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INP

Effects of interactions of axion-like dark matter with Standard Model particles

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OUTLINE

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Pseudoscalar interactions caused by dark matter axions

CP-noninvariant interactions caused by dark matter axions are time-dependent. Like photons, moving axions form a wave which pseudoscalar field reads

$$a(\boldsymbol{r}, t) = a_0 \cos\left(E_a t - \boldsymbol{p}_a \cdot \boldsymbol{r} + \phi_a\right).$$

Here $E_a = \sqrt{m_a^2 + \boldsymbol{p}_a^2}$, $\boldsymbol{p}_a$, and m_a are the energy, momentum, and mass of axions . The Earth motion through our galactic define its velocity relative to dark matter, $V \sim 10^{-3}c$. Therefore, $|\boldsymbol{p}_a| \approx m_a V$ and axions and axion-like particles have momenta of the order of $|\nabla a| \sim 10^{-3}\dot{a}c$.

We suppose that axion-like dark matter interacts like the axion. The Peccei-Quinn theory introduces a new anomalous U(1) symmetry to the Standard Model along with a new pseudoscalar field which spontaneously breaks the symmetry at low energies, giving rise to an axion that suppresses the problematic CP violation.

Strong CP problem and dark matter

QCD Lagrangian:

contains CP violating term:

$$\mathcal{L}_{CP} = -\frac{g^2}{32\pi^2}\Theta\,\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

Neutron electric dipole moment

$d_n \approx \Theta\, 10^{-16}\mathrm{e}\cdot\mathrm{cm} < 10^{-25}\mathrm{e}\cdot\mathrm{cm}$

Problem: why so small?

$\Theta < 10^{-9}$

Peccei&Quinn'77, Wiczeck'78, Weinberg'78

The tilde denotes a dual tensor

Postulate new global U(1) symmetry - Peccei-Quinn symmetry

Re-interpret Θ as a scalar field a - axion - Nambu-Goldstone boson

$$\mathcal{L}_{CP} = -\frac{g^2}{32\pi^2}\Theta\,\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu} \Longrightarrow \mathcal{L}_{CP} = -\frac{g^2}{32\pi^2}\frac{a(x)}{f_a}\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

The result of axion-gluon interactions is an oscillating EDM of a strongly interacting particle like a nucleon:

$$\mathcal{L}_{aEDM} = -\frac{i}{2} g_d a \sigma^{\mu\nu} \gamma^5 F_{\mu\nu}$$

where the EDM is equal to $d_a = g_d a$ and g_d is proportional to g *agg*

The axion-photon interaction leads to mixing of electric and magnetic fields and results in the Lagrangian density

$$\mathcal{L}_\gamma = -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \widetilde{F}^{\mu\nu} = g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B}$$

Another contribution to the total Lagrangian density (Pospelov et al.) is defined by the gradient interaction (axion wind effect):

$$\mathcal{L}_N = g_{aNN} \gamma^\mu \gamma^5 \partial_\mu a$$

M. Pospelov, A. Ritz, and M. Voloshin, Phys. Rev. D **78**, 115012 (2008); V. A. Dzuba, V. V. Flambaum, and M. Pospelov, Phys. Rev. D **81**, 103520 (2010).

The Lagrangian $L=\overline{\psi}\mathcal{L}\psi$ describing electromagnetic interactions of a Dirac particle with allowance for a pseudoscalar axion field is defined by

$$\mathcal{L}=\gamma^\mu(i\hbar\partial_\mu-eA_\mu)-m+\frac{\mu'}{2}\sigma^{\mu\nu}F_{\mu\nu}-i\frac{d}{2}\sigma^{\mu\nu}\gamma^5F_{\mu\nu}+g_{aNN}\gamma^\mu\gamma^5\Lambda_\mu,$$

$$\Lambda_\mu=\partial_\mu a,\qquad \gamma^5=\begin{pmatrix}0&-1\\-1&0\end{pmatrix},$$

where μ' and d are the anomalous magnetic and electric dipole moments. In the last term, $a=a_0\cos{(m_at-\boldsymbol{p}_a\cdot\boldsymbol{r})}$ is the axion field.

