Estimation of the LO hadronic contribution to $g_\mu - 2$ using the IHEP total cross section database

V.V. Bryzgalov, O.V. Zenin

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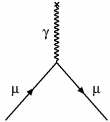
Introduction

$$\vec{\mu}_\mu = -g_\mu \frac{e}{2m_\mu} \vec{S}$$

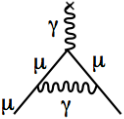
- $a_\mu = (g_\mu - 2)/2$ measured by FNAL Muon g-2 experiment to 0.215 ppm
- $\sim 5\sigma$ theory/experiment tension (with the e^+e^- based HVP estimate)
- ~ 1 ppm precision SM test, sensitive to TeV scale New physics
  - Theory uncertainty mostly due to QCD

Tree level



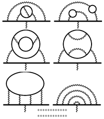
$a_\mu = 0$

Schwinger



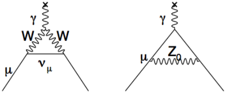
$+\frac{\alpha}{2\pi} = 11614097.3 \times 10^{-10}$

QED 2-5 loops



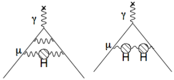
$+44374.6 \times 10^{-10}$

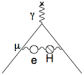
Electroweak



$+15.4 \times 10^{-10}$

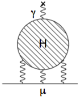
Hadron VP NLO





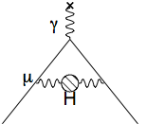
-9.8×10^{-10}

Hadron light-by-light



$+9.2 \times 10^{-10}$

Hadron VP LO



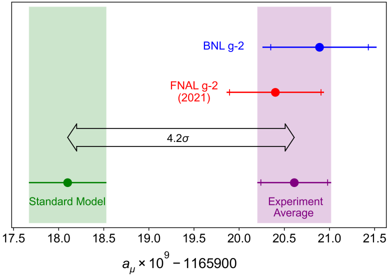
$+695 \times 10^{-10}$

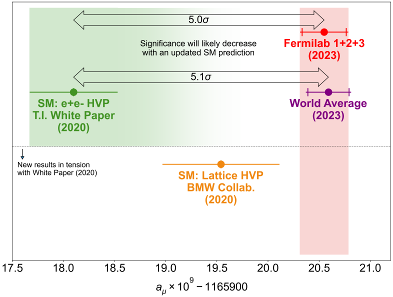
(e^+e^- based)

← **The topic of this talk**

- Controversy in the e^+e^- input
- $\sim$ 3σ tension between e^+e^- and lattice estimates

Experiment vs theory

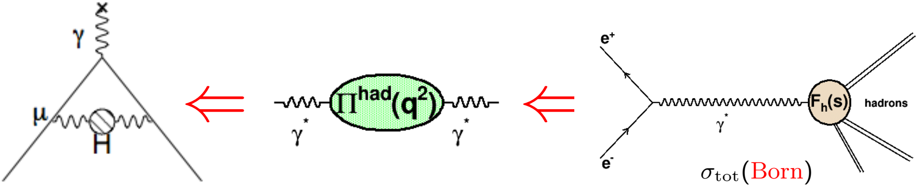




By M. Incagli (Muon g-2 Collaboration), https://indico.cern.ch/event/1312628/

- BNL E821 (2004): 3.7σ experiment/SM tension
- BNL E821 + FNAL g-2 Run-1 (2021, 5% of the full statistics): 4.2σ
- World average including FNAL g-2 Run-1-2-3 (*Muon g-2 Collaboration, arXiv:2308.06230*): **5.1σ tension!**
- SM prediction uncertainty mostly comes from hadron LO VP term:
  - e^+e^- HVP value too low (*the "White Paper": Muon g-2 Theory Initiative, Phys. Rept. 887 (2020) 1*)
  - Lattice HVP calculation gets SM a_μ closer to the experiment (*BMW Collaboration, Nature 593 (2021) 51*)
  - Tension between e^+e^- and lattice HVP
  - New CMD-3 $\pi^+\pi^-$ data $\sim$ 5% higher than the world average (*CMD-3 Collaboration, arXiv:2309.12910*). $\Rightarrow$ Taken alone, CMD-3 puts SM a_μ estimate within $\sim 2\sigma$ from the experiment
  - More e^+e^- data to come: CMD-3 in other channels, SND, Babar, KLOE ($\pi^+\pi^-$), BESIII ($\pi^+\pi^-$, $\pi^+\pi^-\pi^0$), Belle II ...

$a_\mu(\text{had, LO})$ via $\sigma(e^+e^- \to \text{hadrons})$



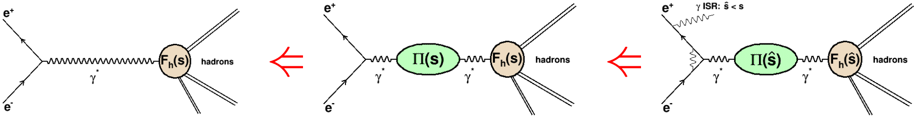
The dispersion relation (A. Petermann, Phys. Rev. 105 (1957) 1931):

$$a_\mu(\text{had, LO}) = 4\alpha_0^2 \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) \frac{1}{\pi} \,\text{Im}\,\Pi^{\text{had}}(s) = \frac{\alpha_0^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{ds}{s} K(s) R^{\text{had}}(s)$$

$$R^{\text{had}}(s) = \sigma_{\text{tot}}(e^+e^- \to hadrons, \text{Born}) \Big/ \frac{4\pi\alpha_0^2}{3s}$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)} \,.$$

a_μ(had, LO) via $\sigma(e^+e^- \to \text{hadrons})$ *(continued)*



Born — Improved Born Approximation — Experiment

- We need Born cross section for the dispersion integral
- All experiments publish cross sections corrected for ISR + e^+e^- vertex loops
  - An extreme case is the radiative return measurements (BaBar, Belle, KLOE . . .)
- Some experiments correct for photon VP, others leave the VP correction to readers
  - *A caveat:* pre-1985 experiments applied only electron VP correction, both in the s-channel hadron production and the t-channel Bhabha scattering, the latter being used for luminosity determination. We roll this partial VP correction back in order to consistently apply the full VP correction.
- FSR correction.
  - Additional hard γ's are rejected in the event selection to suppress backgrounds from other final states. Experimentalists then 'undress' the cross section, i.e. correct it for soft FSR using certain FSR model.
  - Need to add FSR contribution back: $\sigma(\text{hadrons}(+\gamma'\text{s})) = \sigma(\text{hadrons})\left[1 + \eta(s)\frac{\alpha}{\pi}\right]$, where $\eta(s)$ is computed in scalar QED for charged pions and kaons. The FSR correction factor is approximated by $C_{\text{FSR}} = (1 + 0.004 \pm 0.004)^{N_{\text{charged}}}$, where the uncertainty is introduced to estimate the associated systematics.

$a_\mu(\text{had}, \text{LO})$ via $\sigma(e^+e^- \to \text{hadrons})$ *(continued)*

- Thus, we need first to uniformly rescale all published measurements to Born cross section:
  - Need to know photon $\Pi(s)$ including hadronic VP which is yet unknown as we determine it using a dispersion relation with $\sigma_{\rm tot}(e^+e^- \to \textit{hadrons}, \textit{Born})$ as the input
  - Do it iteratively: use simple analytical parameterisation of the hadronic VP as the first aprroximation, rescale published cross sections to Born, substitute them into the dispersion relation to get the hadronic VP, *etc, etc*
- $\sigma_{\rm tot}(e^+e^- \to \textit{hadrons})$ is measured mostly inclusively at $\sqrt{s} > 2$ GeV and for (semi)exclusive final states at $\sqrt{s} < 2$ GeV
- Most final states are measured by multiple experiments
- Parameterise Born cross section in each final state in a model-independent way
- Fit the parameterisation taking into account correlated uncertainties within each experiment and between experiments
- Substitute the parameterised cross section into dispersion relations to find final state's contribution to the photon VP and $a_\mu(\text{had}, \text{LO})$
- Find total hadronic VP and $a_\mu(\text{had}, \text{LO})$ by summing up contributions from individual final states at $0.3 < \sqrt{s} < 11.2$ GeV;
  use ChPT parameterisation of $R^{\rm had}(s)$ at $m_\pi < \sqrt{s} < 0.3$ GeV ($\pi^0\gamma$, $\pi\pi(\gamma)$);
  add contributions from narrow resonances J/Ψ, $\Psi(2S)$, $\Upsilon(1-4S)$;
  insert analytical parameterisation of $R^{\rm had}(s)$ at $\sqrt{s} > 11.2$ GeV into dispersion relations.

So far, one more e^+e^- based HVP estimate:

- Prerequisites and the workflow:
  - The input: ▸ IHEP database of total cross sections
  - Rescale published cross sections to R^{had} (apply/unfold radiative corrections)
    - ★ The list of inputs is given in the ▸ backup.
  - Parameterise and fit R^{had} in each final state
  - Integrate fitted R^{had} with the $K(s)$ kernel to obtain HVP contribution to a_μ from each final state at $0.3 < \sqrt{s} < 11.2$ GeV, outside this range use analytical parameterisations of R^{had}
- Prerequisites in place since 2003 [V.V. Ezhela et al, hep-ph/0312114]
- The code was used for the PDG minireview "σ and R in e^+e^- collisions" [R.L. Workman et al. (Particle Data Group), Review of Particle Physics, PTEP 2022, 083C01 (2022), also in earlier RPP editions since 2002]
- ⇒ All in place, why not making our HVP estimate?
  - No common ▸ code with the Muon g-2 Theory Initiative contributors ⇒ **one more independent cross-check**.

Model-independent parameterisation of R^{had}

- Each final state is typically measured by many independent experiments, need to average them.
- Averaging requires to parameterise R^{had} by some continuous function:
  - No prior assumptions about contributions of various amplitudes to the production of the final state.
- A simple choice: parameterise R^{had} by continuous piecewise linear curve. The optimal number and position of the nodes are determined only by the set of experimental measurements $\{s_i, R_i^{\mathrm{had}}\}$, no signal model is assumed.

$\{s_i, R_i^{\mathrm{had}}\}$ points are clustered as follows:
Define the clusterization radius determined by the size of s interval where R^{had} is compatible with a constant within experimental uncertainties. For each s define sliding intervals of "compatibility with a constant": $[s, s + r^{+}(s)]$, $[s - r^{-}(s), s]$.
For each pair of measurements $\{i, j\}$ ($s_j > s_i$) define the proximity metric:

$$w_{ij} = \min\left\{\frac{1}{\sigma_i^2}, \frac{1}{\sigma_j^2}\right\}\left[\frac{s_j - s_i}{\sqrt{a^2 r^{+}(s_i) r^{-}(s_j)}}\right]^b,$$

where $\sigma_{i,j}$ are the statistical uncertainties of the measurements and $a, b \sim 1$ are fixed parameters (their variation gives us an estimate of the algorithm's systematics).

$\{i, j\}$ pair with the minimum $w_{ij} = w_{\mathrm{min}}$ is merged into a single point as follows:

continued on the next slide . . .

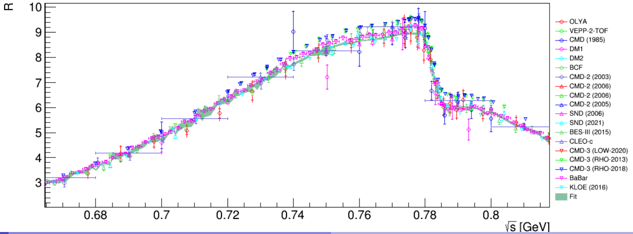
Model-independent parameterisation of R^{had} *(continued)*

1. Set $w_{\min}$ to a value exceeding any possible w_{ij}.
2. For all $\{i,j\}$ pairs:
   1. Find w_{ij} for the $\{i,j\}$ pair.
   2. If for the $\{i,j\}$ pair $w_{ij} \geq 1/\sigma_i^2$ and $w_{ij} \geq 1/\sigma_j^2$ then move on to the next pair of points.
   3. If for the $\{i,j\}$ pair $w_{ij} < w_{\min}$ then $w_{\min} := w_{ij}$, $\{i,j\}_{\min} := \{i,j\}$.
3. If $\{i,j\}_{\min}$ is not found then **stop the clusterization**.
4. Otherwise merge the pair of points $\{i,j\}_{\min}$ into a single point with $s = w_i s_i + w_j s_j$ and $\sigma^2 = \sigma_i^2 + \sigma_j^2$, where weights $w_{i,j} = \frac{1}{\sigma_{i,j}^2} \Big/ \left(\frac{1}{\sigma_i^2} + \frac{1}{\sigma_j^2}\right)$.
5. Return to **step 2**.

