Results on hadronic flux tube from lattice QCD

Vitaly Bornyakov

NRC Kurchatov Institute-IHEP, Protvino

and

Far East University, Vladivostok

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OUTLINE

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- Static potential in gluodynamics at T=0
- String breaking in QCD
- Flux tube in gluodynamics at T=0
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- Multiquark systems

Motivation

- Confinement problem

Cofinement is one of the most fundamental properties of strong interactions

and of the theory describing strong interactions – QCD. Still solid understanding of its microscopic nature is lacking

First-principle Monte Carlo simulations of QCD represent a useful tool to check existing models of confinement, but also to provide new numerical “phenomenology” and to stimulate new ideas about the mechanism of confinement.

- Relevance for charmonium

static quarks potential is used in computations of hyperfine splittings of heavy quarkonium hybrids (Soto, Valls, Phys.Rev.D 108 (2023) 1, 014025)

- Confinement-deconfinement transition

- The bosonic string picture emerged to explain the Regge trajectories

Goddard, Goldstone, Rebbi, and Thorn, 1973; Goto, 1971

- Then Formulation of Lattice QCD by K. Wilson, “Confinement of Quarks”, 1974

- Kogut and Susskind came up with a picture of a flux tube (color string) that connects the quarks while discussing lattice QCD: “Hamiltonian Formulation of Wilson’s Lattice Gauge Theories”, 1975

- Eichten, Gottfried, Kinoshita, Kogut, Lane et al., “The Spectrum of Charmonium” , 1975

$$V(r) = -\frac{\alpha_s}{r}\left(1 - \frac{r^2}{a^2}\right),$$

Cornell potential

- The formation of string-like flux tubes in SU(N) gauge theories is not rigorously proved, but the results obtained in lattice gauge theory provide evidence that this physical picture is basically correct.

- Numerical simulations show that there is a linear confining potential between a static quark and antiquark for distances equal to or larger than about 0.5 fm.

- This linear potential extends to infinite distances in gluodynamics, while in QCD only to distances of about 1.3 fm, where string breaking takes place

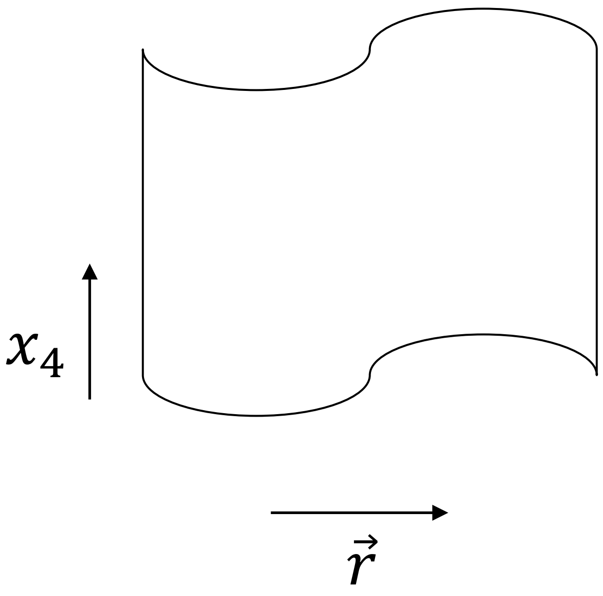
- This long-distance linear quark-antiquark potential is naturally associated with a tube-like structure ("flux tube") of the chromoelectric field in the longitudinal direction, i.e. along the line connecting the static quark and antiquark

Definitions

Wilson loop

$$W(C) = \frac{1}{N_c} Tr\,\{\, P \exp(i \oint_C \; dx_\mu \, A_\mu(x)) \,\}$$

The contour C to compute $V_{\bar{q}q}(r)$:



Spectral representation:

$$< W(r,t) >= \; C_0 e^{-E_0(r)t} + \; C_1 e^{-E_1(r)t} + \; ...$$

$$C_n > 0, \;\; \Sigma\, C_n = 1$$

$$E_0(r) = V_{\bar{q}q}(r) \quad - \quad \text{ground state potential}$$

$$V_{\bar{q}q}(r) = \; -\lim_{t\,\to\,\infty} \frac{1}{\Delta t} \log < W(r, t + \Delta t) >/< W(r,t >$$

The spatial link paths are typically changed using a smearing algorithm to approximate the shape of the flux tube and thus obtain better overlap C_0 with the ground state potential

Definitions

The Polyakov loop on $(L_s/a)^3 \times (L_t/a)$ lattice

$$P(\vec{r}) = \frac{1}{N_c} Tr \prod_{t=a}^{L_t} U_4(\vec{r}, t)$$

its correlator (and respective spectral decomposition):

$$C(r, L_t) = \sum_{\vec{x}} < P(\vec{x})\, P^+(\vec{x}+\vec{r}) > = \sum_n e^{-E_n(r) L_t}$$ Место для уравнения.

Potential

$$V(r) = -\frac{1}{L_t} \log(C(r, L_t))$$

Definitions

To study the flux tube profile the following correlator is computed

$$\langle \mathcal{O}(s) \rangle_W \equiv \frac{1}{3} \frac{\langle \mathrm{Tr}\, \mathcal{O}(s) \mathrm{Tr}\, \mathcal{W}_C \rangle}{\langle \mathrm{Tr}\, \mathcal{W}_C \rangle} - \frac{1}{3} \langle \mathrm{Tr} \mathcal{O} \rangle$$

W_C – Wilson loop

$\mathcal{O}(s)$ - $F_{\mu\nu}^2$

Summary

Since the first studies of the flux tubes the accuracy and reliability of lattice measurements in 3D and 4D SU(N) gauge theories has steadily improved, together with much better control over systematic uncertainties:

- continuum limit $a \to 0$ via extrapolation over few values of a
- finite volume effects
- extraction of the signal for the ground state from the correlator (smoothing, multi level algorithm)

The dominating longitudinal chromoelectric field component E_z in the transverse plane at the midpoint of the line connecting the static quark and antiquark was studied very intensively

The numerical results provide evidence that In the long-distance regime the heavy-quark-antiquark static potential is well described in terms of an Effective String Theory (EST)

Effective bosonic string theory prediction for static potential

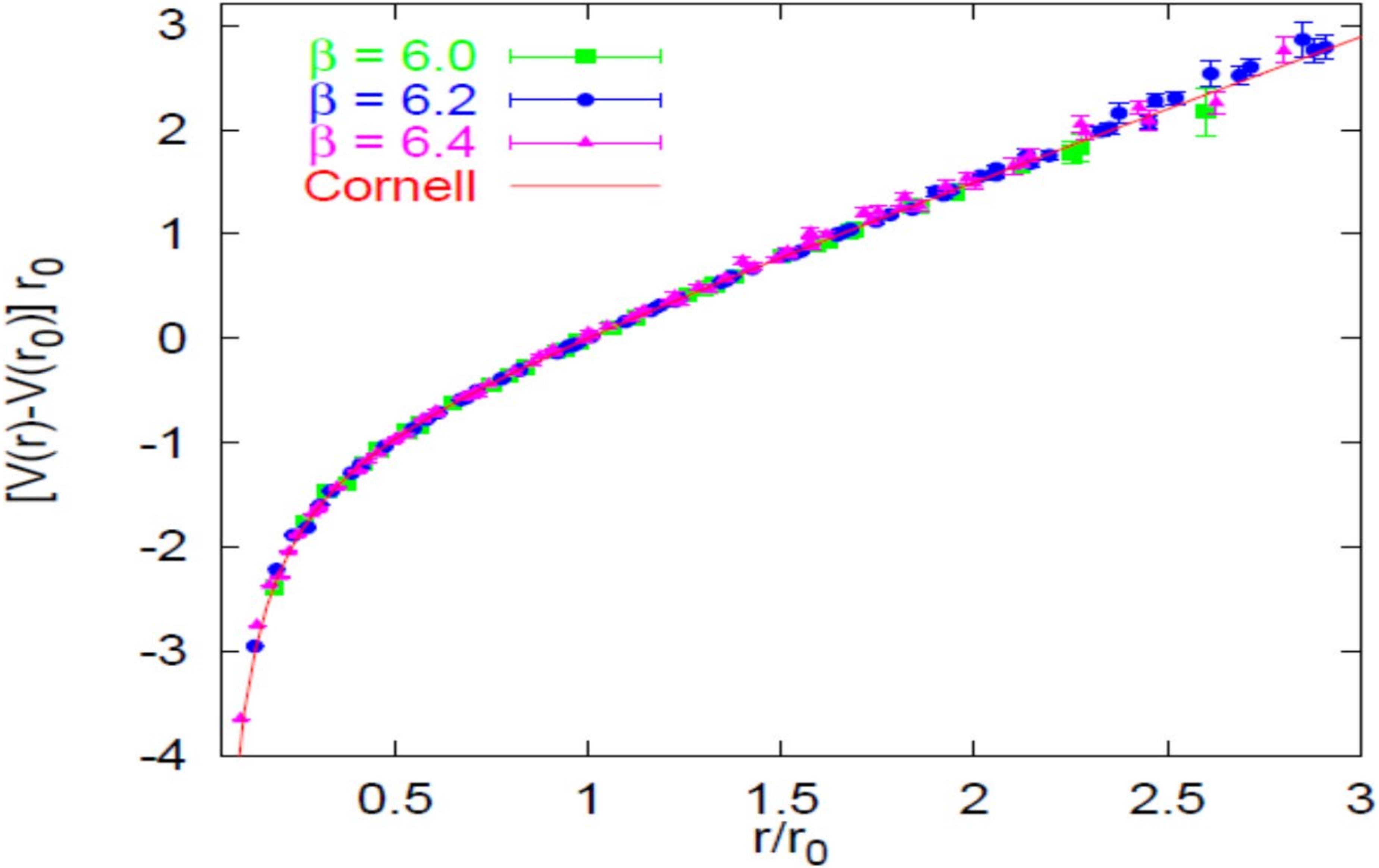
J. F. Arvis, 1983

$$V(r) = \sigma r \sqrt{1 - \frac{(d-2)\pi}{12\,\sigma\, r^2}} = \sigma\, r - \frac{(d-2)\pi}{24\, r} - \frac{(d-2)^2\pi^2}{1152\,\sigma\, r^3} - \cdots$$

