

# Testing General Relativity with Black Hole X-ray Data

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# Motivations

# Tests of General Relativity

- 1915 → General Relativity (Einstein)

- 1919 → Deflection of light by the Sun (Eddington)
- 1960s → Solar System experiments
- 1970s → Binary pulsars

- 2000s → Cosmological tests

Tests in weak  
gravitational fields

Tests on large scales

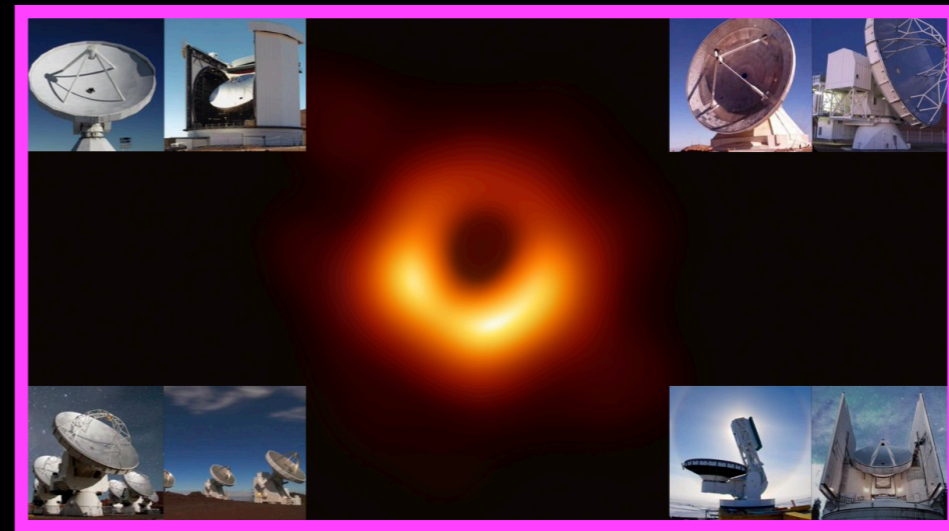
# Tests of General Relativity

- 2010s → Black holes, neutron stars

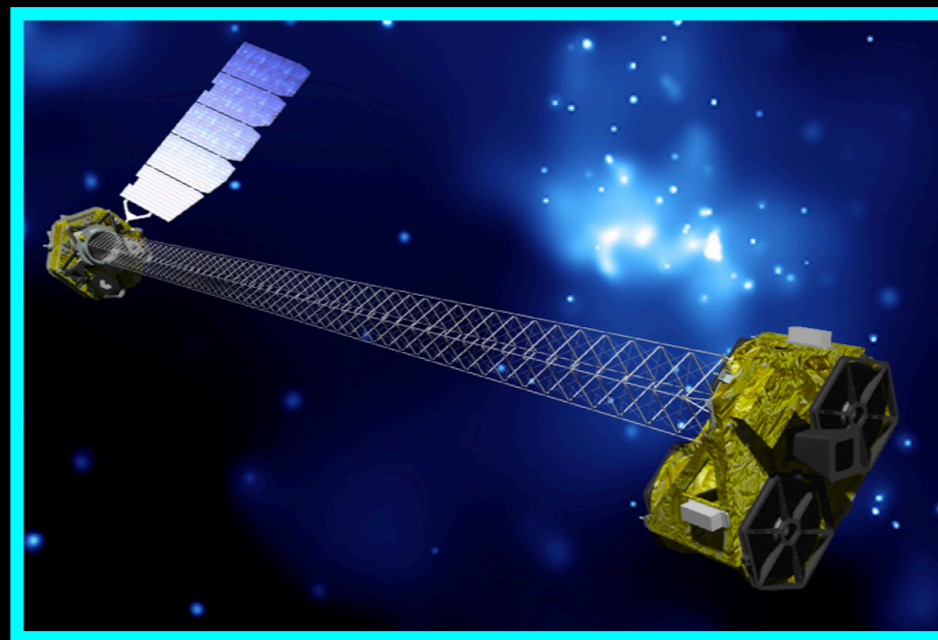
Tests in strong  
gravitational fields



GW Data



VLBI Data



X-ray Data

# Black Holes

# Black Holes in General Relativity

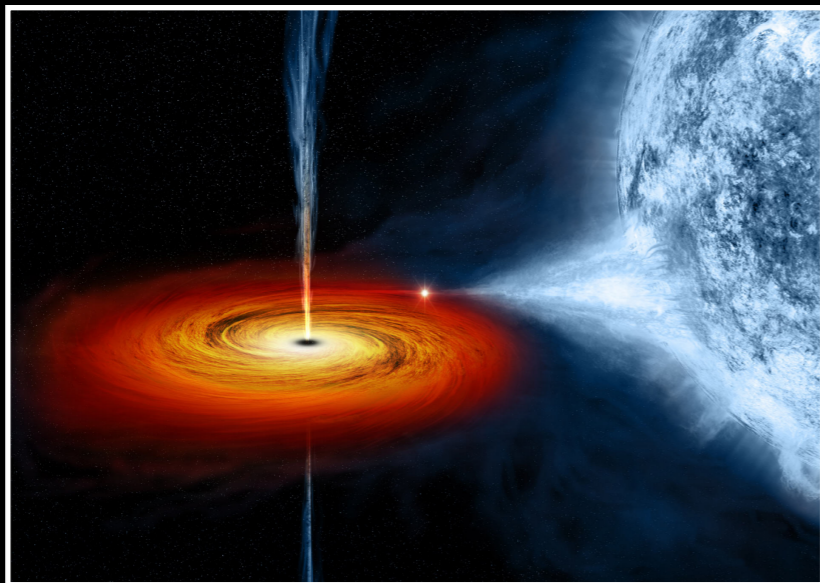
- **No-Hair Theorem**: in 4D General Relativity, black holes are fully characterized by a small number of parameters:  $M$ ,  $J$ ,  $Q$
- **Uniqueness Theorem**: in 4D General Relativity, black holes are only described by the Kerr-Newman solution
- Uncharged black holes are described by the **Kerr solution**

# Beyond General Relativity

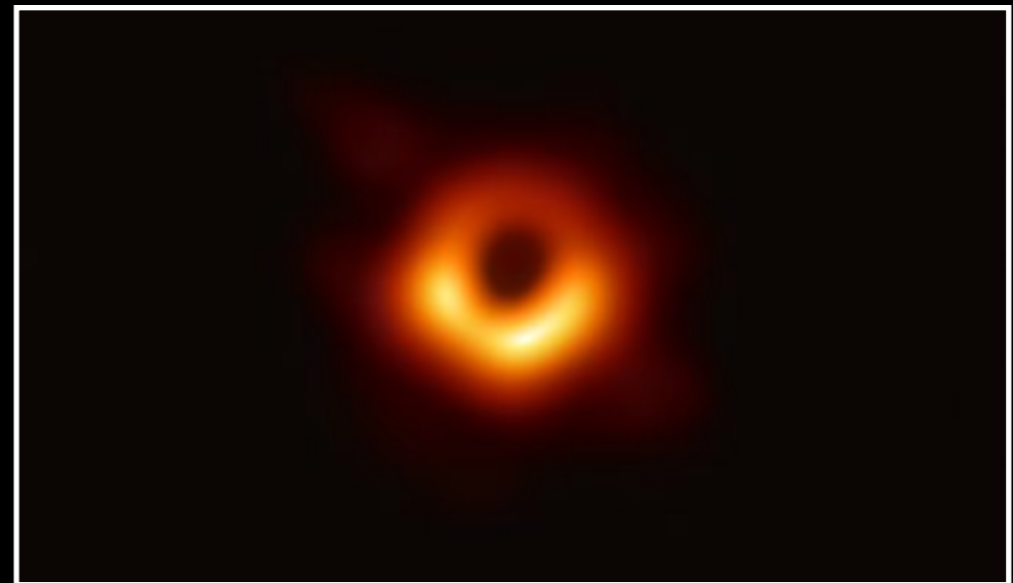
- **Modified theories of gravity:**
  - Einstein-dilaton-Gauss-Bonnet gravity
  - Chern-Simons gravity
  - Lorentz-violating theories
  - ...
- **Macroscopic quantum gravity effects (information paradox):**
  - Mathur (Fuzzballs)
  - Dvali & Gomez
  - Giddings
  - ...
- **Presence of exotic matter:**
  - Hairy black holes (Herdeiro & Radu)
  - ...