The corresponding Hamiltonian in the Dirac representation reads

$$\mathcal{H}=\beta m+\boldsymbol{\alpha}\cdot(\boldsymbol{p}-e\boldsymbol{A})+e\Phi+\mu'(i\boldsymbol{\gamma}\cdot\boldsymbol{E}-\boldsymbol{\Pi}\cdot\boldsymbol{B})$$
$$-d(\boldsymbol{\Pi}\cdot\boldsymbol{E}+i\boldsymbol{\gamma}\cdot\boldsymbol{B})-g_{aNN}(\gamma^5\Lambda_0+\boldsymbol{\Sigma}\cdot\boldsymbol{\Lambda}).$$

Physics – Uspekhi **66** (2) 109 – 147 (2023)

General relativity effects in precision spin experimental tests of fundamental symmetries

S N Vergeles, N N Nikolaev, Yu N Obukhov, A Ya Silenko, O V Teryaev

Attributing the local energy density of dark matter $\rho_{\rm DM} \approx 400$ MeV cm^{-3} [386] to axions in the invisible halo of our Galaxy, the amplitude of the classical axion field $a(x) = a_0 \cos(\omega_{(a)} t - \mathbf{k}_{(a)} \mathbf{x})$ can be evaluated as [56]

$$a_0 = \frac{1}{m_{(a)}} \sqrt{\frac{2\rho_{\rm DM} \hbar}{c^3}}.$$

the oscillating contribution to the EDM of nucleons

$$d_{\rm N}^{(a)}(x) = \eta^{(a)} \frac{\mu_{\rm N}}{c} = \frac{a(x)}{f_{(a)}} \kappa_{(a)} \frac{\mu_{\rm N}}{c},$$

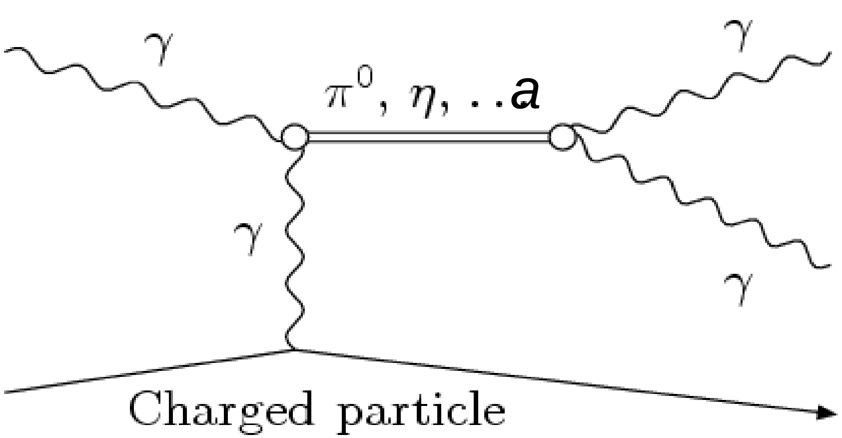
where the chiral suppression of the EDM [101, 102] is shown explicitly:

$$\kappa_{(a)} \sim \frac{m^*}{\Lambda_{\rm QCD}} \approx 10^{-2}.$$

Optical experiments

Primakoff effect

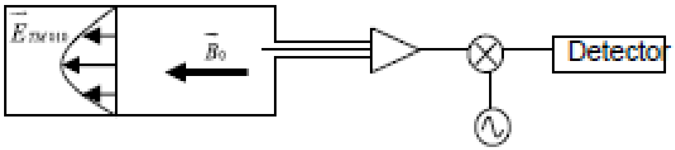
The Primakoff effect is the resonant transformation of a photon in a static electric or magnetic field (for example, in the field of the nucleus) into a massive neutral pseudoscalar particle (in particular, axion). Particles that can be born due to the Primakoff effect are capable of splitting into two photons and converting into a photon in an electromagnetic field (the inverse Primakoff effect); in fact, both the forward and inverse Primakoff effect are described by a vertex on the Feynman diagram connecting a pseudoscalar with two photons.

