Entangled relativity: a new, more parsimonious, general theory of relativity

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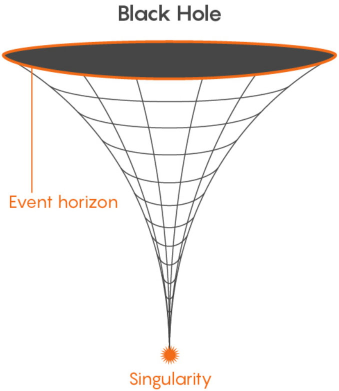
1. Potential issues of general relativity

2. Formulation & phenomenology

General relativity may have the following issues :

- Singularities (black holes, big-bang)
- Spacetime might not have an operational meaning beyond the Planck scale
- Inertia can be defined from nothing, i.e. GR does not satisfy *Mach's principle* of Einstein

General relativity's issues: singularities



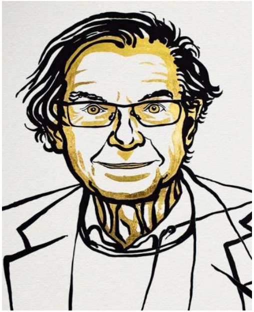


Figure: Penrose's theorem on the inevitabiltiy of singularities inside black holes.

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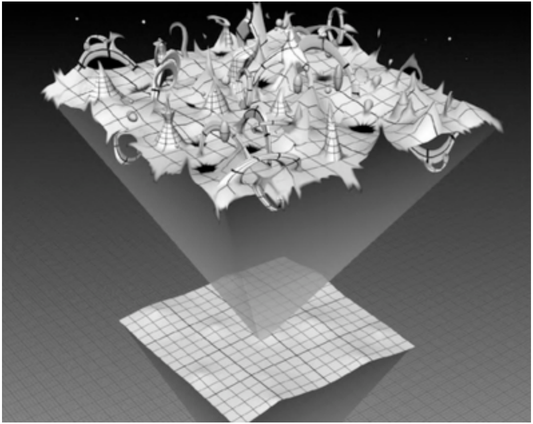
General relativity's issues

Spacetime beyond the Planck scale?



QFT+GR $\Rightarrow$ Planck length and time (at which the smooth and/or continuous structure of spacetime seems to break down)

General relativity may have the following issues :

- Singularities (black holes, big-bang)
- Spacetime might not have an operational meaning beyond the Planck scale
- Inertia can be defined from nothing, i.e. GR does not satisfy *Mach's principle* of Einstein

General relativity's issues

inertia can be defined *ex nihilo* in general relativity: violation of Mach's principle

Einstein beleived in the *relativity of inertia*

"c. Mach's Principle. [Spacetime] is completely determined by [matter] [...]. With (c), according to the field equations of gravitation, **there can be no [spacetime] without matter**. " Einstein [1918a]

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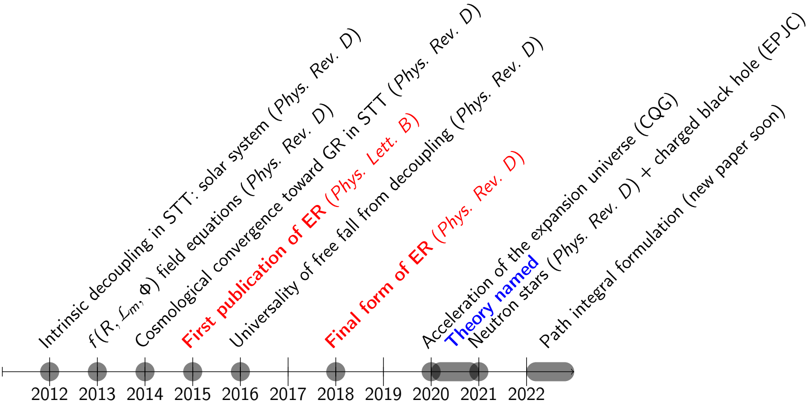
Entangled Relativity $\sim$ General Relativity

Same ingredients: 4D pseudo-Riemmanian manifold coupled to matter fields

It only assumes a difference coupling between R and $\mathcal{L}_m$

$$\frac{R}{2\kappa} + \mathcal{L}_m \rightarrow -\frac{1}{2}\frac{\mathcal{L}_m^2}{R}$$

Chronology of entangled relativity



Arruga and Minazzoli [2021], Arruga et al. [2021], Harko et al. [2013], Ludwig et al. [2015], Minazzoli [2014, 2018, 2021, 2022], Minazzoli and Hees [2013, 2014, 2016], Minazzoli and Santos [2021]

Entangled relativity

Path integral formulation

$$Z_{\rm ER} = \int \mathcal{D}g \prod_i \mathcal{D}f_i \exp\left[-\frac{i}{2\epsilon^2}\int d_g^4x \frac{\mathcal{L}_m^2(f,g)}{R(g)}\right], \qquad (1)$$

2103.05313 & 2206.03824

d_g^4x: infinitesimal spacetime 4-volume
$\mathcal{L}_m$: matter Lagrangian density
f_i: fields, such as gauge bosons, fermions and the Higgs
g: spacetime metric
R: Ricci scalar
ϵ: quantum of energy
c: causal structure constant

Satisfaction of Einstein's 1918 version of Mach's Principle

$$S \propto \int \mathrm{d}^4 x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \qquad \text{hence} \qquad \boxed{\mathcal{L}_m = \emptyset \Rightarrow S = \emptyset} \qquad (2)$$

It does not make sense to consider space-time without considering matter fields in this framework $\Rightarrow$ **Inertia cannot be defined *ex nihilo***!

Matter and spacetime curvature are *entangled* (in the etymological sense) at the level of the action density.