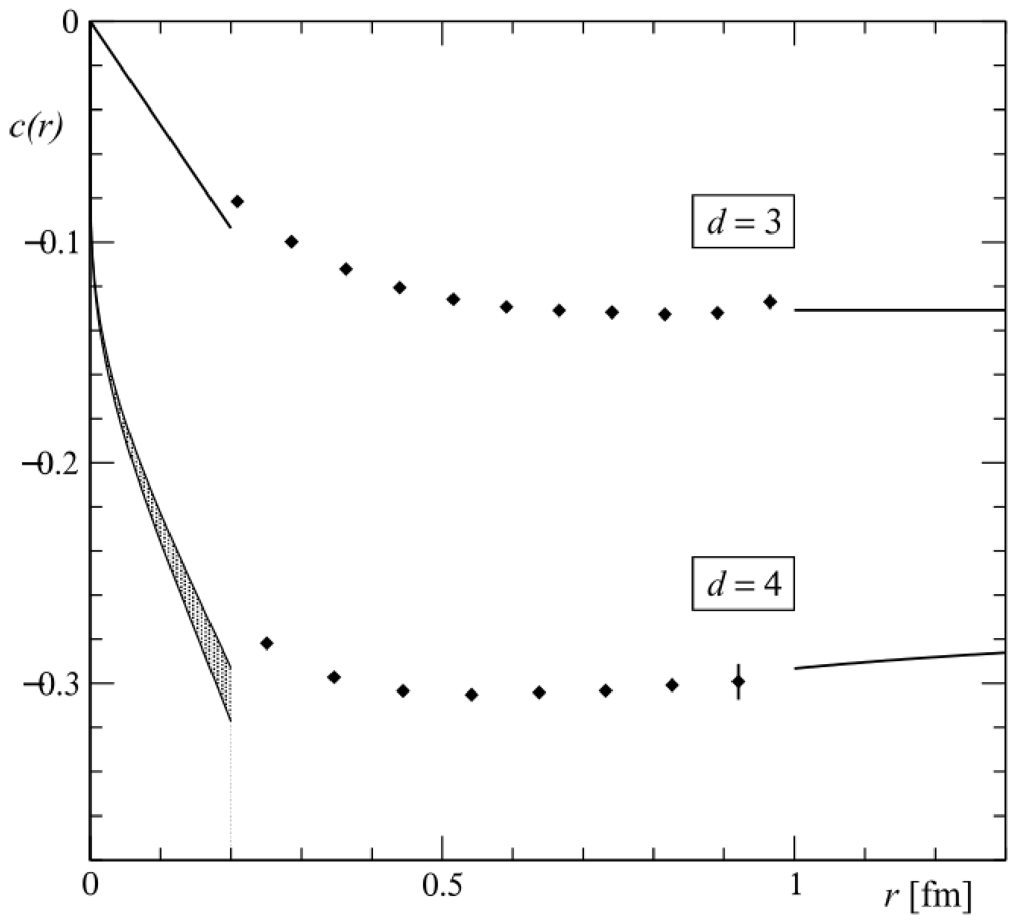
$$V(r) = \sigma r - \frac{\pi(d-2)}{24\, r} + \frac{b}{r^3}$$

Luscher, 1980

Luscher, Weisz, 2004



Static potential in SU(3) gluodynamics at three values of the lattice spacing.
Scale is fixed by Sommer parameter $r_0 \approx 0.5\text{fm}$; G.Bali, Phys.Rep. 2000



$$c(r) = \frac{1}{2} r^3 V''(r) = -\frac{\pi(d-2)}{24} + O(\frac{1}{r^2})$$

Luscher, Weisz, 2002

Morningstar, Juge and Kuti, 2003

SU(3) gluodynamics, improved lattice action

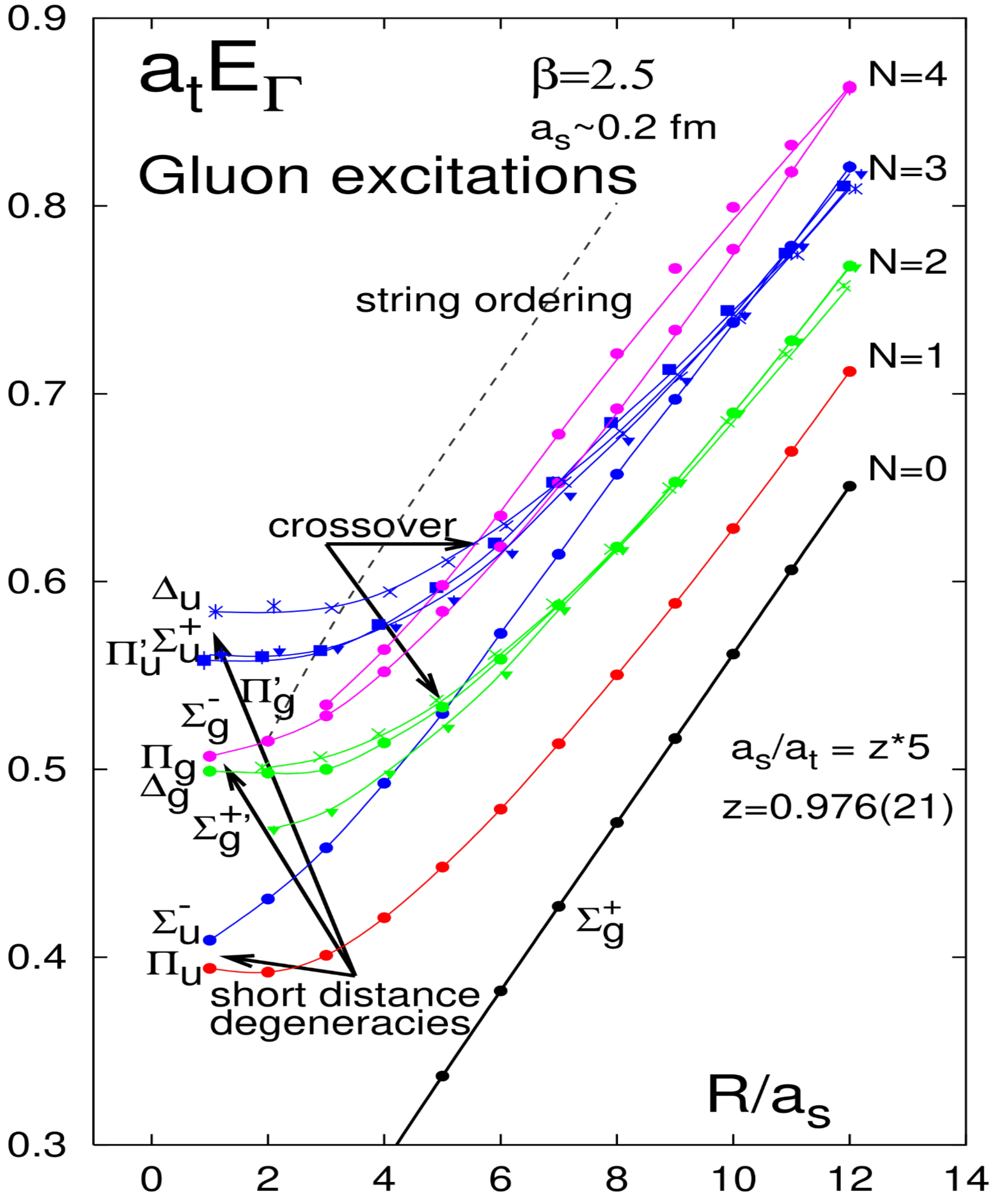
- generalized Wilson loops for excited states,

relevant symmetry group – point group D_{4h}

- standard notation from the physics of diatomic molecules by projection of angular momentum $\boldsymbol{J_g}$ on the string axis

$\Lambda = \boldsymbol{J_g} \cdot \mathbf{R}$ = 0, 1, 2, 3, …,

- respective states are labelled as $\Sigma, \Pi, \Delta, \Phi$, …
- g and u are notations for parity
- Degeneracy at short distance correspond to approximate O(3) symmetry
- Level crossings at intermediate distances $R \sim 1$ fm
- Correct EST ordering at $R > 2$ fm
- Approximate 1/R gaps between levels.



B. Brandt, 2016:

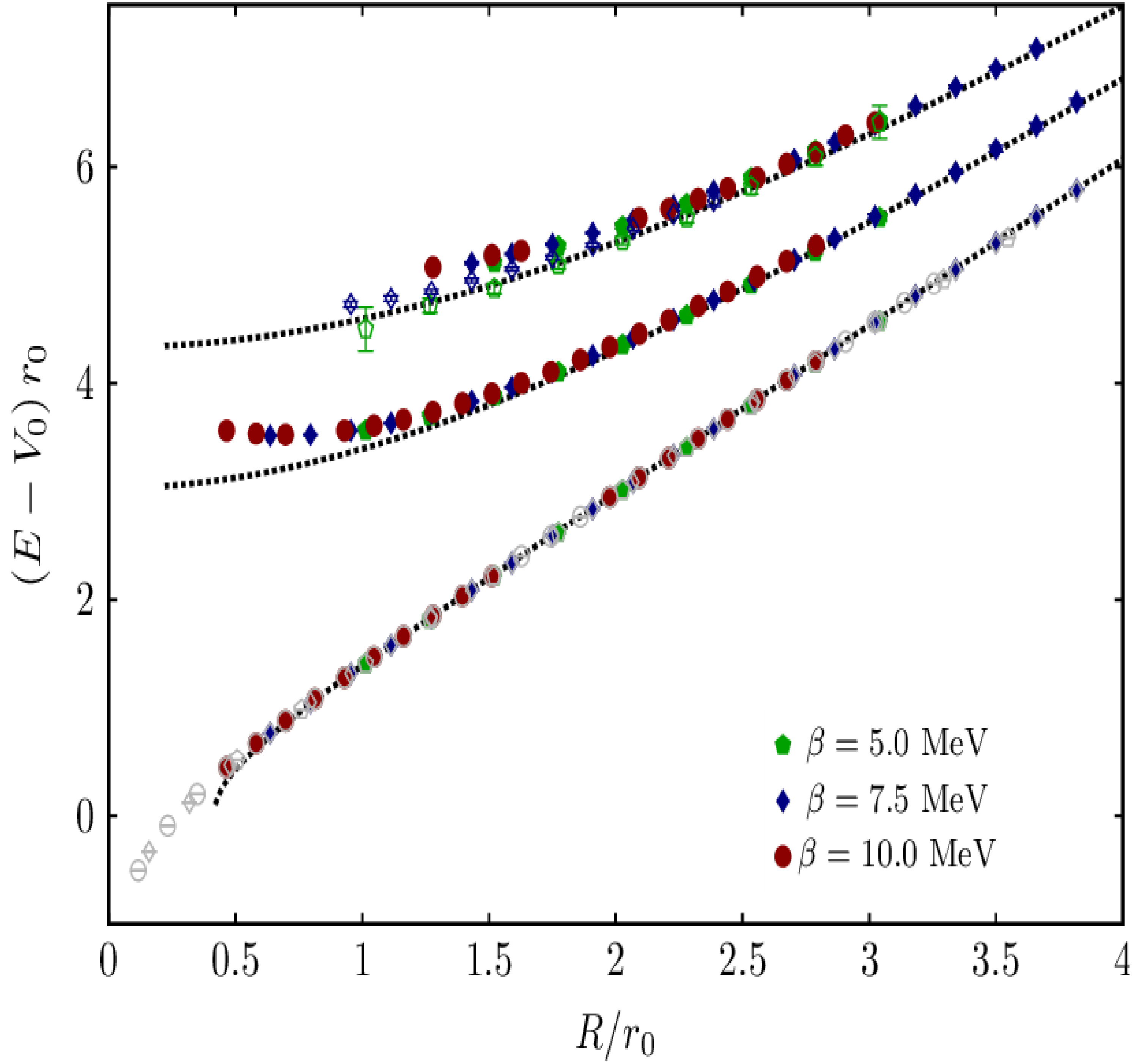
"For (most of) the energy levels the data shows a remarkable agreement with the full LC predictions down to small values of $\sqrt{\sigma}R \ \sim 1$, rather than with its expansion in powers of 1/R, independently of the number of dimensions and the gauge group"

Results from Brandt, 2021

For D=2+1, SU(2)

