Black holes with electroweak hair

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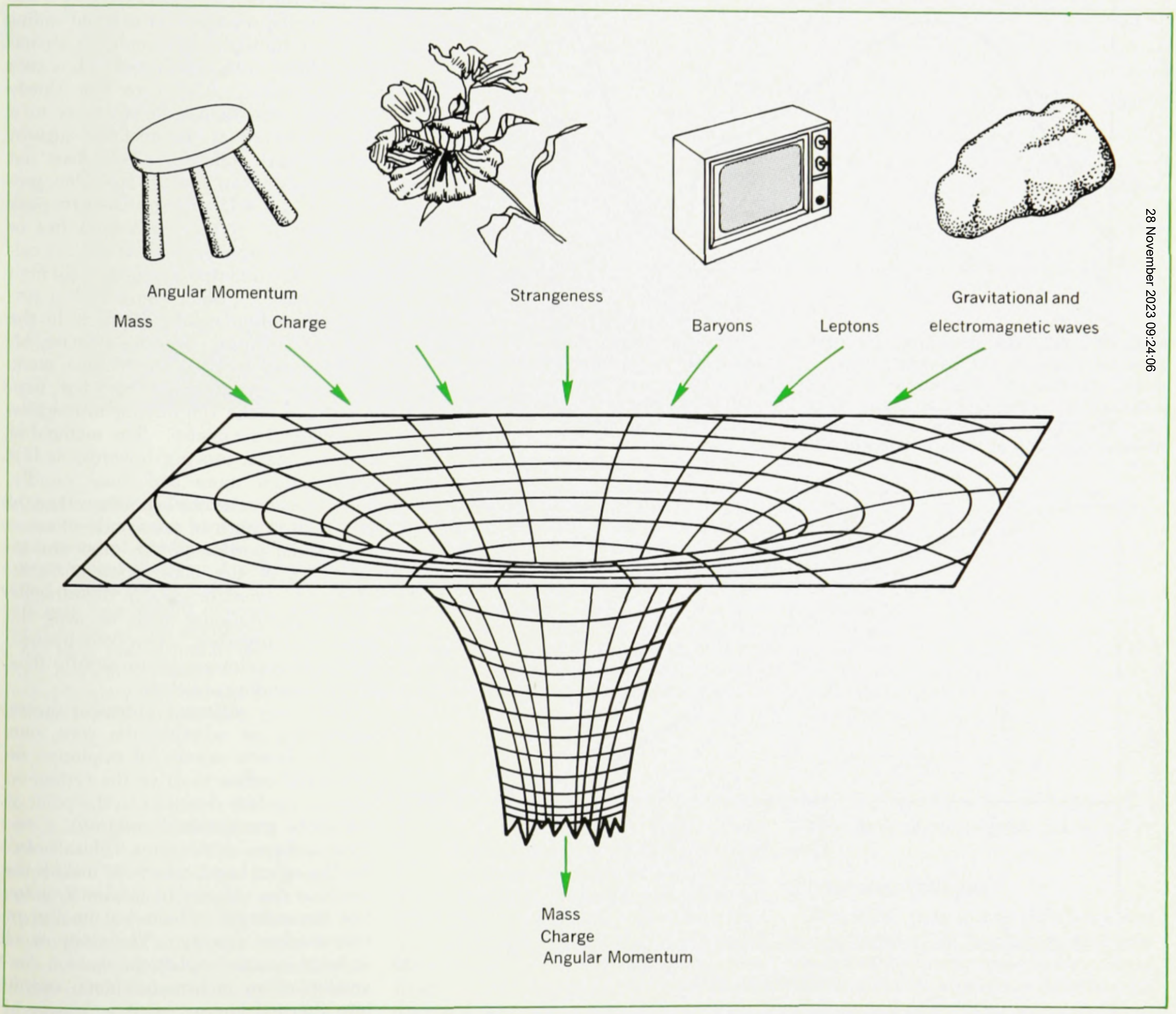
Brief history of hairy black holes

- No-hair conjecture /Ruffini and Wheeler, 1969/: black holes formed by gravitational collapse are characterized by their mass, angular momentum, and electric charge = the only parameters that can survive the collapse $\Rightarrow$ all black holes are described by the Kerr-Newman metrics.

- No-hair theorems /Bekenstein, 1972,.../ confirm the conjecture for a number of special cases. No new black holes holes for gravitating massive scalar, spinor, of vector fields, also for a scalar field with a positive potential, etc.

- First explicit counter-example /M.S.V.+ Gal'tsov, 1989/: static black holes with Yang-Mills hair. Triggered an avalanche of discoveries of other hairy black holes.