Satisfaction of Einstein's 1918 version of Mach's Principle

General relativity may have the following issues:

- Singularities (black holes, big-bang)
- Spacetime might not have an operational meaning beyond the Planck scale
- ~~Inertia can be defined from nothing~~ ✓

Entangled relativity

Path integral formulation

$$Z_{\mathrm{ER}} = \int \mathcal{D}g \prod_i \mathcal{D}f_i \exp\left[-\frac{i}{2\epsilon^2}\int d_g^4x \frac{\mathcal{L}_m^2(f,g)}{R(g)}\right], \qquad (3)$$

2103.05313 & 2206.03824

d_g^4x: infinitesimal spacetime 4-volume
$\mathcal{L}_m$: matter Lagrangian density
f_i: fields, such as gauge bosons, fermions and the Higgs
g: spacetime metric
R: Ricci scalar
ϵ: **quantum of energy**

Classical limit: gravity

Classical equivalence (provided $\mathcal{L}_m \neq \emptyset$)

$$-\frac{1}{2}\int \mathrm{d}^4_g x \frac{\mathcal{L}_m^2}{R} \equiv \int \mathrm{d}^4_g x \frac{1}{\kappa}\left(\frac{R}{2\kappa} + \mathcal{L}_m\right) \quad (4)$$

κ: dimensionfull scalar field ($:= 8\pi G/c^4$)

Classical limit: gravity

Classical equivalence (provided $\mathcal{L}_m \neq \emptyset$)

$$-\frac{1}{2}\int \mathrm{d}_g^4 x \frac{\mathcal{L}_m^2}{R} \equiv \int \mathrm{d}_g^4 x \frac{1}{\kappa}\left(\frac{R}{2\kappa} + \mathcal{L}_m\right) \tag{5}$$

κ: dimensionfull scalar field ($:= 8\pi G/c^4$)

On-shell equivalence: very simple demo

The solution of κ is such that $\kappa = -R/\mathcal{L}_m$
Injecting the solution in the right hand side:

$$\frac{1}{\kappa}\left(\frac{R}{2\kappa} + \mathcal{L}_m\right) = -\frac{1}{2}\frac{{\mathcal{L}_m}^2}{R}$$

One can also check the equivalence of the full set of equations from both side of Eq. (5)

Classical limit: gravity

Classical equivalence (provided $\mathcal{L}_m \neq \emptyset$)

$$-\frac{1}{2}\int \mathrm{d}^4_g x \frac{\mathcal{L}_m^2}{R} \equiv \int \mathrm{d}^4_g x \frac{1}{\kappa}\left(\frac{R}{2\kappa} + \mathcal{L}_m\right) \quad (6)$$

κ: dimensionfull scalar field ⇑ Cauchy well-posed ⇑

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu} + \kappa^2\left[\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right]\frac{1}{\kappa^2} \quad (7)$$

$$\nabla_\sigma\left(\kappa^{-1}T^{\alpha\sigma}\right) = \mathcal{L}_m \nabla^\alpha \kappa^{-1} \quad (8)$$

Trace metric field equation: $$3\kappa^2\Box\frac{1}{\kappa^2} = \kappa\left(T - \mathcal{L}_m\right) \quad (9)$$

The solution of κ is such that $\kappa = -R/\mathcal{L}_m$

Recall that $\kappa = -R/T$ in GR

Charged black hole: Minazzoli and Santos [2021]

$$\boxed{\mathrm{d}s^2 = -\lambda_0^2 \mathrm{d}t^2 + \lambda_r^{-2} \mathrm{d}r^2 + \rho^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right)} \qquad (10)$$

$$\begin{aligned} \rho^2 &= r^2 \left(1 - \frac{r_-}{r}\right)^{6/13} \\ \lambda_0^2 &= \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{15/13} \\ \lambda_r^2 &= \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{7/13} \end{aligned} \qquad (11)$$

$$R = -\frac{Q^2}{(r - r_-)^{6/13} r^{46/13}} \qquad \mathcal{L}_m = \frac{Q^2}{(r - r_-)^{8/13} r^{44/13}} \qquad Q^2 = \frac{12}{13} r_- r_+$$

$$\Rightarrow \boxed{\kappa = -\frac{R}{\mathcal{L}_m}} = \frac{(r - r_-)^{2/13}}{r^{2/13}}$$

$\boxed{\kappa \text{ well defined at } \mathcal{L}_m \to 0 \text{ !!!}}$

Conformal transformation

$$\tilde{g}_{\alpha\beta} = e^{-2\varphi/\sqrt{3}} g_{\alpha\beta}$$

$$\kappa = \bar{\kappa} e^{-\varphi/\sqrt{3}}$$

$\bar{\kappa}$: normalisation dimension-full constant ($[\bar{\kappa}] = [G/c^4]$).

With electromagnetic field

$$S \propto \int d^4_{\tilde{g}} x \times \left[\frac{1}{2\bar{\kappa}} \left(\tilde{R} - 2\tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right) - e^{-2\alpha\varphi} \frac{\tilde{F}^2}{2\mu_0} \right] \qquad (12)$$

$\tilde{F}^2 = \tilde{g}^{\alpha\sigma} \tilde{g}^{\beta\epsilon} \tilde{F}_{\sigma\epsilon} \tilde{F}_{\alpha\beta}$
$\tilde{F}_{\alpha\beta} := F_{\alpha\beta}$

Equivalent to usual **Einstein-Maxwell-dilaton theory** with $\alpha = 1/(2\sqrt{3})$. (e.g $\alpha = 1$ for bosonic string & $\alpha = \sqrt{3}$ for 5D KK).