Three lattice spacings

The dotted lines are the predictions from the LC spectrum

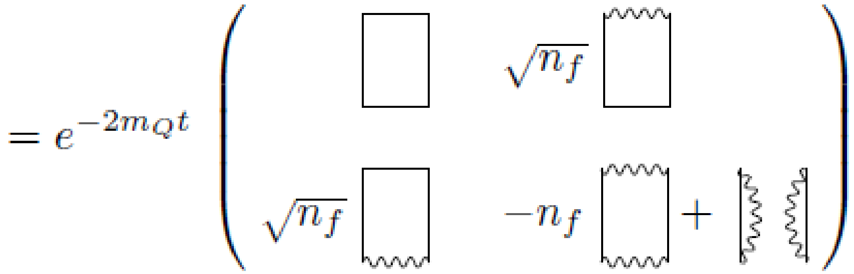


String breaking in QCD

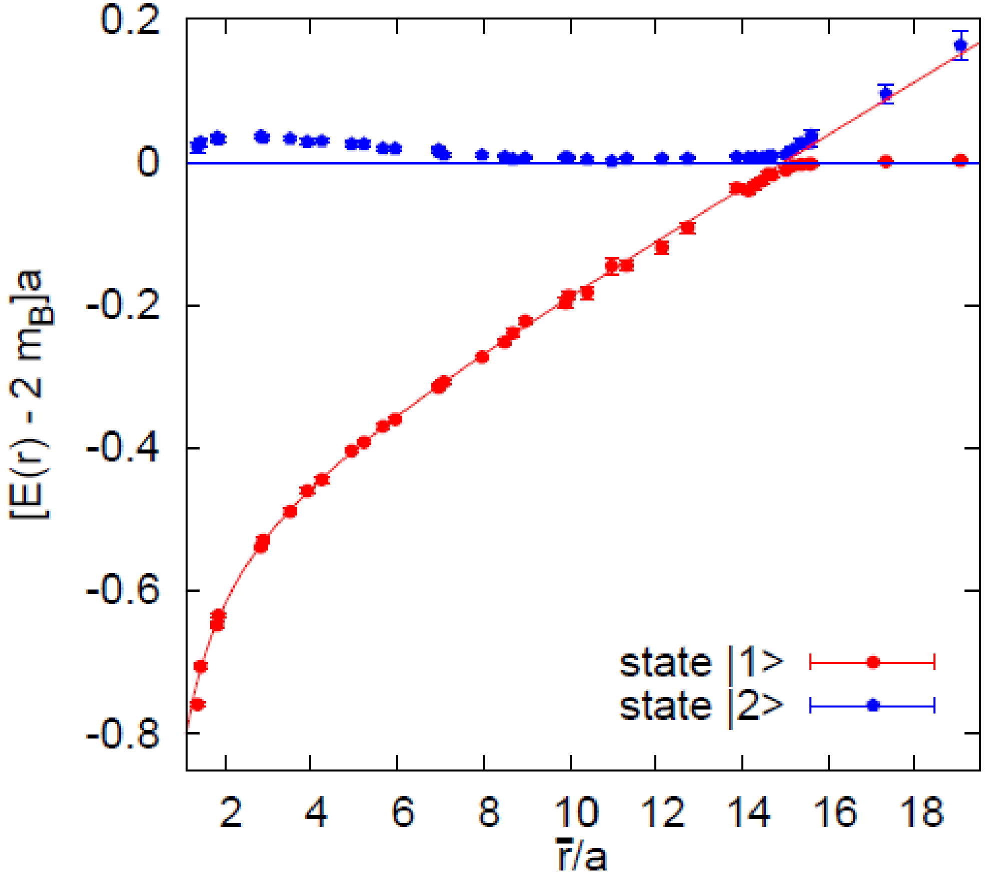
SESAM Collaboration, 2005

$N_f = 2$ QCD

$$C(t) = \begin{pmatrix} C_{QQ}(t) & C_{QB}(t) \\ C_{BQ}(t) & C_{BB}(t) \end{pmatrix}$$



r_{sb} =1.248(13) fm



At T=0 EST predicts a Gaussian profile and a logarithmic broadening of the width

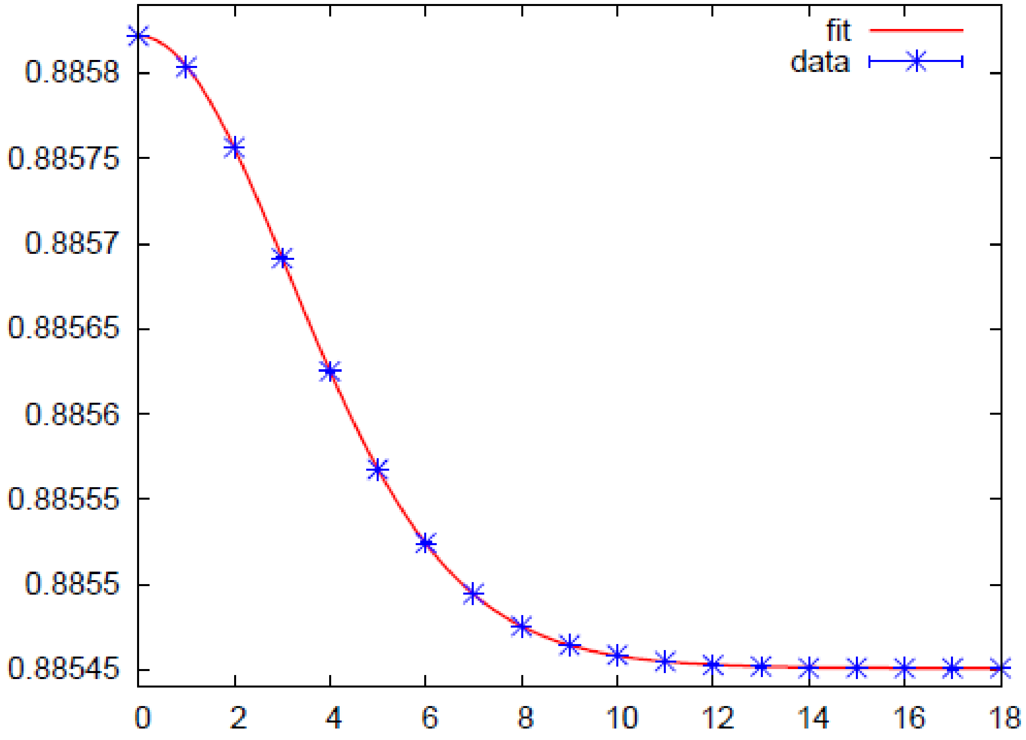
$$w_{lo}^2(r/2) = \frac{d-2}{2\pi\sigma} \log\left(\frac{r}{r_0}\right),$$

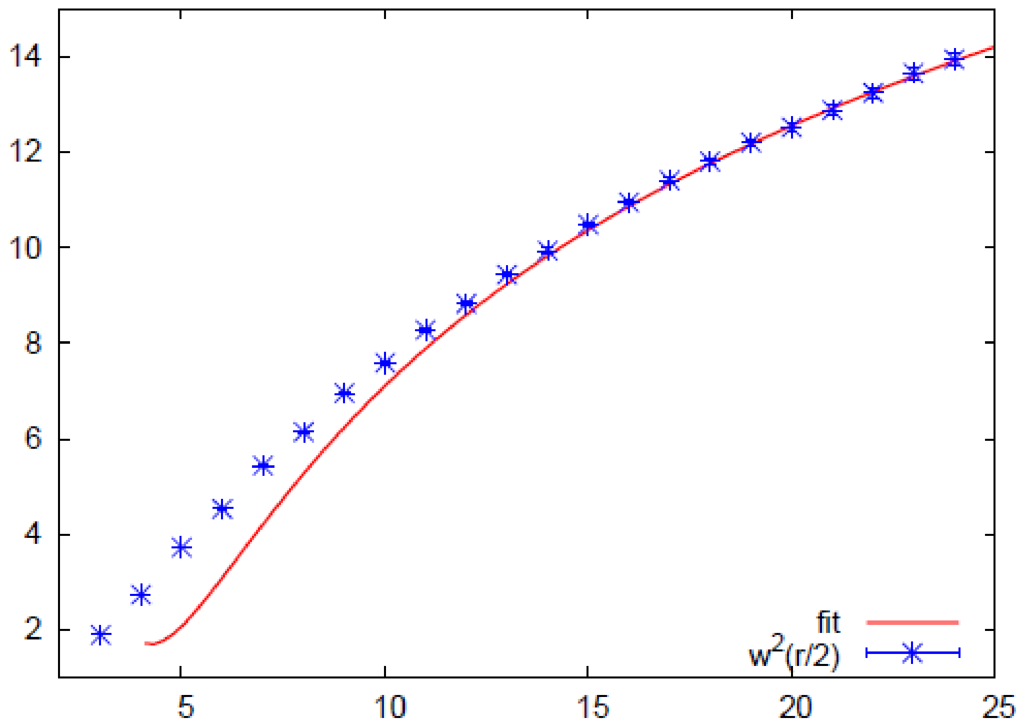
At T>0 EST predicts linear broadening ($\beta = 1/T$):

$$w_{lo}^2(r/2) = \frac{d-2}{2\pi\sigma} \log\frac{\beta}{4r_0} + \frac{d-2}{4\beta\sigma} r + \mathcal{O}(e^{-2\pi r/\beta}).$$

Gliozzi, Pepe, and Wiese, 2010, 2+1 SU(2), T=0

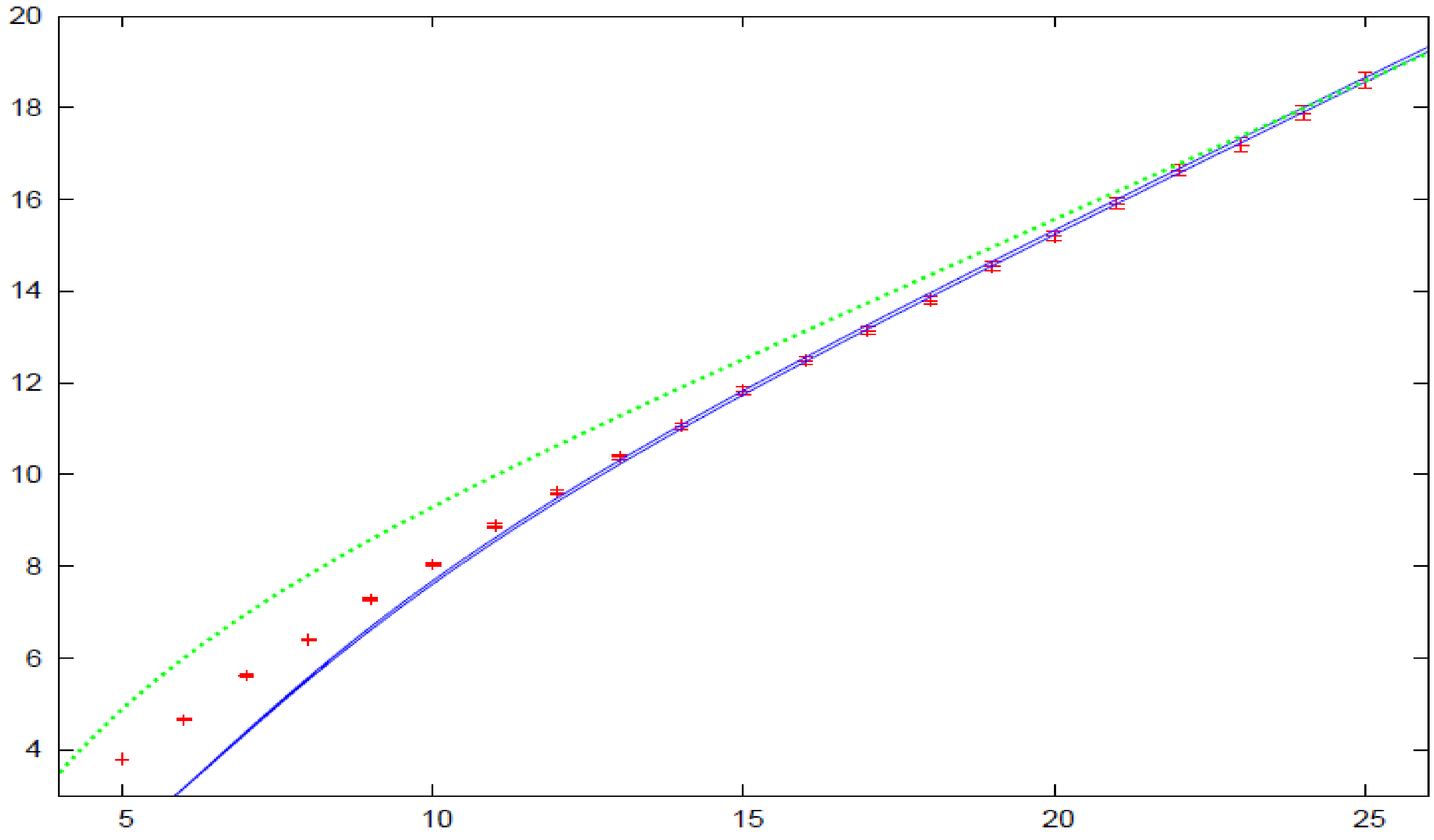
(Left) Gaussian fit to the flux tube profile ; (Right) squared width vs. EST prediction