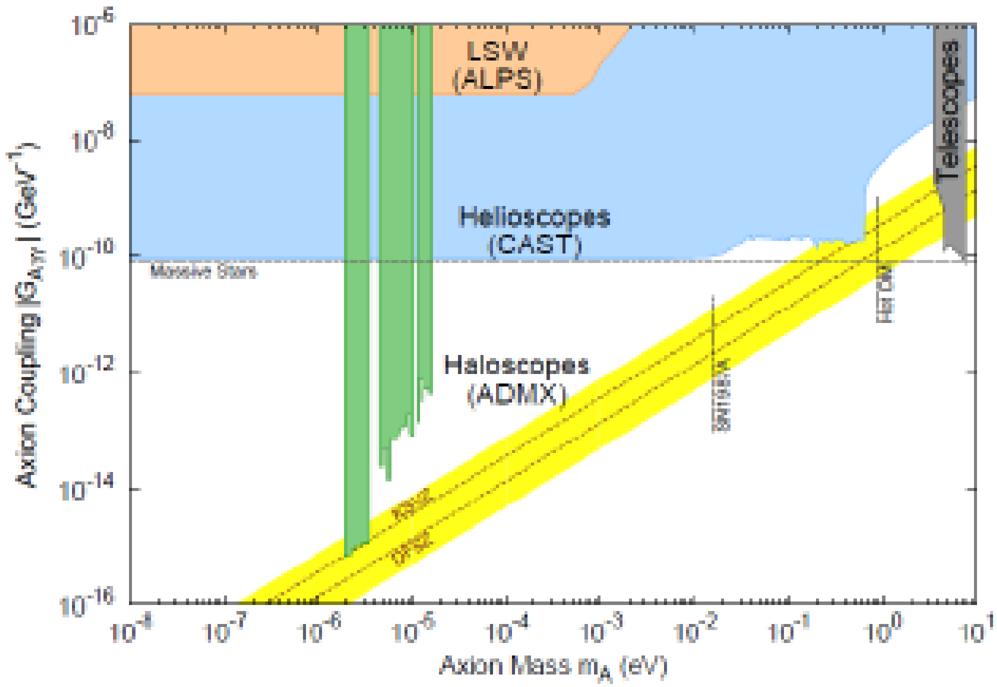


DIRECT AXION DETECTION

- Powerful magnets.
- Very weak signal (~10^{-21} W)
  - ⇒ Cryogenic temperatures.
  - ⇒ Resonant radiofrequency (RF) cavities for enhancement.
  - ⇒ Tunable over a good frequency range.

At the cavity's resonant frequencies the microwaves reinforce to form standing waves in the cavity. Therefore, the cavity functions similarly to an organ pipe or sound box in a musical instrument, oscillating preferentially at a series of frequencies, its resonant frequencies.





Still another application of the direct and inverse Primakoff effect is in the "shining laser light through a wall" approach [396], when an intermediate axion is produced in a magnetic field by the Primakoff mechanism, then penetrates through a wall opaque to the light, and subsequently regenerates back into a photon in the magnetic field [407]. A number of experiments were carried out with helioscopes, which make it possible to detect ultrarelativistic axions emitted by the Sun in the X-ray range (see, for example, [387, 408, 409]).

Relativistic Foldy-Wouthuysen transformation and the relativistic Hamiltonian in the Foldy-Wouthuysen representation

- **For arbitrary-spin particles in external fields, the Foldy-Wouthuysen (FW) representation restores the Schrödinger form of relativistic quantum mechanics (QM) while the Dirac representation corrupts this form (Foldy, Wouthuysen, 1950; Silenko, 2003, 2008).**

The position and spin operators, as well as other operators, are counterparts of the corresponding classical variables only when they are defined in the FW representation but not in the Dirac one (Foldy, Wouthuysen, 1950; Zou, Zhang, Silenko, 2020).

The passage to the classical limit usually reduces to a replacement of the operators in quantum-mechanical Hamiltonians and equations of motion in the FW representatin with the corresponding classical quantities (Silenko, 2013).

The FW transformation belongs to the foundation of QM!