Intrinsic (built-in) decoupling

$$\nabla_\sigma \left(\kappa^{-1} T^{\alpha\sigma}\right) = \mathcal{L}_m \nabla^\alpha \kappa^{-1} \tag{13}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} + \kappa^2 \left[\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box\right] \frac{1}{\kappa^2} \tag{14}$$

The trace of the metric field equation gives an equation on the scalar dof κ

$$3\kappa^2 \Box \frac{1}{\kappa^2} = \kappa \left(T - \mathcal{L}_m\right) \tag{15}$$

$\mathcal{L}_m = T \Rightarrow \kappa = \text{cste}$ **can be solution**
$\Rightarrow$ equations of general relativity

E.g. **Dust with null radiation**
($\mathcal{L}_m = -\rho + F^2/4 = -\rho = T$, since $F^2 \propto E^2 - B^2 = 0$ for null radiation)

Consequences of the intrinsic decoupling

- For a universe made of dust and EM radiation, the scalar degree of freedom ($\propto G$) freezes and one gets GR back at the cosmological level. 2011.14633
- Neutron stars are at max a few percent more massive than the ones of GR. 2011.14629
- Exterior of (spherical) black holes cannot be distinguished from the ones of GR in astrophysical conditions. 2102.10541
- Gravitationnal waves emmited from the fusion of black-holes are indistinguishables from the ones of GR. 1706.09875

More details in backup slides if needed

Consequences of the intrinsic decoupling

Broad consequence of the decoupling

$\kappa = -R/\mathcal{L}_m$ varies <u>much less</u> than the spacetime metric at the classical level.
(Generically, but not always).

Classical limit of entangled relativity ($\mathcal{L}_m \neq \emptyset$)

Quite generically, but not always, one has

$$-\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\frac{\mathcal{L}_m^2}{R} \equiv \int \mathrm{d}^4x\sqrt{-g}\frac{1}{\kappa}\left(\frac{R}{2\kappa}+\mathcal{L}_m\right) \quad (16)$$

$$\approx \frac{1}{\kappa}\int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2\kappa}+\mathcal{L}_m\right) \quad (17)$$

General relativity is a limit of (predicted by) the theory.

Standard QFT on "flat spacetimes" (i.e. gravity neglected)

g is considered to be a constant.

$$Z_{\mathrm{QFT}} = \int \prod_i \mathcal{D}f_i \exp\left[\frac{i}{c\hbar}\int d^4x \mathcal{L}_m(f)\right] \qquad (18)$$

Path integral formulation of ER (gravity neglected)

Both g and $\kappa = -R/\mathcal{L}_m$ are considered to be constant.

$$Z_{\mathrm{ER}} \approx \int \prod_i \mathcal{D}f_i \exp\left[\frac{i}{\kappa\epsilon^2}\int d^4x \mathcal{L}_m(f)\right] \qquad (19)$$

$c\hbar := \kappa\epsilon^2 \Rightarrow \epsilon =: \sqrt{\frac{c\hbar}{\kappa}}$: Planck energy

Path integral formulation (gravity neglected)

Both g and $\kappa = -R/\mathcal{L}_m$ are considered to be constant.

$$Z_{\rm ER} \approx \int \prod_i \mathcal{D}f_i \exp\left[\frac{i}{\kappa\epsilon^2}\int d^4x \mathcal{L}_m(f)\right] \tag{20}$$

$$c\hbar := \kappa\epsilon^2 \Rightarrow \epsilon =: \sqrt{\frac{c\hbar}{\kappa}}: \text{ Planck energy}$$

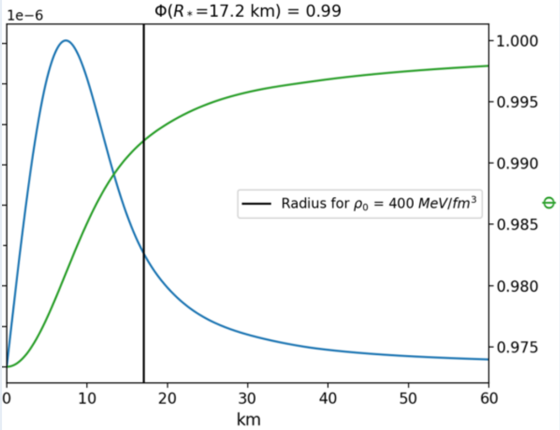
Interpretations

- Standard QFT is a specific semi-classical limit ($(g, \kappa) \approx$ constants) of entangled relativity. (Note that $(g, \kappa) \approx$ constants is a consequence of the classical field equations).
- The only parameter of entangled relativity is the Plank energy.
- $\kappa = 8\pi G/c^4 \Rightarrow \boxed{\hbar \propto G}$ such that $\boxed{G \rightarrow 0 \Leftrightarrow \hbar \rightarrow 0}$.

Neutron stars &/or white dwarfs

Toward experimental tests

A few percent variation of $\kappa \propto \hbar$ for NS 2011.14629



Entangled relativity

Path integral formulation

$$\boxed{Z_{\rm ER} = \int \mathcal{D}g \prod_i \mathcal{D}f_i \exp\left[-\frac{i}{2\epsilon^2}\int d^4_g x \frac{\mathcal{L}_m^2(f,g)}{R(g)}\right]} \quad (21)$$

$$Z_{\rm GR} = \int \mathcal{D}g \exp\left[\frac{i}{2l_P^2}\int d^4_g x R(g)\right], \; l_P\text{: Planck length} \quad (22)$$

Spacetime might not be doomed after all

There are only two universal constants in the definition of the theory:

- **The causal structure constant:** c
- **The Planck energy:** ϵ, whose value is deduced from the $\kappa \approx$ constant limit.

$\Rightarrow$ There is no **Planck length** nor **Planck time** in ER.

Quantum gravity

General relativity may have the following issues:

- Singularities (black holes, big-bang)
- ~~Spacetime might not have an operational meaning beyond the Planck scale~~ ✓(Circumstantial evidence)
- ~~Inertia can be defined from nothing~~ ✓

Repulsive gravity?