Gliozzi, Pepe, and Wiese, 2010, 2+1 SU(2), T>0

squared width vs. EST prediction



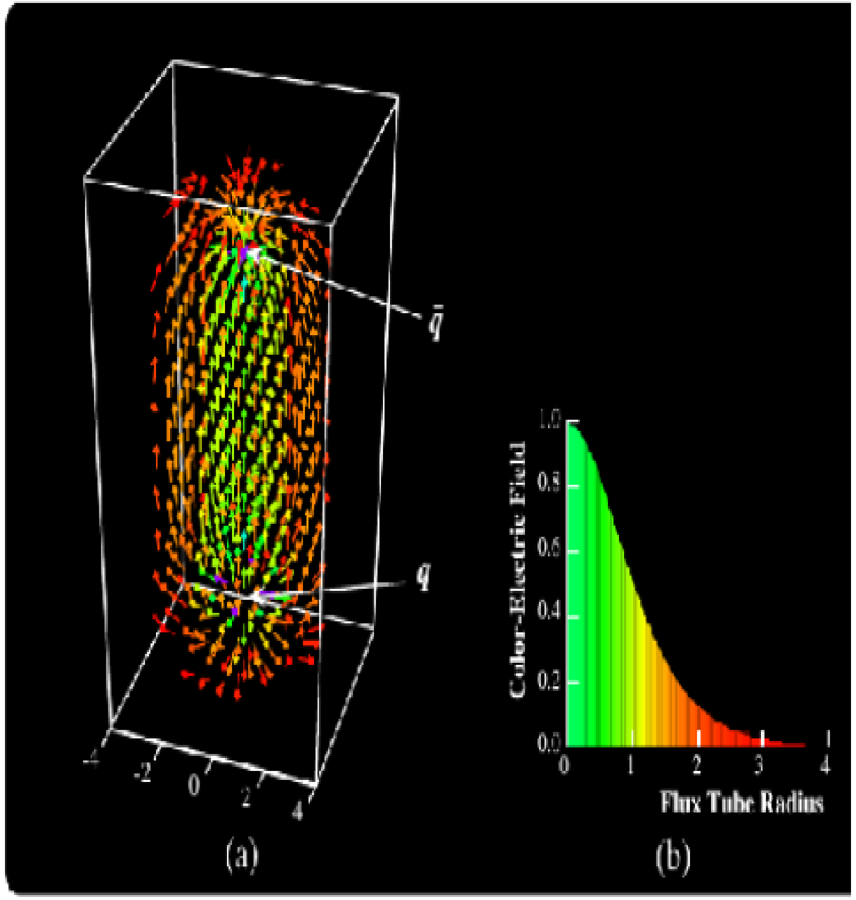
Dual superconductor

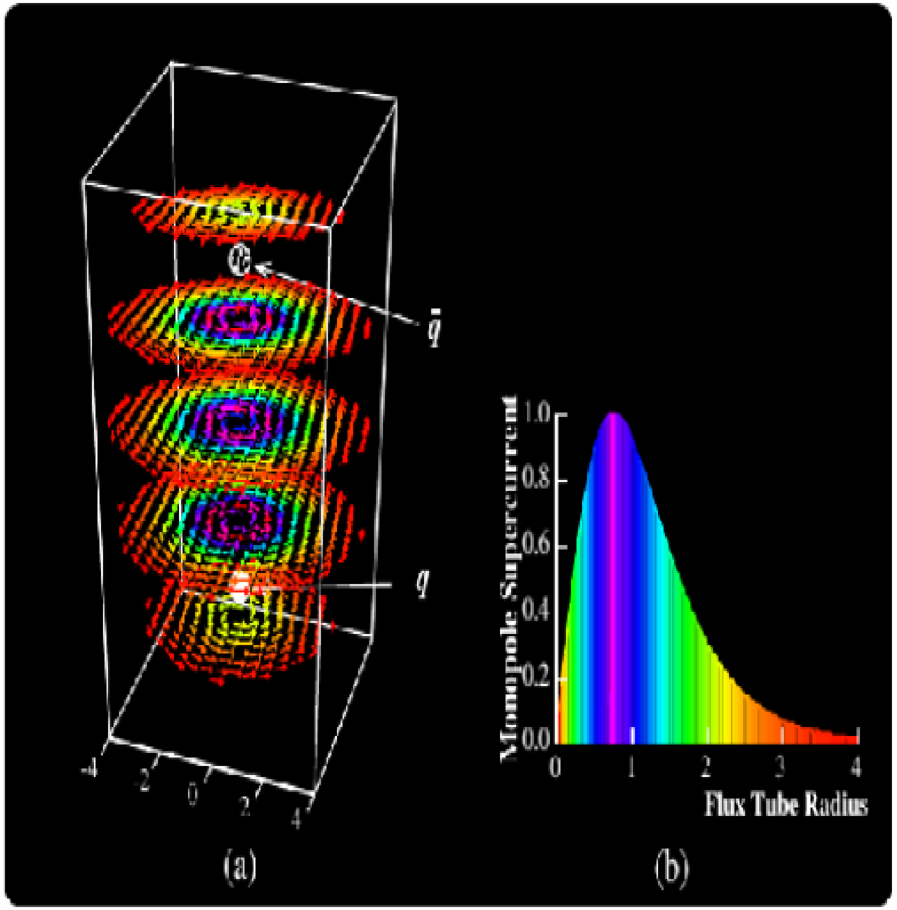
In the dual-superconductor model of confinement the flux tube appears due to condensation of chromomagnetic monopoles as the generalization of Abrikosov vortices of type II superconductors.

Vortex and string pictures have distinct features. The vortex picture predicts an exponential decrease of the field strength at the border of its core with a constant penetration length, while the EST predicts a Gaussian shape and a logarithmic growths of the width with increasing distance between static quarks.

The possibility is that the flux tube consists of a solid, vortex-like, inner core, while its long-distance dynamics is governed by the EST.

Thus, it is important to obtain precise lattice results for the flux tube profile.





profile of the color-electric field(left) and profile of the magnetic currents (right) in DLG .

Koma, 2001

Gubarev, Ilgenfritz, Polikarpov, Suzuki, 1999

By fitting SU(2) results for E_z and k_θ to classical solution of dual Abelian Higgs model they obtained: E_z

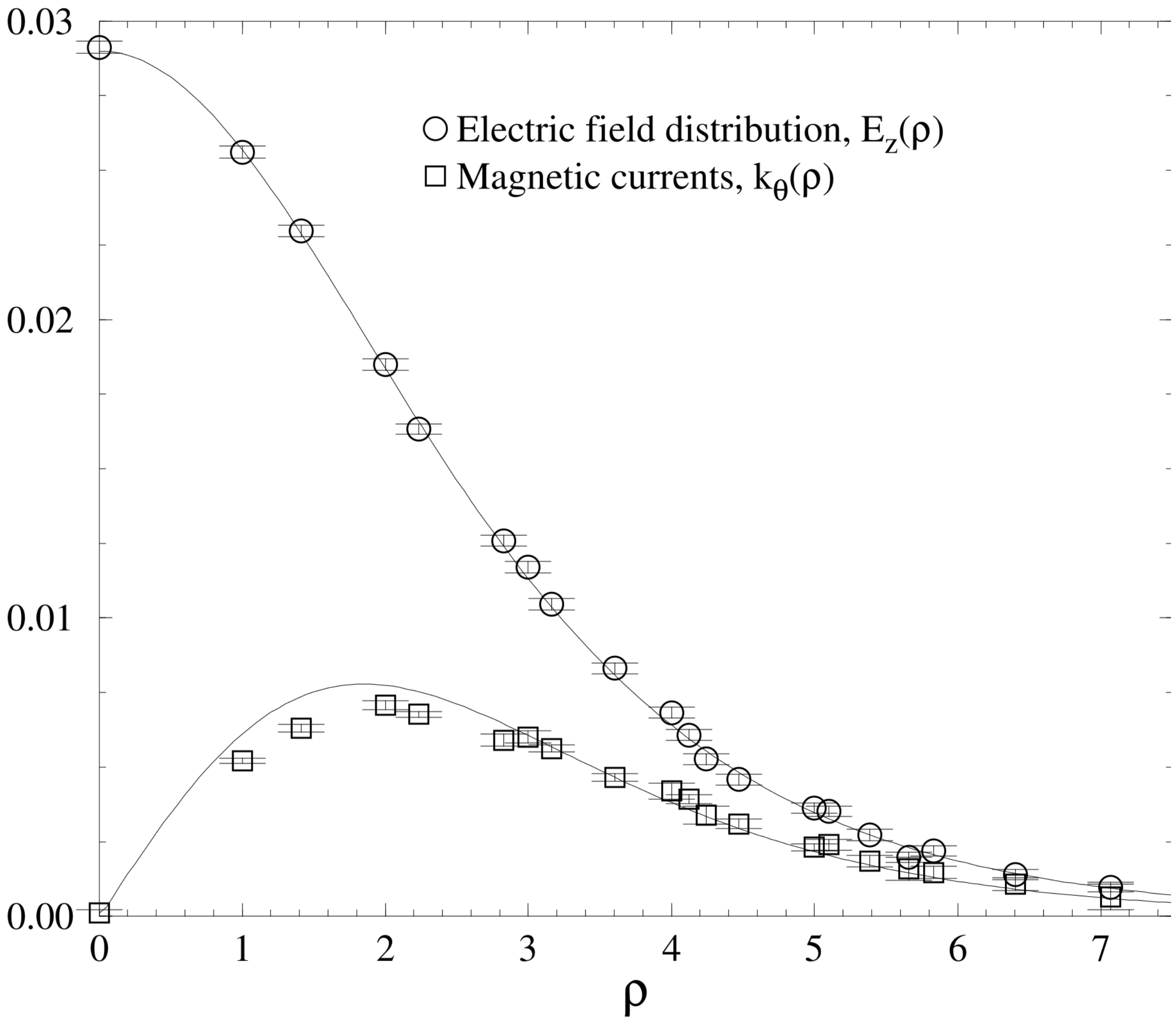
$m_v = 1.31(8)$ GeV

$m_H = 1.36(1)$ GeV

Thus $\lambda \approx \xi$,

border between type-I and

type-II superconductivity



Maximal Abelian gauge *('t Hooft, 1981)*

MA gauge condition

$$\left(\partial_\mu \delta_{kl} + \epsilon_{k3l} A^3_\mu(x)\right) A^l_\mu(x) = 0\,, \quad k = 1,2$$

solutions: extremums over gauge transformations of the functional

$$F[A] = \int d^4x \ \{ (A^1_\mu)^2 + (A^2_\mu)^2 \}$$

Abelian projection:

$$A^a_\mu T^a \to A^3_\mu T^3$$ (in observables)