In result, we get a set of $\{s_k\}$ for the nodes of the piecewise linear curve which will approximate the R^{had}. The corresponding $\{R_k\}$ values are then found by a standard χ^2 fit on the set of experimental measurements $\{s_i, R_i\}$ taking into account their binning and statistical and (correlated) systematic uncertainties.

A typical result of the clusterization ($\rho - \omega$ interference region in $\pi^+\pi^-$):

- Multiple experiments with different binning in $\sqrt{s}$, systematic tension between experiments.
- Too detailed parameterisation leads to unphysical fluctuations in the fitted R^{had} $\Rightarrow$ smoothing/clusterization needed. See the 'Fit' curve.



Fitting the $R^{\rm had}$ data

A standard χ^2 minimization:

$$\chi^2 = \sum_{i,j}\left[\frac{1}{\Delta\sqrt{s_i}}\int_{\Delta\sqrt{s_i}} R^{\rm had}_{fit}(s)d\sqrt{s} - R^{\rm had}_i\right]\times {\rm COV}^{-1}_{ij}\times\left[\frac{1}{\Delta\sqrt{s_j}}\int_{\Delta\sqrt{s_j}} R^{\rm had}_{fit}(s)d\sqrt{s} - R^{\rm had}_j\right],$$

where $R^{\rm had}_{fit}(s)$ is the fitted parameterisation, $R^{\rm had}_i$ are the measurements in $\Delta\sqrt{s_i}$ bins, and ${\rm COV}_{ij}$ is the full covariance matrix between measurements:

$$\begin{aligned}{\rm COV}_{ij} = \delta_{ij}\sigma^2_{{\rm stat},i} \quad + \quad & \frac{1}{\Delta\sqrt{s_i}}\int_{\Delta\sqrt{s_i}} R^{\rm had}_{fit}(s)d\sqrt{s}\times\frac{1}{\Delta\sqrt{s_j}}\int_{\Delta\sqrt{s_j}} R^{\rm had}_{fit}(s)d\sqrt{s}\times \\ & \times\left\{\begin{array}{l}\Delta_{{\rm sys},i}\Delta_{{\rm sys},j},\ \text{if } i,j \text{ are from the same experiment} \\ \Delta_{{\rm sys},i}\Delta_{{\rm sys},j}\times(\text{cross}-\text{experiment covariation}), \\ \qquad\qquad \text{if } i,j \text{ are from different experiments}\end{array}\right\},\end{aligned}$$

where $\Delta_{{\rm sys},i}$ are the relative systematic uncertainties as quoted by the experimentalists.

Why $R^{\rm had}_{fit}(s)$ in the systematic term of ${\rm COV}_{ij}$? Naively taking individual measurements $R^{\rm had}_{i,j}$ for the systematic uncertainty leads to a biased ${\rm COV}_{ij}$ and to a biased fit as $R^{\rm had}_{i,j}$ *are already biased themselves* – a manifestation of the well known *Peele's Pertinent Puzzle (PPP)*: "...a phenomenon exhibiting unexpected mean values for experimental data affected by statistical and systematic errors" [R. Frühwirth *et al*, EPJ Web of Conf., Vol. 27 (2012), 00008]

The problem: $\delta\chi^2/\delta R^{\rm had}_{fit}(s)$ is non-linear w.r.t. $R^{\rm had}_{fit}(s) \Rightarrow$ run the fit iteratively $\longrightarrow$ ...

Fitting the R^{had} data (continued)

... $\longrightarrow$ run the fit iteratively:

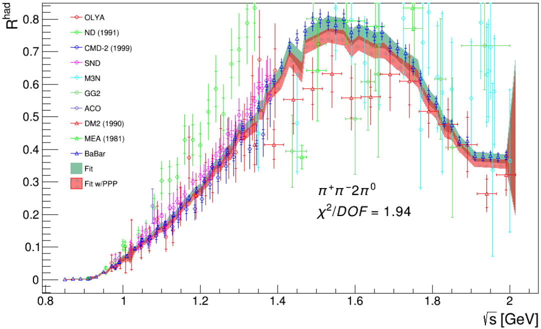
1. Make the fit ignoring the systematic uncertainties to get zeroth approximation for $R^{\text{had}}_{fit}(s)$. Though χ^2/dof is awful, there's no PPP bias in the fit using a diagonal covariance matrix.
2. Rebuild the full covariance matrix using the obtained $R^{\text{had}}_{fit}(s)$.
3. Repeat the fit with the full covariance matrix.
4. Compare just obtained $R^{\text{had}}_{fit}(s)$ with the one from the previous iteration. **Stop** if the convergence condition *(to be refined)* is satisfied, otherwise return to step 2.

In practice, the procedure converges after 2 iterations.

TODO: Estimate the residual bias? Stability w.r.t. the choice of the zeroth approximation for $R^{\text{had}}_{fit}(s)$? Can we start from a non-diagonal covariance matrix using measured R^{had}_i values for its systematic part? ...?

Significance of the PPP effect:
Most prominent in final states with tension between independent experiments, e.g., in $\pi^+\pi^-2\pi^0$. $\rightarrow$
The PPP bias (red band, always negative!) is comparable to the uncertainty of the unbiased fit (green band).
An integral effect is $\delta a_\mu(\text{had, LO})/a_\mu(\text{had, LO}) \sim -1\%$.

More details and pathological examples in ▸ backup



Fitting the R^{had} data: $a_\mu(\mathrm{had,LO})$ integral

- Problematic input data:
  - $\pi^+\pi^-$ with $\chi^2/\mathrm{dof} = 2.18$. χ^2/dof drops to 1.47 upon exclusion of the latest CMD-3 data being in 5σ tension with other measurements. Precision KLOE and BaBar measurements are also in tension (discussed later).
  - $2\pi^+2\pi^-$, $\chi^2/\mathrm{dof} = 2.34$: high precision BaBar measurement in tension with SND and old Orsay data.
  - $\pi^+\pi^-2\pi^0$, $\chi^2/\mathrm{dof} = 1.94$: ND (1991) strongly disagrees with the others, still no reason to exclude.
- We don't drop (imprecise) pre-1990 data: different instrumentation, reconstruction and statistical procedures provide a cross-check with newer experiments.
- In channels with $\chi^2/\mathrm{dof} > 1.5$ the propagated experimental uncertainty of R^{had}_{fit} is scaled by $\sqrt{\chi^2/\mathrm{dof}}$ (cf. Birge factor in PDG).

$$a_\mu(\mathrm{had,LO}) = (696.2 \pm 1.9_{e^+e^-\,\mathrm{exp.}} \pm 2.1_{\mathrm{sys.}}) \times 10^{-10},$$

in agreement with recent results by other groups [Phys. Rept. 887 (2020) 1: $693.1(4.0) \times 10^{-10}$], despite an inclusion of 'high' CMD-3 (2023) $\pi^+\pi^-$ data.
A good channel-by-channel agreement with A. Keshavarzi *et al*, Phys. Rev. D 101 (2020) 1, 014029 (we intentionally chosen identical integration ranges).

Final state	$a_\mu(\mathrm{had,LO}) \times 10^{10}$				$\sqrt{s}$ [GeV]	χ^2/dof
		(exp.)	(par.)	(rad.)		
$\pi^+\pi^-(\gamma)$	505.147	(1.367)	(1.551)	(0.606)	0.3 ÷ 1.937	2.18
$\pi^+\pi^-\pi^0$	48.481	(0.967)	(0.629)	(0.066)	0.66 ÷ 1.937	1.79
$\pi^+\pi^-2\pi^0$	18.778	(0.431)	(0.509)	(0.067)	0.85 ÷ 1.937	1.94
$2\pi^+2\pi^-$	15.397	(0.181)	(0.060)	(0.043)	0.6125 ÷ 1.937	2.34
K^+K^-	23.211	(0.188)	(0.072)	(0.009)	0.985 ÷ 1.937	1.99
K_SK_L	13.188	(0.130)	(0.000)	(0.000)	1.00371 ÷ 1.937	0.95
$\pi^0\gamma$	4.359	(0.093)	(0.049)	(0.000)	0.59986 ÷ 1.38	1.70
$K_SK^+\pi^- + K_SK^-\pi^+$	1.814	(0.100)	(0.000)	(0.000)	1.24 ÷ 1.937	0.99
$2\pi^+2\pi^-\pi^0$	1.746	(0.043)	(0.000)	(0.009)	1.0125 ÷ 1.937	0.00
$2\pi^+2\pi^02\pi^-$	1.728	(0.198)	(0.034)	(0.000)	1.3125 ÷ 1.937	1.99
$2\pi^+2\pi^-3\pi^0$	0.099	(0.013)	(0.002)	(0.001)	1.575 ÷ 1.937	0.57
$3\pi^+3\pi^-$	0.240	(0.014)	(0.000)	(0.012)	1.3125 ÷ 1.937	0.00
$3\pi^+3\pi^-\pi^0$	0.020	(0.004)	(0.001)	(0.000)	1.6 ÷ 1.937	0.65
$\eta\gamma$	0.691	(0.051)	(0.000)	(0.000)	0.6 ÷ 1.354	1.36
$\eta\pi^+\pi^-$	0.575	(0.019)	(0.000)	(0.000)	1.15 ÷ 1.937	1.18
$K^+K^-\pi^0$	0.202	(0.050)	(0.000)	(0.001)	1.44 ÷ 1.937	0.54
$K^+K^-\pi^0\pi^0$	0.100	(0.011)	(0.000)	(0.000)	1.5 ÷ 1.937	1.32
$K^+K^-\pi^+\pi^-$	0.799	(0.033)	(0.000)	(0.000)	1.4 ÷ 1.937	0.00
$K^+K^-\pi^+\pi^-\pi^0$	0.129	(0.024)	(0.000)	(0.000)	1.6125 ÷ 1.937	1.63
$K_SK_L\eta$	0.238	(0.059)	(0.000)	(0.000)	1.575 ÷ 1.937	1.31
$K_SK_L\pi^0$	0.839	(0.114)	(0.000)	(0.000)	1.425 ÷ 1.937	1.50
$K_SK_L\pi^0\pi^0$	0.137	(0.043)	(0.000)	(0.000)	1.35 ÷ 1.937	0.00
$K_SK_L\pi^+\pi^-$	0.166	(0.028)	(0.000)	(0.000)	1.425 ÷ 1.937	0.00
$K_SK^+\pi^-\pi^0 + K_SK^-\pi^+\pi^0$	0.640	(0.044)	(0.000)	(0.000)	1.51 ÷ 1.937	1.08
$K_SK_S\pi^+\pi^-$	0.066	(0.007)	(0.000)	(0.000)	1.63 ÷ 1.937	1.37
$\omega(783)\eta$	0.035	(0.002)	(0.000)	(0.000)	1.34 ÷ 1.937	0.85
$\omega(783) < \pi^0\gamma > \pi^0$	0.894	(0.021)	(0.000)	(0.000)	0.75 ÷ 1.937	1.56
$\omega(783) < \pi^+\pi^-\pi^0 > \pi^+\pi^-$	0.098	(0.005)	(0.000)	(0.000)	1.15 ÷ 1.937	1.10
$\omega\eta\pi^0$	0.055	(0.043)	(0.000)	(0.000)	1.5 ÷ 1.937	1.16
$\phi(1020)\eta$	0.067	(0.003)	(0.000)	(0.000)	1.56 ÷ 1.937	0.98
$\pi^+\pi^-2\pi^0\eta$	0.117	(0.019)	(0.000)	(0.000)	1.625 ÷ 1.937	0.85
$\pi^+\pi^-3\pi^0$	1.067	(0.112)	(0.000)	(0.000)	1.125 ÷ 1.937	0.68
$\pi^+\pi^-\pi^0\eta$	0.663	(0.075)	(0.000)	(0.000)	1.394 ÷ 1.937	0.82
$p\bar{p}$	0.030	(0.001)	(0.000)	(0.000)	1.889 ÷ 1.937	1.24
$n\bar{n}$	0.028	(0.006)	(0.000)	(0.000)	1.89 ÷ 1.937	1.24
$2hadron(hadrons)$	43.509	(0.722)	(0.661)	(0.000)	1.937 ÷ 11.199	1.35
pQCD	2.065		(0.002)		> 11.1990	
ChPT $\pi\pi, \pi^0\gamma$	0.538		(0.013)		0.2792 ÷ 0.3000	
$\Psi(1S)$	6.495		(0.124)		3.0969	
$\Psi(2S)$	1.631		(0.057)		3.6861	
$\Upsilon(1S)$	0.054		(0.002)		9.4604	
$\Upsilon(2S)$	0.021		(0.003)		10.0234	
$\Upsilon(3S)$	0.014		(0.002)		10.3551	
$\Upsilon(4S)$	0.010		(0.001)		10.5794	
Total	**696.181**	(1.925)	(1.953)	(0.813)		