# Astrophysical Black Holes

Stellar-mass black holes  
( $\sim 3-100 M_{\odot}$ )



Supermassive black holes  
( $\sim 10^5-10^{10} M_{\odot}$ )



Intermediate-mass  
black holes





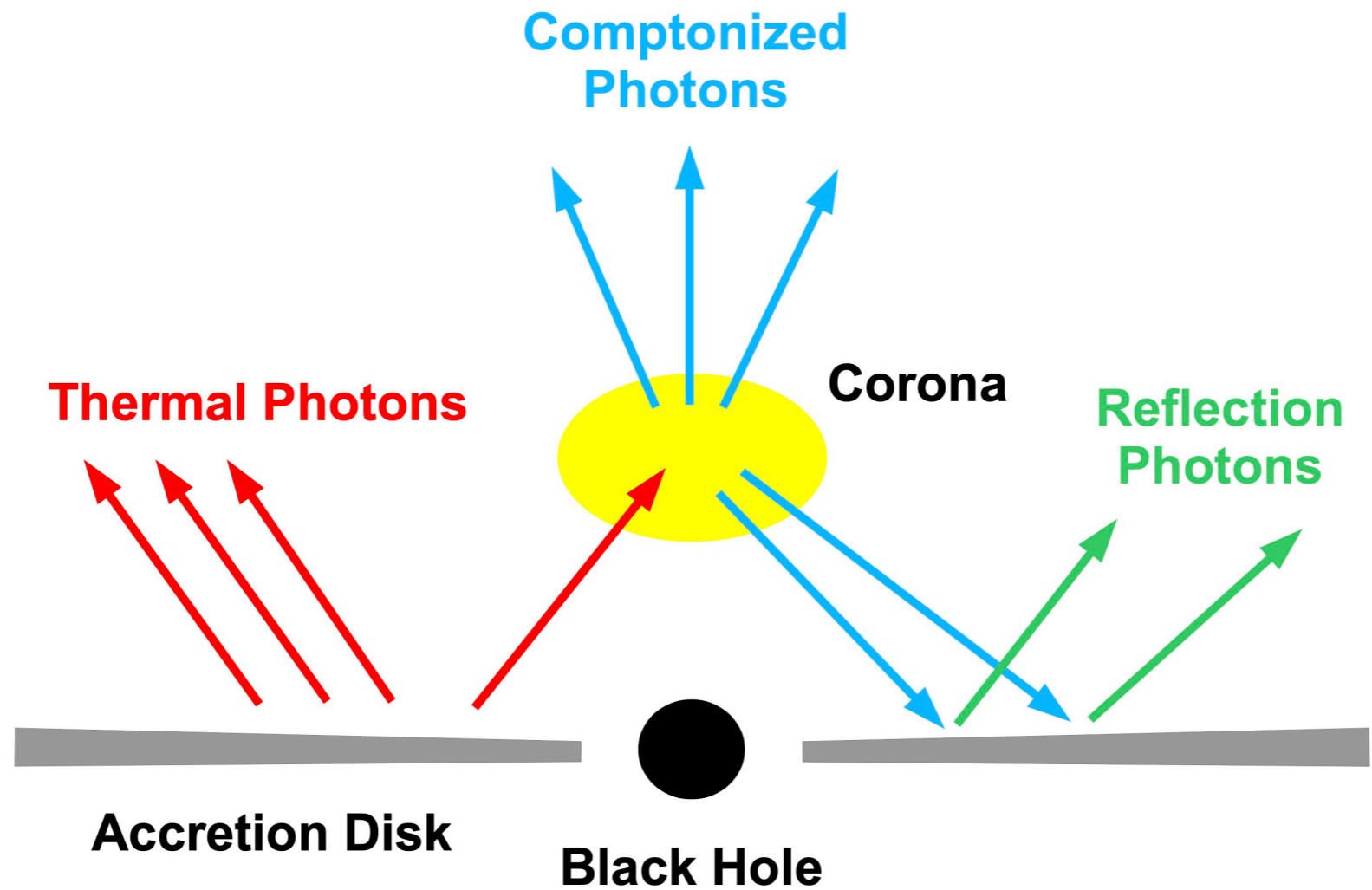
# Astrophysical Black Holes

It is remarkable that the spacetime metric around astrophysical black holes formed from gravitational collapse of stars/clouds should be approximated well by the "ideal" Kerr metric

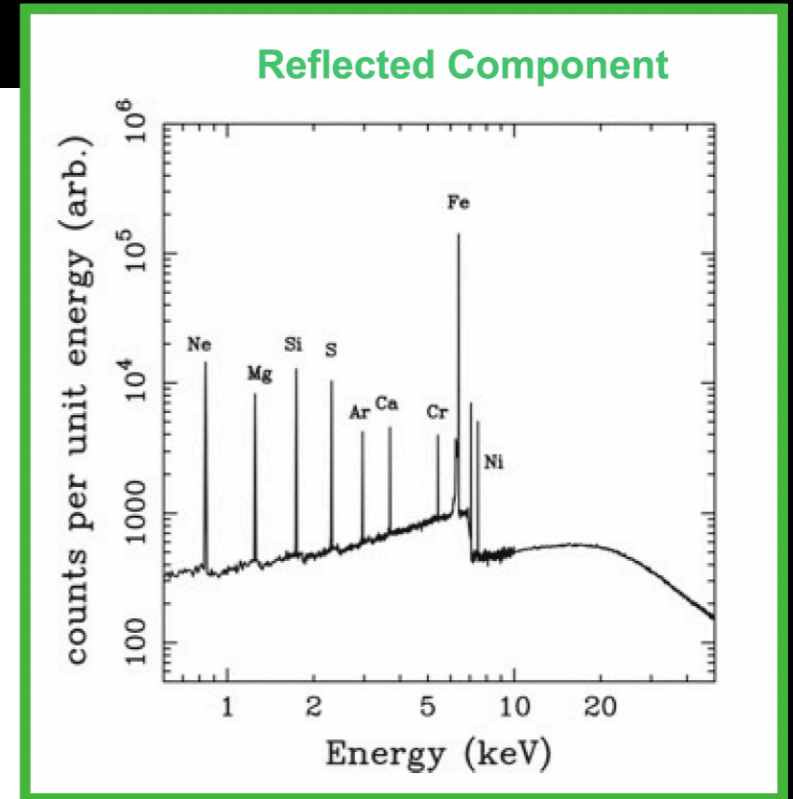
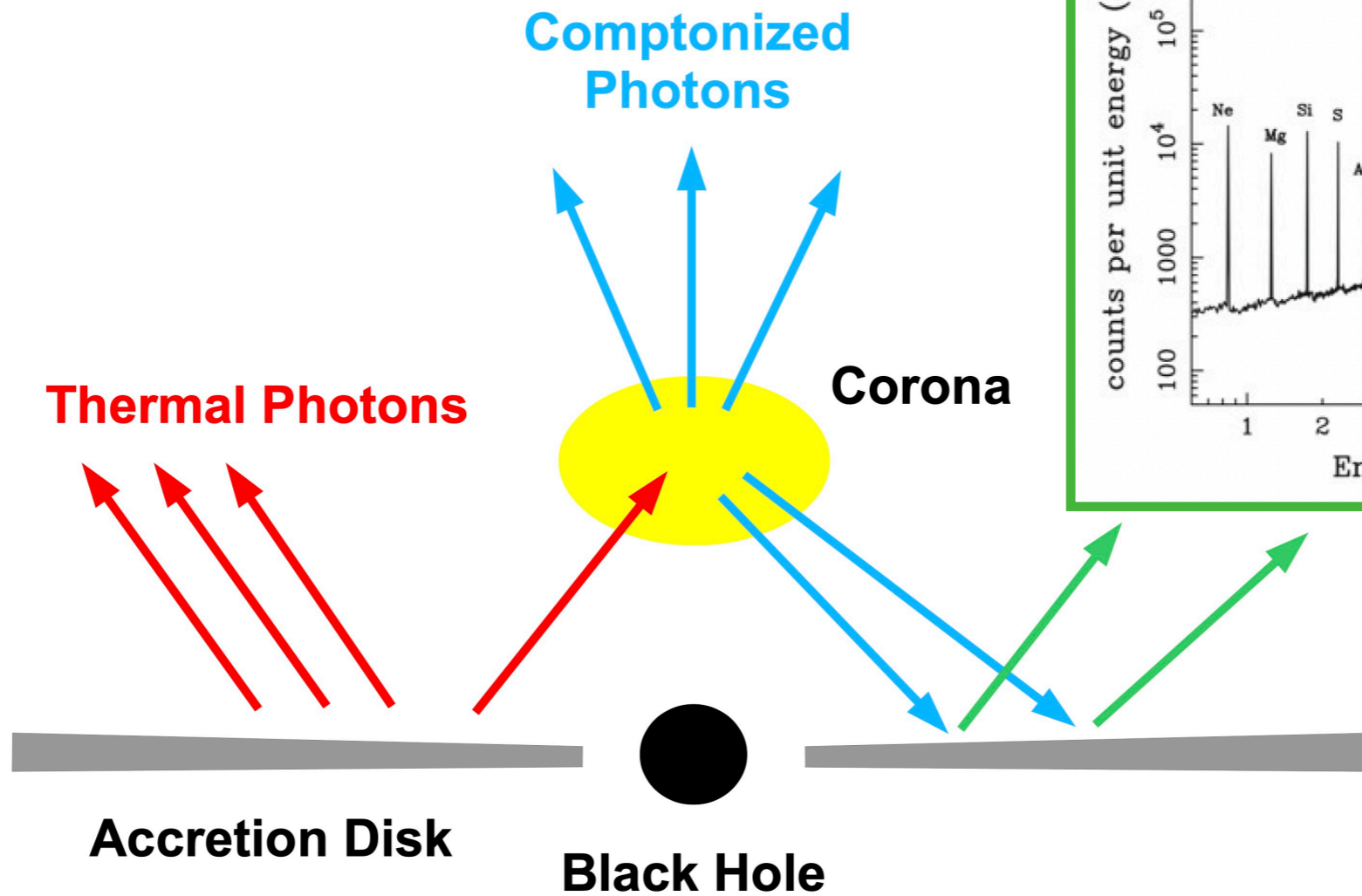
- Initial deviations → Quickly radiated away by GWs
- Accretion disk, nearby stars → Negligible
- Electric charge → Negligible