Lattice formulation - by Kronfeld, Laursen, Schierholz, Wiese, 1989

Decomposition of the gauge field

- usual decomposition:

$$U_\mu(x) = C_\mu(x)u_\mu(x),$$

$$C_\mu(x) \ni SU(2)/U(1), \quad u_\mu(x) \ni U(1)$$

further decomposition of the Abelian field:

$$u_\mu(x) = u_\mu^{mon}(x)u_\mu^{ph}(x)$$

definition of magnetic currents: (DeGrand, Toussaint, 1980)

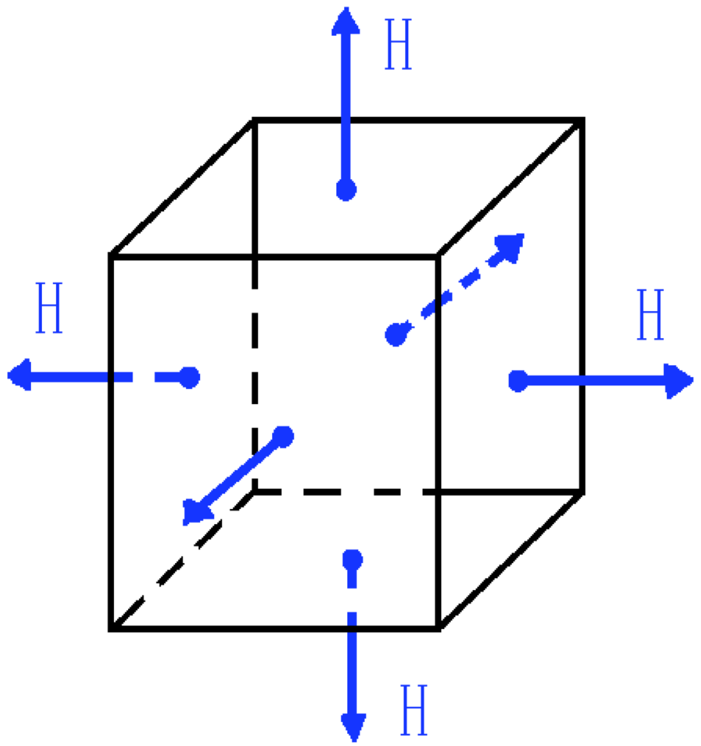
$$u_\mu(x) = e^{i\theta_\mu(x)}$$

$$\theta_{\mu\nu}(x) = \partial_\mu\theta_\nu(x) - \partial_\mu\theta_\nu(x)$$

physical flux:

$$\bar{\theta}_{\mu\nu}(x) = \theta_{\mu\nu}(x) + 2\pi\, m_{\mu\nu}(x)$$

$$-\pi \le \bar{\theta}_{\mu\nu}(x) < \pi\,,\, m_{\mu\nu}(x) = 0, \pm 1, \cdots$$



$$k_\mu(x^*) = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\partial_\nu\, m_{\alpha\beta}(x) = \frac{1}{4\pi}\varepsilon_{\mu\nu\alpha\beta}\partial_\nu\, \bar{\theta}_{\alpha\beta}(x)$$

Conservation law:

$$\sum_{\mu} \partial_\mu k_\mu(s^*) = 0$$

s^* — site on a dual lattice

Magnetic currents k_μ form closed loops,
these loops are combined into clusters

One can decompose the Abelian vector potential into monopole and photon parts

$$A_\mu^{mon}(x) = 2\pi \sum_{y,\nu} D(x-y)\partial_\nu m_{\mu\nu}(x)$$

$$A_\mu^{phot}(x) = A_\mu(x) - A_\mu^{mon}(x)$$

$$u_\mu^{mon}(x) = exp(iA_\mu^{mon}(x))$$

$$u_\mu^{ph}(x) = exp(iA_\mu^{ph}(x))$$

$$U_\mu^{mod}(x) = U_\mu(x)u_\mu^{mon,\dagger}(x)$$

U_μ^{mod} - nonabelian gauge field with monopoles removed (modified)

New decomposition:

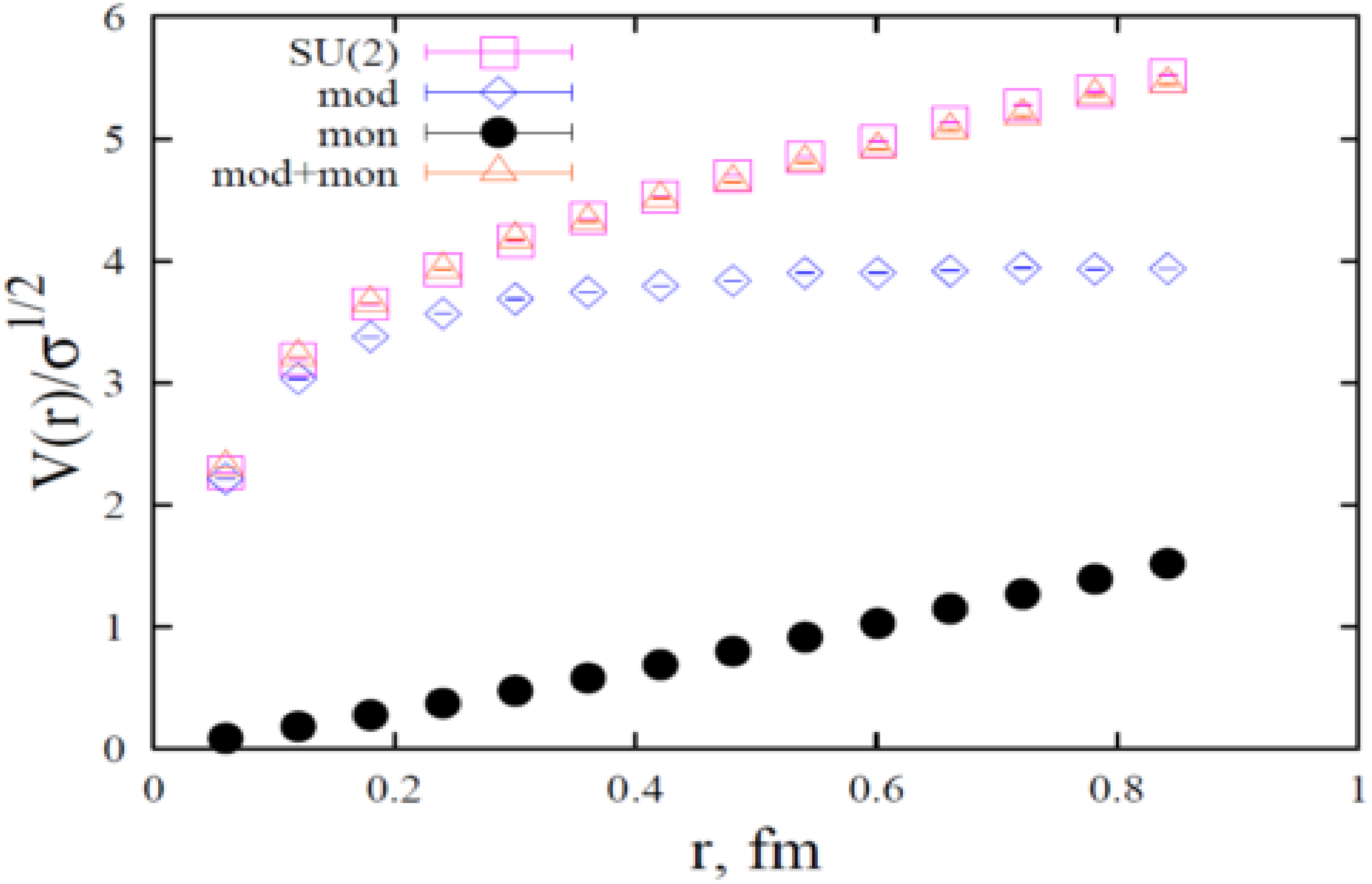
$$U_\mu(x) = U_\mu^{mod}(x) u_\mu^{mon}(x) \quad ,$$

One can study respective decomposition for physical observables, e.g.

$$V(r) \quad vs. \qquad V_{mon}(r) + V_{mod}(r)$$

$V^{mon} + V^{mod}$ approximates the nonabelian static potential with high accuracy at all distances.
SU(2) gluodanamics, $24^4, a = 0.08$fm

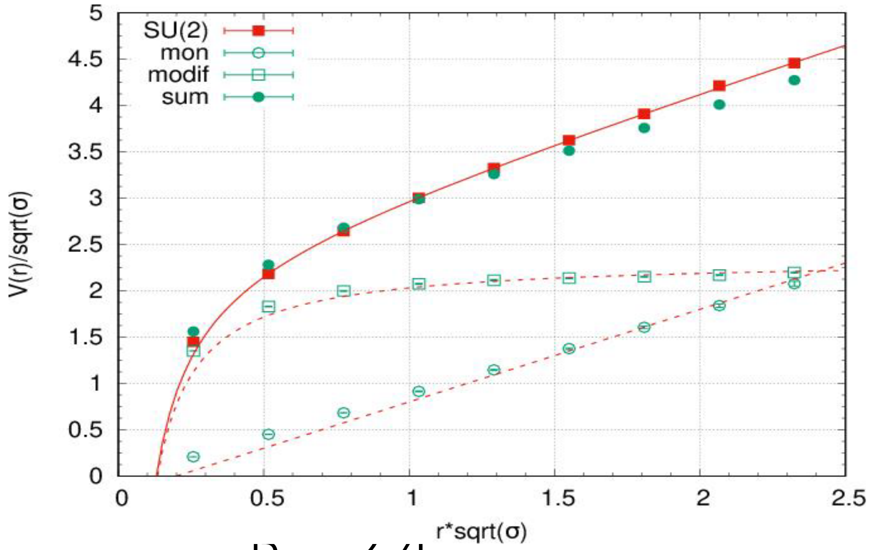
VB, Polikarpov, Schierholz, Suzuki, Syritsyn 2005



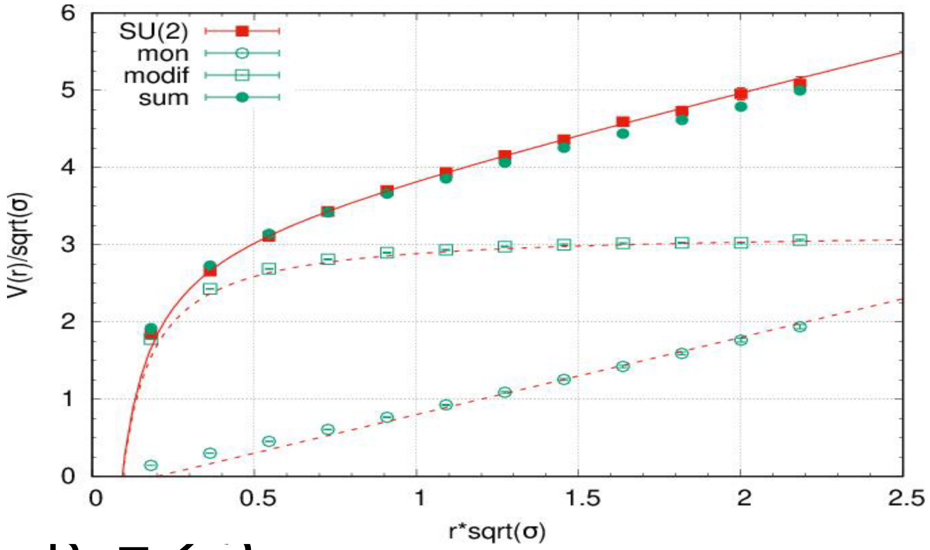
Recently we extended study of this decomposition:

- Continuum limit in SU(2) gluodynamics
- Universality
- SU(3) gluodynamics
- SU(2) QCD