Classical equivalence (provided $\mathcal{L}_m \neq \emptyset$)

$$-\frac{1}{2}\int \mathrm{d}^4_g x \frac{\mathcal{L}_m^2}{R} \equiv \int \mathrm{d}^4_g x \frac{1}{\kappa}\left(\frac{R}{2\kappa} + \mathcal{L}_m\right) \quad (23)$$

κ: dimensionfull scalar field

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} + \kappa^2 \left[\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box\right] \frac{1}{\kappa^2} \quad (24)$$

$$\nabla_\sigma \left(\kappa^{-1} T^{\alpha\sigma}\right) = \mathcal{L}_m \nabla^\alpha \kappa^{-1} \quad (25)$$

Trace metric field equation: $3\kappa^2 \Box \frac{1}{\kappa^2} = \kappa \left(T - \mathcal{L}_m\right)$ (26)

$\kappa < 0$ is not prohibited a priori! $\Rightarrow$ repulsive at high density?

Quantum gravity

General relativity may have the following issues:

- Singularities (black holes, big-bang) ✓?
- ~~Spacetime might not have an operational meaning beyond the Planck scale~~ ✓(Circumstantial evidence)
- ~~Inertia can be defined from nothing~~ ✓

Entangled relativity: wrap up

A simple reformulation that reduces the # of univ. constants

$$\frac{1}{c\hbar}\left(\frac{R}{2\kappa}+\mathcal{L}_m\right)\rightarrow-\frac{1}{2\epsilon^2}\frac{\mathcal{L}_m^2}{R} \tag{27}$$

Which recovers general relativity and standard QFT in a generic limit!

(Which has no elementary length and time scales
and no free parameter at all!)

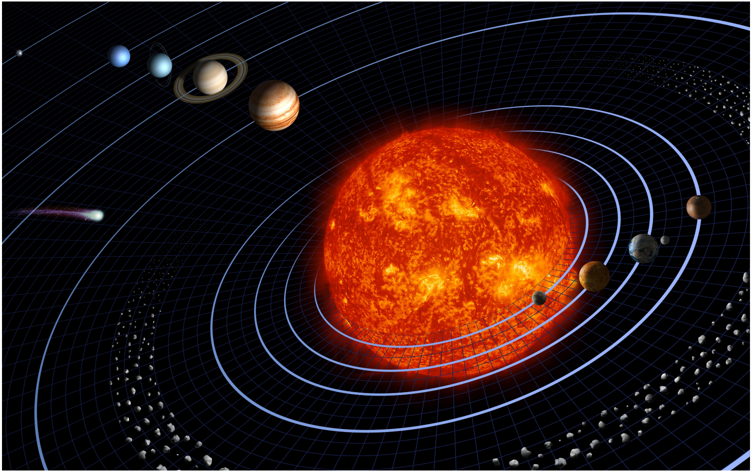
2103.05313 & 2206.03824 & 2304.09482

Spacetime might not be doomed after all

It is an entirely new direction to explore and evaluate.
(Without anything exotic!)

Prediction: $\hbar$ **can vary depending on gravitational phenomena**

Post-Newtonian solution: e.g. Solar System



An ambiguity in the field equations

One needs to know the value of the on-shell matter Lagrangian

$$3\kappa^2 \Box \kappa^{-2} = \kappa \left(T - \mathcal{L}_m \right) \quad (28)$$

- if $\boxed{\mathcal{L}_m = T}$, as argued in e.g. Avelino and Azevedo [2018], then the scalar degree of freedom has no source at all $\Rightarrow$ GR.
- if $\boxed{\mathcal{L}_m = -\rho}$, as argued in e.g. Minazzoli and Harko [2012], then the scalar degree of freedom is sourced by pressure only $\rightarrow$ *Pressuron*. Minazzoli and Hees [2014].
- if $\boxed{\mathcal{L}_m = P}$, one **does not** have general relativity at leading post-Newtonian order. Should only be valid for exotic objects such as *fuzzy dark matter* Arruga et al. [2021]. (Because $\mathcal{L}_m = K - V = P$ for scalar fields).

Post-Newtonian solutions assuming $\nabla_\sigma(\rho_0 u^\sigma) = 0$

No source $\Rightarrow$ GR post-Newtonian phenomenology

$$\mathcal{L}_m = T \qquad \Rightarrow \qquad \Box\phi^2 = 0. \tag{29}$$

Or "Pressuron" $\rightarrow$ name given in Minazzoli and Hees [2014]

$$\mathcal{L}_m = -\rho \qquad \Rightarrow \qquad \frac{1}{\phi^2}\Box\phi^2 = -\frac{\tilde{\kappa}}{\phi}P, \tag{30}$$

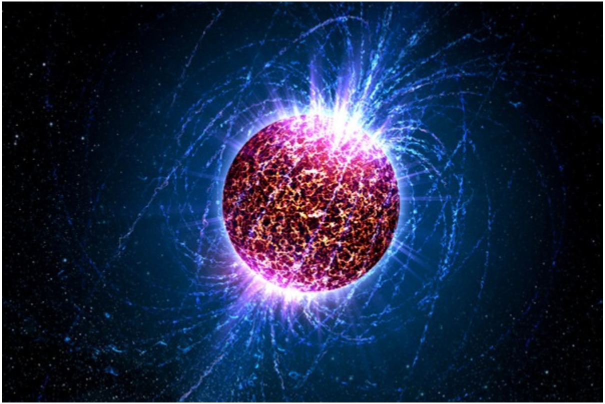
Solution for *pressuron* Minazzoli and Hees [2013]

$$\boxed{g_{\alpha\beta}^{ER} = g_{\alpha\beta}^{GR} + O\left(P/(\rho c^2), 1/c^4\right)} \qquad \Rightarrow PPN : \gamma = \beta = 1 \tag{31}$$