# Disk-Corona Model

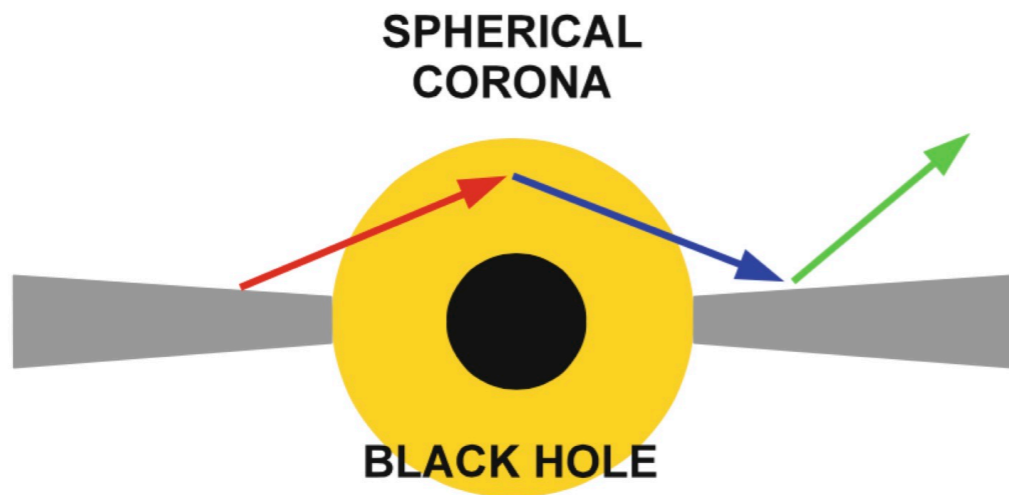
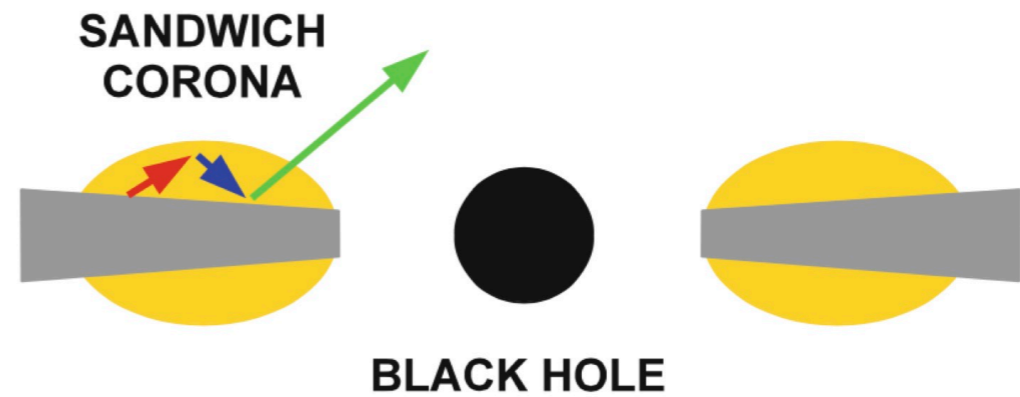
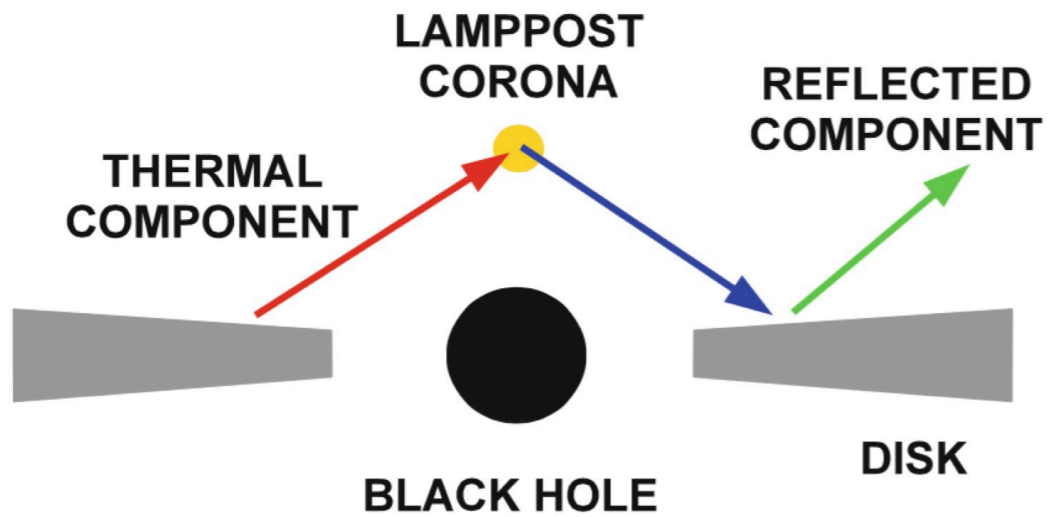
# Disk-Corona Model