Figurative representation of a black hole in action. All details of the infalling matter are washed out. The final configuration is believed to be uniquely determined by mass, electric charge, and angular momentum. Figure 1

Black holes with Yang-Millas hair

Non-Abelian Einstein-Yang–Mills black holes

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Solutions of the self-consistent system of Einstein-Yang–Mills equations with the SU(2) group are derived to describe black holes with a non-Abelian structure of gauge fields in the external region.

In the case of the electrovacuum, the most general family of solutions describing spherically symmetric black holes is the two-parameter Reissner–Nordström family, which is characterized by a mass M and an electric charge Q. It was recently shown for the Einstein-Yang–Mills systems of equations with the SU(2) group that a corresponding assertion holds when the hold has a nonvanishing color-magnetic charge. In this case the structure of the Yang–Mills hair is effectively Abelian.[1] In the present letter we numerically construct a family of definitely non-Abelian solutions for Einstein-Yang–Mills black holes in the case of zero magnetic charge. These solutions are characterized by metrics which asymptotically approach the Schwarzschild metric far from the horizon but are otherwise distinct from metrics of the Reissner–Nordström family. In addition to the complete Schwarzschild metric, the family of solutions is parametrized by a discrete value of n; the number of nodes of the gauge function. For a

Zoo of hairy black holes

- <u>before 2000</u>: Einstein-Yang-Mills black holes and their generalizations – higher gauge groups, additional fields (Higgs, dilaton), non-spherical solutions, stationary generalizations, Skyrme black holes, Gauss-Bonnet, ...
  /M.S.V.+Gal'tsov, Phys.Rep. 319 (1999) 1/
- <u>after 2000</u>: black holes with scalar hair – engineered potential, spinning clouds of massive complex scalar /Herdeiro-Radu/, Horndeski black holes, metric-affine theories, higher dimensions, stringy corrections, hairy black holes with massive gravitons /Gervalle+M.S.V., 2020/, etc, ...
  /M.S.V., 1601.0823/
- Which of these solutions are physical ?

Present status of hairy black holes

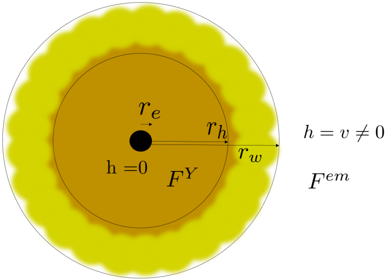
- All known solutions have been obtained within simplified theoretical models. They are nice theoretically but their physical relevance is not obvious.

- To be physically relevant, the solution should be obtained within the context of the physical theory = Einstein's gravity +Standard Model of fundamental interactions (QCD+electroweak).

- Classical configurations in the QCD sector are destroyed by large quantum corrections $\Rightarrow$ useless to study. There remains the gravitating electroweak theory = Einstein-Weinberg-Salam. This describes the Kerr-Newman black holes. Does it describe other black holes ?

- Only unphysical limits of the electroweak theory have been analyzed in the black hole context, since in the full theory the spherical symmetry is lost.

Magnetic electroweak black hole /Maldacena 2020/



The U(1) hypermagnetic field near the horizon + electroweak "corona" made of Z,W,Higgs fields + radial magnetic field in the far field. No symmetry.

Magnetic monopoles in gauge field theories

Dirac monopole /1930/

$$\vec{\mathcal{B}} = \frac{P\vec{r}}{r^3}, \quad \Rightarrow \quad \vec{\nabla}\cdot\vec{\mathcal{B}} \neq 0, \qquad \text{nevertheless} \quad \vec{\mathcal{B}} = \vec{\nabla}\times\vec{\mathcal{A}}_\pm$$

where the vector potential contais the Dirac string singularity, but this can be excluded by using two local gauges,

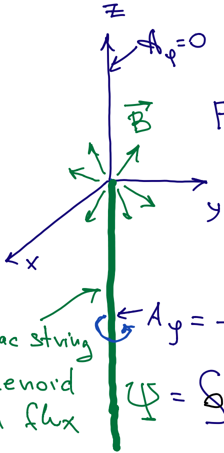
$$\begin{aligned} \mathcal{A}_- &= -P(\cos\vartheta - 1)d\varphi \ \text{ in northern hemisphere} \quad \vartheta &\in [0, \pi/2+\epsilon) \\ \mathcal{A}_+ &= -P(\cos\vartheta + 1)d\varphi \ \text{ in southern hemisphere} \quad \vartheta &\in (\pi/2-\epsilon, \pi] \end{aligned}$$

The two gauges are related in the equatorial region,

$$\mathcal{A}_- = \mathcal{A}_+ + d\,(2P\varphi)\,, \qquad \psi_+ = \exp\left(ie\,2P\varphi\right)\psi_-$$

hence $2eP = n \in \mathbb{Z} \ \Rightarrow \boxed{P = \dfrac{n}{2e}}$ /n is called "magnetic charge"/