$P/(\rho c^2) = \mathcal{O}(10^{-10})$ for the Earth

However $P/(\rho c^2)$ not negligible for neutron stars

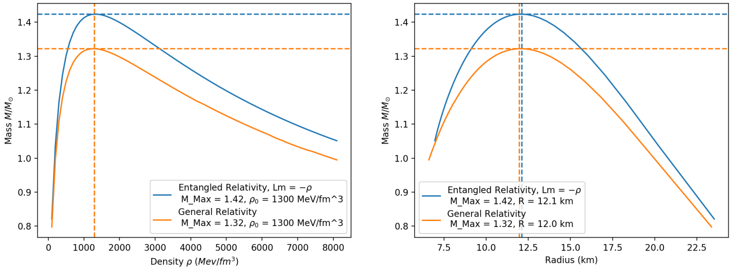
Neutron stars in entangled relativity



Neutron stars

Always heavier in entangled relativity w.r.t. general relativity

Assuming $P = K\rho^\gamma$, with $\gamma = 5/3$, $K = 1.5 \times 10^{-3}(fm^3/MeV)^{2/3}$.
And $\mathcal{L}_m = -\rho$. (Otherwise, same result as GR for $\mathcal{L}_m = T$).



Max: 8% more massive Arruga et al. [2021]

No free theoretical parameter!

Neutron stars
Birkoff's theorem no longer valid

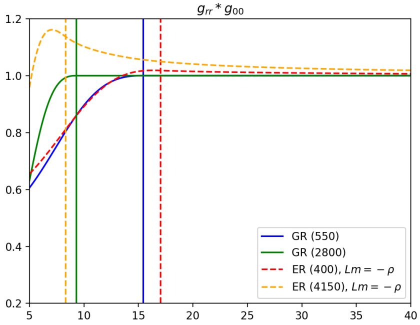
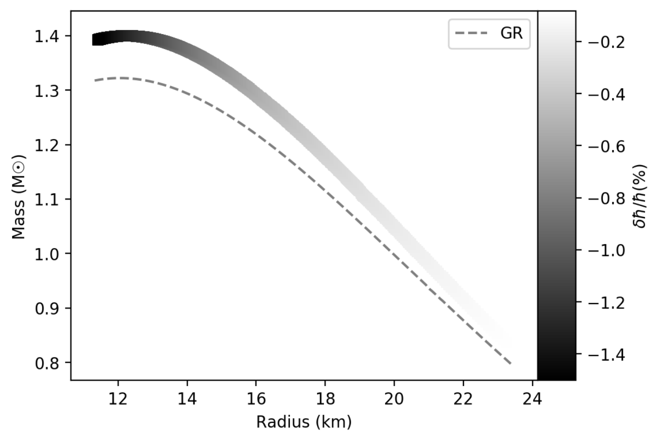


Figure: $g_{00}g_{rr}$ of the two solutions of an object of mass $M = 1.25 M_\odot$ in general relativity and entangled relativity. The vertical: radius of the solutions. (Central density for each solution, in MeV/fm^3). Arruga et al. [2021]

Neutron stars&/or white dwarfs

Toward experimental tests

A few percent variation of $\kappa \propto \hbar$ (not published yet)



Neutron stars &/or white dwarfs

Toward experimental tests

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Measuring the fine-structure constant on a white dwarf surface; a detailed analysis of Fe V absorption in G191−B2B

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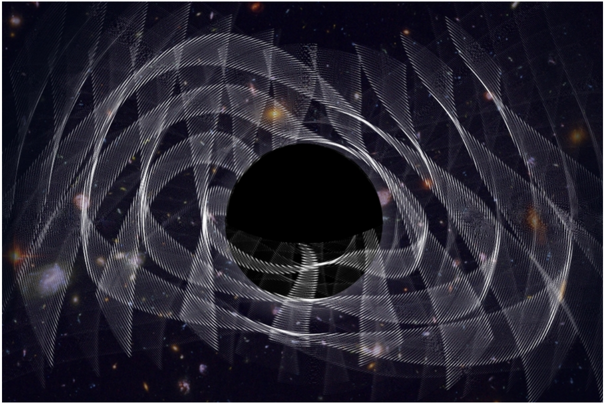
ABSTRACT

The gravitational potential $\phi = GM/Rc^2$ at the surface of the white dwarf G191−B2B is 10 000 times stronger than that at the Earth's surface. Numerous photospheric absorption features are detected, making this a suitable environment to test theories in which the fundamental constants depend on gravity. We have measured the fine-structure constant, α, at the white dwarf surface, used a newly calibrated *Hubble Space Telescope* (*HST*) Space Telescope Imaging Spectrograph spectrum of G191−B2B, two new independent sets of laboratory Fe V wavelengths, and new atomic calculations of the sensitivity parameters that quantify Fe V wavelength dependency on α. The two results obtained are: $\Delta\alpha/\alpha_0 = (6.36 \pm 0.35_{\rm stat} \pm 1.84_{\rm sys}) \times 10^{-5}$ and $\Delta\alpha/\alpha_0 = (4.21 \pm 0.48_{\rm stat} \pm 2.25_{\rm sys}) \times 10^{-5}$. The measurements hint that the fine-structure constant increases slightly in the presence of strong gravitational fields. A comprehensive search for systematic errors is summarized, including possible effects from line misidentifications, line blending, stratification of the white dwarf atmosphere, the quadratic Zeeman effect and electric field effects, photospheric velocity flows, long-range wavelength distortions in the *HST* spectrum, and variations in the relative Fe isotopic abundances. None fully account for the observed deviation but the systematic uncertainties are heavily dominated by laboratory wavelength measurement precision.

Key words: white dwarfs – cosmological parameters

Black holes in entangled relativity

Unlike in general relativity: cannot be vacuum solutions



Black holes

Charged black hole: special case of string dilaton charged black hole

With conformal transformation $\tilde{g}_{\alpha\beta} = e^{-2\varphi/\sqrt{3}} g_{\alpha\beta}$, and $\phi = e^{-\varphi/\sqrt{3}}$, one recovers an usual dilaton theory for wich charged black hole solutions have been found during the first superstring revolution. Garfinkle et al. [1991], Gibbons and Maeda [1988]

$$S \propto \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\tilde{\kappa}} \left(\tilde{R} - 2\tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right) - e^{-\varphi/\sqrt{3}} \tilde{F}^2 \right] \tag{32}$$

One only needs to make a conformal tansformation back to the original frame.