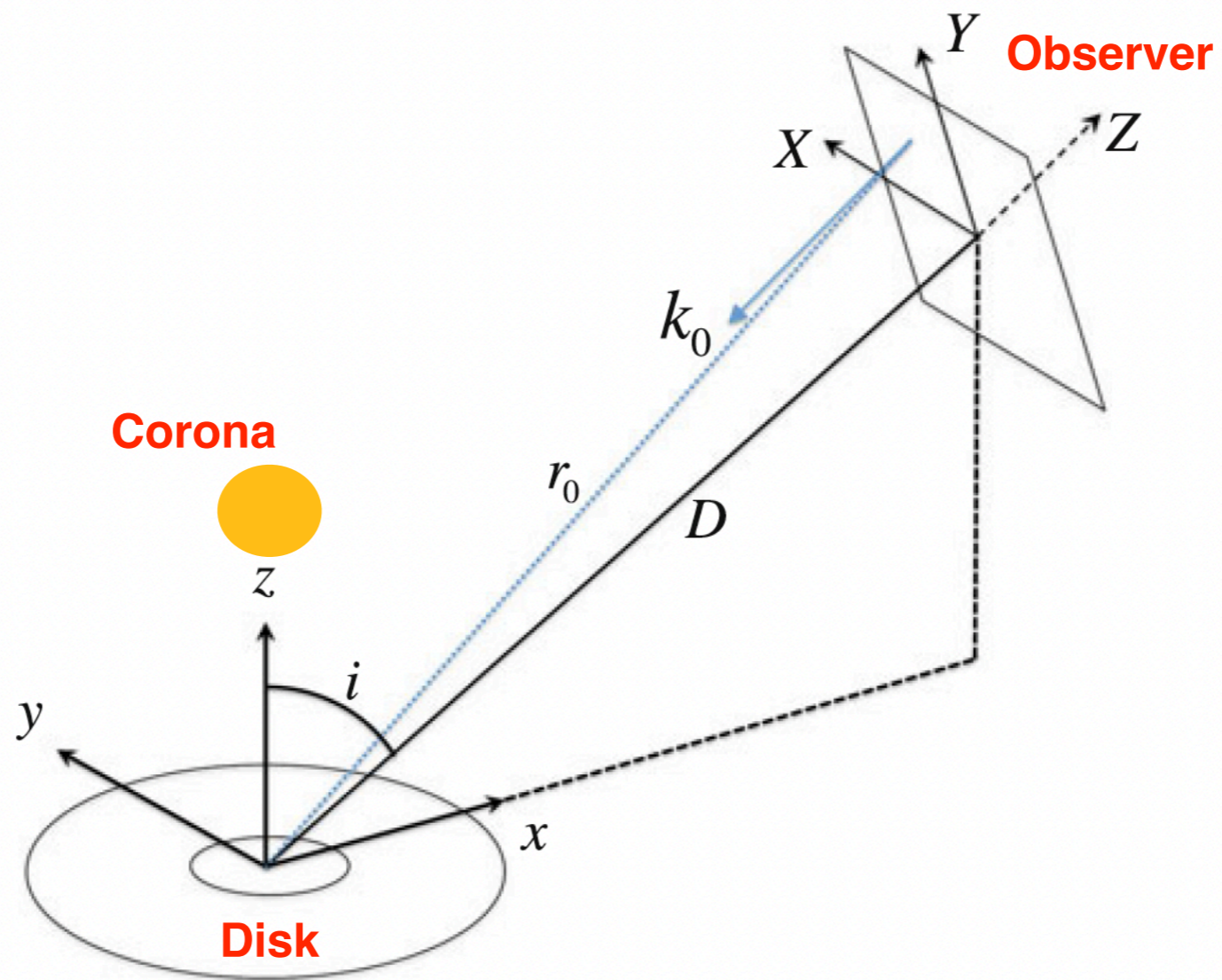
# Disk-Corona Model



# Coronal Geometries



# Synthetic Spectra



# Synthetic Spectra

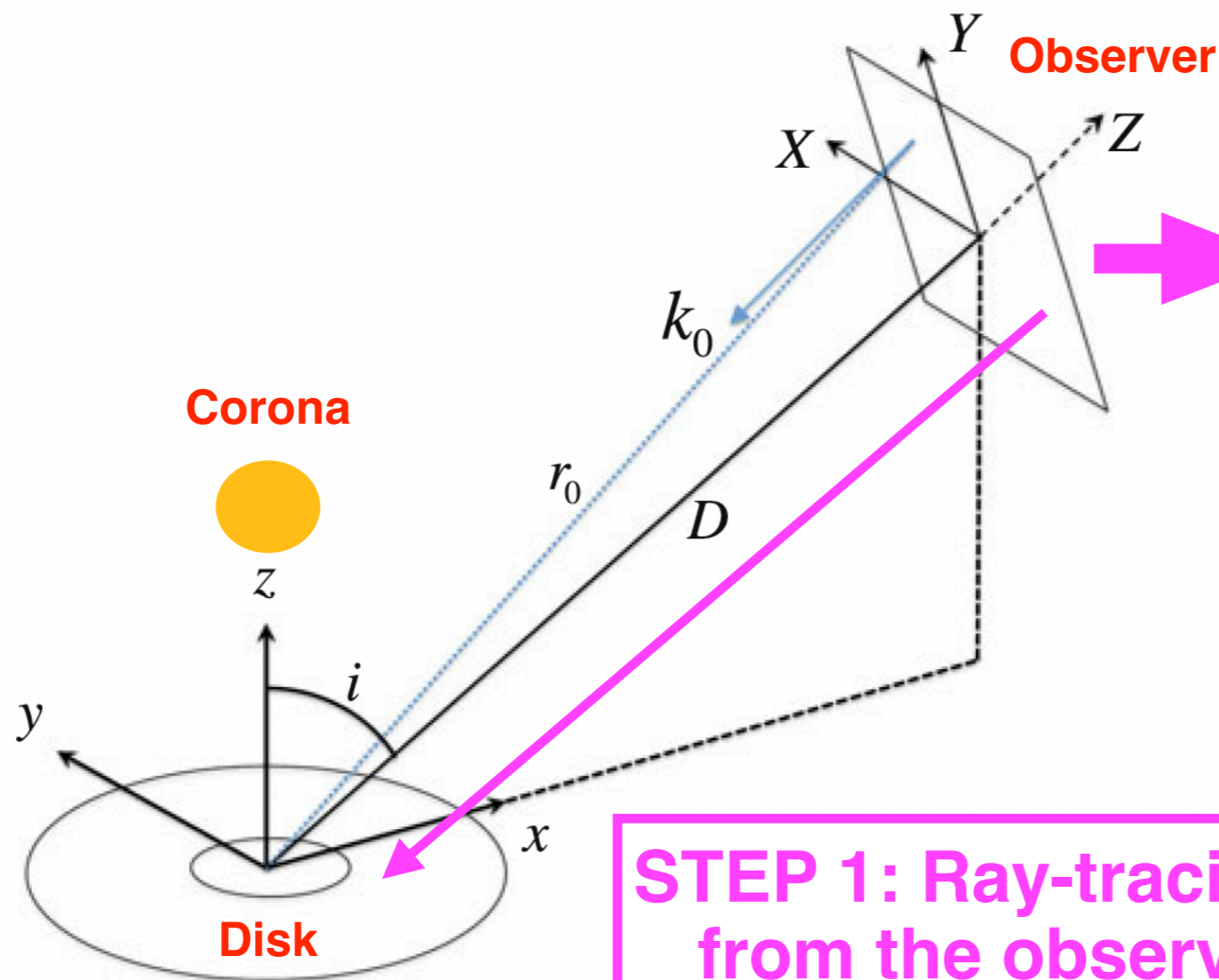
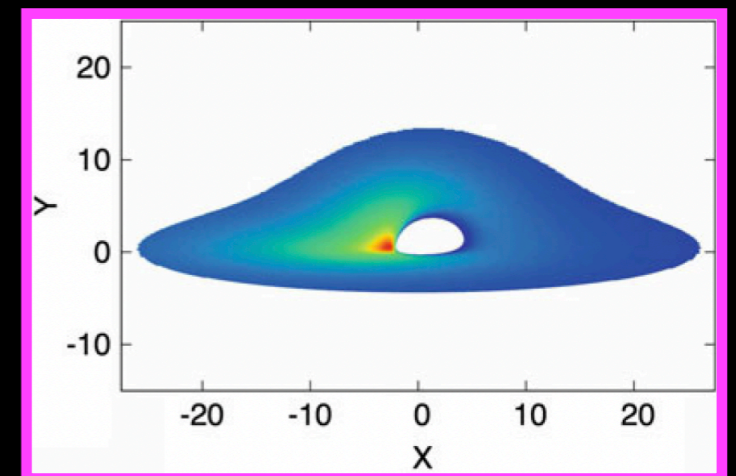


Image of the accretion disk

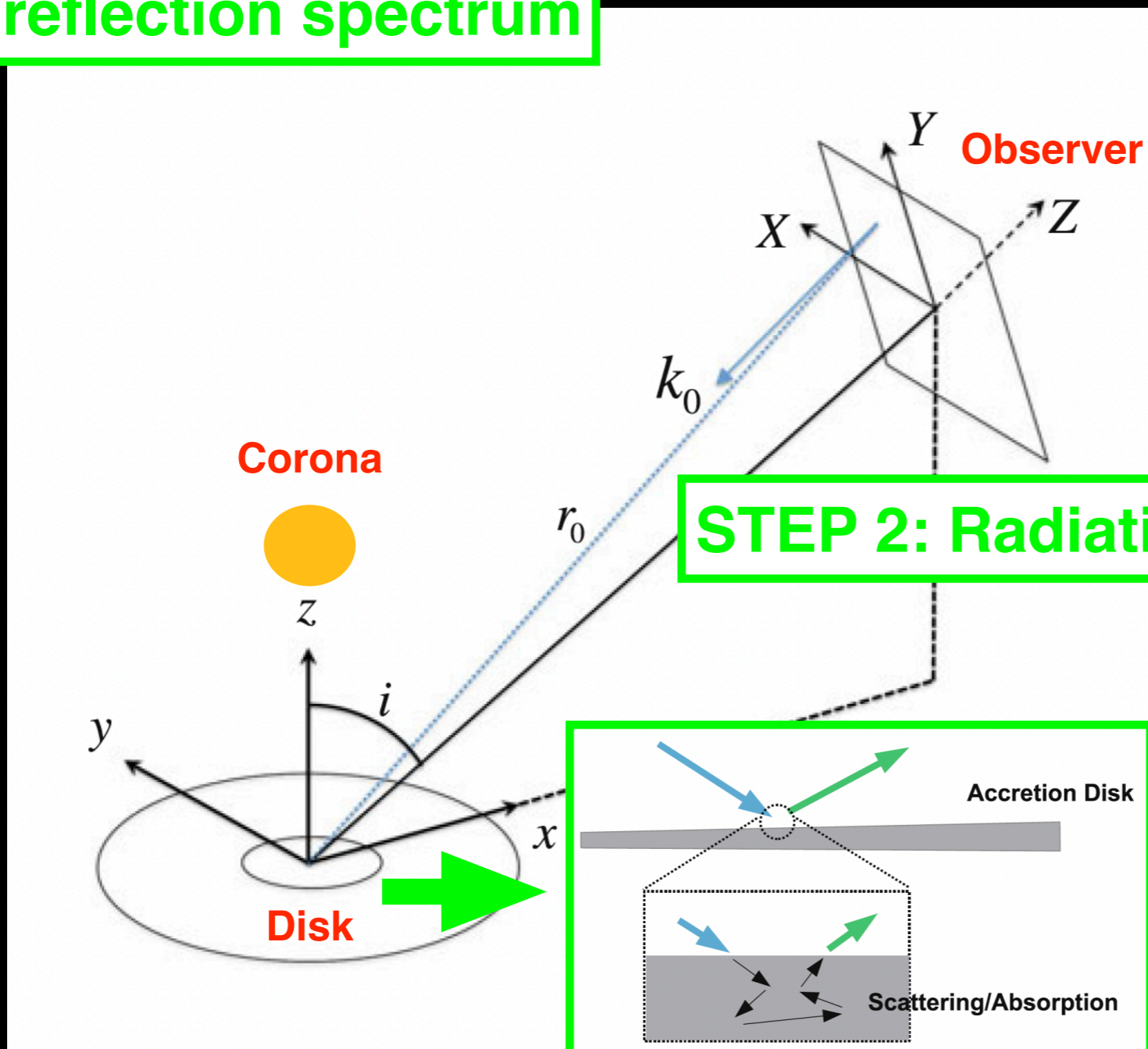


Every point on the image is characterized by its redshift factor  $g$

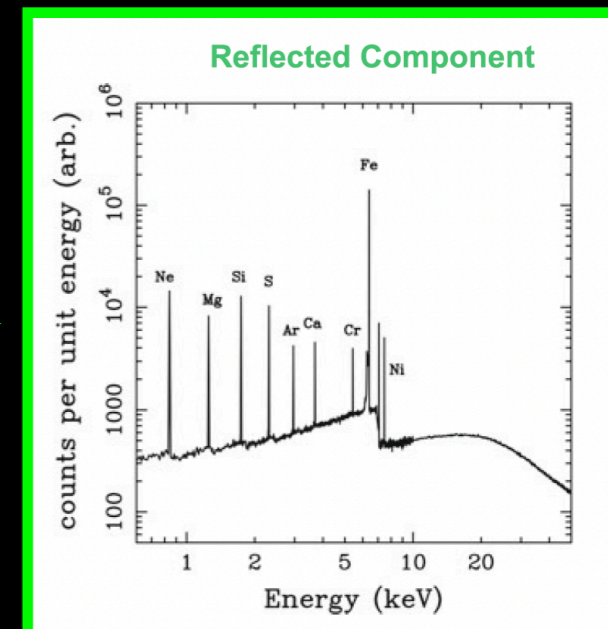
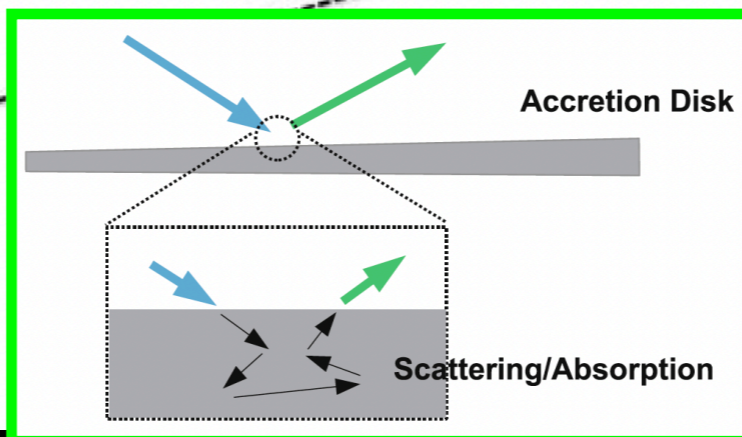
**STEP 1: Ray-tracing calculations from the observer to the disk**