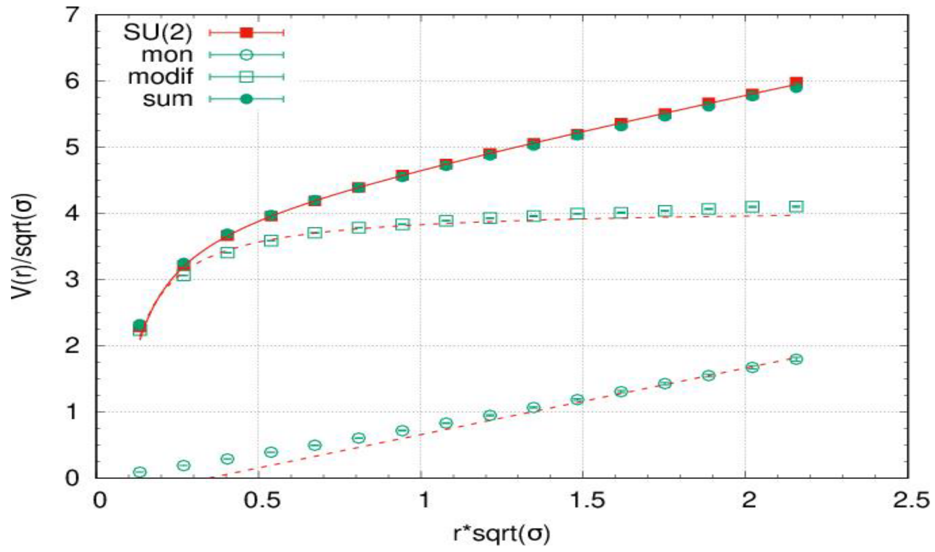
SU(2) gluodynamics with Wilson action (VB, Kudrov, Rogalyov)



β = 2.4



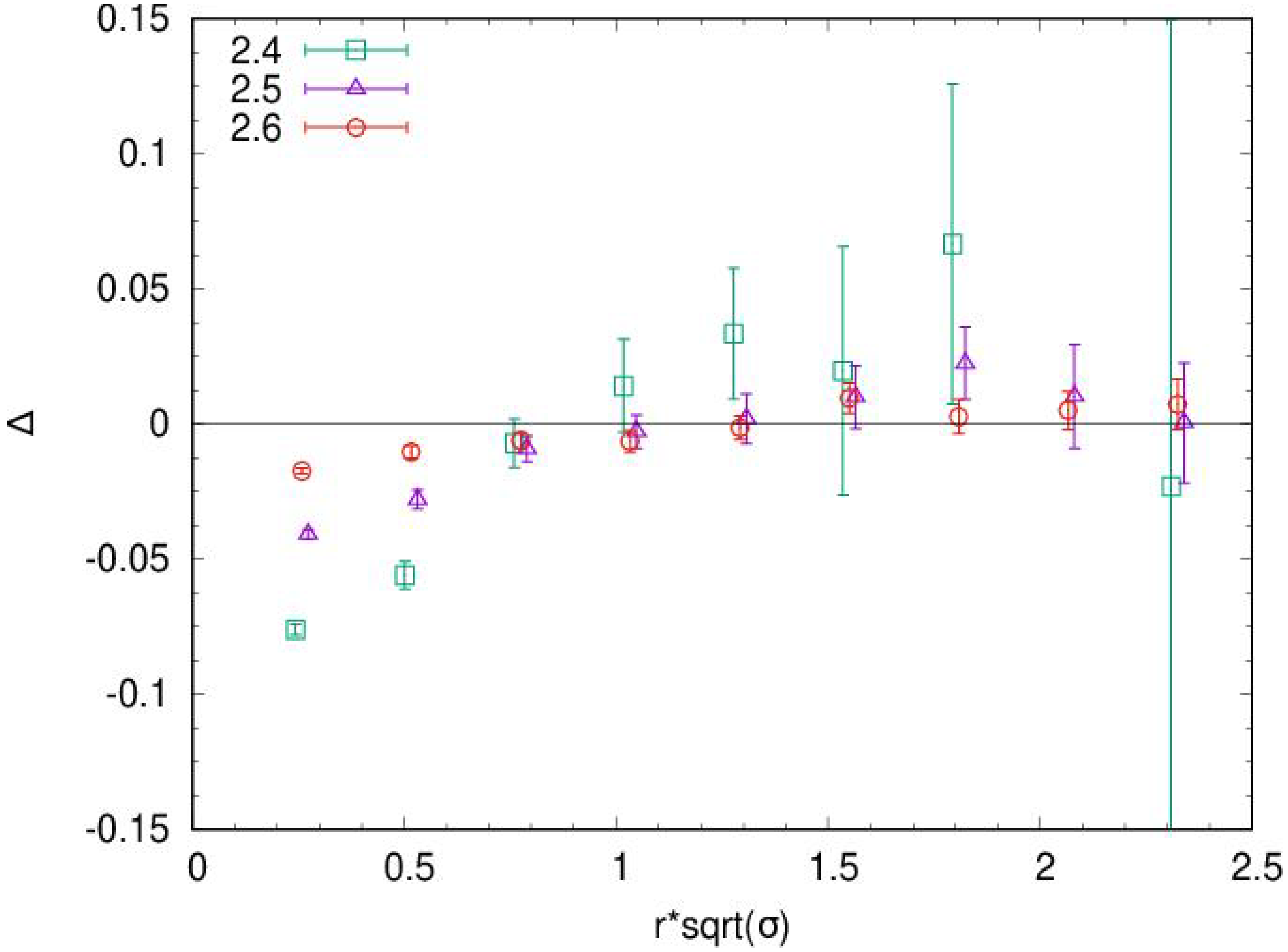
β = 2.5



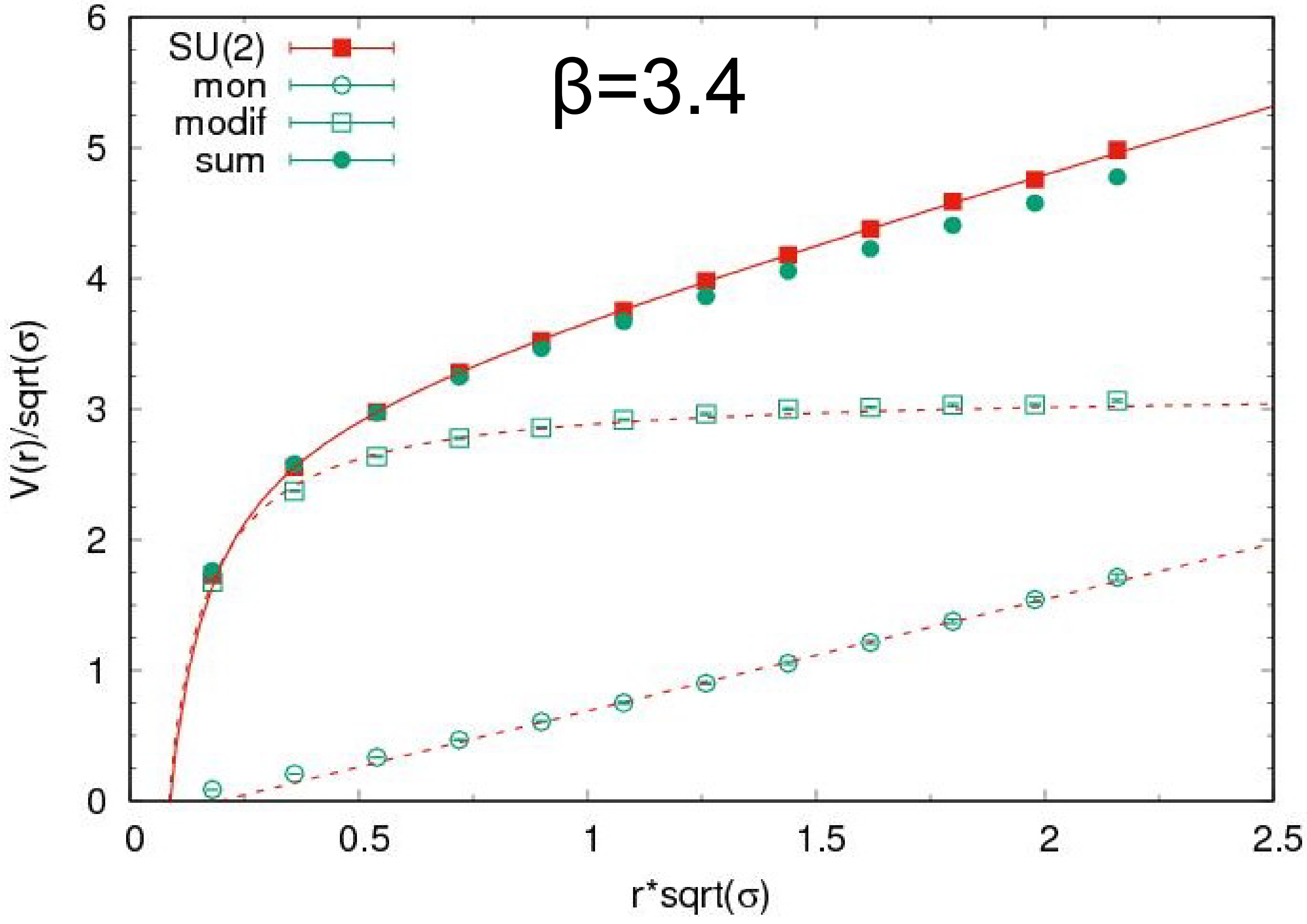
β = 2.6

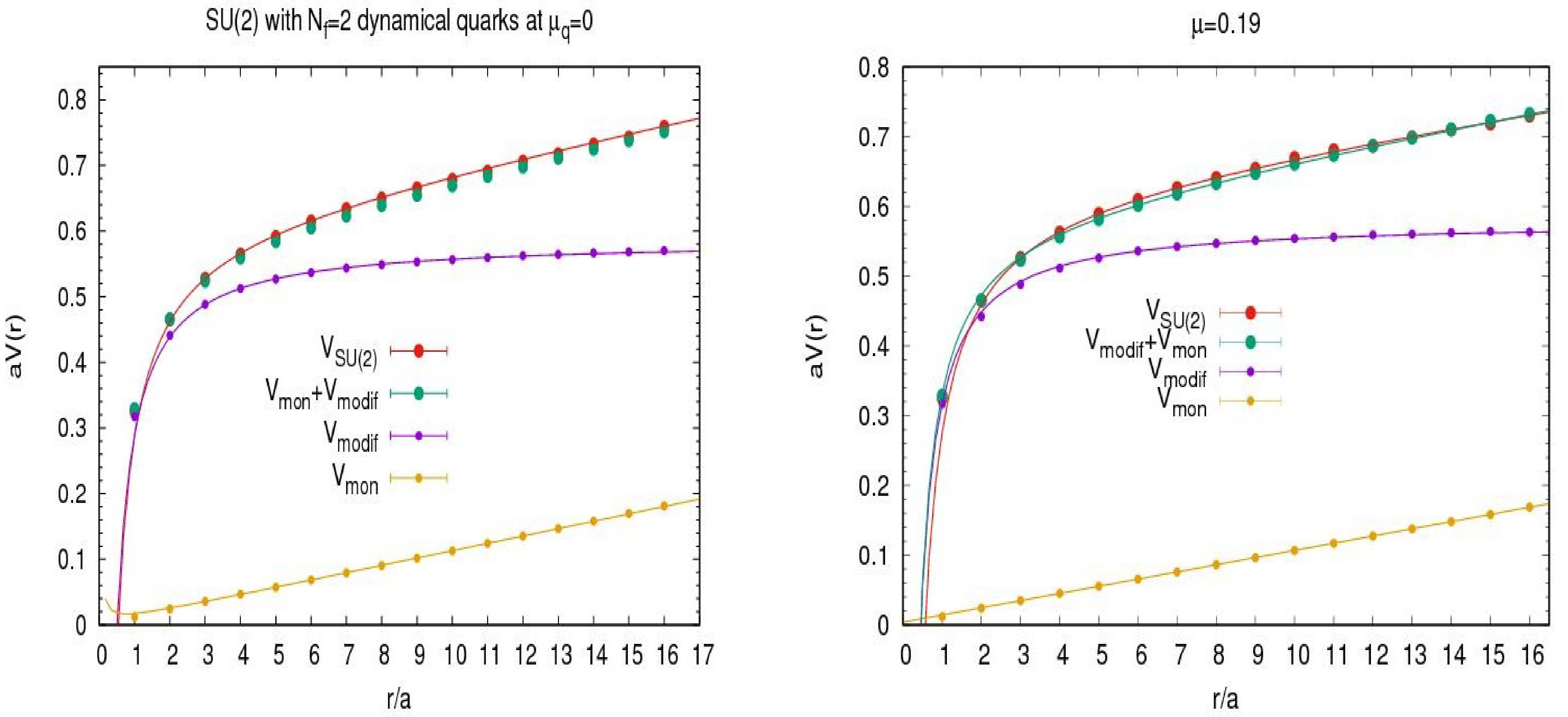
Relative deviation

$$\Delta(r) = \frac{V(r) - (V_{mon}(r) + V_{mod}(r))}{V(r)}$$



SU(2) gluodynamics with tadpole improved action
Universality





SU(2) with N_f=2 dynamical quarks at μ_q=0
aV(r)
r/a
$V_{SU(2)}$
$V_{mon}+V_{modif}$
V_{modif}
V_{mon}
μ=0.19
aV(r)
r/a
$V_{SU(2)}$
$V_{modif}+V_{mon}$
V_{modif}
V_{mon}