$R^{\rm had}$ outside the experimental range

- No $\sigma(e^+e^- \to \pi^+\pi^-)$ measurements at $2m_\pi < \sqrt{s} < 0.3$ GeV $\Rightarrow$ use ChPT parameterisation of the pion formfactor:

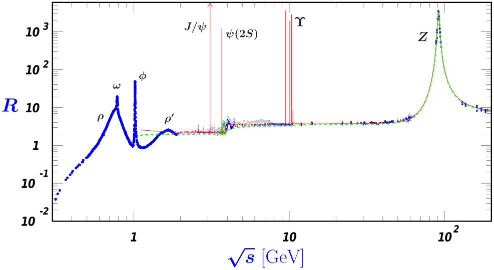
$$F_\pi^{\rm ChPT}(s) = 1 + \frac{< r^2 >_\pi}{6} s + c_1 s^2 + c_2 s^3 + \mathcal{O}(s^4),$$

  where the pion charge radius $< r^2 >_\pi = (11.27 \pm 0.21)\,{\rm GeV}^{-2}$ is extracted from the t-channel $\pi - e$ scattering [Nucl. Phys. B 277 (1986) 168] and $c_{1,2}$ are from the $\sigma(\pi\pi)$ fit at $0.4 < \sqrt{s} < 0.6$ GeV. Though we didn't update the parameters since 2003, the impact would be at $\sim 0.05 \times 10^{-10}$ level
- No $\sigma(e^+e^- \to \pi^0\gamma)$ data at $\sqrt{s} < 0.6$ GeV $\Rightarrow$ parameterise using the $\pi^0 \to \gamma^*\gamma$ transition formfactor [Phys. Rev. D 65 (2002) 073034]. Much smaller than $\pi\pi$ in the same range.
- Narrow $\Psi(1,2S)$, $\Upsilon(1-4S)$ resonances: the relativistic Breit–Wigner σ parameterisation with undressed Γ_{ee}, $\Gamma_{\rm tot}$, M values. *A caveat*: due to V–γ interference we can't use $R^{\rm had}(s)$ in the otherwise convenient form $\sigma^{\rm had}_{\rm IBA}(s)/\sigma^{\mu\mu}_{\rm IBA}(s)$, instead use an explicit *Born* parameterisation, $R^{\rm res}(s) = \sigma^{\rm res}_{\rm BW}(s)/\sigma^{\mu\mu}_0(s)$ (see, e.g., *S. Eidelman, F. Jegerlehner, Z. Phys. C 67 (1995) 585*).
- $R^{\rm had}$ at $\sqrt{s} > 11.2$ GeV: measurements do exist up to LEP II energies, still use the 3-loop pQCD expression [K.G. Chetyrkin *et al.*, Phys. Rept 277 (1996) 189]:

$$R^{\rm had}(s) = 3 \sum_{2m_q < \sqrt{s}} Q_q^2 \left(1 - \frac{4m_q^2}{s}\right)^{1/2} \left(1 + \frac{2m_q^2}{s}\right) \left[1 + \frac{\alpha_S(s)}{\pi} + \ldots\right]$$

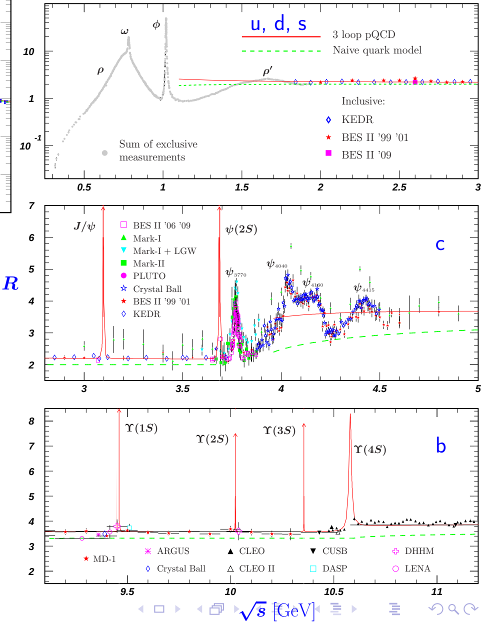
  Switching between data/pQCD in the $11.2 < \sqrt{s} < 40$ GeV range gives a negligible uncertainty on $a_\mu(\rm had, LO)$.

R^{had}: overall picture



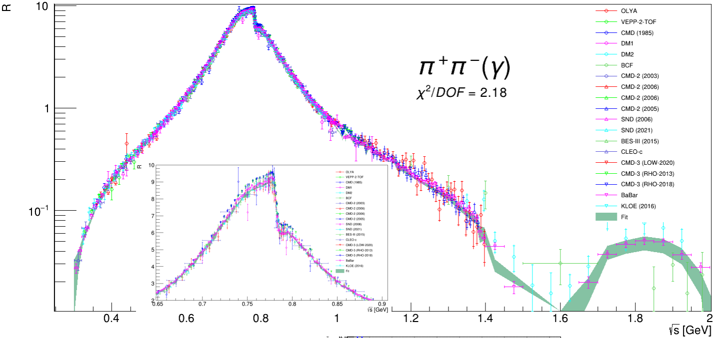
R.L. Workman et al., Review of Particle Physics, PTEP 2022, 083C01 (2022) (our contribution)

- New CMD-3 and BES III (2023) data not included (the difference would be hardly visible).
- Good agreement between inclusive $e^+e^- \to 2\text{hadron(hadrons)}$ and the sum of exclusive measurements at $\sqrt{s} \sim 2$ GeV. This indicates that we didn't miss (semi)exclusive final states with a non-negligible cross section.
- Good agreement between data and pQCD prediction for R^{had} outside $q\bar{q}$ threshold regions.



$\pi^+\pi^-$ channel

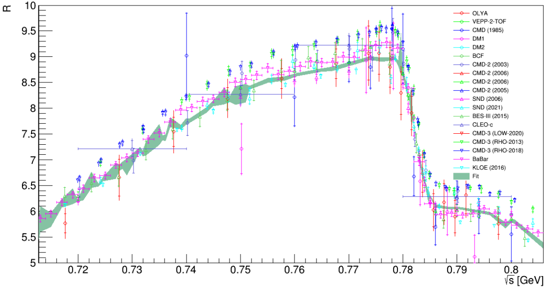
- $\sim$ 70% contribution to $a_\mu(\mathrm{had,LO})$ in $0.3 \div 1.937$ GeV range
- CMD-3 (Novosibirsk) 2013-2020 data $\sim$ 5% higher than others, including CMD-2.
- BaBar/KLOE tension (both using radiative return).
- Fit dominated by KLOE with its $\sim$ 1% uncertainty.
- Don't drop anything, just rescale the fit uncertainty.

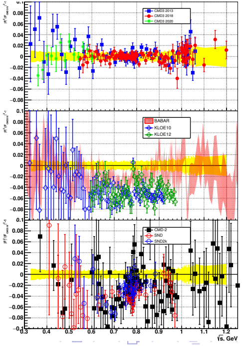


$\longrightarrow$ $a_\mu(\pi\pi, \mathrm{LO}) \times 10^{10} = 505.1 \pm 1.4_{\mathrm{exp}}$

CMD-3 excluded: $503.0 \pm 1.4_{\mathrm{exp}}$, $\chi^2/\mathrm{dof} = 1.45$

▸ What if ...?





$\Leftarrow$

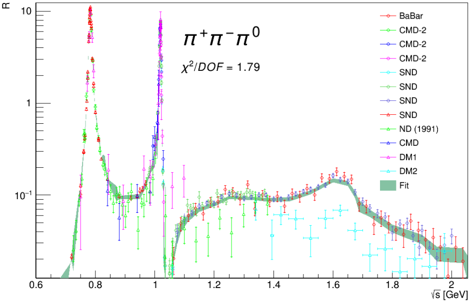
From CMD-3 Coll., arXiv:2309.12910

Yellow band is the fit to CMD-3 data.

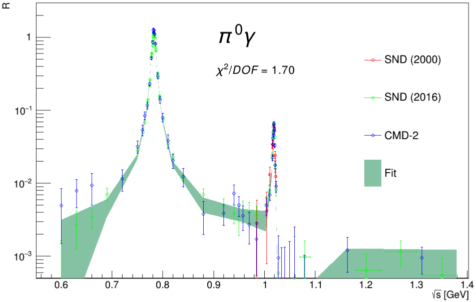
BaBar, KLOE to CMD-3 ratios.

CMD-2, SND, SND2k to CMD-3 ratios.

$\pi^+\pi^-\pi^0$ and $\pi^0\gamma$ channels

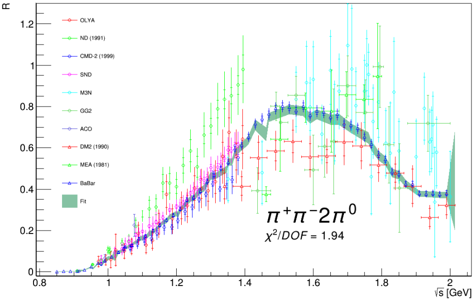


$$\Delta a_\mu(\mathrm{had,LO}) \times 10^{10} = 48.48 \pm 0.96_{\mathrm{exp}}$$

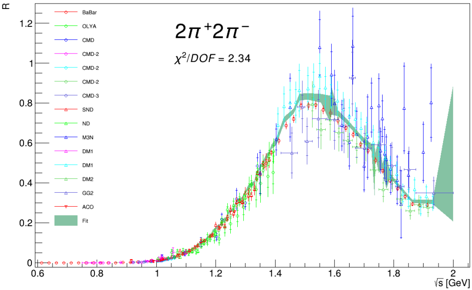


$$\Delta a_\mu(\mathrm{had,LO}) \times 10^{10} = 4.36 \pm 0.09_{\mathrm{exp}}$$

4π channels

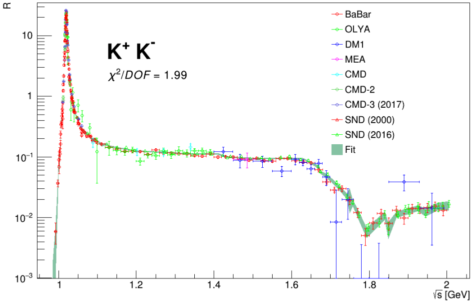


$\Delta a_\mu(\mathrm{had, LO}) \times 10^{10} = 18.78 \pm 0.44_{\mathrm{exp}}$

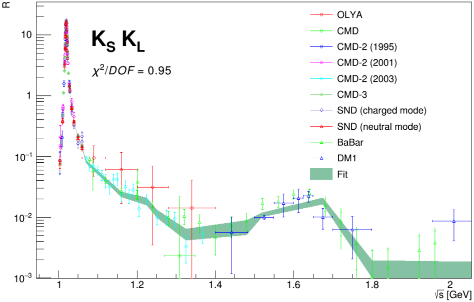


$\Delta a_\mu(\mathrm{had, LO}) \times 10^{10} = 15.40 \pm 0.18_{\mathrm{exp}}$

$K\bar{K}$ channels

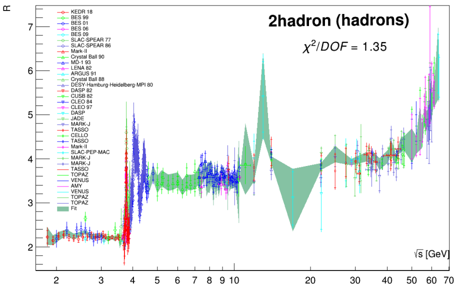


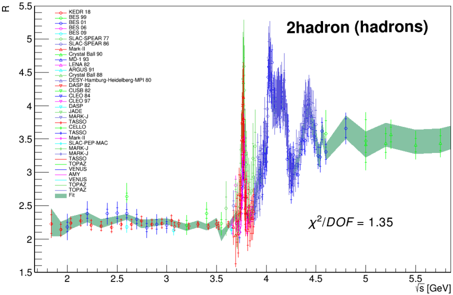
$$\Delta a_\mu(\mathrm{had,LO}) \times 10^{10} = 23.21 \pm 0.19_{\mathrm{exp}}$$