# Synthetic Spectra

For reflection spectrum



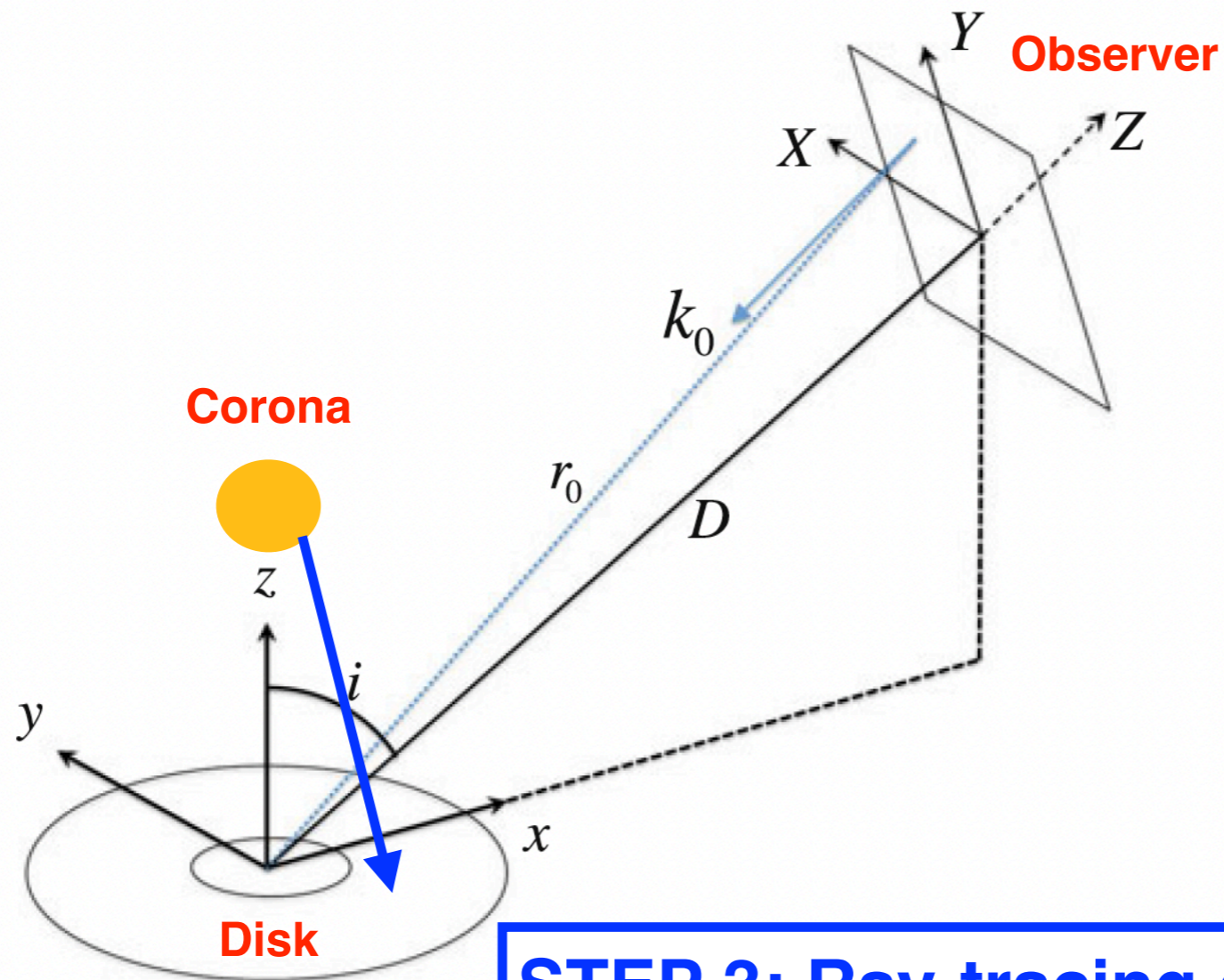
STEP 2: Radiative transfer equations





# Synthetic Spectra

For reflection spectrum



**STEP 3: Ray-tracing calculations from the corona to the disk**

# Synthetic Spectra

- Photon motion (from the observer to the disk)
  - Particle motion on the disk
  - Radiative transfer equations (atomic physics, fundamental constants)
  - Photon motion (from the corona to the disk)
  - Particle motion on the disk
- 
- Accretion disk model
  - Corona model

# **Testing Black Holes with X-ray Data**

# Synthetic Spectra

- Photon motion (from the observer to the disk)
- Particle motion on the disk
- Radiative transfer equations (atomic physics, fundamental constants)

- Photon motion (from the observer to the disk)
- Particle motion on the disk

- Accretion disk model
- Corona model

**We can test the Kerr metric**

**We can test the geodesic motion**

# Synthetic Spectra

- Photon motion (from the observer to the disk)
  - Particle motion on the disk
  - Radiative transfer equations (atomic physics, fundamental constants)
  - Photon motion (from the observer to the disk)
  - Particle motion on the disk
  - Accretion disk model
  - Corona model
- We can test atomic physics and the values of fundamental constants in strong gravitational fields**

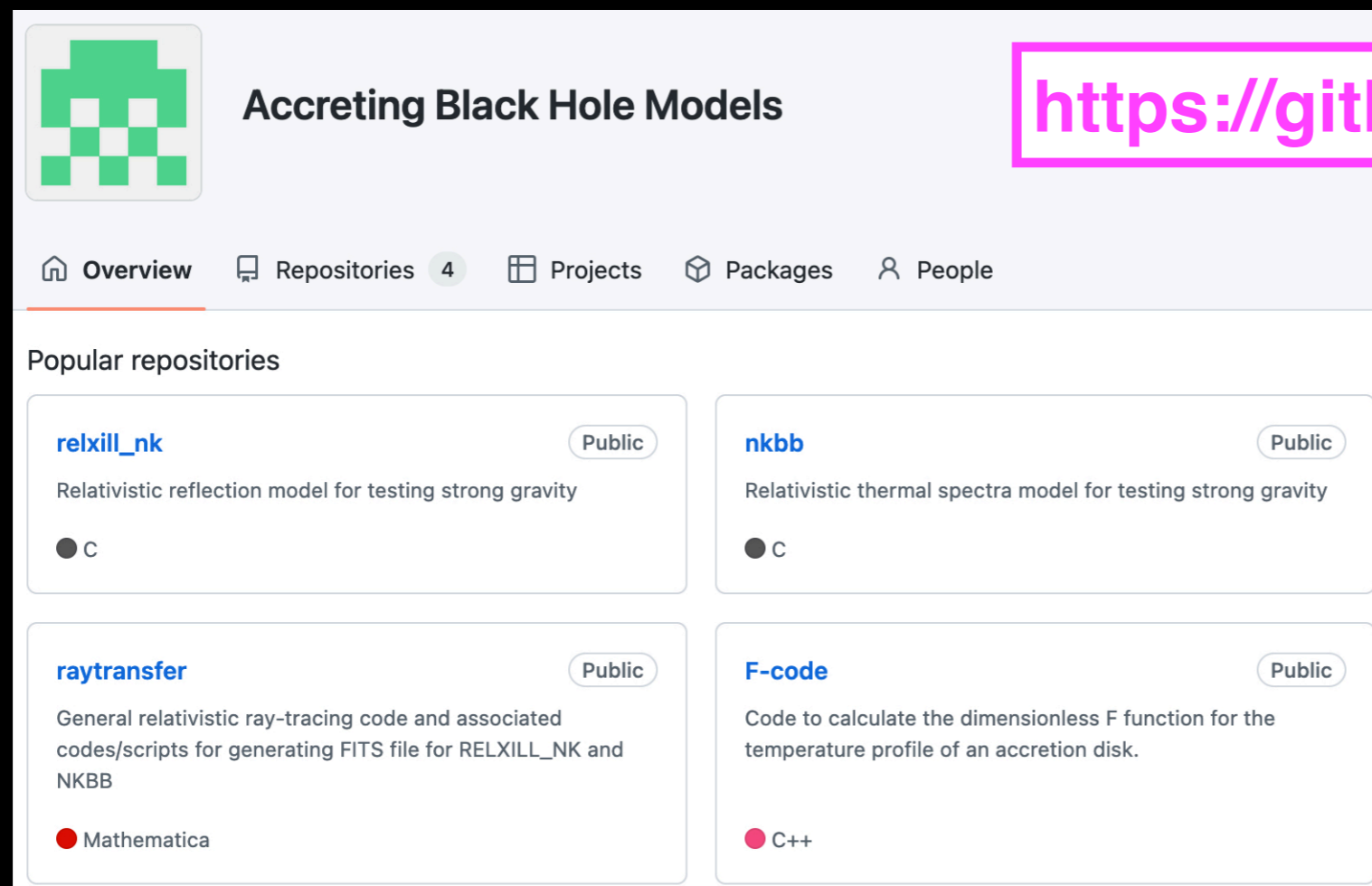
# XSPEC Models

- `relxill_nk` (Bambi et al. 2017; Abdikamalov et al. 2019)

Reflection spectrum for thin accretion disks in stationary, axisymmetric, and asymptotically-flat spacetimes

- `nkbb` (Zhou et al. 2019)

Thermal spectrum for thin accretion disks in stationary, axisymmetric, and asymptotically-flat spacetimes



The screenshot shows the GitHub repository page for 'Accreting Black Hole Models'. The repository is public and contains four popular repositories:

- `relxill_nk`: Relativistic reflection model for testing strong gravity. Language: C.
- `nkbb`: Relativistic thermal spectra model for testing strong gravity. Language: C.
- `raytransfer`: General relativistic ray-tracing code and associated codes/scripts for generating FITS file for RELXILL\_NK and NKBB. Language: Mathematica.
- `F-code`: Code to calculate the dimensionless F function for the temperature profile of an accretion disk. Language: C++.

