Kerr black holes and massive higher spins

From Quarks to Galaxies: Elucidating Dark Sides

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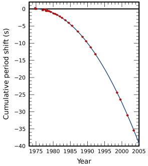
FREEDOM TO RESEARCH

Main Messages/Plan

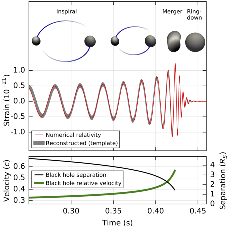
- The era of the Gravitational waves physics, challenge for developing theoretical tools for future detectors
- Effective field theory together with advanced QFT/Amplitude tools entered the competition recently
- Kerr Black Holes should correspond to the simplest theory of massive higher-spin fields interacting with gravity
- Old problem: how to make fields with spin $s > 2$ interact ...
- Recent ideas massive higher-spin symmetry (Zinoviev) and chiral approach (Ochirov, E.S.) allowed to advance (Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, E.S)

General Relativity in few lines

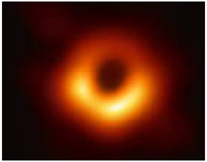
- Born in 1915, Perihelion precession of Mercury, gravitational waves predicted in 1916



- Evidence in 1974 from Hulse-Taylor pulsar;
- Many other tests, including GPS
- Detection of gravitational waves by LIGO/Virgo in 2016 **just 100 years delay**



- 'Photo' of a black hole by Event Horizon in 2017



Challenge: to develop theoretical tools to solve GR for two (and more) compact objects orbiting/scattering each other and emitting gravitational waves.

It is surprisingly hard to solve Einstein equations for two compact objects orbiting each other and emitting gravitational waves

Apart from numerically solving Einstein's equations, one can try post-Newtonian approximation (PN); gravitational self-force; effective one body (Buonanno, Damour); post-Minkowskian approximation (PM); non-relativistic general relativity formalism; ...

True numerics is needed at the last stage only

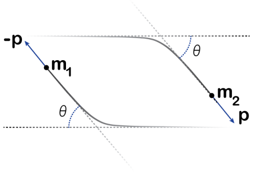
From Einstein to Amplitudes

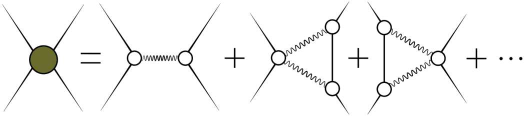
Amplitude methods have come into the game recently (Bjerrum-Bohr,...; Bern, Zeng,...; Cristofoli,...; Di Vecchia,...; Guevara,...; Maybee,...; Moynihan,...; Parra-Martinez,...; Plefka,...; Vines,...;...)

Everything compact and rotating from a distance looks like (and can be modeled as) a higher-spin massive particle

Therefore, instead of solving Einstein equations one can look for a theory where a massive spin-s field interacts with gravity.

Kerr black hole is 'very simple'





Physics of Spin

What can we see in (gedanken) "colliders"?

Apart from a small chance of producing a black hole that swallows the Earth and 'man eating dragons' ©

optimistic options are constrained by Special Relativity and Quantum Mechanics

(Wigner, 1939) explained that Quantum Mechanics (Hilbert space, superposition, ...) together with Special Relativity imply that particles = unitary irreducible representations of Poincare group $ISO(1,3)$

What can we see in (gedanken) "colliders"?

Massless. in $4d$ they can have $s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots$ and carry two degrees of freedom $\pm s$ for $s > 0$ (physical polarizations). Light (photon) is $s = 1$, graviton is $s = 2$. **Mediators of long-range forces: electromagnetism and gravity**

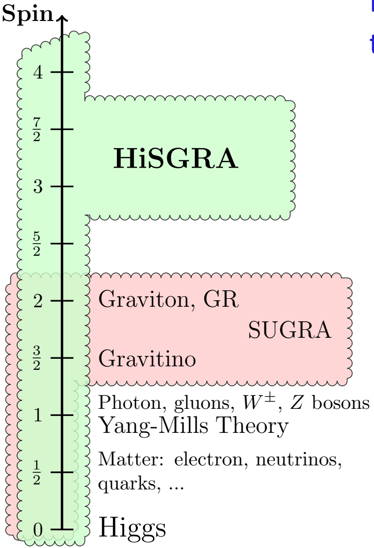
Massive. in $4d$: the same range of s and carry $2s+1$ degrees of freedom, travel slower than light. Masses via Higgs?

Exotic cases. Continuous spin: ∞-many degrees of freedom per point; Tachyons, $m^2 < 0$

Any updates since 1939? $\rightarrow$ mixed-symmetry fields in $d > 4$ and partially-massless fields when $\Lambda \neq 0$

These are free options ... what are possible interactions?