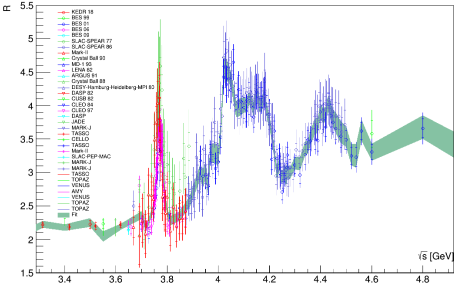
$$\Delta a_\mu(\mathrm{had,LO}) \times 10^{10} = 13.19 \pm 0.13_{\mathrm{exp}}$$

Inclusive measurements at $\sqrt{s} > 2$ GeV





uds continuum and $c\bar{c}$ threshold region ↑
(J/Ψ and $\Psi(2S)$ not shown)

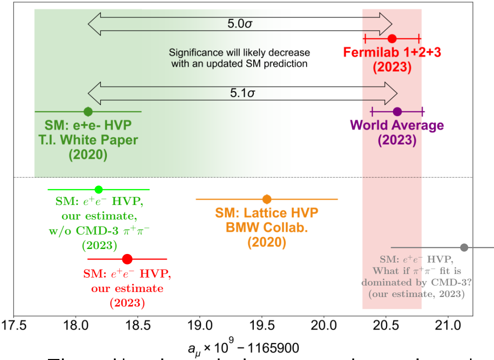


← Region above $D\bar{D}$ threshold

- Inclusive data above Υ's are well described by pQCD ⇒ the data are used (with the correction for $\gamma^*{-}Z$ interference) for $a_\mu(\mathrm{had, LO})$ at $1.937 < \sqrt{s} < 11.2$ GeV:
  $\Delta a_\mu(\mathrm{had, LO}) \times 10^{10} = 43.51 \pm 0.72$
- Negligible $a_\mu(\mathrm{had, LO})$ uncertainty due to variation of the integration upper limit within $11.2 \div 40$ GeV range (the correction for $\gamma^*{-}Z$ interference is taken into account).

Results

Final state	$a_\mu(\text{had, LO}) \times 10^{10}$	(exp.)	(par.)	(rad.)	$\sqrt{s}$ [GeV]	χ^2/dof
$\pi^+\pi^-(\gamma)$	505.147	(1.367)	(1.551)	(0.606)	0.3 ÷ 1.937	2.18
$\pi^+\pi^-\pi^0$	48.481	(0.967)	(0.629)	(0.066)	0.66 ÷ 1.937	1.79
$\pi^+\pi^-2\pi^0$	18.778	(0.431)	(0.509)	(0.067)	0.85 ÷ 1.937	1.94
$2\pi^+2\pi^-$	15.397	(0.181)	(0.060)	(0.043)	0.6125 ÷ 1.937	2.34
K^+K^-	23.211	(0.188)	(0.072)	(0.009)	0.985 ÷ 1.937	1.99
K_SK_L	13.188	(0.130)	(0.000)	(0.000)	1.00371 ÷ 1.937	0.95
$\pi^0\gamma$	4.359	(0.093)	(0.049)	(0.000)	0.59986 ÷ 1.38	1.70
$K_SK^+\pi^- + K_SK^-\pi^+$	1.814	(0.100)	(0.000)	(0.000)	1.24 ÷ 1.937	0.99
$2\pi^+2\pi^-\pi^0$	1.746	(0.043)	(0.000)	(0.009)	1.0125 ÷ 1.937	0.00
$2\pi^+2\pi^02\pi^-$	1.728	(0.198)	(0.034)	(0.000)	1.3125 ÷ 1.937	1.99
$2\pi^+2\pi^-3\pi^0$	0.099	(0.013)	(0.002)	(0.001)	1.575 ÷ 1.937	0.57
$3\pi^+3\pi^-$	0.240	(0.014)	(0.000)	(0.012)	1.3125 ÷ 1.937	0.00
$3\pi^+3\pi^-\pi^0$	0.020	(0.004)	(0.001)	(0.000)	1.6 ÷ 1.937	0.65
$\eta\gamma$	0.691	(0.051)	(0.000)	(0.000)	0.6 ÷ 1.354	1.36
$\eta\pi^+\pi^-$	0.575	(0.019)	(0.000)	(0.000)	1.15 ÷ 1.937	1.18
$K^+K^-\pi^0$	0.202	(0.050)	(0.000)	(0.001)	1.44 ÷ 1.937	0.54
$K^+K^-\pi^0\pi^0$	0.100	(0.011)	(0.000)	(0.000)	1.5 ÷ 1.937	1.32
$K^+K^-\pi^+\pi^-$	0.799	(0.033)	(0.000)	(0.000)	1.4 ÷ 1.937	0.00
$K^+K^-\pi^+\pi^-\pi^0$	0.129	(0.024)	(0.000)	(0.000)	1.6125 ÷ 1.937	1.63
$K_SK_L\eta$	0.238	(0.059)	(0.000)	(0.000)	1.575 ÷ 1.937	1.31
$K_SK_L\pi^0$	0.839	(0.114)	(0.000)	(0.000)	1.425 ÷ 1.937	1.50
$K_SK_L\pi^0\pi^0$	0.137	(0.043)	(0.000)	(0.000)	1.35 ÷ 1.937	0.00
$K_SK_L\pi^+\pi^-$	0.166	(0.028)	(0.000)	(0.000)	1.425 ÷ 1.937	0.00
$K_SK^+\pi^-\pi^0 + K_SK^-\pi^+\pi^0$	0.640	(0.044)	(0.000)	(0.000)	1.51 ÷ 1.937	1.08
$K_SK_S\pi^+\pi^-$	0.066	(0.007)	(0.000)	(0.000)	1.63 ÷ 1.937	1.37
$\omega(783)\eta$	0.035	(0.002)	(0.000)	(0.000)	1.34 ÷ 1.937	0.85
$\omega(783) < \pi^0\gamma > \pi^0$	0.894	(0.021)	(0.000)	(0.000)	0.75 ÷ 1.937	1.56
$\omega(783) < \pi^+\pi^-\pi^0 > \pi^+\pi^-$	0.098	(0.005)	(0.000)	(0.000)	1.15 ÷ 1.937	1.10
$\omega\eta\pi^0$	0.055	(0.043)	(0.000)	(0.000)	1.5 ÷ 1.937	1.16
$\phi(1020)\eta$	0.067	(0.003)	(0.000)	(0.000)	1.56 ÷ 1.937	0.98
$\pi^+\pi^-2\pi^0\eta$	0.117	(0.019)	(0.000)	(0.000)	1.625 ÷ 1.937	0.85
$\pi^+\pi^-3\pi^0$	1.067	(0.112)	(0.000)	(0.000)	1.125 ÷ 1.937	0.68
$\pi^+\pi^-\pi^0\eta$	0.663	(0.075)	(0.000)	(0.000)	1.394 ÷ 1.937	0.82
$p\bar{p}$	0.030	(0.001)	(0.000)	(0.000)	1.889 ÷ 1.937	1.24
$n\bar{n}$	0.028	(0.006)	(0.000)	(0.000)	1.89 ÷ 1.937	1.24
2hadron(hadrons)	43.509	(0.722)	(0.661)	(0.000)	1.937 ÷ 11.199	1.35
pQCD	2.065		(0.002)		> 11.1990	
ChPT $\pi\pi, \pi^0\gamma$	0.538		(0.013)		0.2792 ÷ 0.3000	
$\Psi(1S)$	6.495		(0.124)		3.0969	
$\Psi(2S)$	1.631		(0.057)		3.6861	
$\Upsilon(1S)$	0.054		(0.002)		9.4604	
$\Upsilon(2S)$	0.021		(0.003)		10.0234	
$\Upsilon(3S)$	0.014		(0.002)		10.3551	
$\Upsilon(4S)$	0.010		(0.001)		10.5794	
Total	**696.181**	(1.925)	(1.953)	(0.813)		



← The table shows both propagated experimental uncertainties (exp.) and the systematic uncertainties due to cross section parameterisation (par.) *(technically, due to $E_{\text{c.m.}}$ binning)* and radiative corrections (rad.).

Our estimate,

$$a_\mu(\text{had, LO}) = (696.2 \pm 1.9_{\text{exp.}} \pm 1.9_{\text{par.}} \pm 0.8_{\text{rad.}}) \times 10^{-10}$$

is consistent with results obtained by dispersive method by other authors before 2021, though we included 2021-2023 data. The *Muon g − 2 Theory Initiative group* quoted an average value of $(693.1 \pm 4.0_{\text{tot}}) \times 10^{-10}$ obtained by merging the recent results [Davier 20, Keshavarzi 20, Colangelo 19, Hoferichter 19, Keshavarzi 18, Davier 17]. We also have a good per final state agreement with [Keshavarzi 20]. With our $a_\mu(\text{had, LO})$ estimate, the $a_\mu^{\text{SM}} - a_\mu^{\text{exp}}$ disagreement remains at $\sim 5\sigma$ level.

Open issues & prospects

Experimental inputs:

- Controversy between experiments:
  - CMD-3 (2023) $\pi^+\pi^-$ cross section is $\sim 5\%$ ($\sim 4\sigma$) higher than the others at 600–800 MeV. Waiting for their final $\pi^+\pi^-$ results. ▸ more details ...
    Is there an excess in CMD-3 data in other final states? SND2k full statistics?
  - KLOE vs BaBar tension in $\pi^+\pi^-$. More ISR data to arrive: BaBar, Belle, KLOE2
- All-neutral final states in inclusive measurements?
- Unexpected states? Low-mass New Physics?
- Using space-like data to evaluate $a_\mu(\mathrm{had,LO})$: MUonE μe scattering experiment
- Hadronic form-factors from τ decays ...
- *New Physics affecting a_μ^{exp} measurement itself? (cf. talk by Alexander Silenko)*

Hadronic VP from lattice QCD:

- BMW Collaboration (2021) estimated $a_\mu(\mathrm{had,LO})$ to sub-percent precision (a_μ^{SM} uncertainty is comparable to the one of a_μ^{exp}). The resulting a_μ^{SM} value is *consistent* with a_μ^{exp} ▸ more on this ...

Questions to our procedure:

- Systematics associated with the unfolding of radiative corrections applied by experimentalists?
- Building a non-biased global covariance matrix?
- Cross section parameterisation for the fit.
- ...?

Summary

- Using an up-to-date as of November 2023 compilation of the world data on $\sigma_{\rm tot}(e^+e^- \to {\rm hadrons})$ we independently estimated the leading order hadronic contribution to the muon anomalous magnetic moment:

$$a_\mu({\rm had,LO}) = (696.2 \pm 1.9_{\rm exp.} \pm 2.1_{\rm syst.}) \times 10^{-10} \ ,$$

consistent with the *Muon $g-2$ Theory Initiative (2020)* average $(693.1 \pm 4.0_{\rm tot}) \times 10^{-10}$, despite we included 'high' CMD-3 (2023) $\pi^+\pi^-$ data.