<https://github.com/ABHModels>

# Strategies

# Strategies

- **Top-down approach**: we test a specific alternative theory of gravity against Einstein's theory of General Relativity. Problems:
  - There are many theories of gravity...
  - Usually we do not know their rotating black hole solutions...
- **Bottom-up approach**: we parametrize possible deviations from General Relativity with a number of phenomenological "deformation parameters"



# Example: PPN Formalism

- Parametrized Post-Newtonian (PPN) formalism
- Weak field limit:  $M/r \ll 1$
- Solar System experiments

$$ds^2 = - \left( 1 - \frac{2M}{r} + \beta \frac{2M^2}{r^2} + \dots \right) dt^2 + \left( 1 + \gamma \frac{2M}{r} + \dots \right) (dx^2 + dy^2 + dz^2)$$

$$|\beta - 1| < 2.3 \cdot 10^{-4} \quad (\text{Lunar Laser Ranging experiment})$$

$$|\gamma - 1| < 2.3 \cdot 10^{-5} \quad (\text{Cassini spacecraft})$$

- In General Relativity (Schwarzschild metric):  $\beta = \gamma = 1$

# Example: Johannsen Metric

- Several parametric black hole spacetimes proposed in the literature
- Johannsen metric (Johannsen 2013):

$$ds^2 = -\frac{\tilde{\Sigma} (\Delta - a^2 A_2^2 \sin^2 \theta)}{B^2} dt^2 + \frac{\tilde{\Sigma}}{\Delta A_5} dr^2 + \tilde{\Sigma} d\theta^2$$

$$- \frac{2a [(r^2 + a^2) A_1 A_2 - \Delta] \tilde{\Sigma} \sin^2 \theta}{B^2} dt d\phi$$

$$+ \frac{[(r^2 + a^2)^2 A_1^2 - a^2 \Delta \sin^2 \theta] \tilde{\Sigma} \sin^2 \theta}{B^2} d\phi^2,$$

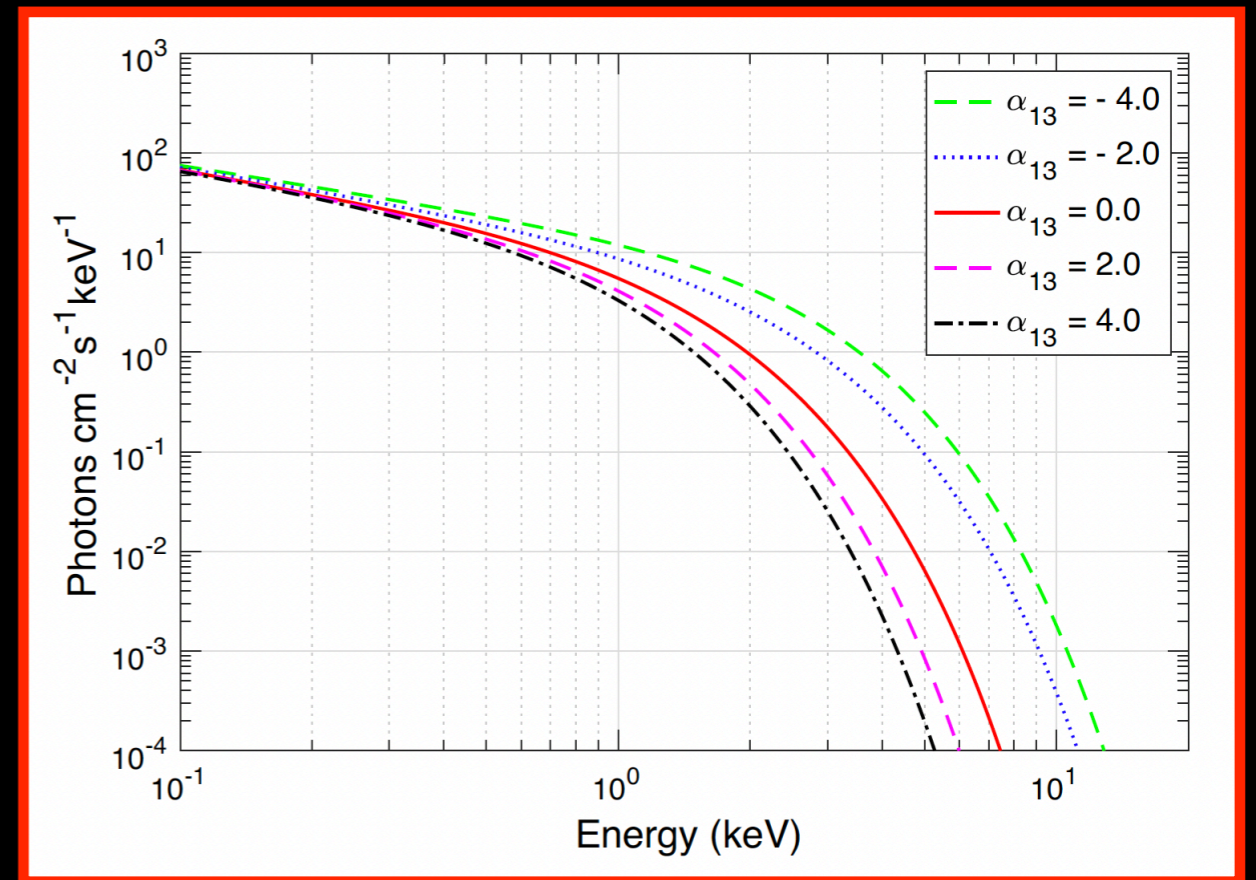
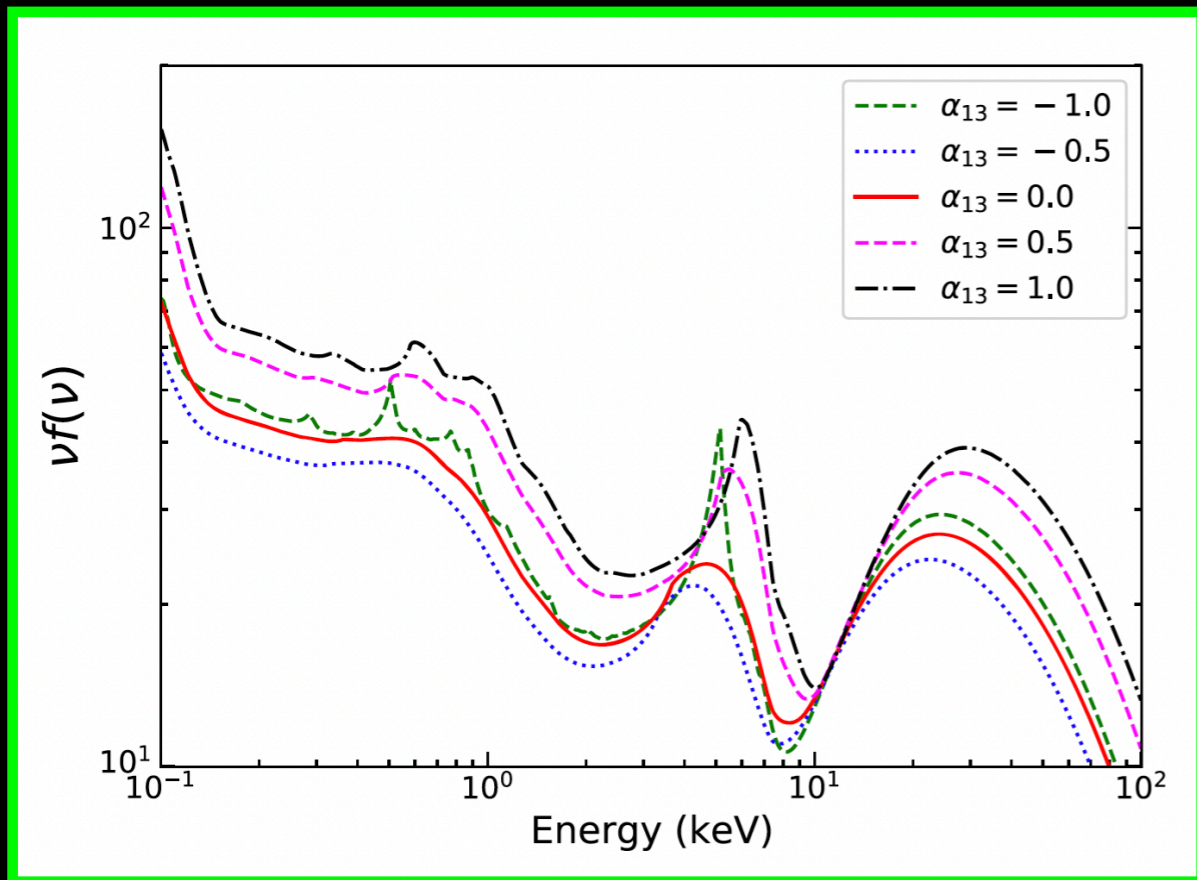
$$\tilde{\Sigma} = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - 2Mr + a^2,$$

$$B = (r^2 + a^2) A_1 - a^2 A_2 \sin^2 \theta$$

$$f = \sum_{n=3}^{\infty} \epsilon_n \frac{M^n}{r^{n-2}}, \quad A_1 = 1 + \sum_{n=3}^{\infty} \alpha_{1n} \left(\frac{M}{r}\right)^n,$$

$$A_2 = 1 + \sum_{n=2}^{\infty} \alpha_{2n} \left(\frac{M}{r}\right)^n, \quad A_5 = 1 + \sum_{n=2}^{\infty} \alpha_{5n} \left(\frac{M}{r}\right)^n$$