Charged black hole: Minazzoli and Santos [2021]

$$\boxed{\mathrm{d}s^2 = -\lambda_0^2 \mathrm{d}t^2 + \lambda_r^{-2} \mathrm{d}r^2 + \rho^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right)} \quad (33)$$

$$\rho^2 = r^2 \left(1 - \frac{r_-}{r}\right)^{6/13}$$
$$\lambda_0^2 = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{15/13} \quad (34)$$
$$\lambda_r^2 = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{7/13}$$

- Tends to **usual Schwarzschild metric** for $r_- \to 0$ ($F^2 \to 0$).
- Tends to **BH with scalar hair** for $r_+ \to 0$ ($F^2 \to 0$). But scalar charge gets radiated away via GWs during collapse (general theorem).

$\Rightarrow$ **Usual Schwarzschild metric is a good approximation of spherical black holes in entangled relativity for $F^2 \to 0$**

Charged black hole: Minazzoli and Santos [2021]

$$\boxed{ds^2 = -\lambda_0^2 dt^2 + \lambda_r^{-2} dr^2 + \rho^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)} \tag{35}$$

$$\begin{aligned} \rho^2 &= r^2 \left(1 - \frac{r_-}{r}\right)^{6/13} \\ \lambda_0^2 &= \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{15/13} \\ \lambda_r^2 &= \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{7/13} \end{aligned} \tag{36}$$

$$R = -\frac{Q^2}{(r - r_-)^{6/13} r^{46/13}} \qquad \mathcal{L}_m = \frac{Q^2}{(r - r_-)^{8/13} r^{44/13}} \qquad Q^2 = \frac{12}{13} r_- r_+$$

$$\Rightarrow \boxed{\kappa = -\frac{R}{\mathcal{L}_m}} = \frac{(r - r_-)^{2/13}}{r^{2/13}}$$

$\boxed{\kappa \text{ well defined at } \mathcal{L}_m \to 0 \text{ !!!}}$

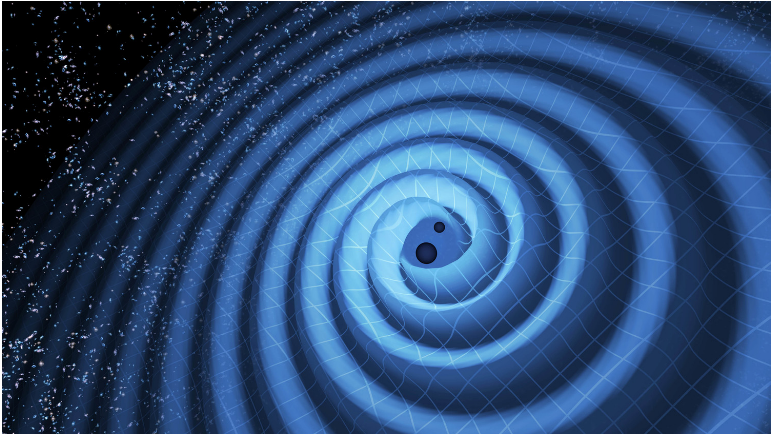
Charged black hole with rotation

With conformal transformation $\tilde{g}_{\alpha\beta} = e^{-2\varphi/\sqrt{3}} g_{\alpha\beta}$, and $\phi = e^{-\varphi/\sqrt{3}}$, one recovers an usual dilaton theory for wich charged black hole solutions have been found during the first superstring revolution.

$$S \propto \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\tilde{\kappa}} \left(\tilde{R} - 2\tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right) - e^{-\varphi/\sqrt{3}} \tilde{F}^2 \right] \qquad (37)$$

Solution valid for slow rotation found.
(Not published yet).

Gravitational waves



Gravitational waves: from the literature

From numerical studies in Einstein-Maxwell-dilaton theory

$$S \propto \int d^4x \sqrt{-g} \left[R - 2(\nabla\phi)^2 - e^{-2\alpha_0\phi} F^2 \right]. \tag{38}$$

"*Finally, an immediate conclusion of our work is that for small charges differences with respect to waveforms in [general relativity] and [Einstein-Maxwell-dilaton theory] are quite small*"
Hirschmann et al. [2018]

Confirmed with analytical study Khalil et al. [2018].

Reminder: entangled relativity (with $\tilde{\kappa} := 1$)

$$S \propto \int d^4x \sqrt{-\tilde{g}} \left[\tilde{R} - 2 \left(\tilde{\nabla}\varphi \right)^2 - e^{-\varphi/\sqrt{3}} \tilde{F}^2 \right] \tag{39}$$

Gravitational waves: from the literature

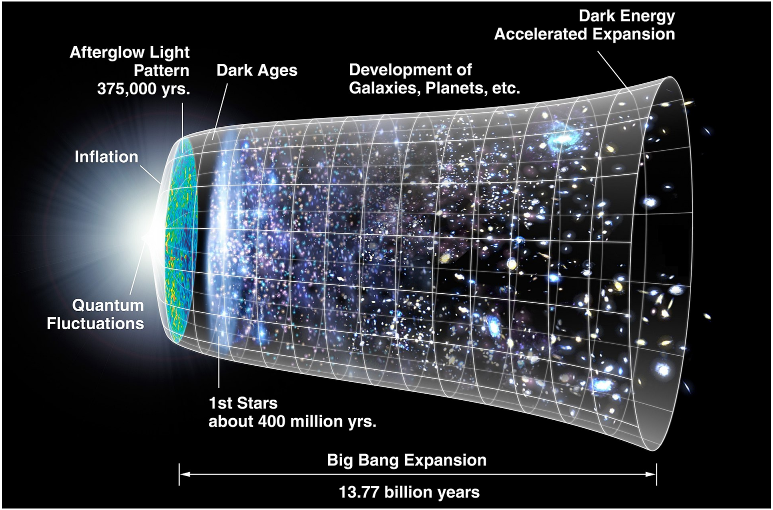
Binary black holes mergers

From studies for general dilaton theories, it appears that **gravitational waves from the merger of binary black holes in entangled relativity are indistiguishable from the ones of general relaitvity**—at least, provided that the matter part does not correspond to very specific types of dark matter candidates (like dark photons). Khalil et al. [2018]