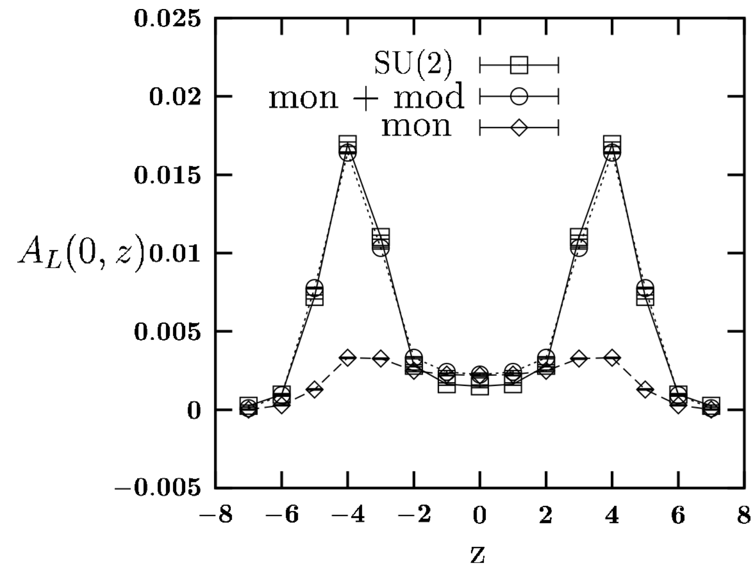
Implications:

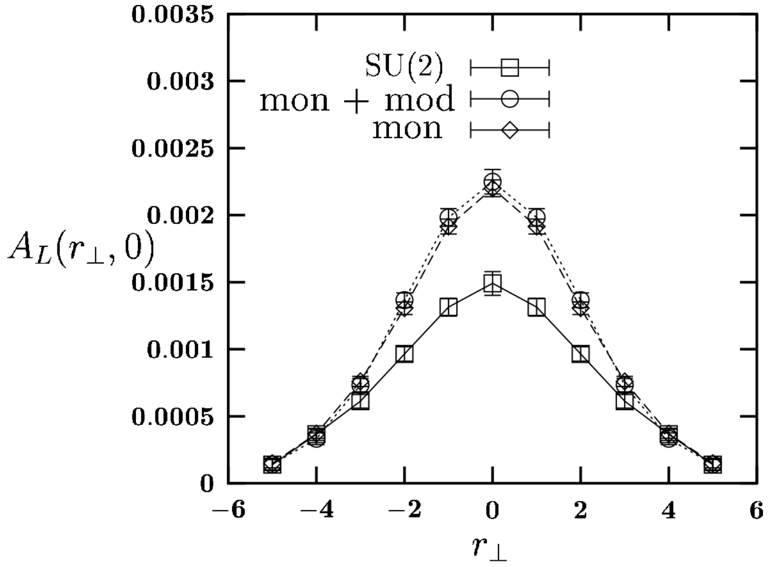
- Agreement at small distances implies that $A_\mu^{mod}(x)$ reproduces perturbative part of V(r)

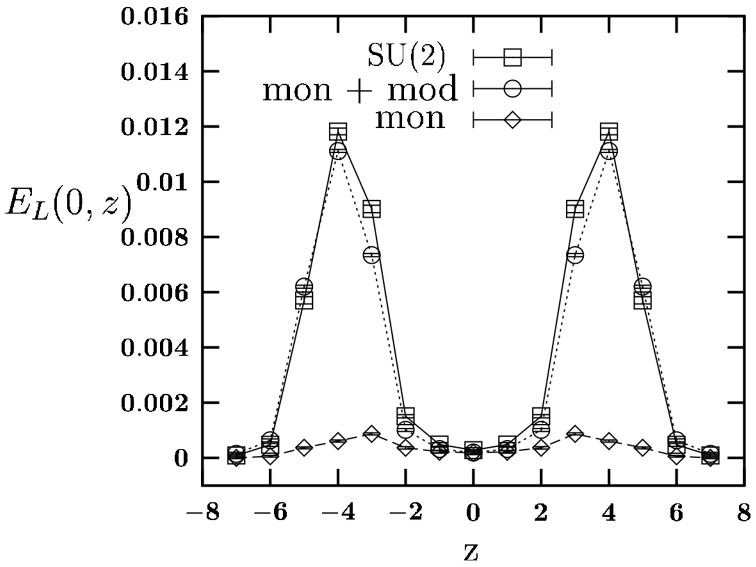
-The classical part of the hadron string action (or flux tube core) is described by $A_\mu^{mon}(x)$ while string vibrations (Luescher term) are described by $A_\mu^{mod}(x)$

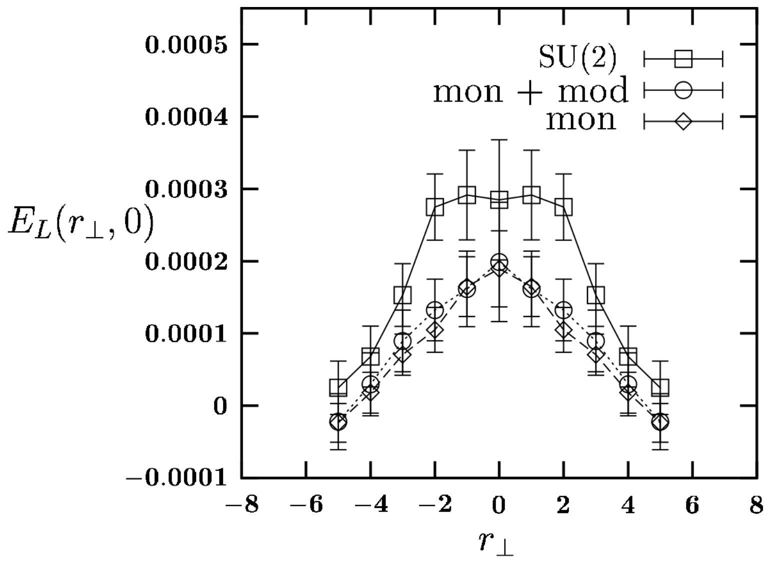
- These two components of $A_\mu(x)$ are not correlated – this should be demonstrated

Decomposition for action and energy density (from 2005 paper)