In the general case, it is convenient to present the Dirac Hamiltonian as follows:

$$\mathcal{H} = \beta\mathcal{M} + \mathcal{E} + \mathcal{O}, \quad \beta\mathcal{M} = \mathcal{M}\beta, \quad \beta\mathcal{E} = \mathcal{E}\beta, \quad \beta\mathcal{O} = -\mathcal{O}\beta.$$

The even operators $\mathcal{M}$ and $\mathcal{E}$ and the odd operator $\mathcal{O}$ are diagonal and off-diagonal in two spinors, respectively.

FW transformation for relativistic arbitrary-spin particles in arbitrarily strong external fields – final results and the proof of validity:

A.J. Silenko, General method of the relativistic Foldy-Wouthuysen transformation and proof of validity of the Foldy-Wouthuysen Hamiltonian, Phys. Rev. A **91**, 022103 (2015).

If one holds only terms proportional to the zero and first powers of ħ, the final FW Hamiltonian takes the form

$$\mathcal{H}_{FW} = \beta\epsilon + \mathcal{E} + \frac{1}{4}\left\{\frac{1}{2\epsilon^2 + \{\epsilon,\mathcal{M}\}}, (\beta[\mathcal{O},[\mathcal{O},\mathcal{M}]] - [\mathcal{O},[\mathcal{O},\mathcal{F}]])\right\},$$

$$\epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2}.$$

This is the only relativistic method which gives the *exact* form of leading terms in the FW Hamiltonian of the zero and first orders in $\hbar$ and does not need cumbersome derivations. The first-order terms in $\hbar$ describe spin effects.

In the considered case,

$$\mathcal{M} = m - \mu' \boldsymbol{\Sigma} \cdot \boldsymbol{B} - d \boldsymbol{\Sigma} \cdot \boldsymbol{E}, \qquad \mathcal{E} = e\Phi - g_{aNN} \boldsymbol{\Sigma} \cdot \boldsymbol{\Lambda},$$

$$\mathcal{O} = \boldsymbol{\alpha} \cdot (\boldsymbol{p} - e\boldsymbol{A}) + i\mu' \boldsymbol{\gamma} \cdot \boldsymbol{E} - id\boldsymbol{\gamma} \cdot \boldsymbol{B} - g_{aNN} \gamma^5 \Lambda_0.$$

The FW Hamiltonian has the form

$$\mathcal{H}_{FW} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3,$$

$$\mathcal{H}_1 = \beta\epsilon' + e\Phi - \frac{1}{2}\left\{\left(\frac{\mu_0 m}{\epsilon'} + \mu'\right), \boldsymbol{\Pi} \cdot \boldsymbol{B}\right\}$$

$$+\frac{1}{4}\left\{\left(\frac{\mu_0 m}{\epsilon' + m} + \mu'\right)\frac{1}{\epsilon'}, \left(\boldsymbol{\Sigma} \cdot [\boldsymbol{\pi} \times \boldsymbol{E}] - \boldsymbol{\Sigma} \cdot [\boldsymbol{E} \times \boldsymbol{\pi}] - \nabla \cdot \boldsymbol{E}\right)\right\}$$

$$+\frac{\mu'}{4}\left\{\frac{1}{\epsilon'(\epsilon' + m)}, \left[(\boldsymbol{B} \cdot \boldsymbol{\pi})(\boldsymbol{\Pi} \cdot \boldsymbol{\pi}) + (\boldsymbol{\Pi} \cdot \boldsymbol{\pi})(\boldsymbol{\pi} \cdot \boldsymbol{B}) + 2\pi(\boldsymbol{\pi} \cdot \boldsymbol{j} + \boldsymbol{j} \cdot \boldsymbol{\pi})\right]\right\},$$

$$\mathcal{H}_2 = -d\mathbf{\Pi}\cdot\boldsymbol{E} + \frac{d}{4}\left\{\frac{1}{\epsilon'(\epsilon'+m)}, \left[(\boldsymbol{E}\cdot\boldsymbol{\pi})(\mathbf{\Pi}\cdot\boldsymbol{\pi}) + (\mathbf{\Pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{E})\right]\right\}$$
$$-\frac{d}{4}\left\{\frac{1}{\epsilon'}, \left(\mathbf{\Sigma}\cdot[\boldsymbol{\pi}\times\boldsymbol{B}] - \mathbf{\Sigma}\cdot[\boldsymbol{B}\times\boldsymbol{\pi}]\right)\right\},$$