- The SM prediction of a_μ including our $a_\mu({\rm had,LO})$ estimate $a_\mu^{\rm SM} = 11\ 659\ 184(4) \times 10^{-10}$ is in $\sim 4.7\sigma$ tension with the experimental value $a_\mu^{\rm exp} = 11\ 659\ 205.9(2.2) \times 10^{-10}$ [*FNAL g-2 Coll., Phys. Rev. Lett. 131, 161802 (2023)*].

Thank you!

Backup

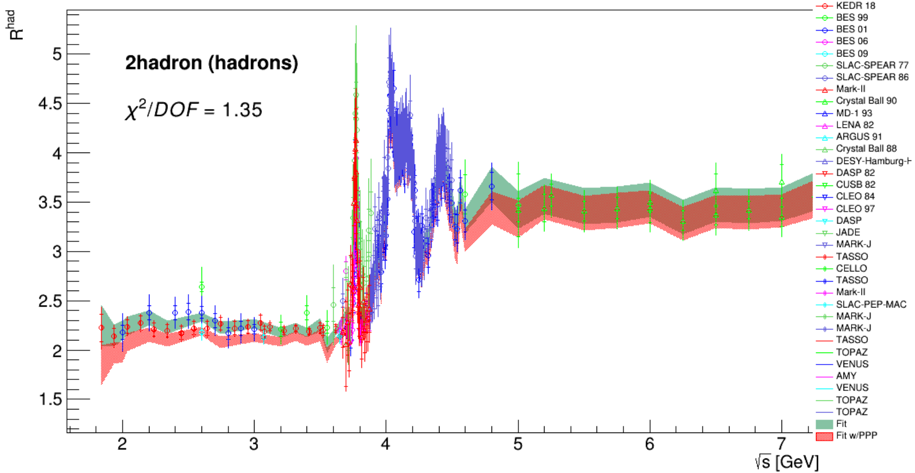
IHEP PPDS CS total cross section database

(where we store the input data)

- Originates from the PPDS CrossSection database maintained at IHEP (Protvino) since 1980s.
- Implemented from scratch for Unix in 2017-2020 (no code from the old BDMS based version).
- Covers total cross section measurements published since 1947. Contains 22146 data records, each comprising cross section measurements for a single reaction published in a single paper (i.e. one paper may be split into several records).
- The data are encoded in a language with a strict grammar (an automatic protection against meaningless content and input mistakes).
- Flexible query language (not SQL).
- Web-based command line interface http://hera.ihep.su:4200/cs/ with basic plotting.
- Coverage of world data is fragmentary since 1990s, still PPDS CS is actively used to maintain our compilations of $e^+e^- \to hadrons$ total cross sections and total (inelastic) cross sections with hadron-hadron beams (cf. the reviews on total cross sections in the Review of Particle Physics before 2023).

Back

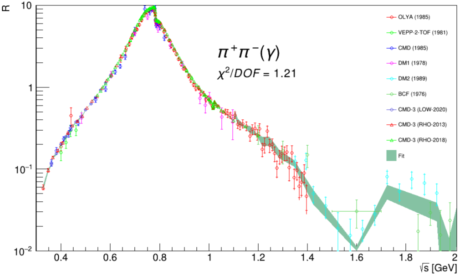
PPP bias: pathological examples

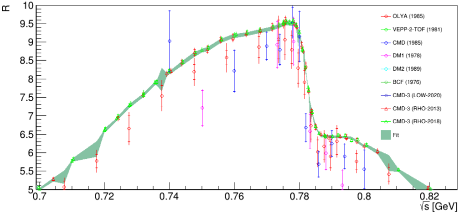


A naive construction of the systematic part of the covariance matrix using inputs (*biased a priori*) from individual experiments leads to PPP bias while fitting correlated data by the least squares method. Generally speaking, the fit can be systematically lower than *any* of the individual measurements, see the example above. [**Yes:** the red curve is the global χ^2 minimum with $\chi^2/\mathrm{dof} = 1.25$]

Back

What if ... ? $\pi^+\pi^-$ fit dominated by CMD-3:





Exclude precise post-1990 $\pi\pi$ data except CMD-3.
$a_\mu(\pi\pi, \mathrm{LO})$ gets $+27.3 \times 10^{-10}$ boost making the SM prediction $a_\mu^{\mathrm{SM}} = 11\ 659\ 211.3(6.0) \times 10^{-10}$ consistent with $a_\mu^{\mathrm{exp}} = 11\ 659\ 205.9(2.2) \times 10^{-10}$.
(Our nominal estimate $11\ 659\ 184(4) \times 10^{-10}$ is $\sim 4.7\sigma$ away from a_μ^{exp}.)

Back

Final state	$a_\mu(\mathrm{had, LO}) \times 10^{10}$	(exp.)	(par.)	(rad.)	$\sqrt{s}$ [GeV]	χ^2/dof
$\pi^+\pi^-(\gamma)$	529.580	(2.832)	(3.272)	(3.323)	0.32698 ÷ 1.937	1.21
$\pi^+\pi^-\pi^0$	48.481	(0.967)	(0.629)	(0.066)	0.66 ÷ 1.937	1.79
$\pi^+\pi^-2\pi^0$	18.778	(0.431)	(0.509)	(0.067)	0.85 ÷ 1.937	1.94
$2\pi^+2\pi^-$	15.397	(0.181)	(0.060)	(0.043)	0.6125 ÷ 1.937	2.34
K^+K^-	23.211	(0.188)	(0.072)	(0.009)	0.985 ÷ 1.937	1.99
K_SK_L	13.188	(0.130)	(0.000)	(0.000)	1.00371 ÷ 1.937	0.95
$\pi^0\gamma$	4.359	(0.093)	(0.049)	(0.000)	0.59986 ÷ 1.38	1.70
$K_SK^+\pi^- + K_SK^-\pi^+$	1.814	(0.100)	(0.000)	(0.000)	1.24 ÷ 1.937	0.99
$2\pi^+2\pi^-\pi^0$	1.746	(0.043)	(0.000)	(0.009)	1.0125 ÷ 1.937	0.00
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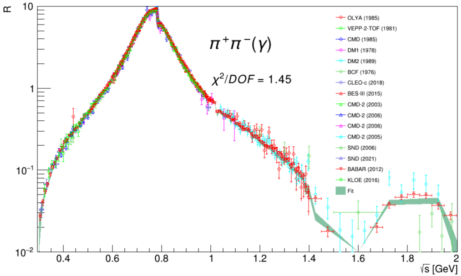
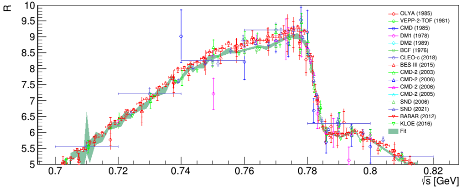
$2\pi^+2\pi^-3\pi^0$	0.099	(0.013)	(0.002)	(0.001)	1.575 ÷ 1.937	0.57
$3\pi^+3\pi^-$	0.240	(0.014)	(0.000)	(0.012)	1.3125 ÷ 1.937	0.00
$3\pi^+3\pi^-\pi^0$	0.020	(0.004)	(0.001)	(0.000)	1.6 ÷ 1.937	0.65
$\eta\gamma$	0.691	(0.051)	(0.000)	(0.000)	0.6 ÷ 1.354	1.36
$\eta\pi^+\pi^-$	0.575	(0.019)	(0.000)	(0.000)	1.15 ÷ 1.937	1.18
$K^+K^-\pi^0$	0.202	(0.050)	(0.000)	(0.001)	1.44 ÷ 1.937	0.54
$K^+K^-\pi^0\pi^0$	0.100	(0.011)	(0.000)	(0.000)	1.5 ÷ 1.937	1.32
$K^+K^-\pi^+\pi^-$	0.799	(0.033)	(0.000)	(0.000)	1.4 ÷ 1.937	0.00
$K^+K^-\pi^+\pi^-\pi^0$	0.129	(0.024)	(0.000)	(0.000)	1.6125 ÷ 1.937	1.63
$K_SK_L\eta$	0.238	(0.059)	(0.000)	(0.000)	1.575 ÷ 1.937	1.31
$K_SK_L\pi^0$	0.839	(0.114)	(0.000)	(0.000)	1.425 ÷ 1.937	1.50
$K_SK_L\pi^0\pi^0$	0.137	(0.043)	(0.000)	(0.000)	1.35 ÷ 1.937	0.00
$K_SK_L\pi^+\pi^-$	0.166	(0.028)	(0.000)	(0.000)	1.425 ÷ 1.937	0.00
$K_SK^+\pi^-\pi^0 + K_SK^-\pi^+\pi^0$	0.640	(0.044)	(0.000)	(0.000)	1.51 ÷ 1.937	1.08
$K_SK_S\pi^+\pi^-$	0.066	(0.007)	(0.000)	(0.000)	1.63 ÷ 1.937	1.37
$\omega(783)\eta$	0.035	(0.002)	(0.000)	(0.000)	1.34 ÷ 1.937	0.85
$\omega(783) < \pi^0\gamma > \pi^0$	0.894	(0.021)	(0.000)	(0.000)	0.75 ÷ 1.937	1.56
$\omega(783) < \pi^+\pi^-\pi^0 > \pi^+\pi^-$	0.098	(0.005)	(0.000)	(0.000)	1.15 ÷ 1.937	1.10
$\omega\eta\pi^0$	0.055	(0.043)	(0.000)	(0.000)	1.5 ÷ 1.937	1.16
$\phi(1020)\eta$	0.067	(0.003)	(0.000)	(0.000)	1.56 ÷ 1.937	0.98
$\pi^+\pi^-2\pi^0\eta$	0.117	(0.019)	(0.000)	(0.000)	1.625 ÷ 1.937	0.85
$\pi^+\pi^-3\pi^0$	1.067	(0.112)	(0.000)	(0.000)	1.125 ÷ 1.937	0.68
$\pi^+\pi^-\pi^0\eta$	0.663	(0.075)	(0.000)	(0.000)	1.394 ÷ 1.937	0.82
$p\bar{p}$	0.030	(0.001)	(0.000)	(0.000)	1.889 ÷ 1.937	1.24
$n\bar{n}$	0.028	(0.006)	(0.000)	(0.000)	1.89 ÷ 1.937	1.24
$2hadron(hadrons)$	43.509	(0.722)	(0.661)	(0.000)	1.937 ÷ 11.199	1.35
pQCD	2.065		(0.002)		> 11.1990	
ChPT $\pi\pi, \pi^0\gamma$	3.364		(0.106)		0.2792 ÷ 0.3270	
$\Psi(1S)$	6.495		(0.124)		3.0969	
$\Psi(2S)$	1.631		(0.057)		3.6861	
$\Upsilon(1S)$	0.054		(0.002)		9.4604	
$\Upsilon(2S)$	0.021		(0.003)		10.0234	
$\Upsilon(3S)$	0.014		(0.002)		10.3551	
$\Upsilon(4S)$	0.010		(0.001)		10.5794	
Total	723.440	(3.139)	(3.122)	(3.530)		

What if ...? CMD-3 excluded from $\pi^+\pi^-$ fit





Exclude CMD-3 $\pi\pi$ data.

$a_\mu(\text{had, LO}) = (694.0 \pm 2.5) \times 10^{-10}$ is close to the White Paper average $(693.1 \pm 4.0) \times 10^{-10}$.

The SM estimate $a_\mu^{\text{SM}} = 11\ 659\ 182(5) \times 10^{-10}$ is $\sim 5\sigma$ away from $a_\mu^{\text{exp}} = 11\ 659\ 205.9(2.2) \times 10^{-10}$.