Magnetic field produced by a solenoid



$$A = P(\cos\theta - 1)\, d\varphi$$

$$F = dA = -P \sin\theta \, d\theta \wedge d\varphi$$

$$\boxed{B = \star F = -\frac{P}{r^2}\, dr}$$

radial magnetic field

Dirac string = solenoid with flux $\Psi = -4\pi P$

$$\Psi = \oint A_\varphi \, d\varphi = -4\pi P$$

t'Hooft-Polyakov monopole /1974/

Gauge fiel theory with a triplet Higgs field

$$\mathcal{L} = -\frac{1}{4e^2}\, F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2} D_\mu \Phi^a D^\mu \Phi^a - \frac{\lambda}{4}\left(\Phi^a\Phi^a - \Phi_0^2\right)^2$$

with $D_\mu \Phi^a = \partial_\mu \Phi^a + \epsilon_{abc} A^b_\mu \Phi^c$.

A globally regular solution with a finite energy and magnetic charge $P = 1/e \quad \Rightarrow \quad n = 2$. Enormously popular theoretically, but no observational evidence: does not belong to the Standard Model.

What is known about monopoles in the Standard Model ?

Electroweak monopoles in flat space

SU(2)×U(1) electroweak theory of Weinberg-Salam

$$\mathcal{L}_{\rm WS} = -\frac{1}{4g^2}\,\mathrm{W}^a_{\mu\nu}\mathrm{W}^{a\mu\nu} - \frac{1}{4g'^2}\,B_{\mu\nu}B^{\mu\nu} - (D_\mu\Phi)^\dagger D^\mu\Phi - \frac{\beta}{8}\left(\Phi^\dagger\Phi - 1\right)^2$$

where Higgs is a complex doublet, $\Phi^{\rm tr} = (\phi_1, \phi_2)$,

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu\,, \qquad \mathrm{W}^a_{\mu\nu} = \partial_\mu \mathrm{W}^a_\nu - \partial_\nu \mathrm{W}^a_\mu + \epsilon_{abc}\mathrm{W}^b_\mu \mathrm{W}^c_\nu\,, \\ D_\mu\Phi &= \left(\partial_\mu - \frac{i}{2}\,B_\mu - \frac{i}{2}\,\tau^a \mathrm{W}^a_\mu\right)\Phi \end{aligned}$$

Couplins $g'^2 = 0.23$, $g^2 = 1 - g'^2$, $\beta = 1.88$. Electron charge $e = gg'$ defines $\boldsymbol{g}_0 = \sqrt{4\pi\alpha/(\boldsymbol{\hbar c}e^2)}$ ⇒ length and mass scales

$$\boldsymbol{l}_0 = \frac{1}{\boldsymbol{g}_0\boldsymbol{\Phi}_0} = 1.5\times 10^{-16}\ \mathrm{cm}, \quad \boldsymbol{m}_0 = \frac{\boldsymbol{\hbar}}{\boldsymbol{c}}\,\boldsymbol{g}_0\boldsymbol{\Phi}_0 = 128.6\ \mathrm{GeV}$$

The Z, W, Higgs masses expressed in in unites of $\boldsymbol{m}_0$ are $m_{\rm z} = 1/\sqrt{2}$, $m_{\rm w} = g m_{\rm z}$, $m_{\rm h} = \sqrt{\beta}\, m_{\rm z}$.

Dirac monopole

$$
\begin{aligned}
B = W^3 &= \frac{n}{2}(\cos\vartheta \pm 1)\, d\varphi, \qquad W^1 = W^2 = 0, \qquad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\
\mathcal{A} &= \frac{1}{e} B, \qquad \vec{\mathcal{B}} = \frac{P\vec{r}}{r^3}, \qquad P = \frac{n}{2e} = \frac{n}{2gg'}, \qquad n \in \mathbb{Z}
\end{aligned}
$$

Energy is infinite. Remarque:

- Dirac monopole is stable within the U(1) electrodynamics.
- It should be unstable within the electroweak theory because the magnetic field $\vec{\mathcal{B}} = P\vec{r}/r^3$ becomes very strong as $r \to 0$ and the electroweak vacuum becomes unstable with respect to condensation. Nobody studied this.