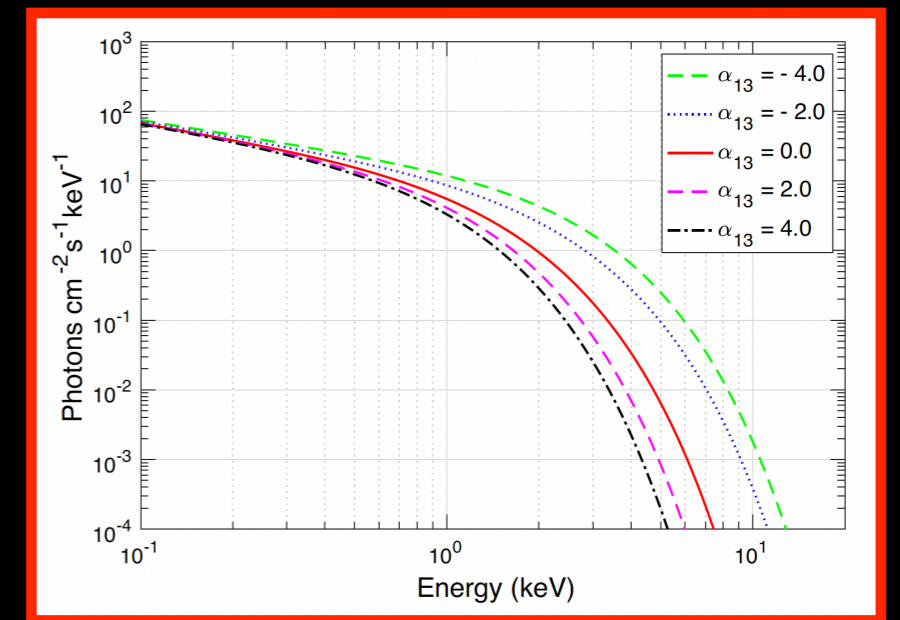
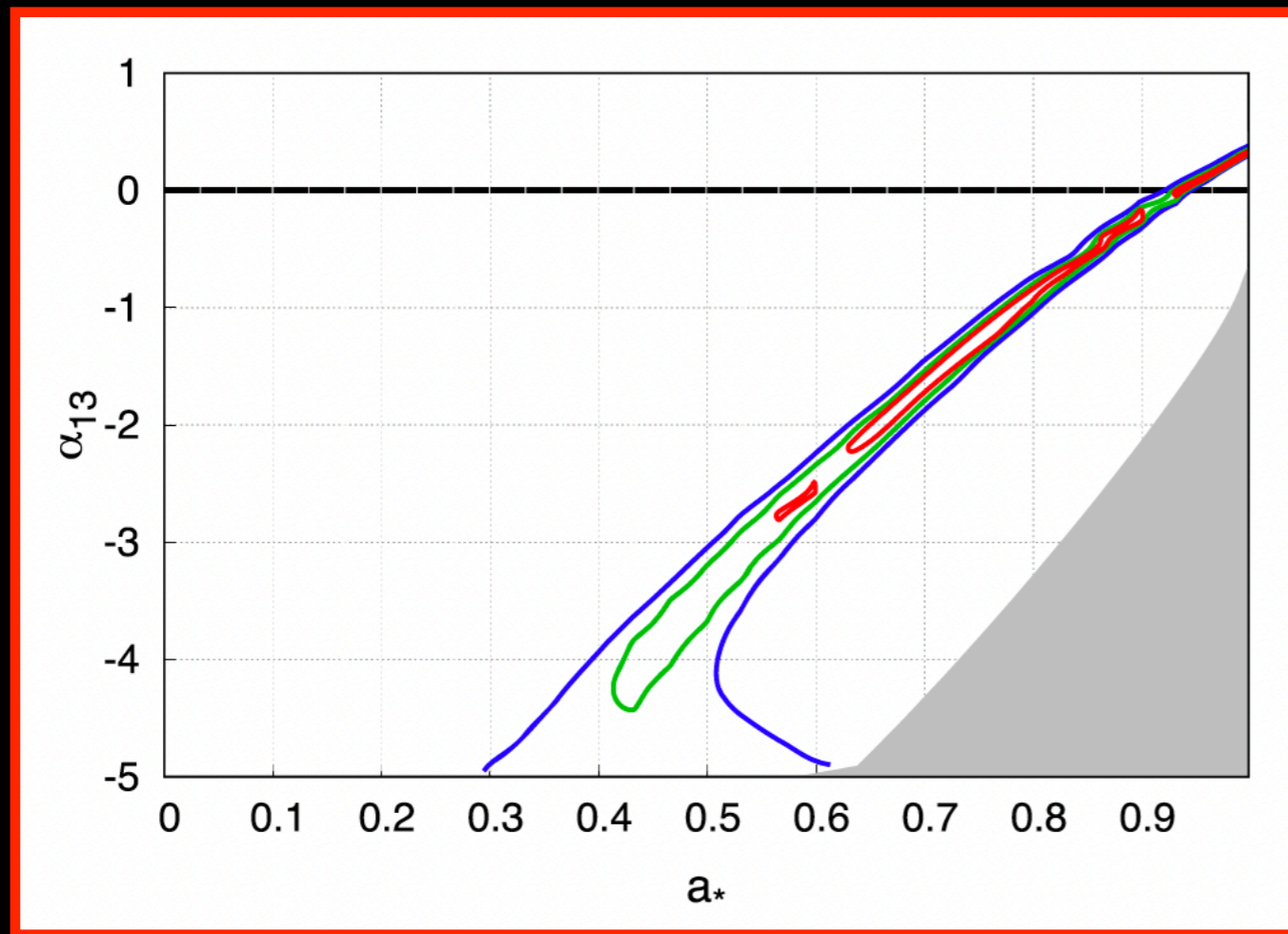
# Example: Johannsen Metric