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Final state	$a_\mu(\text{had, LO}) \times 10^{10}$	(exp.)	(par.)	(rad.)	$\sqrt{s}$ [GeV]	χ^2/dof
$\pi^+\pi^-(\gamma)$	502.997	(1.429)	(1.398)	(0.209)	0.3 ÷ 1.937	1.45
$\pi^+\pi^-\pi^0$	48.481	(0.967)	(0.781)	(0.066)	0.66 ÷ 1.937	1.79
$\pi^+\pi^-2\pi^0$	18.778	(0.431)	(0.099)	(0.067)	0.85 ÷ 1.937	1.94
$2\pi^+2\pi^-$	15.397	(0.181)	(0.072)	(0.043)	0.6125 ÷ 1.937	2.34
K^+K^-	23.211	(0.188)	(0.073)	(0.009)	0.985 ÷ 1.937	1.99
K_SK_L	13.188	(0.130)	(0.000)	(0.000)	1.00371 ÷ 1.937	0.95
$\pi^0\gamma$	4.359	(0.093)	(0.041)	(0.000)	0.59986 ÷ 1.38	1.70
$K_SK^+\pi^- + K_SK^-\pi^+$	1.814	(0.100)	(0.000)	(0.000)	1.24 ÷ 1.937	0.99
$2\pi^+2\pi^-\pi^0$	1.746	(0.043)	(0.000)	(0.009)	1.0125 ÷ 1.937	0.00
$2\pi^+2\pi^02\pi^-$	1.728	(0.198)	(0.033)	(0.000)	1.3125 ÷ 1.937	1.99
$2\pi^+2\pi^-3\pi^0$	0.099	(0.013)	(0.003)	(0.001)	1.575 ÷ 1.937	0.57
$3\pi^+3\pi^-$	0.240	(0.014)	(0.000)	(0.012)	1.3125 ÷ 1.937	0.00
$3\pi^+3\pi^-\pi^0$	0.020	(0.004)	(0.002)	(0.000)	1.6 ÷ 1.937	0.65
$\eta\gamma$	0.691	(0.051)	(0.000)	(0.000)	0.6 ÷ 1.354	1.36
$\eta\pi^+\pi^-$	0.575	(0.019)	(0.000)	(0.000)	1.15 ÷ 1.937	1.18
$K^+K^-\pi^0$	0.202	(0.050)	(0.000)	(0.001)	1.44 ÷ 1.937	0.54
$K^+K^-\pi^0\pi^0$	0.100	(0.011)	(0.000)	(0.000)	1.5 ÷ 1.937	1.32
$K^+K^-\pi^+\pi^-$	0.799	(0.033)	(0.000)	(0.000)	1.4 ÷ 1.937	0.00
$K^+K^-\pi^+\pi^-\pi^0$	0.129	(0.024)	(0.000)	(0.000)	1.6125 ÷ 1.937	1.63
$K_SK_L\eta$	0.238	(0.059)	(0.000)	(0.000)	1.575 ÷ 1.937	1.31
$K_SK_L\pi^0$	0.839	(0.114)	(0.000)	(0.000)	1.425 ÷ 1.937	1.50
$K_SK_L\pi^0\pi^0$	0.137	(0.043)	(0.000)	(0.000)	1.35 ÷ 1.937	0.00
$K_SK_L\pi^+\pi^-$	0.166	(0.028)	(0.000)	(0.000)	1.425 ÷ 1.937	0.00
$K_SK^+\pi^-\pi^0 + K_SK^-\pi^+\pi^0$	0.640	(0.044)	(0.000)	(0.000)	1.51 ÷ 1.937	1.08
$K_SK_S\pi^+\pi^-$	0.066	(0.007)	(0.000)	(0.000)	1.63 ÷ 1.937	1.37
$\omega(783)\eta$	0.035	(0.002)	(0.000)	(0.000)	1.34 ÷ 1.937	0.85
$\omega(783) < \pi^0\gamma > \pi^0$	0.894	(0.021)	(0.000)	(0.000)	0.75 ÷ 1.937	1.56
$\omega(783) < \pi^+\pi^-\pi^0 > \pi^+\pi^-$	0.098	(0.005)	(0.000)	(0.000)	1.15 ÷ 1.937	1.10
$\omega\eta\pi^0$	0.055	(0.043)	(0.000)	(0.000)	1.5 ÷ 1.937	1.16
$\phi(1020)\eta$	0.067	(0.003)	(0.000)	(0.000)	1.56 ÷ 1.937	0.98
$\pi^+\pi^-2\pi^0\eta$	0.117	(0.019)	(0.000)	(0.000)	1.625 ÷ 1.937	0.85
$\pi^+\pi^-3\pi^0$	1.067	(0.112)	(0.000)	(0.000)	1.125 ÷ 1.937	0.68
$\pi^+\pi^-\pi^0\eta$	0.663	(0.075)	(0.000)	(0.000)	1.394 ÷ 1.937	0.82
$p\bar{p}$	0.030	(0.001)	(0.000)	(0.000)	1.889 ÷ 1.937	1.24
$n\bar{n}$	0.028	(0.006)	(0.000)	(0.000)	1.89 ÷ 1.937	1.24
$2hadron(hadrons)$	43.509	(0.722)	(0.779)	(0.000)	1.937 ÷ 11.199	1.35
pQCD	2.065		(0.002)		> 11.1990	
ChPT $\pi\pi, \pi^0\gamma$	0.538		(0.013)		0.2792 ÷ 0.3000	
$\Psi(1S)$	6.495		(0.124)		3.0969	
$\Psi(2S)$	1.631		(0.057)		3.6861	
$\Upsilon(1S)$	0.054		(0.002)		9.4604	
$\Upsilon(2S)$	0.021		(0.003)		10.0234	
$\Upsilon(3S)$	0.014		(0.002)		10.3551	
$\Upsilon(4S)$	0.010		(0.001)		10.5794	
Total	694.030	(1.969)	(1.396)	(0.416)		

Experimental inputs

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$\pi^+\pi^-(\gamma)$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2012)	PR D86, 032013	σ	0.3 – 3.0	ISR, VP		Normalisation *in situ* to $\sigma(e^+e^- \to \mu^+\mu^-)$ data
BCF (1976)	LNC 15, 393	σ	1.2 – 3	ISR, lep. VP	1.008	
BES-III (2015)	hepex-150708188	σ	0.6025 – 0.8975	ISR, VP		
CLEO-c (2018)	PR D97, 032012	σ	0.300 – 1.000	ISR, VP		
CMD (1985)	NP B256, 365	σ	0.36 – 0.82	ISR, lep. VP	1.008	
CMD-2 (2003)	hepex-0308008	σ	0.6105 – 0.9615	ISR, VP		
CMD-2 (2005)	ZETFP 82, 841	R	0.98 – 1.38	ISR	1.008	
CMD-2 (2006)	hepex-0610016	σ	0.37 – 0.52	ISR		
CMD-2 (2006)	hepex-0610021	σ	0.6 – 0.97	ISR, VP		
CMD-3 (LOW-2020)	hepex-2302.08834	Formfactor	0.360352 – 0.601222	ISR, VP	1.008	
CMD-3 (RHO-2013)	hepex-2302.08834	Formfactor	0.326980 – 1.060255	ISR, VP	1.008	
CMD-3 (RHO-2018)	hepex-2302.08834	Formfactor	0.547784 – 1.199168	ISR, VP	1.008	
DM1 (1978)	PL 76B, 512	σ	0.483 – 1.096	ISR, lep. VP	1.008	
DM2 (1989)	PL 220B, 321	σ	1.35 – 2.12	ISR, VP	1.008	
KLOE (2016)	JHEP 1803, 173	σ	0.32 – 0.97	ISR, VP		Combination of 2008, 2010, 2012 runs
OLYA (1985)	NP B256, 365	σ	0.4 – 1.397	ISR, lep. VP	1.008	Radiative corrections discussed in BudkerINP-2002-74
SND (2006)	hepex-0605013	σ	0.39 – 0.97	ISR	1.008	
SND (2021)	JHEP 01 (2021), 113	σ	0.5251 – 0.8832	ISR	1.008	
VEPP-2-TOF (1981)	SJNP 33, 368	σ	0.4 – 0.46	ISR, lep. VP	1.008	

$\pi^+\pi^-\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2005)	PR D70, 072004	σ	0.6125 – 4.45	ISR		
CMD (1989)	NOVO-89-15	σ	0.84 – 1.013	ISR	1.008	
CMD-2 (1995)	BUDKERINP-95-35	σ	1.008 – 1.027	ISR		
CMD-2 (1998)	BUDKERINP-98-30	σ	0.994 – 1.040	ISR	1.008	
CMD-2 (2000)	hepex-0308008	σ	0.78 – 0.80	ISR		
DM1 (1980)	NP B172, 13	σ	0.483 – 1.098	ISR	1.008	
DM2 (1992)	ZP C56, 15	σ	1.34 – 2.4	ISR, VP	1.008	
ND (1991)	PRPL 202, 99	σ	0.66 – 1.38	ISR	1.008	
SND (2000)	PR D63, 072002	σ	0.98402 – 1.05966	ISR		
SND (2002)	hepex-0201040	σ	0.98 – 1.38	ISR		
SND (2003)	PR D68, 052006	σ	0.44 – 0.98	ISR		
SND (2015)	ZETF 148, 34	σ	1.05 – 2.00	ISR		

Uncommented multiplicative factors account for the FSR correction.

Journal abbreviations:

EPJ ***Eur. Phys. J.***
JETP ***J. Exp. Theor. Phys.***
JETPL ***JETP Letters***
JHEP ***J. of High Energy Phys.***
LNC ***Lettere al Nuovo Cimento***
NP ***Nuclear Physics***
PL ***Physics Letters***
PR ***Physical Review***
PRPL ***Physics Reports***
SJNP ***Sov. J. Nucl. Phys.***
ZETF ***Zh. Eksp. Teor. Fiz.***
ZETFP ***Pisma Zh. Eksp. Teor. Fiz.***
ZP ***Zeitschrift für Physik***

Experimental inputs

$\pi^+\pi^-2\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
ACO (1976)	PL 63B, 349	σ	0.915 – 1.076	ISR, lep. VP	1.008	
BABAR (2017)	PR D96, 092009	σ	0.85 – 4.49	ISR		
CMD-2 (1999)	PL 466B, 392	σ	0.98 – 1.4	ISR	1.008	
DM2 (1990)	LAL-90-35	σ	1.34 – 2.40	ISR, VP	1.008	
GG2 (1981)	NP B184, 31	σ	1.44 – 2.20	ISR, lep. VP	1.008	
M3N (1979)	NP B152, 215	σ	1.35 – 2.125	ISR, lep. VP	1.008	
MEA (1981)	LNC 31, 445	σ	1.45 – 1.80	ISR, lep. VP	1.008	
ND (1991)	PRPL 202, 99	σ	0.91 – 1.395	ISR	1.008	
OLYA (1986)	ZETFP 43, 497	σ	0.97 – 1.4	ISR, lep. VP	1.008	
SND (2001)	BUDKERINP-2001-34	σ	0.98 – 1.38	ISR	1.008	

$2\pi^+2\pi^-$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
ACO (1976)	PL 63B, 349	σ	0.915 – 1.076	ISR, lep. VP	1.008	
BABAR (2012)	PR D85, 112009	σ	0.6125 – 4.4875	ISR		
CMD (1988)	SJNP 47, 248	σ	1.019 – 1.403	ISR, lep. VP	1.016	
CMD-2 (2000)	PL 475B, 190	σ	0.75 – 0.97	ISR	1.016	
CMD-2 (2000)	PL 491B, 81	σ	0.984 – 1.060	ISR	1.016	
CMD-2 (2004)	PL 595B, 101	σ	0.98 – 1.38	ISR	1.016	
CMD-3 (2017)	PL 768B, 345	σ	0.92235 – 1.05995	ISR		
DM1 (1979)	PL 81B, 389	σ	0.893 – 1.098	ISR, lep. VP	1.016	
DM1 (1982)	PL 109B, 129	σ	1.41 – 2.166	ISR, lep. VP	1.016	
DM2 (1990)	LAL-90-35	σ	1.34 – 2.26	ISR, VP	1.016	
GG2 (1980)	PL 95B, 139	σ	1.2 – 2.4	ISR, lep. VP	1.016	
M3N (1979)	NP B152, 215	σ	1.35 – 2.125	ISR, lep. VP	1.016	
ND (1991)	PRPL 202, 99	σ	1.005 – 1.395	ISR	1.016	
OLYA (1988)	ZETFP 47, 432	σ	1.051 – 1.384	ISR, lep. VP	1.016	
SND (2001)	BUDKERINP-2001-34	σ	0.98 – 1.38	ISR	1.016	

Experimental inputs

$\pi^0\gamma$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
CMD-2 (2005)	PL 605B, 26	σ	0.59938 – 1.31	ISR		
SND (2000)	EPJ C12, 25	σ	0.98513 – 1.03930	ISR		
SND (2016)	PR D93, 092001	σ	0.6 – 1.4	ISR		

$\pi^+\pi^-\pi^0\eta$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
CMD-3 (2017)	PL 773B, 150	σ	1.4 – 2.0	ISR	0.7708	$\pi^+\pi^-\pi^0\eta\langle\pi^+\pi^-\pi^-\rangle$ is counted in the $2\pi^+2\pi^-2\pi^0$ channel, hence the cross section is multiplied by $1 - \mathrm{Br}(\eta \to \pi^+\pi^-\pi^0) = 0.7708$.