Spin by spin



Different spins lead to very different types of theories/physics:

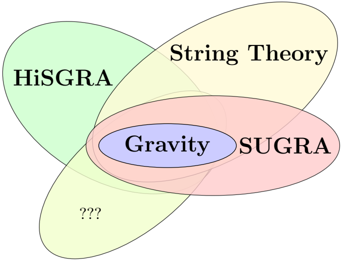
- $s = 0$: Higgs
- $s = 1/2$: Matter

- $s = 1$: Yang-Mills, Lie algebras
- $s = 3/2$: SUGRA and supergeometry, graviton $\in$ spectrum
- $s = 2$ (graviton): GR and Riemann Geometry, no color

- $s > 2$: HiSGRA and String theory, ∞ states, graviton is there too!

Various examples (not all)

- string theory
- divergences in (SU)GRA's
- Quantum Gravity via AdS/CFT



seem to indicate that quantization of gravity requires

- infinitely many fields
- higher spin particles must be part of the spectrum

On a different note: everything from far away is massive, rotating and maybe charged, i.e. is a massive higher-spin particle

Massive Higher Spins

Where? Baryons known with $S \leq 15/2$; nuclei, say, Tantalum has $S = 9$; string's spectrum is full of higher spins ... now Black holes

Massive higher spins are complicated: 2nd-class constraints, Boulware-Deser ghosts, even free actions are not easy (Singh, Hagen; Zinoviev)

$$(\Box - m^2)\Phi_{\mu_1\dots\mu_s} = 0 \qquad\qquad \boldsymbol{\partial^\nu \Phi_{\nu\mu_2\dots\mu_s} = 0}$$

Low spins: $\boldsymbol{s = 1}$ **spontaneously broken Yang-Mills**; $\boldsymbol{s = 3/2}$; $s = 2$ massive (bi)-gravity (dRGT; Hassan, Rosen); $\boldsymbol{s = 5/2}$ (Chiodaroli, Johansson, Pichini)

Interactions:
- Zinoviev's massive gauge symmetry;
- Chiral approach (Ochirov, E.S.);
- Light-cone (Metsaev); Covariant (Buchbinder, ...)

Even at the cubic level there are many $s - s - h^{\pm}$ amplitudes. (Arkani-Hamed, Huang[2]) selected a family with the best high energy behavior

$$\mathcal{A}(s,s,h^+) \sim \mathcal{A}(0,0,h^+)\langle \mathbf{12}\rangle^{2s}$$
$$\mathcal{A}(s,s,h^-) \sim \mathcal{A}(0,0,h^-)[\mathbf{12}]^{2s}$$

where $\mathcal{A}(0,0,h^{\pm})$ is the scalar amplitude for $h = \pm 1, \pm 2$. For $s = 0, \frac{1}{2}, 1$ these are as in "Standard model".

(Guevara, Ochirov, Vines) showed that these amplitude reproduce the classical behavior of a rotating Kerr black hole for $h = \pm 2$.

The dictionary is "roughly" $s \to \infty$ for $a = s\hbar$, $\hbar \to 0$.

Double-copy construction (Bern, Carrasco, Johansson) allows us to think of $\sqrt{\text{Kerr}}$, where $h = \pm 1$.

We can get an EM interaction of massive spin-one via Higgs

$$\mathcal{L} = -\tfrac{1}{2}|D_{[\mu}W_{\nu]}|^2 + |mW_\mu|^2 - iQ\boldsymbol{\overline{W}_\mu F^{\mu\nu} W_\nu} + \ldots$$

for $SO(3)$, where one boson A_μ remains massless, $D = \partial - iQA$.

Instead we can start from the gauge invariant free action

$$\mathcal{L}_2 = -\frac{1}{2}|\partial_{[\mu}W_{\nu]}|^2 + |mW_\mu - \partial_\mu \varphi|^2\,,$$

that is invariant under $\delta W_\mu = \partial_\mu \xi$, $\delta\phi = m\xi$ (Stueckelberg).

Upon $\partial \to D$ we find that the gauge symmetry is lost, but it gets restored by adding the **non-minimal term**. $\delta W_\mu = D_\mu \xi$, $\delta\phi = m\xi$. This gives $\mathcal{A}(1,1,h)$!