Cho-Maison monopole /1996/

U(1) hypercharge field $B = (\cos\vartheta - 1)\, d\varphi$ as for the Dirac monopole with $n = 2$, combined with non-Abelian

$$W^a_\mu\, dx^\mu \;=\; (1 - f(r))\, \epsilon_{aik} \frac{x^i dx^k}{r^2}\,, \quad \Phi = \phi(r) \begin{pmatrix} \sin\frac{\vartheta}{2}\, e^{-i\varphi} \\ -\cos\frac{\vartheta}{2} \end{pmatrix}$$

= extended non-Abelian core with a pointlike U(1) hypermagnetic charge in the center. Energy is a sum of a divergent U(1) part and a finite SU(2) part,

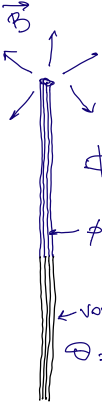
$$E \equiv E_{\mathrm{U}(1)} + E_{\mathrm{SU}(2)} = \frac{2\pi}{g'^2} \int_0^\infty \frac{dr}{r^2} + E_{\mathrm{SU}(2)} \qquad /E_{\mathrm{SU}(2)} = 15.76/$$

The total magnetic charge

$$P = \frac{1}{e} = \frac{1}{gg'} = \frac{g'}{g} + \frac{g}{g'} \equiv P_{\mathrm{U}(1)} + P_{\mathrm{SU}(2)}$$

where $P_{\mathrm{U}(1)}$ is pointlike and located at the origin and $P_{SU(2)}$ is distributed over the space.

Nambu monopole

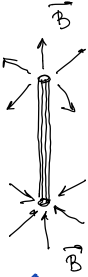


$$P = \frac{\sin^2\theta_W}{e}$$

$$\Phi = \phi \begin{pmatrix} \sin\frac{\theta}{2}\, e^{-i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

$\phi = 0$

vortex, $\theta = \pi$



monopole – antimonopole pair

time-dependent solutions

Summary

Electroweak theory contains two types of static, spherically symmetric monopole solutions, both with infinite energy:

- Pointlike Dirac monopole for any value of the magnetic charge $n = \pm 1, \pm 2, \ldots$
- Non-Abelian monopole of Cho-Maison for $n = \pm 2 \Rightarrow$ superposition of a pointlike hypermagnetic U(1) monopole and a regular SU(2) field.

New come new results

Part I: Stability of electroweak monopoles

R.Gervalle and M.S.V., Nucl.Phys. B 984 (2022) 115937

Generic perturbations

$$W_\mu^a \to W_\mu^a + \delta W_\mu^a, \qquad B_\mu \to B_\mu + \delta B_\mu, \qquad \Phi \to \Phi + \delta\Phi$$

Linearizing the equations with respect to δW_μ^a, δB_μ, $\delta\Phi$, using the null spacetime tetrad approach and separating the angular variables in terms of the spin-weighted spherical harmonics, assuming the $e^{\pm i\omega t}$ time dependence, the perturbation equations reduce to

$$\left(-\frac{d^2}{dr^2} + \hat{U}\right)\Psi = \omega^2\Psi\,,$$

where Ψ is a 16-component vector and $\hat{U}$ is the symmetric 16×16 matrix. If there are bound states with $\omega^2 < 0$ then the background is unstable.

Stability of Cho-Maison monopole – Jacobi criterion

$$\left(-\frac{d^2}{dr^2} + \hat{U}\right)\Psi = \omega^2\Psi\,, \qquad \Psi^{\rm tr} = (\Psi_1, \ldots, \Psi_{16}) \equiv \Psi_k$$

One sets $\omega = 0$, finds 16 regular at the origin solutions $\Psi_k^{(a)}(r)$, and computs the determinant

$$\Delta(r) = \left|\Psi_k^{(a)}(r)\right| \qquad a, k = 1, \ldots, 16$$

If $\Delta(r) > 0$ then all eigenvalues $\omega^2 > 0$. This was checked for the Cho-Maison monopole in sectors with $j = 0, 1, 2, 3, 4$. For higher j the bound states are unlikely dues to the high centrifugal barrier $\Rightarrow$

The non-Abelian monopole of Cho-Maison is stable with respect to all small perturbations

Stability of Dirac monopole

One perturbative channe decouples

$$\left(-\frac{d^2}{dr^2}+\frac{g^2}{2}-\frac{|n|}{2r^2}\right)\psi=\omega^2\psi \quad \text{if} \quad \boxed{j=\left|\frac{n}{2}\right|-1}, \qquad |n|>1,$$

solution oscillates infinitely many times for $r\to 0$,

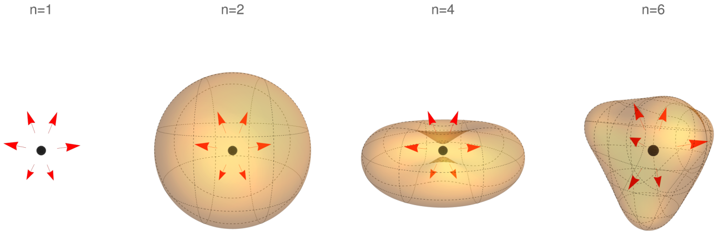
$$\psi=\sqrt{r}\,\cos\left(\frac{\sqrt{2n-1}}{2}\ln\frac{r}{r_0}\right)$$

$\Rightarrow$ all Dirac monopoles with $|n|>1$ are unstable.