$\pi^+\pi^-2\pi^0\eta$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2018)	PR D98, 112015	σ	1.625 – 4.325	ISR		

$\pi^+\pi^-3\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2018)	PR D98, 112015	σ	1.125 – 4.325	ISR		

Experimental inputs

$2\pi^+2\pi^0 2\pi^-$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2006)	hepex-0602006	σ	1.3125 – 4.4875	ISR	1.145	The multiplicative factor accounts for the $2\pi^+2\pi^-2\pi^0$ term in the expression for the missing $\pi^+\pi^-4\pi^0$ channel: $\sigma(\pi^+\pi^-4\pi^0) = 0.0625\sigma(3\pi^+3\pi^-) + 0.145(\sigma(2\pi^+2\pi^-2\pi^0) - \sigma(\pi^+\pi^-\pi^0\eta\langle\pi^+\pi^-\pi^0\rangle)$ $)$ $\pm 100\%$ [see M. Davier *et al.*, Eur. Phys. J C71 (2011) 1515)]
CMD (1988)	SJNP 47, 248	σ	1.403	ISR	1.16332	An FSR correction is applied on top of the factor accounting for missing 6π channels.
DM2 (1986)	ROMA-THESIS-1986-SCHIOPPA	σ	1.32 – 2.24	ISR, VP	1.16332	The radiative correction applied by the authors is questionable.

$2\pi^+2\pi^-3\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2021)	arxiv:2102.01314	σ	1.575 – 4.475	ISR	1.016	

$2\pi^+2\pi^-\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2007)	PR D76, 092005	σ	1.0125 – 4.4875	ISR	1.0	
CMD (1988)	SJNP 47, 248	σ	1.019 – 1.403	ISR	1.016	
M3N (1979)	NP B152, 215	σ	1.35 – 2.125	ISR, lep. VP	1.016	

Experimental inputs

$3\pi^+3\pi^-$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2006)	hepex-0602006	σ	1.3125 – 4.4875	ISR	1.0625	The multiplicative factor accounts for the $3\pi^+3\pi^-$ term in the expression for the missing $\pi^+\pi^-4\pi^0$ channel: $\sigma(\pi^+\pi^-4\pi^0)$ $= 0.0625\sigma(3\pi^+3\pi^-) + 0.145(\sigma(2\pi^+2\pi^-2\pi^0) - \sigma(\pi^+\pi^-\pi^0\eta\langle\pi^+\pi^-\pi^0\rangle))$ $\pm100\%$ [see M. Davier *et al.*, Eur. Phys. J C71 (2011) 1515)]
CMD (1988)	SJNP 47, 248	σ	1.403	ISR	1.088	FSR correction is applied on top of 1.0625 factor (see above).
CMD-3 (2013)	PL B723, 82	σ		ISR	1.0625	
DM1 (1981)	PL 107B, 145	σ	1.45 – 2.455	ISR, lep. VP	1.088	
DM2 (1986)	ROMA-THESIS-1986-SCHIOPPA	σ	1.57 – 2.25	ISR, VP	1.088	

$3\pi^+3\pi^-\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
CMD-3 (2019)	PL 792B, 419	σ	1.60 – 2.0075	ISR		

Experimental inputs

$\eta\gamma$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
ACO (1976)	PL 63B, 352	σ	1.01525 – 1.02325	ISR		
CMD-2 (1995)	BUDKERINP-95-35	σ	1.008 – 1.027	ISR		
CMD-2 (2001)	PL 509B, 217	σ	0.6 – 1.354	ISR		
SND (2000)	EPJ C12, 25	σ	0.98513 – 1.03930	ISR		

$\eta\pi^+\pi^-$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2008)	PR D76, 092005	σ	1.025 – 2.975	ISR	0.4440	
BABAR (2018)	PR D97, 052007	σ	1.15 – 3.5	ISR	0.4440	
CMD-2 (2000)	PL 489B, 125	σ	1.285 – 1.38	ISR	0.447552	
ND (1991)	PRPL 202, 99	σ	1.075 – 1.375	ISR	0.447552	
SND (2015)	PR D91, 052013	σ	1.225 – 2.000	ISR	0.4440	

$\phi(1020)\eta$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2008)	PR D77, 092002	σ		ISR, VP	0.168	Measurement in the $\phi\langle K^+K^-\rangle\eta\langle 2\gamma\rangle$ mode. $\phi\langle KK\rangle\pi^+\pi^-\pi^0$ contribution is already counted in $K^+K^-\pi^+\pi^-\pi^0$ and $K_SK_L\eta(\pi^+\pi^-\pi^0)$ final states, hence we apply the multiplicative factor $1 - \mathrm{Br}(\phi \to KK) = 1 - 0.492 - 0.34 = 0.168$.
BABAR (2008)	PR D77, 119902	σ		ISR	0.168	Measured in $\eta\langle\pi^+\pi^-\pi^0\rangle$ mode.
CMD-3 (2019)	hepex-1906.08006	σ		ISR	0.168	

Experimental inputs

K^+K^-

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2013)	PR D88, 032013	σ	0.985000 – 5.000000	ISR, VP		
CMD (1983)	NOVO-83-85	σ	1.088 – 1.34	ISR	1.008	
CMD-2 (2008)	arXiv:0804.0178v1	σ	1.01136 – 1.03406	ISR		
CMD-3 (2017)	arXiv:1710.02989	σ	1.01 – 1.06	ISR		
DM1 (1981)	PL 99B, 257	σ	1.4245 – 2.03	ISR, lep. VP	1.008	
MEA (1980)	LNC 28, 337	σ	1.45 – 1.52	ISR, lep. VP	1.008	
OLYA (1981)	PL 107B, 297	σ	1.017 – 1.4	ISR	1.008	
SND (2000)	hepex-0009036	σ	1.01017 – 1.05966	ISR		
SND (2016)	PR D94, 112006	σ	1.047 – 2.005	ISR, VP		

$K^+K^-\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
DM2 (1990)	LAL-90-71	σ		ISR, VP	1.008	
DM2 (1991)	ZP C52, 227	σ		ISR	1.008	

$K^+K^-\pi^0\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2012)	PR D86, 012008	σ	1.5 – 4.02	ISR		

$K^+K^-\pi^+\pi^-$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2012)	PR D86, 012008	σ	1.4125 – 4.9875	ISR		
CMD-3 (2016)	PL 756B, 153	σ	1.4349 – 2.0046	ISR, VP		
DM1 (1981)	PL 110B, 335	σ	1.45 – 2.14	ISR, lep. VP		
DM2 (1990)	lal-90-71	σ		ISR, VP		

Experimental inputs

$K^+K^-\pi^+\pi^-\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2008)	PR D77, 119902	σ		ISR	2.19	This final state is used to estimate $\sigma(KK\pi^+\pi^-\pi^0)+\sigma(KK\pi^0\gamma)$. As $\pi^+\pi^-\pi^0$ in $K^+K^-\pi^+\pi^-\pi^0$ is dominated by ω contribution, we find $\sigma(KK\pi^+\pi^-\pi^0)+\sigma(KK\pi^0\gamma) \simeq 2\sigma(K^+K^-\pi^+\pi^-\pi^0) \cdot \left(1+\text{Br}(\omega\to\pi^0\gamma)/\text{Br}(\omega\to 3\pi)\right) \simeq 2.19\sigma(K^+K^-\pi^+\pi^-\pi^0)$.

K_SK_L

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2014)	PR D89, 092002	σ	1.08 – 2.16	ISR		
CMD (1983)	NOVO-83-85	σ	1.088 – 1.309	ISR, lep. VP		
CMD-2 (1995)	BUDKERINP-95-35	σ	1.008 – 1.027	ISR		
CMD-2 (2001)	hepex-9906032	σ	1.00402 – 1.03965	ISR		
CMD-2 (2003)	PL 551B, 27	σ	1.05 – 1.368	ISR		
CMD-3 (2016)	PL 760B, 314	σ	1.004058 – 1.059962	ISR		
DM1 (1981)	PL 99B, 261	σ	1.4415 – 2.14	ISR, lep. VP	1.0	
OLYA (1982)	ZETFP 36, 91	σ	1.09 – 1.34	ISR, lep. VP		
SND (2000, charged mode)	hepex-0009036	σ	1.00371 – 1.05966	ISR		
SND (2000, neutral mode)	hepex-0009036	σ	1.00371 – 1.05966	ISR		

$K_SK_L\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2017)	PR D95, 052001	σ	1.425 – 3.975	ISR		

Experimental inputs

$K_S K_L \eta$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2017)	PR D95, 052001	σ	1.575 – 3.975	ISR	1.5416	The $KK\eta\langle\pi^+\pi^-\pi^0\rangle$ modes are counted in the $KK\pi^+\pi^-\pi^0$ final state, hence $K_S K_L \eta$ final state is used to extract $\sigma(KK\eta) \cdot \left(1 - \mathrm{Br}(\eta \to \pi^+\pi^-\pi^0)\right) \simeq 2\sigma(K_S K_L \eta)\cdot(1-0.2292) = 1.5416\sigma(K_S K_L \eta)$.

$K_S K_L \pi^0 \pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2017)	PR D95, 052001	σ	1.35 – 3.95	ISR		

$K_S K_L \pi^+ \pi^-$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2014)	PR D89, 092009	σ	1.63 – 3.38	ISR		

Experimental inputs

$K_S K^+ \pi^- \pi^0 + K_S K^- \pi^+ \pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2017)	PR D95, 092005	σ	1.51 – 3.99	ISR	2.0	The multiplicative factor follows from symmetry relation: $\sigma(K^0K^+\pi^0\pi^-) = \sigma(K_SK^+\pi^0\pi^-) + \sigma(K_SK^-\pi^0\pi^+) \Rightarrow \sigma(K^0K^+\pi^0\pi^-) + \sigma(\bar{K}^0K^-\pi^0\pi^+) = 2\sigma(K_SK^+\pi^-\pi^0 + K_SK^-\pi^+\pi^0)$ [see M. Davier *et al.*, Eur. Phys. J C71 (2011) 1515)]

$K_S K^+ \pi^- + K_S K^- \pi^+$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2008)	PR D77, 092002	σ	1.24 – 4.70	ISR, VP	2.0	The multiplicative factor is derived from symmetry relation: $\sigma(K^0K^+\pi^-) + \sigma(\bar{K}^0K^-\pi^+) = 2\sigma(K_SK^+\pi^+K_SK^-\pi^+)$.

$K_S K_S \pi^+ \pi^-$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2014)	PR D89, 092009	σ	1.63 – 3.38	ISR	2.0	The multiplicative factor accounts for missing $K_LK_L\pi^+\pi^-$ channel, $\sigma(K_LK_L\pi^+\pi^-) = \sigma(K_SK_S\pi^+\pi^-)$.

Experimental inputs

$\omega(783) < \pi^0\gamma > \pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
CMD-2 (2003)	hepex-0304009	σ	0.92 – 1.38	ISR		
CMD-2 (2004)	PL 580B, 119	σ	0.60 – 0.97	ISR		
DM2 (1990)	LAL-90-35	σ	1.34 – 2.40	ISR, VP	0.098	$\omega\langle\pi^+\pi^-\pi^0\rangle\pi 0$ measurement scaled by $Br(\omega \to \pi^0\gamma)/Br(\omega \to \pi^+\pi^-\pi^0)$, where the latter branching is the one used in the original paper.
KLOE (2008)	arXiv:0807.4909	σ	1.00010 – 1.02995	ISR		
ND (1986)	PL 174B, 453	σ	1.02 – 1.39	ISR	0.087	
SND (2000)	BUDKERINP-2000-35	σ	0.92 – 1.38	ISR		
SND (2000)	NP B569, 158	σ	0.984 – 1.060	ISR		
SND (2011)	JETPL 94, 734	σ		ISR		2009 data
SND (2016)	PR D94, 112001	σ		ISR		Reprocessed 2010-2011 data, 2012 data added.