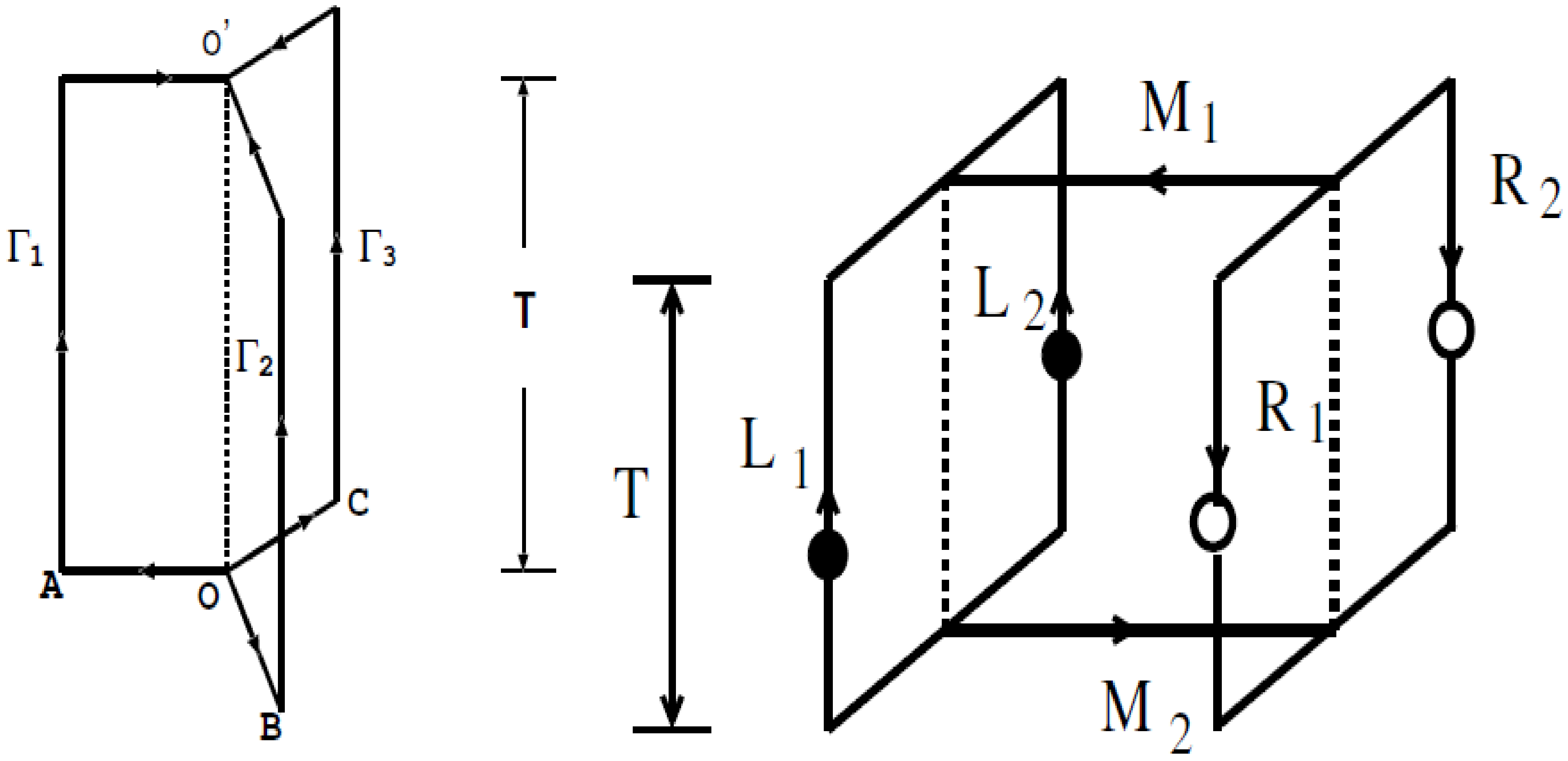








Multiquark systems



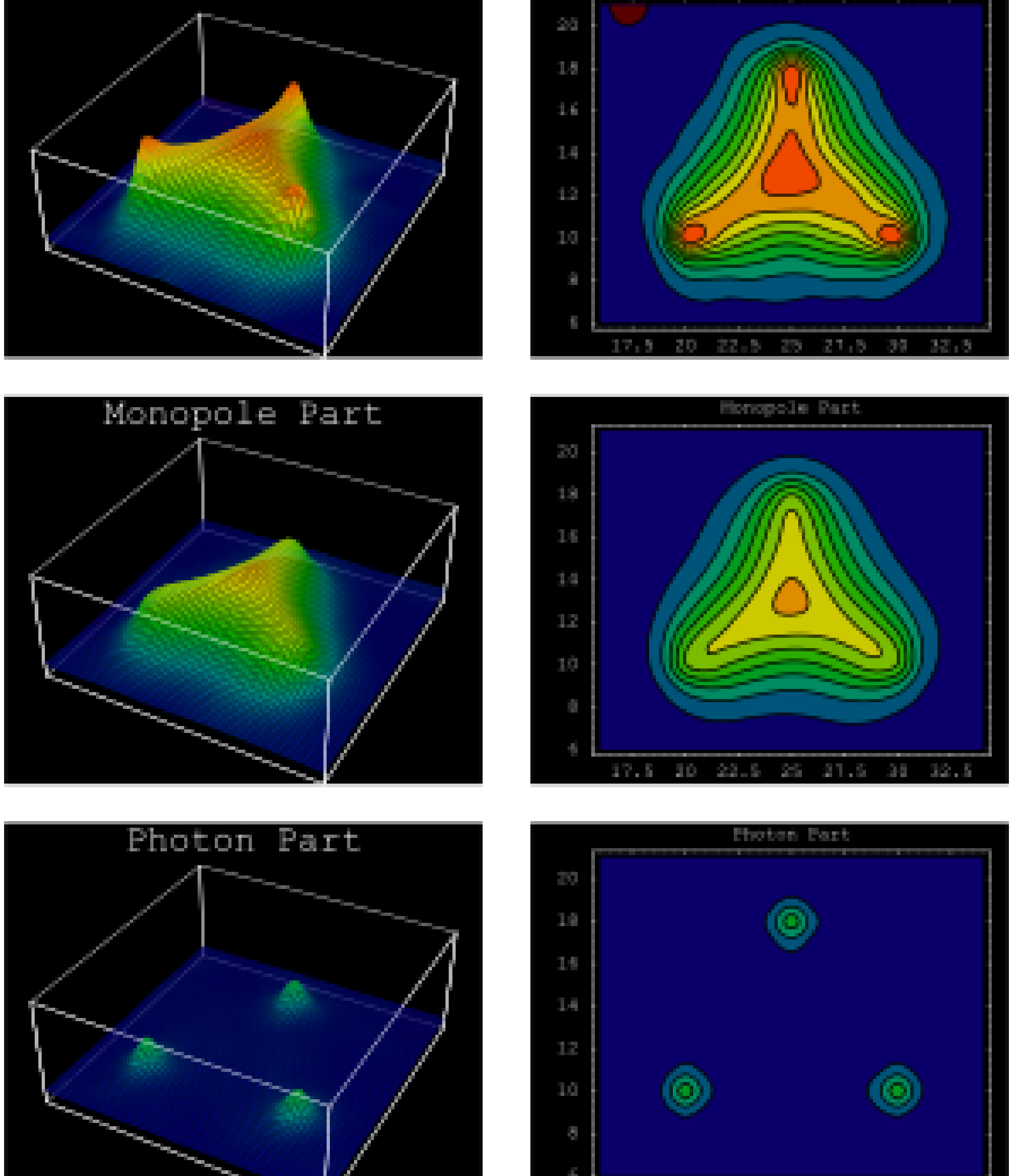
Wilson loops for static baryon (operator with one junction) and tetraquark (operator with two junctions) Bicudo, 2022

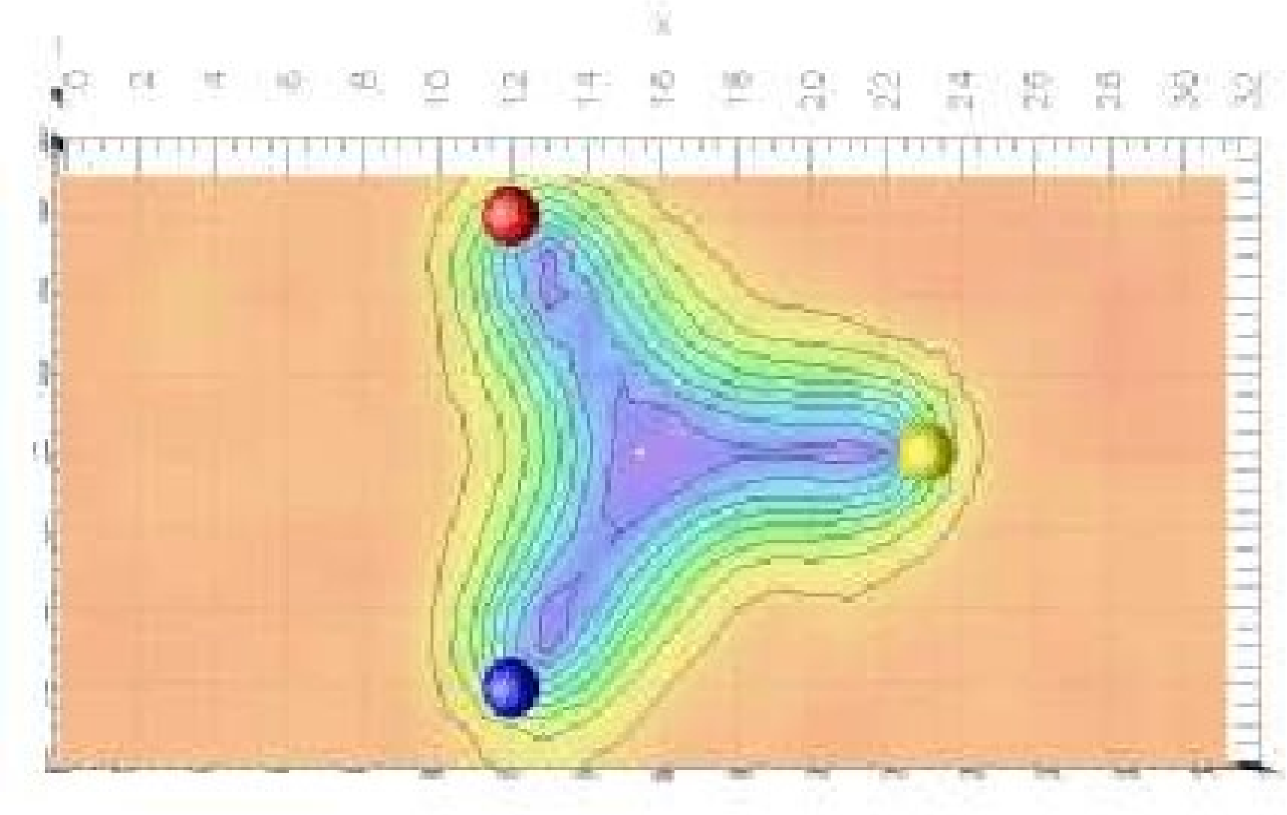
Abelian projected action density for 3Q configuration (top)

Its monopole component (middle)

and photon component (bottom)

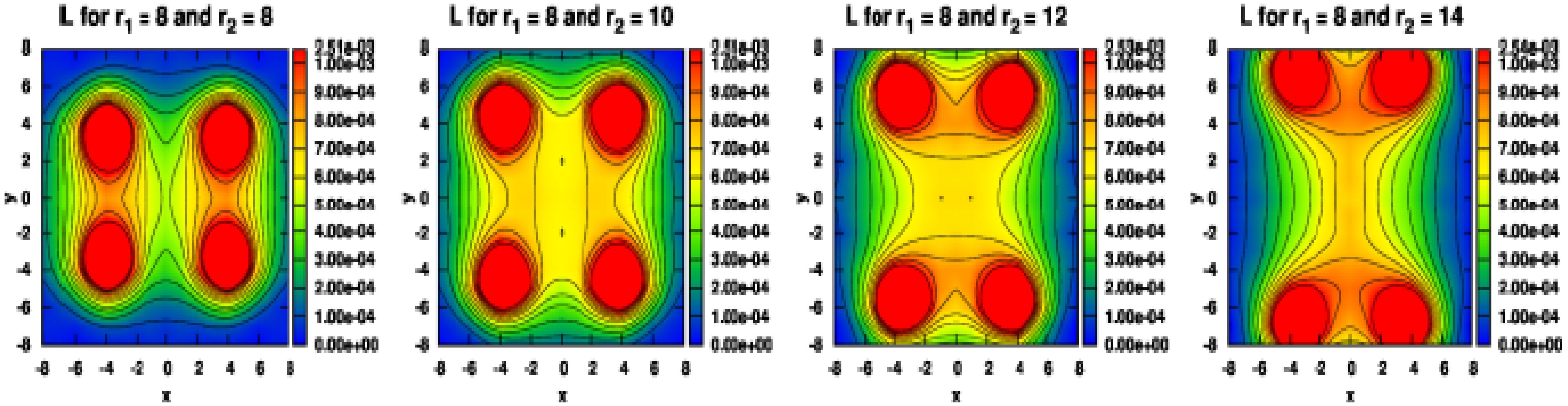
DIK collaboration, 2004





transition to flux-tube formation occurs when the distance of the quarks from the centre of the triangle is greater than 0.5 fm. The diameter of the flux tubes remains approximately constant as the quarks move to large separations

Leinweber, Williams et al., 2007



Evidence of flip-flop of the flux tube in the four static quarks system [151].

Depending on positions of the quarks and antiquarks, there are either flux tubes of two mesons (left most) or the flux tube of a diquark-antidiquark tetraquark (right most).

Cardoso, Cardoso, Bicudo, 2012

$$V_{Q\bar{Q}} = -\frac{\alpha_{Q\bar{Q}}}{r} + \sigma_{Q\bar{Q}} r + C_{Q\bar{Q}},$$

$$V_{3Q} = -\alpha_{3Q} \sum_{i<j} \frac{1}{|\mathrm{r}_i - \mathrm{r}_j|} + \sigma_{3Q} L_{\min} + C_{3Q}.$$

$Q\bar{Q}$ and QQQ potentials

$$V_{4Q} = -\alpha_{4Q}\{(\frac{1}{r_{12}} + \frac{1}{r_{34}}) + \frac{1}{2}(\frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} + \frac{1}{r_{24}})\} + \sigma_{4Q} L_{\min} + C_{4Q}$$

$QQ\,\bar{Q}\,\bar{Q}$ potential

$$V_{5Q} = -\alpha_{5Q}\{(\frac{1}{r_{12}} + \frac{1}{r_{34}}) + \frac{1}{2}(\frac{1}{r_{15}} + \frac{1}{r_{25}} + \frac{1}{r_{35}} + \frac{1}{r_{45}}) + \frac{1}{4}(\frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} + \frac{1}{r_{24}})\} + \sigma_{5Q} L_{\min} + C_{5Q},$$

$QQQQ\bar{Q}$ potential

$$\alpha_{Q\bar{Q}} \simeq 2\alpha_{3Q} \simeq 2\alpha_{4Q} \simeq 2\alpha_{5Q} \simeq 0.27$$

$$\sigma_{Q\bar{Q}} \simeq \sigma_{3Q} \simeq \sigma_{4Q} \simeq \sigma_{5Q}$$