The $n=2$ Dirac is unstable in the $j=0$ sector: not splitting. The non-Abelian Cho-Maison monopole also has $n=2$ and is stable $=$ it should be remnant of Dirac's monopole decay.

Dirac monopoles with $|n|>2$ decay in sectors with $j>0$ and should condense to non spherically-symmetric non-Abelian states.

Conjecture



Dirac monopoles with $|n| \geq 2$ are conjectured to condense to non-spherically symmetric non-Abelian states. The magnetic charge splits into the pointlike $ng/(2g')$ at the origin and $ng'/(2g)$ smoothly distributed in space. The energy is infinite due to the central singularity. (Perhaps the latter can be shielded by an event horizon?)

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Part II: Non-Abelian multi-monopoles for $|n| > 2$

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Axial symmetry

Rebbi-Rossi ansatz (even parity), with $\mathrm{T}_a = \tau_a/2$,

$$W = \mathrm{T}_a \mathrm{W}^a_\mu dx^\mu \;=\; \mathrm{T}_2 \left(F_1\, dr + F_2\, d\vartheta \right) + \frac{n}{2} \left(\mathrm{T}_3\, F_3 - \mathrm{T}_1 F_4 \right) d\varphi$$

$$B_\mu dx^\mu \;=\; \frac{n}{2}\, Y\, d\varphi\,, \qquad \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \qquad n \in \mathbb{Z},$$

$F_1, F_2, F_3, F_4, Y, \phi_1, \phi_2$ are 7 real-valued functions of r, ϑ.

- System of 7 elliptic PDE's in the domain $r \in [0, \infty)$, $\vartheta \in [0, \pi/2]$, assuming the invariance under $\vartheta \to \pi - \vartheta$.
- For $n = \pm 2$ solution is spherically symmetric = the Cho-Maison monopole. Iteratively increasing n gives axially-symmetric monopoles.

Energy

Splits into an infinite U(1) part and a finite SU(2) part

$$E = \int T_{00}\sqrt{-g}\, d^3x = \frac{2\pi\nu^2}{g'^2}\int_0^\infty \frac{dr}{r^2} + E_{\mathrm{reg}}.$$

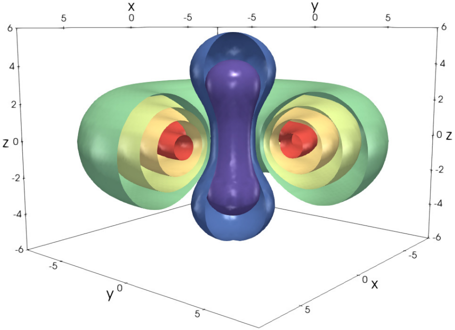
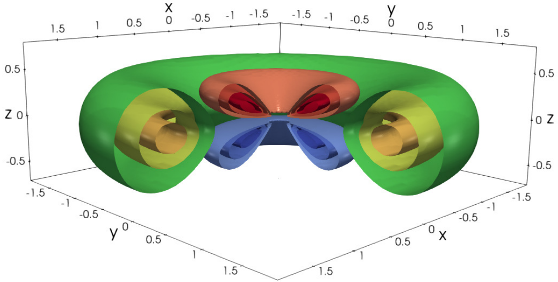


Figure: Surfaces of constant T_{00} for $n = 10$.

Magnetic charge and electric current isosurfaces



Magnetic charge density $\rho_{SU(2)}$ (green) and positive J_φ and negative J_φ densities of the azumuthal electric current for the $n = 4$ monopole. The magnetic charge forms a ring whose magnetic field forces the charged W-bosons to Larmore-orbit, creating two electric currents. These currents create the magnetic field which squeezes the magnetic charge toward equatorial plane.

Large charge limit

For large n the U(1) field B becomes very strong and drives to zero all other fields in the central monopole region thus restoring the full gauge symmetry. This creates the vacuum bubble in the monopole center.

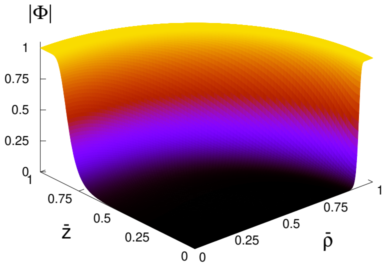
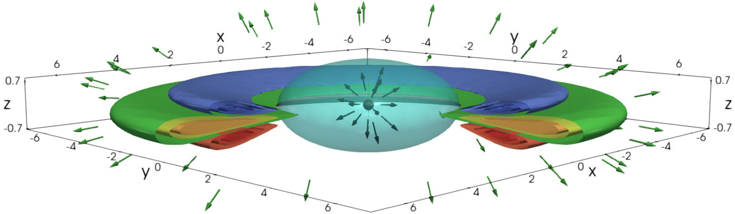


Figure: The norm of the Higgs field $|\Phi|$ for $n = 100$.