$\omega(783) < \pi^+\pi^-\pi^0 > \pi^+\pi^-$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2008)	PR D76, 092005	σ	1.15 – 2.525	ISR	0.13380	$\omega\langle\pi^+\pi^-\pi^0\rangle\pi^+\pi^-$ contribution is already contained in the $2\pi^+2\pi^-\pi^0$ channel. Here we derive $\sigma(\omega\langle\pi^0\gamma\rangle 2\pi) = (1.008_{fsr} + \frac{1}{2}) \times \Big(Br(\omega \to \pi^0\gamma)/Br(\omega \to 3\pi)\Big) \times \sigma(\omega\langle 3\pi\rangle\pi^+\pi^-)$. (BABAR published the cross section already divided by $Br(\omega \to 3\pi)$, hence BABAR multiplicative factor differs from the others).
CMD-2 (2000)	PL 489B, 125	σ	1.285 – 1.38	ISR	0.1509	
DM1 (1981)	PL 106B, 155	σ	1.4425 – 2.145	ISR, lep. VP	0.1509	
DM2 (1992)	ZP C56, 15	σ	1.34 – 2.4	ISR, VP	0.1509	

Experimental inputs

$\omega(783)\eta$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
CMD-3 (2017)	PL 773B, 150	σ	1.4 – 2.0	ISR	0.107	$\omega\langle\pi^+\pi^-\pi^0\rangle\eta$ is counted in $2\pi^+2\pi^-2\pi^0$ and $\pi^+\pi^-\pi^0\eta$ channels, hence this channel is used to derive only $\sigma\left(\omega(\text{non}-3\pi)\eta\right) = \left(1 - \text{Br}(\omega \to \pi^+\pi^-\pi^0)\right) \times \sigma(\omega\eta) = 0.107\sigma(\omega\eta)$.
SND (2016)	PR D94, 092002	σ	1.36 – 2.00	ISR	0.107	

$\omega\eta\pi^0$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
SND (2016)	PR D94, 032010	σ		ISR	0.792	$\eta\langle\pi^+\pi^-\pi^0\rangle\omega\langle\pi^+\pi^-(\pi^0)\rangle\pi0$ final states are counted in the $2\pi^+2\pi^-(2,3)\pi^0$ channel, hence the multiplicative factor $1 - \text{Br}(\eta \to \pi^+\pi^-\pi^0)\text{Br}(\omega \to \pi^+\pi^-(\pi^0))$ is applied.

Experimental inputs

$p\bar{p}$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
BABAR (2013)	PR D87, 092005	σ	1.877 – 4.500	ISR		
CMD-3 (2016)	PL 759B, 634	σ	1.9 – 2.0	ISR		
DM1 (1979)	PL 86B, 395	σ	1.937 – 2.135	ISR, lep. VP	1.008	Radiative correction?
DM2 (1983)	NP B224, 379	σ	2.0 – 2.2375	ISR, VP	1.008	Radiative correction? systematics?

$n\bar{n}$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
Fenice (1998)	NP B517, 3	σ	1.99 – 2.44	ISR, VP		
SND (2011)	PR D90, 112007	σ	1.89 – 2.00	ISR		
SND (2012)	PR D90, 112007	σ	1.90 – 1.98	ISR		

Experimental inputs

$2hadron(hadrons)$

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	Radiative correction	Mult. factor	Comment
AMY (1990)	PR D42, 1339	R	50.0 – 61.40	ISR, VP		
ARGUS (1991)	ZP C54, 13	R	9.360 – 0.000511	ISR, VP		
BES (1999)	PRL 84, 594	R	2.60 – 5.0	ISR, VP		
BES (2001)	hepex-0102003	R	2.0 – 4.80	ISR, VP		
BES (2006)	hepex-0612054	R	3.6500 – 3.8720	ISR, VP		
BES (2009)	arXiv:0903.0900	R	2.60 – 3.65	ISR, VP		
CELLO (1987)	PL 183B, 400	R	14.0 – 46.60	ISR, VP		
CLEO (1984)	PR D29, 1285	R	10.490 – 10.49	ISR, VP		
CLEO (1997)	PR D57, 1350	R	10.520 – 10.52	ISR, VP		
Crystal Ball (1988)	ZP C40, 49	R	9.390 – 9.460	ISR, VP		
Crystal Ball (1990)	SLAC-PUB-5160	R	5.0 – 7.40	ISR, VP		
CUSB (1982)	PRL 48, 906	R	10.430 – 11.090	ISR, VP		
DASP (1980)	ZP C4, 87	R	12.0 – 31.250	ISR, VP		
DASP (1982)	PL 116B, 383	R	9.510 – 9.51	ISR, VP		
DESY-Hamburg-Heidelberg-MPI (1980)	ZP C6, 125	R	9.450 – 10.040	ISR, VP		
JADE (1987)	PRPL 148, 67	R	12.0 – 46.470	ISR, VP		
KEDR (2018)	PL 788B, 42	R	1.841 – 3.7201	ISR, VP		
LENA (1982)	ZP C15, 299	R	7.440 – 9.4150	ISR, VP		
Mark-II (1979)	SLAC-219	R	3.670 – 3.8720	ISR, VP		
Mark-II (1990)	PR D43, 34	R	29.0 – 29.	ISR, VP		
MARK-J (1979)	PL 85B, 463	R	31.570 – 31.57	ISR, VP		
MARK-J (1982)	PL 108B, 63	R	34.850 – 34.85	ISR, VP		
MARK-J (1986)	PR D34, 681	R	12.0 – 46.470	ISR, VP		
MD-1 (1993)	ZP C70, 31	R	7.30 – 10.290	ISR, VP		
SLAC-PEP-MAC (1985)	PR D31, 1537	R	29.0 – 29.	ISR, VP		
SLAC-SPEAR (1977)	PRL 39, 526	R	3.5980 – 3.8860	ISR, VP		
SLAC-SPEAR (1986)	SLAC-PUB-4160	R	3.670 – 4.4960	ISR, VP		
TASSO (1984)	PL 138B, 441	R	41.450 – 44.20	ISR, VP		
TASSO (1984)	ZP C22, 307	R	12.0 – 41.40	ISR, VP		
TASSO (1990)	ZP C47, 187	R	14.030 – 43.70	ISR, VP		
TOPAZ (1990)	PL 234B, 525	R	50.0 – 61.40	ISR, VP		
TOPAZ (1993)	PL 304B, 373	σ	57.370 – 59.840	ISR		
TOPAZ (1995)	PL 347B, 171	σ	57.770 – 57.77	ISR		
VENUS (1987)	PL 198B, 570	R	50.0 – 52.0	ISR, VP		
VENUS (1990)	PL 246B, 297	R	63.60 – 64.0	ISR, VP		

CMD-3 vs CMD-2

F. Ignatov (CMD-3 Coll.), $e^+e^- \to \pi^+\pi^-$ at CMD-3, 6th Plenary Workshop on the Muon g-2 Theory Initiative, Sep 4-8, 2023
I. Logashenko (CMD-3 Coll.), CMD-2 vs. CMD-3 and future plans at VEP2000, ibid.:

- $\sim 3-5\%$ discrepancy with SND (VEPP-2M) and SND2k (VEPP-2000). No discrepancy between the latter. SND2k will process full statistics soon.
- $\sim 5\%$ discrepancy with ISR experiments: BaBar, BES-III, CLEO, KLOE.
- Most of experiments claim 0.5–1.0% systematics

- CMD-3 vs CMD-2:
  - Similarities: Z-chamber, analysis strategy.
  - Differences: drift chamber (DC), calo., readout electronics, DC resolution, CMD-3 statistics is 30 $\times$ CMD-2, analysis implementation. *"CMD-2 and CMD-3 are very different realization of the same-type measurement" [I. Logashenko]*
  - Momentum resolution: $\sim 1.3\%$ (CMD-3) vs $\sim 3\%$ (CMD-2) at $p = 400$ MeV.

- Possible unaccounted sources of systematics for CMD-2 and CMD-3:
  - Cosmics (counted as $\pi^\pm$): unlikely, CMD-2 1994/95/98 data consistent.
  - Detector efficiencies: unlikely, good agreement between different CMD-2 runs, same for CMD-3.
  - Trigger efficiency: unlikely, same reason
  - $\pi/\mu/e$ separation: missing systematics?
  - Event separation: systematics underestimated in CMD-2?
  - Fiducial volume: θ-dependence of efficiency in CMD-2 not studied; CMD-3 compares Z-chamber vs LXe calorimeter, θ-distribution analyzed.
- No plans to reanalyze CMD-2 data.

- CMD-3 will collect dedicated data for additional systematics study in $\sim$1 year:
  - Select $E_{c.m.}$ points around 700 MeV (largest discrepance of CMD-3 vs others)
  - Data with CsI calo only (CMD-2 like)
  - Data with lower lumi. and shorter bunches – effects of z cut and cosmics
  - Data with higher amplitudes in the drift chamber (with lower beams) – fiducial volume systematics
  - Data with full beams and no collisions – beam-induced backgrounds
  - Different triggers
- Major upgrade of CMD-3 by 2028: the goal is $\sim 0.2-0.3\%$ accuracy in $\sigma(\pi\pi)$

back

$a_\mu(\text{had, LO})$ from lattice

- (Euclidean)time-momentum representation for $a_\mu(\text{had, LO})$ [1]:

  $a_\mu(\text{had, LO}) = \alpha_0^2 \int_0^\infty dt\, K(t) G_{1\gamma\text{I}}(t)\,,$

  where $G_{1\gamma\text{I}}(t)$ is the 1-photon-irreducible part of the two-point function

  $G(t) = \frac{1}{3e^2} \sum_{\mu=1,2,3} \int d^3x \, \langle J_\mu(t,\vec{x}) J_\mu(0,0) \rangle\,,$

  with the quark EM current

  $J_\mu = e\left[\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \ldots\right]$

  and the weight function:

  $K(t) = \int_0^\infty \frac{dQ^2}{m_\mu^2} \omega\left(\frac{Q^2}{m_\mu^2}\right)\left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Q5}{2}\right)\right]\,,$

  with $\omega(r) = \left[r + 2 - \sqrt{r(r+4)}\right]^2 / \sqrt{r(r+4)}$.

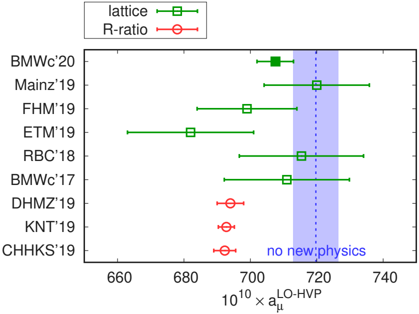
- Lattice calculation of $G(t)$ gives [1]:

  $$a_\mu(\text{had, LO}) = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}$$

- Reaching the sub-percent precision is a huge challenge:
  - Choosing an optimum lattice spacing
  - Numerical noise reduction for large t separations in the $G(t)$ correlator
  - QED and strong-isospin breaking
  - Infinite volume and continuum extrapolations

  See [1] for details.

[1] *Sz. Borsanyi et al., Nature 593 (2021) 7857, 51*



For a review see Section 3 in *T. Aoyama et al, Phys. Rept. 887 (2020) 1*.

For recent updates see the HVP lattice section of Sixth Plenary Workshop of the Muon g-2 Theory Initiative.

back

The source code

```
git clone https://glab.ihep.su/zenin_o/compas_users.git
cd compas_users/
git checkout master
cd ee/
cat README
# Good luck!
#
# Yes, the input cross section data are already checked into the tree.
# Just use this as a starting point.
```

Browse the code online at https://glab.ihep.su/zenin_o/compas_users/

Questions, bugreports: zenin_o@ihep.ru

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