} \cdot \hat{\vec{M}} - \vec{a}\hat{\vec{K}} \text{ (rotation and acceleration)}$$

where $\vec{a}$ is acceleration, $\hat{\vec{K}}$ - boost operator

The last line is far from being trivial since $[\hat{K}, \hat{H}_0] \neq 0$
F. Becattini (2017)

Equilibrium theory and Feynman graphs

Amusingly enough, one can start from $\hat{H}_{eff}$ and develop diagrammar for the "Equilibrium theory"
(....F.Becattini + coauthors, Dubna group...)

Analog of "ground state "is the
state with maximal entropy

One-loop quantum graphs can be consistently treated

For example, for massless fermions one finds a closed form:

$$T_{00} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2}$$

no UV divergencies, $T_{00}(T_{Unruh} = 0)$

Cont'd

As a result of calculations we get matrix elements of various operators as funcrions of $(\vec{a}, \vec{\Omega})$

$< T^{\alpha\beta}(\vec{a}, \vec{\Omega}) >, < J_5^{\alpha}(\vec{a}, \vec{\Omega})) >$

where,in case of hydrodynamics, acceleration and rotation 4-vectors are given by:

$$a_\mu \;=\; u^\alpha \partial_\alpha u_\mu, \qquad \omega_\mu \;=\; (1/2)\epsilon_{\mu\nu\rho\sigma} u^\nu \partial^\rho u^\sigma$$

where u^ν is the fluid velocity

Note: energy of the center of mass is removed from $T^{\alpha\beta}$

Gravitational interaction (the other approach)

In field theory, gravitational interaction is described by fundamental interaction Lagrangian:

$$\delta L \; = \; -\tfrac{1}{2} T^{\alpha\beta} h_{\alpha\beta}$$

where $T^{\alpha\beta}$ is the energy-momentum tensor of matter, $h_{\alpha\beta}$ is the gravitational potential, also accommodating $\vec{\Omega}_{grav}, \vec{a}_{grav}$.

$$\vec{a}_{grav} \; \sim \; \vec{\nabla} h_{00}, \qquad \vec{\omega}_{grav} \; \sim \; \vec{\nabla} \times \vec{h}$$

$$(\vec{h})_i \equiv h_{0i}$$

Evaluate loop correction to $T^{\alpha\beta}$ pertubatively in external grav. field

Generalization of equivalence principle

Central point (Dubna group):
kinematical $(\vec{a}_{kin}, \vec{\omega}_{kin})$ and dynamical $(\vec{a}_{grav}, \vec{\omega}_{grav})$ introduced

Furthermore, one evaluates "external probes",

$$< T^{\alpha\beta} >, \qquad < J_5^{\alpha} > .....$$

within both approaches, statistical and gravitational.
The results compared for the same values of $\vec{a}, \vec{\omega}$. and are to be he same in both cases

tA generalization of the equivalence principle to higher orders in acceleration, or
duality between thermodynamic and geometric approaches

Duality is confirmed numerically on a number of examples.

The reason for duality

The reason for results identity is that both theories are built on conserved quantities
Standard (Landau+Lifshitz):
conservation of energy and angular momentum

Novel (Becattini) :
conservation of energy, angular momentum, boost
Much less obvious since energy and boost do not commute

No analytic proof

The results are identical but derivations are not.

A problem:
In field theory: spin is allowed but
dipole gravitational moment is forbidden by CP-invariance

In statistics:

angular momentum $\vec{M}$ and boost $\vec{K}$ enter symmetrically.

Dipole gravitational momentum cancelled later, between particles and antiparticles in equilibrium.

(similar to Sakharov's conditions for baryon number violations)

Anomalous axial current, IR sensitive evaluation

Our aim is to evaluate axial current J_5^α which is anomalous in presence of certain external grav. field:

$$\nabla^\alpha J_{\alpha,5} \ = \ C^{grav} R\tilde{R}$$

Use for this purpose equilibrium theory in non-inertial frames in absence of external field

Consider third order perturbation theory in

$$\delta L \sim (\vec{a}\vec{K}), (\vec{\omega}\vec{M})$$

Find non-vanishing J_5^α:

$$J_5^{\alpha, kinematic} \ = \ -\frac{1}{24\pi^2}\left(\lambda_1 \omega_\mu^2 + \lambda_2 a_\mu^2\right)\omega^\alpha$$

Kinematical Vortical Effect

Evaluate covariant derivative of the axial current obtained

Find out that difference of the coefficients λ_1 and λ_2 is related to the coefficient C_5^{grav} in front of the gravitational anomaly:

$$\frac{\lambda_1 - \lambda_2}{32} = C_5^{grav}$$

is called kinematical vortical effect effect (KVE)
(G. Prokhorov, O. Teryaev, V.Z.)

Cont'd

By a direct clculation, the relation has been verified both in case of spin-1/2 and spin-3/2 constituents. Namely:

$$\lambda_1 = -1 \quad \lambda_2 = -3 \quad C_5^{grav} = 1/(384\pi^2) \qquad (spin\ 1/2)$$

$$\lambda_1 = -53, \quad \lambda_2 = -15, \quad C_5^{grav} = -19/(384\pi^2) \quad (spin\ 3/2)$$

Significance of all this

First step:
Start from empty Minkowski vacuum $\vec{J}_5^{tot} \equiv 0$
This is a general-relativity covariant statement

Division of $\vec{J}_{tot}^5$ into currents of real (thermal) and virtual (vacuum) particles is frame/gauge dependent

Second Go to accelerated frame, as fixation of gauge.
In accelerated frame $T = T_{Unruh}$ and evaluate $J_5^{kinematic}$
Everything is UV stable, since current is evaluated over Unruh sample of particles

Third: compare with $\nabla_\alpha J^\alpha = \mathit{standard\ anomaly}$,
as evaluated via gravitational interactions.
Observe agreement

Cont'd

We have found that we can mix up UV- finite calculations of equilibrium theory and UV sensitive calculations of the standard field theory. They are absolutely consistent with each other.

$G_N \to 0$ limit

there exists a kind of theory which unifies effects, or interactions, surviving in the limit of the Newton constant tending to zero, $G_N \to 0$.

It includes, preliminary :
– theory of equilibrium
– using accelerated frames, or covariant derivatives ∇_α
– certain class of Feynman graphs for loop effects for matter in external grav. field, calculated to lowest order in gravity and in appproximation of flat space

Concluding remark

The term *limit of* $G_N \to 0$ is used by Witten in series of articles around (2022) who argued that, no matter how small G_N is, gravity in the bulk triggers a dual, entropic picture on the boundary. This idea is applied mostly to de Sitter space

We probably have a simplest example of this types (with more detailed calcuations

Perspectives

Thermodynamic ultraviolet regularization

Phase transitions in (a, T) plane