Large charge monopole, $n = 40$.



The vacuum bubble contains the U(1) hypermagnetic field of the pointlike charge $P_{\mathrm{U}(1)} = ng/(2g')$. Outside the massive fields condense to a ring of magnetic charge $P_{\mathrm{SU}(2)} = ng'/(2g)$ squeezed between two superconducting electric currents. Still farther away there remains only the field of the Dirac monopole of charge $P_{\mathrm{U}(1)} + P_{\mathrm{SU}(2)} = n/(2e)$. The energy is infinite due to the central pointlike charge. Perhaps the latter can be shielded by an event horizon ?

Part III: Black holes with electroweak hair

Einstein-Weinberg-Salam theory

$$\mathcal{L} = \frac{1}{2\kappa} R + \mathcal{L}_{\rm WS}$$

with

$$\mathcal{L}_{\rm WS} = -\frac{1}{4g^2} \mathrm{W}^a_{\mu\nu}\mathrm{W}^{a\mu\nu} - \frac{1}{4g'^2} B_{\mu\nu}B^{\mu\nu} - (D_\mu\Phi)^\dagger D^\mu\Phi - \frac{\beta}{8}\left(\Phi^\dagger\Phi - 1\right)^2$$

the length scale and mass scale are electroweak, same as before: $\boldsymbol{l}_0 =$1.5 $\times$ 10^{-16} cm and $\boldsymbol{m}_0 =$128.6 GeV. The couplings

$$g^2 = 0.77, \quad g'^2 = 0.23, \quad \beta = 1.88, \quad \kappa = \frac{8\pi\mathbf{G}\boldsymbol{\Phi}_0^2}{\boldsymbol{c}^4} = 5.42 \times 10^{-33}.$$

Equations to solve

Electroweak:

$$\nabla^\mu B_{\mu\nu} = g'^2 \frac{i}{2}(\Phi^\dagger D_\nu \Phi - (D_\nu\Phi)^\dagger \Phi),$$
$$\mathcal{D}^\mu \mathrm{W}^a_{\mu\nu} = g^2 \frac{i}{2}(\Phi^\dagger \tau^a D_\nu \Phi - (D_\nu\Phi)^\dagger \tau^a \Phi),$$
$$D_\mu D^\mu \Phi - \frac{\beta}{4}(\Phi^\dagger\Phi - 1)\Phi = 0,$$

Einstein:

$$\begin{aligned} G_{\mu\nu} &= \kappa T_{\mu\nu} \qquad \text{where } \kappa \sim 10^{-33} \quad \text{is very small and} \\ T_{\mu\nu} &= \frac{1}{g^2}\mathrm{W}^a{}_{\mu\sigma}\mathrm{W}^a{}_\nu{}^\sigma + \frac{1}{g'^2} B_{\mu\sigma}B_\nu{}^\sigma + 2D_{(\mu}\Phi^\dagger D_{\nu)}\Phi + \mathrm{g}_{\mu\nu}\mathcal{L}_{\mathrm{WS}} \end{aligned}$$

=30 coupled equations. A simple solution:

Magnetically charged Reissner-Nordstrom

Same electroweak fields as for the Dirac monopole,

$$B = W^3 = \frac{n}{2}(\cos\vartheta \pm 1)\, d\varphi, \quad W^1 = W^2 = 0, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and the RN metric,

$$ds^2 = -N(r)\, dt^2 + \frac{dr^2}{N(r)} + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right),$$

$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \qquad Q^2 = \frac{\kappa n^2}{8e^2}, \qquad n \in \mathbb{Z}.$$

The event horizon is at $r_H = M + \sqrt{M^2 - Q^2}$.

This solution is stable at large r_H but becomes unstable at small r_H

Stability of Reissner-Nordstrom

The same instability as for the Dirac monopole: for $\boxed{j = |n|/2 - 1, \quad |n| > 1)}$ one obtains the one-channel problem

$$\left(-\frac{d^2}{dr_\star^2} + N(r)\left[\frac{g^2}{2} - \frac{|n|}{2r^2}\right]\right)\psi(r) = \omega^2\psi(r)$$

with $dr_\star = dr/N(r)$. In flat space $N(r) = 1$ and there are infinitely many bound states with $\omega^2 < 0 \Rightarrow$ Dirac monopoles are unstable.