Binary neutron stars mergers

Studies remain to be done. However, it might be degenerate with the various (unknown) equations of state one can consider to describe nuclear matter inside neutron stars. **Also: critically depends on on-shell matter Lagrangian!**

Cosmological evolution of the universe



Cosmological matter era and the Equivalence Principle

Assuming a flat Friedman-Lemaître-Robsertson-Walker metric, **the phenomenology of entangled relativity converges toward the one of general relativity without a cosmological constant during the matter era**. Minazzoli [2014], Minazzoli and Hees [2014]

That is $\boxed{\kappa(t) \to \kappa_0}$.

It means Newton's constant G is an assymptotic value of an actual field.

$$\text{Because dust + EM radiation} \Rightarrow \mathcal{L}_m = T.$$
$$\Rightarrow \boxed{\ddot{\varphi} + 3H\dot{\varphi} = 0} \qquad (\text{with } \varphi \propto \kappa^{-2}).$$

($\mathcal{L}_m^{\mathrm{EM}} \propto E^2 - B^2 = 0 = T^{\mathrm{EM}}$ for radiation).
NLO remains to be studied.

Cosmological matter era and the Equivalence Principle

The acceleration of the expansion of the universe is not Λ

$$\ddot{\varphi} + 3H\dot{\varphi} \propto \mathcal{L}_m - T \qquad (G \propto 1/\varphi)$$

CC: $\mathcal{L}_{DE} = \Lambda$ & $T = 4\Lambda \Rightarrow$ no decoupling $\Rightarrow \dot{G}/G \neq 0$

DE: if $\mathcal{L}_{DE} \neq T \Rightarrow$ no decoupling $\Rightarrow \dot{G}/G \neq 0$

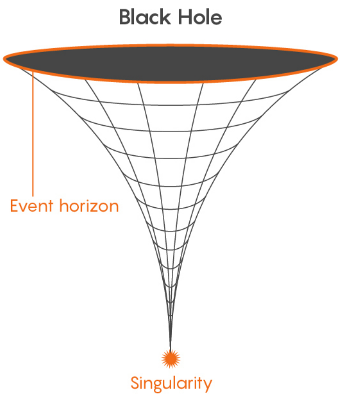
DE: if $\mathcal{L}_{DE} \approx T \Rightarrow$ decoupling $\Rightarrow \dot{G}/G \approx 0$

Really rigid framework as most (classical) dark energy candidates (including Λ) would induce a variation of G (and $\hbar$).

My take: the CC is likely UV sensitive anyway

The issue of dark energy can likely not be understood properly until quantum gravity is not understood well enough.

Singularities



Repulsive gravity?

Classical equivalence (provided $\mathcal{L}_m \neq \emptyset$)

$$-\frac{1}{2}\int d_g^4x \frac{\mathcal{L}_m^2}{R} \equiv \int d_g^4x \frac{1}{\kappa}\left(\frac{R}{2\kappa} + \mathcal{L}_m\right) \quad (40)$$

κ: dimensionfull scalar field

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu} + \kappa^2\left[\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right]\frac{1}{\kappa^2} \quad (41)$$

$$\nabla_\sigma\left(\kappa^{-1}T^{\alpha\sigma}\right) = \mathcal{L}_m\nabla^\alpha\kappa^{-1} \quad (42)$$

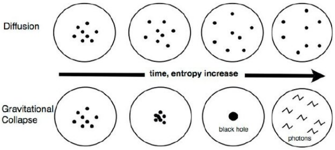
Trace metric field equation: $$3\kappa^2\Box\frac{1}{\kappa^2} = \kappa\left(T - \mathcal{L}_m\right) \quad (43)$$

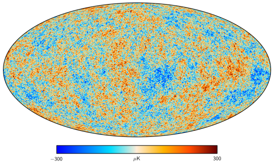
$\kappa < 0$ is not prohibited a priori! $\Rightarrow$ repulsive gravity?

Repulsive gravity?

General relativity may have the following issues:

- Singularities (black holes, big-bang) $\kappa < 0$ not prohibited (could also potentially explain the apparently low entropy of the primordial universe)
- ~~Spacetime might not have an operational meaning beyond the Planck scale~~ ✓(Circumstantial evidence)
- ~~Inertia can be defined from nothing~~ ✓





Conclusion: simply a non-linear reformulation of GR

$$\boxed{\boxed{\text{GR:}} \quad \theta = \int \frac{d_g^4 x}{c\hbar} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right)} \longrightarrow \boxed{\theta = -\int \frac{d_g^4 x}{2\epsilon^2} \frac{\mathcal{L}_m^2}{R} \quad \boxed{\text{:ER}}}$$

- More economical than GR: only 2 universal parameters.
- Seems to satisfy the 3 principles demanded by Einstein in 1918.
- Has both GR and standard QFT in the same (predicted) limit.
- Predicts that $\hbar$ varies in (or next to) dense environments.
- Does not have an elementary length or time scale:
  - The smooth/continuous space-time likely fundamental.
  - Quantum gravity should likely be obtained from the path integral.
  - The effect of the matter fields quantum vacuum on space-time should be pretty different. But possibly computable without the need of a cutoff at the Planck energy!