where $\mathcal{H}_1$ defines the CP-conserving part of the total Hamiltonian $\mathcal{H}_{FW}$, $\mu_0 = e\hbar/(2m)$ is the Dirac magnetic moment, $\epsilon' = \sqrt{m^2+\boldsymbol{\pi}^2}$, and $\boldsymbol{j} = \frac{1}{4\pi}\left(c\nabla\times\boldsymbol{B} - \frac{\partial \boldsymbol{E}}{\partial t}\right)$ is the density of external electric current.

The terms describing the direct interaction with the axion field are given by

$$\mathcal{H}_3 = \frac{g_{aNN}}{2}\left\{\frac{\mathbf{\Pi}\cdot\boldsymbol{p}}{\epsilon'}, \Lambda_0\right\}$$
$$-\frac{g_{aNN}}{2}\left[\left\{\frac{m}{\epsilon'}, \mathbf{\Sigma}\cdot\mathbf{\Lambda}\right\} + \frac{(\mathbf{\Sigma}\cdot\boldsymbol{p})}{\epsilon'(\epsilon'+m)}(\boldsymbol{p}\cdot\mathbf{\Lambda}) + (\mathbf{\Lambda}\cdot\boldsymbol{p})\frac{(\mathbf{\Sigma}\cdot\boldsymbol{p})}{\epsilon'(\epsilon'+m)}\right].$$

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Regular Article - Theoretical Physics

Relativistic spin dynamics conditioned by dark matter axions

A. J. Silenko[1,2,3,a]

Corrections to the spin motion including corrections caused by the axion-induced EDM

In the semiclassical approximation, the angular velocity of the spin rotation has the form

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{TBMT} + \boldsymbol{\Omega}_{EDM} + \boldsymbol{\Omega}_{axion},$$

$$\boldsymbol{\Omega}_{TBMT} = -\frac{e}{2m}\left\{\left(g-2+\frac{2}{\gamma}\right)\boldsymbol{B} - \frac{(g-2)\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{B}) - \left(g-2+\frac{2}{\gamma+1}\right)(\boldsymbol{\beta}\times\boldsymbol{E})\right\},$$

$$\boldsymbol{\Omega}_{EDM} = -\frac{e\eta}{2m}\left[\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}) + \boldsymbol{\beta}\times\boldsymbol{H}\right],$$

$$\boldsymbol{\Omega}_{axion} = 2g_{aNN}\left(\Lambda_0\boldsymbol{\beta} - \frac{\boldsymbol{\Lambda}}{\gamma} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{\Lambda})\boldsymbol{\beta}\right),$$

where $\boldsymbol{\Omega}_{TBMT}$ is determined by the Thomas-Bargmann-Michel-Telegdi equation and the factors $g = 4(\mu_0 + \mu')m/e$ and $\eta = 4dm/e$ are introduced.

The equation describes the spin motion relative to the Cartesian coordinate axes. In accelerators and storage rings, the spin dynamics is determined relative to the radial, longitudinal and vertical axes. The angular velocity of the spin motion relative to the latter axes is given by

$$\boldsymbol{\Omega}' = \boldsymbol{\Omega} - \boldsymbol{\omega}_c,$$

where $\boldsymbol{\omega}_c$ is the vector which is parallel to the vertical axis and defines

the angular velocity of the cyclotron motion.

$\mathcal{H}_3 = -g_{aNN}\boldsymbol{\Sigma}\cdot\boldsymbol{\Lambda}$ in the nonrelativistic limit. However, a particle

motion can be relativistic.

The newly added first term in Ω_{axion} is three orders of magnitude larger than the second term. This fact significantly increases an importance of a search for a possible manifestation of the axion field in storage ring experiments.