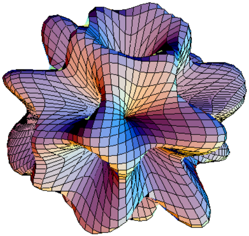
In curved space $N(r) \leq 1 \Rightarrow$ a finite number of bound states if $r_H < r_H^0$ and no bound states if $r_H > r_H^0$. For $r_H = r_H^0$ the first bound state appears as a static zero mode $\psi_0(r)$ which approximates the W-condensate $=$ black hole hair.

Perturbative black hole hair

Values $r_H^0(n)$ for which the zero mode appears

n	2	4	6	10	20	40	100	200
r_H^0	0.89	1.47	1.93	2.69	4.12	6.19	10.33	15.03

The mode is maximal at the horizon and proportional to $Y_{jm}(\vartheta,\varphi)$ with $j = |n/2| - 1$, describes the W-current tangential to the horizon. This current produce magnetic and Z-fluxes orthogonal to the horizon= vortices of finite length =corona. Schematically,



Non-perturvative solutions

Hairy black holes cannot be spherically symmetric for $|n| > 2$ but can be axially symmetric:

$$ds^2 = -e^{2U} N(r)\, dt^2 + e^{2k-2U} \left(\frac{dr^2}{N(r)} + r^2 d\vartheta^2 + e^{2w} r^2 \sin^2\vartheta d\varphi^2 \right),$$

$$W = \mathrm{T}_a \mathrm{W}^a_\mu dx^\mu = \mathrm{T}_2 \left(F_1\, dr + F_2\, d\vartheta \right) + \frac{n}{2} \left(\mathrm{T}_3\, F_3 - \mathrm{T}_1 F_4 \right) d\varphi$$

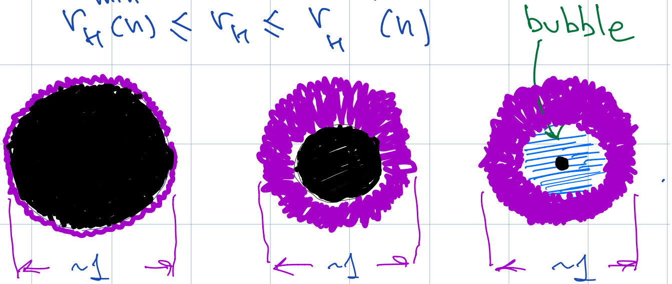
$$B_\mu dx^\mu = \frac{n}{2}\, Y d\varphi\,, \qquad \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad \mathrm{T}_a = \tau_a/2\,,$$

where $U, k, w, F_1, F_2, F_3, F_4, Y, \phi_1, \phi_2$ are 10 real functions of r, ϑ and $N(r) = 1 - r_H/r$ where r_H is the black hole "size".

10 coupled PDE's to solve. For $n = \pm 2$ the solution is spherically symmetric. Iteratively increasing n gives axially-symmetric black holes. For $r_H \approx r_H^0$ they are slightly hairy, more hair grows as r_H decreases.

Decreasing the horizon size r_H

$$r_H^{min}(n) \leq r_H \leq r_H^{max}(n)$$

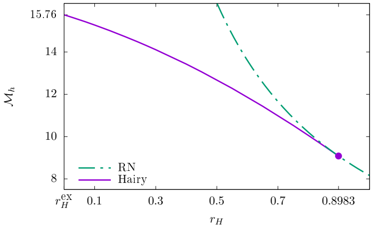


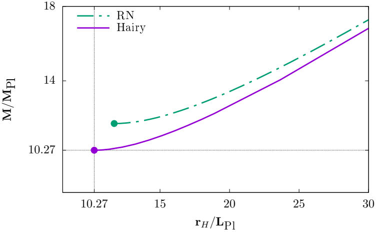
when horizon dicreases, the hair grows, until a bubble of symmetric phase appears. Inside the buble – RN-de Sitter geometry for magnetic charge $P_{U(1)} = \frac{n}{2g'} < P = \frac{n}{2gg'}$. The rest of the charge is in the hair.

ADM mass

$$
\begin{aligned}
-g_{00} &= 1 - \frac{2M}{r} + \ldots, \\
M &= \frac{\mathrm{k_H A_H}}{4\pi} + \frac{\kappa}{4\pi}\int_{r>r_{\mathrm{H}}} (2T_{\hat{0}\hat{0}} + T)\sqrt{-g}\, d^3x
\end{aligned}
$$

Hairy solutions are less energetic than the RN of the same size.
Their minimal size is also smaller, as $P_{\mathrm{U(1)}} < P = P_{\mathrm{U(1)}} + P_{\mathrm{SU(2)}}$.