Conclusion: Many (difficult) things remain to be studied

- How to explain the acceleration of the expansion of the universe? tentative solution given in Minazzoli [2021].
  - Possibly related to quantum gravity.
- Detailed cosmological & astrophysical studies are needed.
  - Effect of cosmic magnetic fields? ($\mathcal{L}_m \propto B^2 \neq T = 0$).
- On-shell Lagrangian for realistic fluids to be derived from first principles!
- What does the theory predict inside black holes, or near the big bang?
- How to evaluate the path integral? (Still need for lattice approximation schemes (Regge, causal dynamical triangulation)?)
- related: **What is $\mathcal{L}_m$ in the UV?** (Standard model of particles built upon the flat spacetime approximation $\rightarrow$ new ideas are likely required here due to pure nonlinear coupling).

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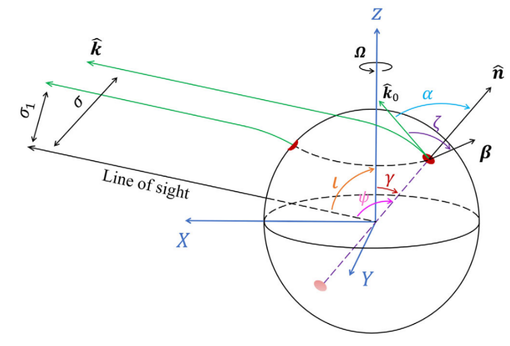
Neutron stars

Toward experimental tests

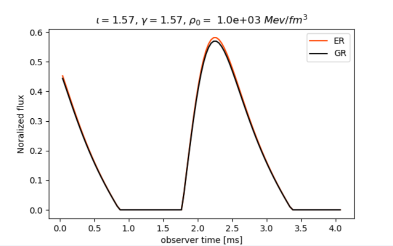
Schematic illustration for the x-rays emitted from a hot spot on a rotating NS and reaching the observer at infinity

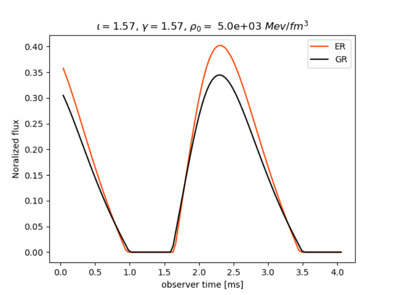


2007.10080

Next step: simulate X-ray pulse profiles (to be eventually tested with the NICER instrument)

Preview: unpublished results of Denis Arruga





Takes into account the non-conservation of the photon number close to the neutron star, which follows from $\nabla_\nu \left(\sqrt{\Phi} F^{\mu\nu}\right) = 0$.
Same assumptions as in 2007.10080 otherwise.

Path integral formulation ($\kappa \approx$ constant only)

Only κ is considered to be constant (as a consequence of the intrinsic decoupling)

$$Z_{\rm ER} \approx \int \mathcal{D}g \prod_i \mathcal{D}f_i \exp\left[\frac{i}{\kappa\epsilon^2}\int d^4_g x\left(\frac{R}{2\kappa} + \mathcal{L}_m\right)\right] \quad (44)$$

Interpretations

- The Core theory of physics can be a limit of entangled relativity when $\kappa \approx$ constant.
- $\mathcal{L}_m$ is unknown at the fundamental level but must be such that it is the standard model of particles when $\kappa = 8\pi G/c^4$, with G being Newton's constant measured on Earth.

Classical limit: gravity

Classical equivalence (provided $\mathcal{L}_m \neq \emptyset$)

$$-\frac{1}{2}\int \mathrm{d}_g^4 x \frac{\mathcal{L}_m^2}{R} \equiv \frac{1}{\bar{\kappa}}\int \mathrm{d}_g^4 x \left(\frac{\varphi^2 R}{2\bar{\kappa}} + \varphi \mathcal{L}_m\right) \qquad (45)$$

φ: dimension-less scalar field ⇑ Cauchy well-posed ⇑

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{\bar{\kappa}}{\varphi}T_{\mu\nu} + \varphi^{-2}\left[\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box\right]\varphi^2 \qquad (46)$$

$$\nabla_\sigma\left(\varphi T^{\alpha\sigma}\right) = \mathcal{L}_m \nabla^\alpha \varphi \qquad (47)$$

Trace metric field equation: $3\varphi^{-2}\Box\varphi^2 = \bar{\kappa}\left(T - \mathcal{L}_m\right)$ (48)

$$\varphi = -\bar{\kappa}\frac{\mathcal{L}_m}{R} \qquad (49)$$

Origins

Class of theories with intrinsic decoupling found with Aurélien

$$S = \frac{1}{c} \int \mathrm{d}_g^4 x \left[\frac{1}{2\alpha} \left(\Phi R - \frac{\omega(\Phi)}{\Phi} (\partial_\sigma \Phi)^2 \right) + \sqrt{\Phi} \mathcal{L}_m \right] \qquad (50)$$

$$R^{\mu\nu} = \alpha \frac{1}{\sqrt{\Phi}} \left[T^{\mu\nu} - \frac{1}{2} g^{\mu\nu} T \right] + \frac{1}{\Phi} \left[\nabla^\mu \partial^\nu \Phi + \frac{1}{2} g^{\mu\nu} \Box \Phi \right] + \frac{\omega(\Phi)}{\Phi^2} \partial^\mu \Phi \partial^\nu \Phi, \qquad (51)$$

and

$$\frac{2\omega(\Phi) + 3}{\Phi} \Box \Phi + \frac{\omega_\Phi(\Phi)}{\Phi} (\partial_\sigma \Phi)^2 = \alpha \frac{1}{\sqrt{\Phi}} [T - \mathcal{L}_m] \qquad (52)$$

Entangled relativity corresponds to $\omega(\Phi) = 0$.
$(\gamma = \beta = 1 \ \forall \ \omega \neq -3/2)$