Instead of the tedious Hamiltonian analysis of (second class) constraints to make sure that interactions of $\Phi_{\mu_1\dots\mu_s}$ do not activate unphysical degrees of freedom, one can convert all second class to first class. The latter can be taken care of by gauge symmetries:

$$\text{massive spin-}s = \bigoplus_{k=0}^{k=s} \text{massless spin-}k$$

which leads to

$$\delta\Phi_k = \partial\xi_{k-1} + \boldsymbol{\xi_k} + \boldsymbol{g_{..}\xi_{k-2}} + \mathcal{O}(\Phi\xi)$$

For the steps along the Zinoviev approach see 'Kerr Black Holes Enjoy Massive Higher-Spin Gauge Symmetry', Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, E.S.; arxiv: 2212.06120 & 2311.14668

We need to write down the most general ansatz for interactions and gauge transformations ... lots of coefficients

An equivalent approach is to use Ward identities for the massive higher-spin gauge symmetry ... which is closer to amplitudes

There is a unique solution with the lowest number of derivatives extending the "minimal coupling" $\partial \to \partial - iQA$

There is a simple generating function for on-shell cubic vertices, e.g.

$$\tilde{\mathcal{L}}^{(1)} = -\,\bar{\Phi}_{\mu} F^{\mu\nu} \Phi_{\nu}\,,$$

$$\tilde{\mathcal{L}}^{(2)} = \bar{\Phi}_{\mu\nu} F^{\nu}{}_{\rho} \Phi^{\rho\mu} + \frac{4}{m^2} \overline{D_{[\mu} \Phi_{\nu]\rho}} F^{\rho\sigma} D^{[\mu} \Phi^{\nu]}{}_{\sigma} + \frac{4}{m^2} \overline{D_{[\mu} \Phi_{\nu]\rho}} F^{\nu}{}_{\sigma} D^{[\sigma} \Phi^{\mu]\rho}\,,$$

The quartic analysis is more complicated ...

Chiral approach

The problem is to embed physical degrees of freedom into a Lorentz covariant field

$$\text{degrees of freedom} \rightsquigarrow \text{Lorentz (spin-)tensor}\,, \Phi_{A_1\ldots A_n,A'_1\ldots A'_m}$$

In principle, any $\Phi_{A(n),A'(m)}$ of $sl(2,\mathbb{C})$ with $n+m=2s$ is good enough and

$$\Phi_{\mu_1\ldots\mu_s} \sim \Phi_{A(s),A'(s)} \sim (s,s)$$

Simple idea in $4d$ (Ochirov, E.S.): let's take

$$\Phi_{A_1\ldots A_{2s}} \sim (2s,0)$$

It does not have any longitudinal unphysical modes.

No auxiliary fields are needed. Parity is not easy ... but everything is a consistent interaction

Chiral approach: Massive low spins

Chalmers and Siegel, '97-98, mapped "Standard model" to its chiral version, i.e. "chiralized" massive $s = 1/2, 1$

For $s = 1/2$ we just integrate out half of the Majorana spinor $\psi_A\,, \psi_{A'}$:

$$\mathcal{L}_M = i\psi^A \nabla_{AA'} \psi^{A'} + \tfrac{1}{2} m(\psi_A \psi^A + \psi_{A'} \psi^{A'})$$

to get some **nonminimal interactions** as well

$$\mathcal{L} = \psi^A (\Box - m^2) \psi_A + \psi^A F_{AB} \psi^B + R \psi^A \psi_A$$

where F_{AB} is the self-dual part of the (non)-abelian gauge background field

$$F_{\mu\nu} = F_{AB} \epsilon_{A'B'} + \epsilon_{AB} F_{A'B'} = F_- + F_+$$

Wanted: Black hole Lagragian,

$$\mathcal{L}_{\text{BH}} = \Phi_{A(2s)}(\Box - m^2)\Phi^{A(2s)} + \mathcal{L}_{\text{non-min}}$$

i.e. the simplest parity-invariant theory that couples HS to photons/gluons ($\sqrt{\textbf{Kerr}}$) or to gravitons (**Kerr**) that starts with (Arkani-Hamed, Huang^2; Guevara, Ochirov, Vines; Chung, Huang, Kim, Lee)

$$\mathcal{A}(s,s,+) \sim \langle \mathbf{1}\,\mathbf{2}\rangle^{2s}\mathcal{A}(0,0,+) \quad \mathcal{A}(s,s,-) \sim [\mathbf{1}\,\mathbf{2}]^{2s}\mathcal{A}(0,0,-)$$

How far away are we?