Extreme limit

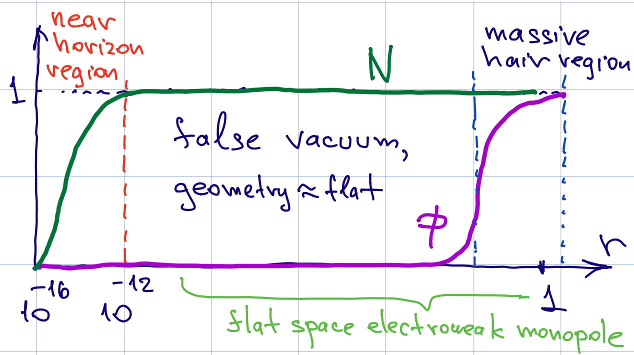
- As r_H approaches the lower bound $r_H \to r_H^{\min}$, the horizon becomes degenerate and the temperature vanishes.

- The extreme hairy black hole contains inside only a part of the magnetic charge, $P_{\mathrm{U}(1)} = ng/(2g')$, and is smaller than the extreme RN black hole containing inside the total charge $P = n/(2gg')$.

- Close to the horizon the hypermagnetic field $B \propto |n|/r^2$ is very strong and drives to zero the SU(2) and Higgs fields. This creates a bubble of symmetric phase where the geometry is the extreme RN-de Sitter for the charge $P_{\mathrm{U}(1)} = ng/(2g')$ and cosmological constant $\Lambda = \kappa\beta/8$. The horizon size

$$r_H^{\min} = \frac{\sqrt{\kappa}|n|}{2\sqrt{2}g'} \lll r_{\mathrm{hair}} \sim \sqrt{|n|}$$

Horizon size vs hair size



horizon region size is parametrically small as compared to the hair region size.
The hair decouples from the horizon and lives in flat geometry and reduces to the flat space monopole solution.

Structure of extreme solutions

- Inside the central bubble: a tiny RNdS black hole supporting the hypermagnetic field of charge $P_{\mathrm{U}(1)} = ng/(2g')$ which suppresses the other fields and restores the full electroweak symmetry:

$$B = \frac{n}{2r^2}, \quad W^a_\mu = 0, \quad \Phi = 0, \quad \text{RNdS geometry}$$

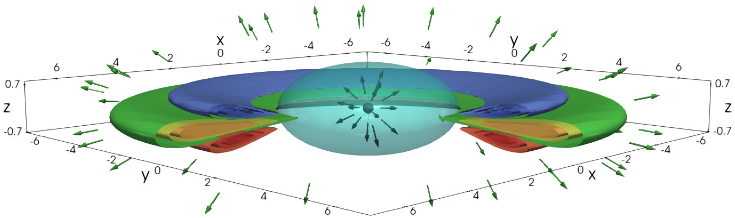
- Bubble wall $=$ hair: a condensate of massive W, Z, Φ carrying the magnetic charge $P_{\mathrm{SU}(2)} = ng'/(2g)$.
- Far field region: All massive fields vanish, there remains radial magnetic field of charge $P_{\mathrm{U}(1)} + P_{\mathrm{SU}(2)}$,

$$\vec{\mathcal{B}} = \frac{n}{2e}\frac{\vec{r}}{r^3}, \qquad Z = W = 0, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Bubble$=$hair size, energy, and quadrupole moments:

$$r_{\mathrm{hair}} \sim \sqrt{|n|}, \quad M_{\mathrm{hair}} \sim 10|n|^{3/2}, \quad Q_{\mathrm{WS}} \sim n^2, \quad Q_{\mathrm{G}} \sim \kappa Q_{\mathrm{WS}}$$

Extreme black hole for $n = 40$.



At the center – a tiny extreme RNdS black hole surrounded by the vacuum bubble. Outside the bubble – a condense of the massive fields forming rings. Far away – the radial magnetic field.

Increasing the magnetic charge n

- Maximal charge n: The minimal value of the event horizon $r_H^{\min} \propto \sqrt{\kappa}\,|n|$ increases with n faster than the maximal value $r_H^{\max} \propto \sqrt{n}$. The two merge for $\boxed{n \sim 1/\kappa \approx 10^{32}}$. Then

$$r_H \approx 1 \text{ cm}, \qquad M \approx M_{\mathrm{U(1)}} \approx M_{\mathrm{SU(2)+Higgs}} \approx 10^{25} \text{ kg},$$

  typical for planetary size black holes $\Rightarrow$ very very large for hairy black holes.

- For a given n there should be also $|n|$ different non-axially symmetric hairy black holes with symmetries of $Y_{jm}(\vartheta, \varphi)$, $j = |n/2| - 1 \Rightarrow$ CORONA. Their number is the same as the black hole entropy $S \sim r_H^2 \sim |n|$ = /also entropy?/.

Conclusion

Solutions describing black hole with electroweak hair are constructed. They can be large and perhaps astrophysically relevant