Witten effect and similar effects

DYONS OF CHARGE $e\theta/2\pi$

E. WITTEN [1]
CERN, Geneva, Switzerland

If a non-zero vacuum angle *θ* is the only mechanism for *CP* violation, the electric charge of the monopole is exactly calculable and is *–θe/2π*, plus an integer:

$q = ne - \theta e/2\pi$

It has been found much later

ChunJun Cao and A. Zhitnitsky, Axion detection via topological Casimir effect, Phys. Rev. D 96, 015013 (2017);
A. Zhitnitsky, A few thoughts on and the electric dipole moments, Phys. Rev. D 108, 076021 (2023).

that magnetic dipole moment μ of any microscopical configuration in the background of $\theta_{\rm QED}$ generates the electric dipole moment $\langle d_{\rm ind}\rangle$ proportional to $\theta_{\rm QED}$, i.e., $\langle d_{\rm ind}\rangle = -\frac{\theta_{\rm QED}\cdot\alpha}{\pi}\mu$. We also argue that many $\mathcal{CP}$ odd correlations such as $\langle \vec{B}_{\rm ext}\cdot\vec{E}\rangle = -\frac{\alpha\theta_{\rm QED}}{\pi}\vec{B}^2_{\rm ext}$ will be generated in the background of an external magnetic field $\vec{B}_{\rm ext}$ as a result of the same physics.

There is also the new idea to use electric-magnetic duality to motivate the possible existence of non-standard axion couplings, which can both violate the usual quantization rule and exchange the roles of electric and magnetic fields in axion electrodynamics.

In this case, an electrically charged particle acquires also a magnetic charge and becomes a dyon.

B. Heidenreich, J. McNamara, and M. Reece, Non-standard axion electrodynamics and the dual Witten effect, arXiv: 2309.07951 [hep-ph] (2023).

We can use this idea and find equations of motion of a particle with electric and magnetic charges and dipole moments (dyon) in electromagnetic fields.

A. J. Silenko, Equation of spin motion for a particle with electric and magnetic charges and dipole moments, arXiv: 2309.04985 [hep-ph] (2023).

The Lorentz force F acting on the electric charge e **and the Lorentz-like force F* acting on the magnetic charge** e* **are given by**

$$\frac{dp}{dt} = \boldsymbol{F} + \boldsymbol{F}^* = e(\boldsymbol{E} + \boldsymbol{\beta} \times \boldsymbol{B}) + e^*(\boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{E}).$$

The method of derivation has been presented in

J. D. Jackson, Classical Electrodynamics, 3rd ed. (Wiley, New York, 1998); T. Fukuyama and A. J. Silenko, Int. J. Mod. Phys. A 28, 1350147 (2013); **A. J. Silenko, Phys. Scripta 90,** 065303 (2015).

Equation of motion:

$$m\frac{du^\mu}{d\tau} = eF^{\mu\nu}u_\nu + e^*\widetilde{F}^{\mu\nu}u_\nu$$

Equation for the angular velocity of spin motion:

$$\boldsymbol{\Omega} = -\frac{e}{m}\left[\left(G+\frac{1}{\gamma}\right)\boldsymbol{B} - \frac{G\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{B})\boldsymbol{\beta} - \left(G+\frac{1}{\gamma+1}\right)\boldsymbol{\beta}\times\boldsymbol{E}\right] + \frac{e^*}{m}\left[\left(G^*+\frac{1}{\gamma}\right)\boldsymbol{E} - \frac{G^*\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{E})\boldsymbol{\beta} + \left(G^*+\frac{1}{\gamma+1}\right)\boldsymbol{\beta}\times\boldsymbol{B}\right].$$

Here $G = (g-2)/2$, $g = 2mc\mu/(es)$, $G^* = (g^*-2)/2$, $g^* = -2mcd/(e^*s)$.

Summary

- The Dirac Hamiltonian with an allowance for the field of dark matter axions has been written down and its relativistic Foldy-Wouthuysen transformation has been carried out. We take into account the axion-photon and axion-gluon coupling and direct axion-particle coupling (axion wind effect).
- Axion-induced interactions lead to an appearance of oscillating EDMs and, possibly, oscillating effective magnetic charges. We obtain related equations of motion.
- The axion wind term results in the spin rotation about the radial axis.


Thank you for your attention