Wanted: Black hole Lagragian,

$$\mathcal{L}_{\text{BH}} = \langle\Phi|(\Box - m^2)|\Phi\rangle + \mathcal{L}_{\text{non-min}}$$

that starts with (Arkani-Hamed, Huang²; Guevara, Ochirov, Vines; Chung, Huang, Kim, Lee)

$$\mathcal{A}(s,s,+) \sim \langle \mathbf{1}\,\mathbf{2}\rangle^{2s}\mathcal{A}(0,0,+) \quad \mathcal{A}(s,s,-) \sim [\mathbf{1}\,\mathbf{2}]^{2s}\mathcal{A}(0,0,-)$$

The minimal coupling correctly reproduces all-plus amplitudes obtained from unitarity (Aoude, Haddad, Helset; Lazopoulos, Ochirov, Shi):

$$\mathcal{A}(s,s,+,\ldots,+) \sim \langle \mathbf{1}\,\mathbf{2}\rangle^{2s}\mathcal{A}(0,0,+,\ldots,+)$$

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To restore parity at the cubic level we need

$$\mathcal{L}_{\text{non-min}} \sim \sum_{k=0}^{2s-1} \frac{1 \text{ or } (2s-k-1)}{m^{2k}} \langle\Phi|\left(\overleftarrow{D}\overrightarrow{D}\right)^k F_- \text{ or } R_-|\Phi\rangle$$

Synergy

Usually, analysis at the n-th order leaves some coefs free that can be fixed at higher orders. Therefore, at the quartic order we have some ambiguities to be fixed in some way.

We used some additional assumptions, e.g. parity, unitarity, classical limit $s \to \infty$, massive higher-spin gauge symmetry, certain already known low order terms, ... to land on a Compton amplitude $A(\mathbf{1}^s, \bar{\mathbf{2}}^s, 3^-, 4^+)$

Inverting the statement, this also gives us a theory of a single massive higher-spin field interacting with photons/gluons up to the quartic order.

Summary

- GW is a new window to the secrets of the Universe
- Future: LISA, Einstein; Black-hole, neutron star systems, ...
- Precision calculation are in demand
- Massive higher spins have direct applications to Gravitational Wave physics: we need the simplest theory of this sort to model Kerr black holes and a bit more complicated to model other objects
- With the help of massive higher spin symmetry and chiral approach we were able to find a quartic (Compton) amplitude that passess all the tests
- A complete theory of Black holes is yet to be constructed... should look like a string theory provided mass- and spin-changing process are included...

Thank you for your attention!

… backup slides …

The up to the quartic order action for $\sqrt{Kerr}$ black hole has a relatively simple form

$$\mathcal{L} = \langle D_\mu \Phi | D^\mu \Phi \rangle - m^2 \langle \Phi | \Phi \rangle + \sum_{k=0}^{2s-1} \frac{ig}{m^{2k}} \langle \Phi | \Big\{ |\overleftarrow{D}|\overrightarrow{D}|^{\odot k} \odot |F_-| \Big\} | \Phi \rangle + \mathcal{O}(|F_-|^2)$$

$$- \sum_{k \le l = 0}^{2s-4} \sum_{j=0}^{2s-3-l} \frac{g^2}{m^{2(j+l)+6}} \times$$

$$\times \langle \Phi | \Big\{ \big(|\overleftarrow{D}|\overrightarrow{D}| + m^2 \big) \odot |\overleftarrow{D}|\overrightarrow{D}|^{\odot j} \odot |\overleftarrow{D}|\overrightarrow{D}_+|^{\odot k} \odot |\overleftarrow{D}_+|\overrightarrow{D}|^{\odot (l-k)} \odot \mathfrak{F}_6 \Big\} | \Phi \rangle$$

where

$$\mathfrak{F}_6 = \frac{1}{4} \{ T^c, T^{c'} \} |F_-^c| \odot |\overleftarrow{D}| F_+^{c'} |\overrightarrow{D}|$$

Quartic (Compton) amplitude

The associated quartic amplitude that does not have any spurious poles is

$$A(\mathbf{1}^s, \bar{\mathbf{2}}^s, 3^-, 4^+) = \frac{\langle 3|1|4]^2 (U+V)^{2s}}{m^{4s} t_{13} t_{14}} - \frac{\langle \mathbf{1}3\rangle \langle 3|1|4][4\mathbf{2}]}{m^{4s} t_{13}} P_2^{(2s)} + \frac{\langle \mathbf{1}3\rangle \langle 3\mathbf{2}\rangle [\mathbf{1}4][4\mathbf{2}]}{m^{4s}} P_2^{(2s-1)} - \frac{\langle \mathbf{1}3\rangle \langle 3\mathbf{2}\rangle [\mathbf{1}4][4\mathbf{2}]}{m^{4s-2}} \langle \mathbf{12}\rangle [\mathbf{12}] P_4^{(2s-1)} + C^{(s)} \,.$$

where

$$U = \frac{1}{2}\big(\langle \mathbf{1}|4|\mathbf{2}] - \langle \mathbf{2}|4|\mathbf{1}]\big) - m[\mathbf{12}] \,, \qquad V = \frac{1}{2}\big(\langle \mathbf{1}|4|\mathbf{2}] + \langle \mathbf{2}|4|\mathbf{1}]\big) \,.$$

and P are symmetric homogeneous polynomials in certain variables