# **Results: Agnostic Tests**

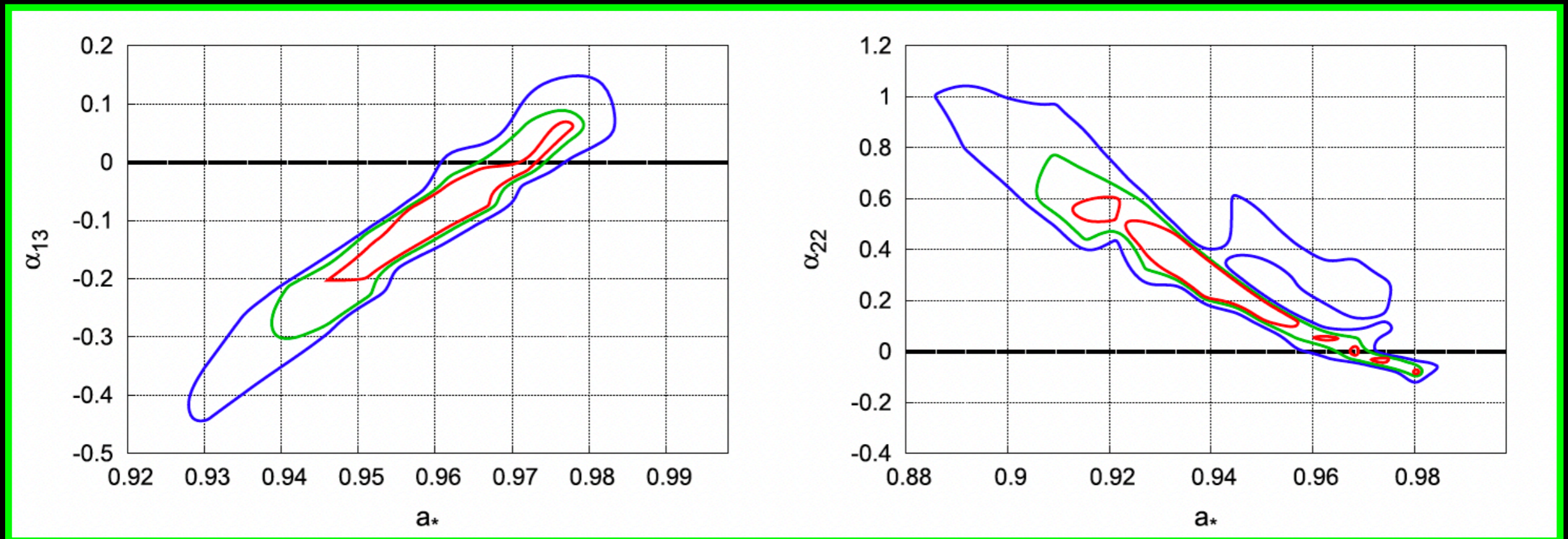
# Analysis of Thermal Spectra

- LMC X-1 (Tripathi et al. 2020), RXTE

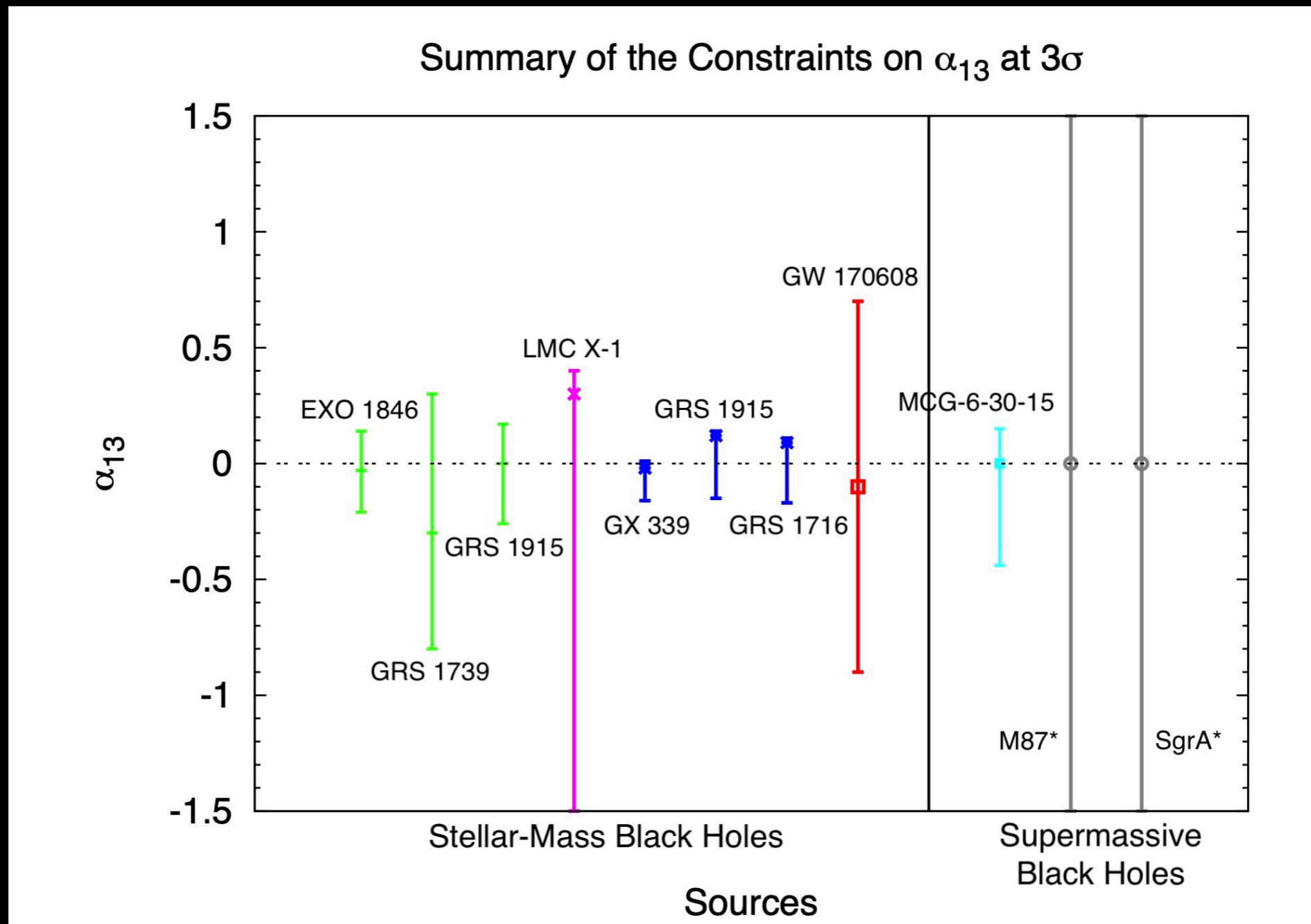


# Analysis of Reflection Features

- MCG-6-30-15 (Tripathi et al. 2019), XMM+NuSTAR



# Summary: X-ray, GWs, VLBI



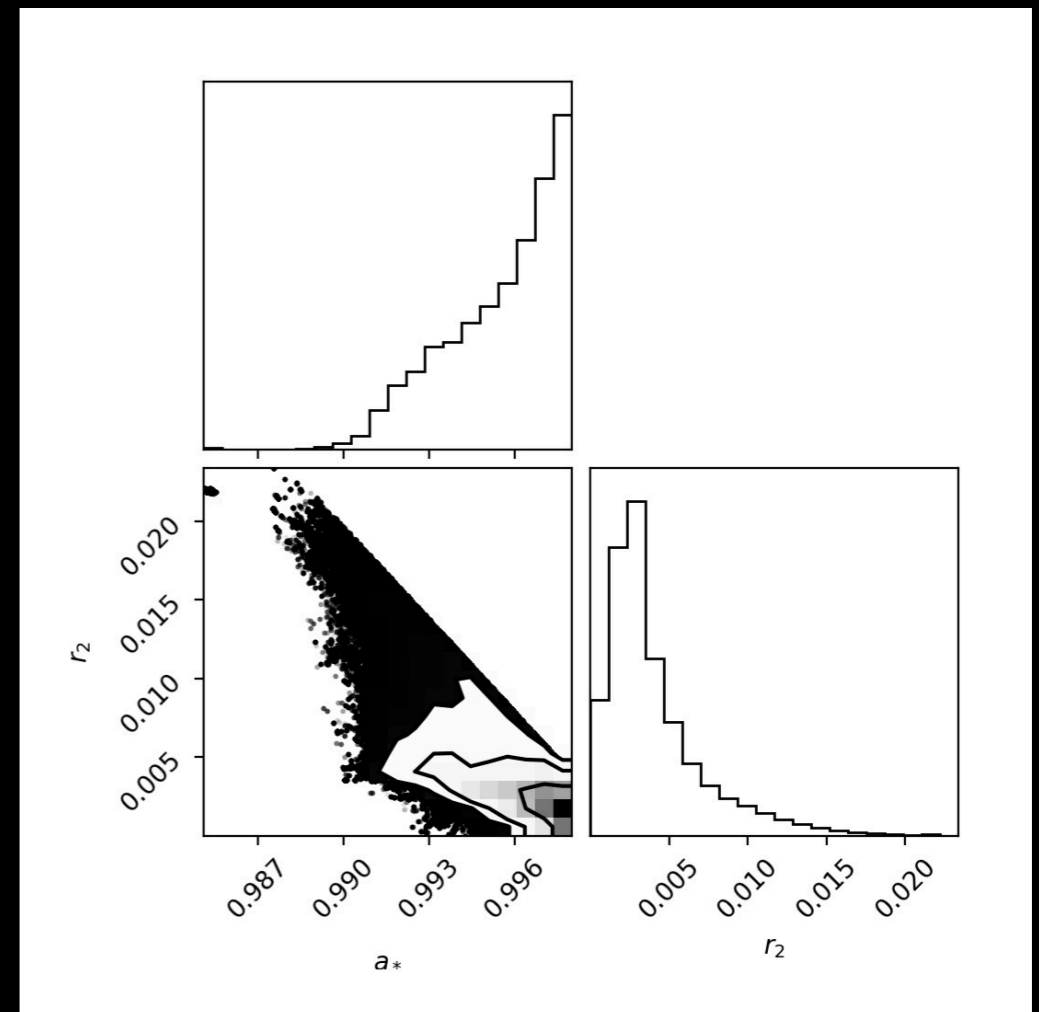
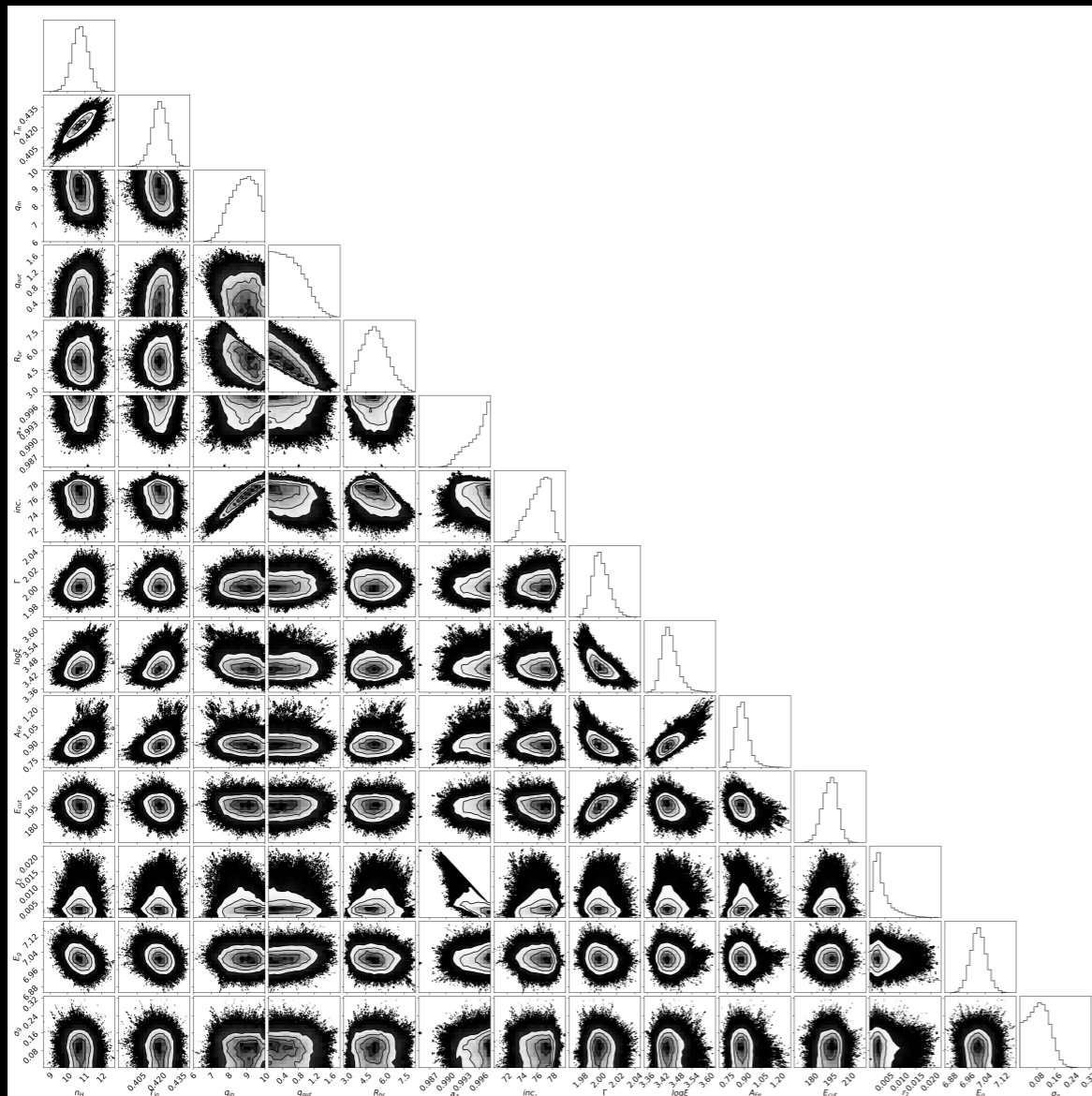
**Results:**

**Tests of Specific Gravity Models**



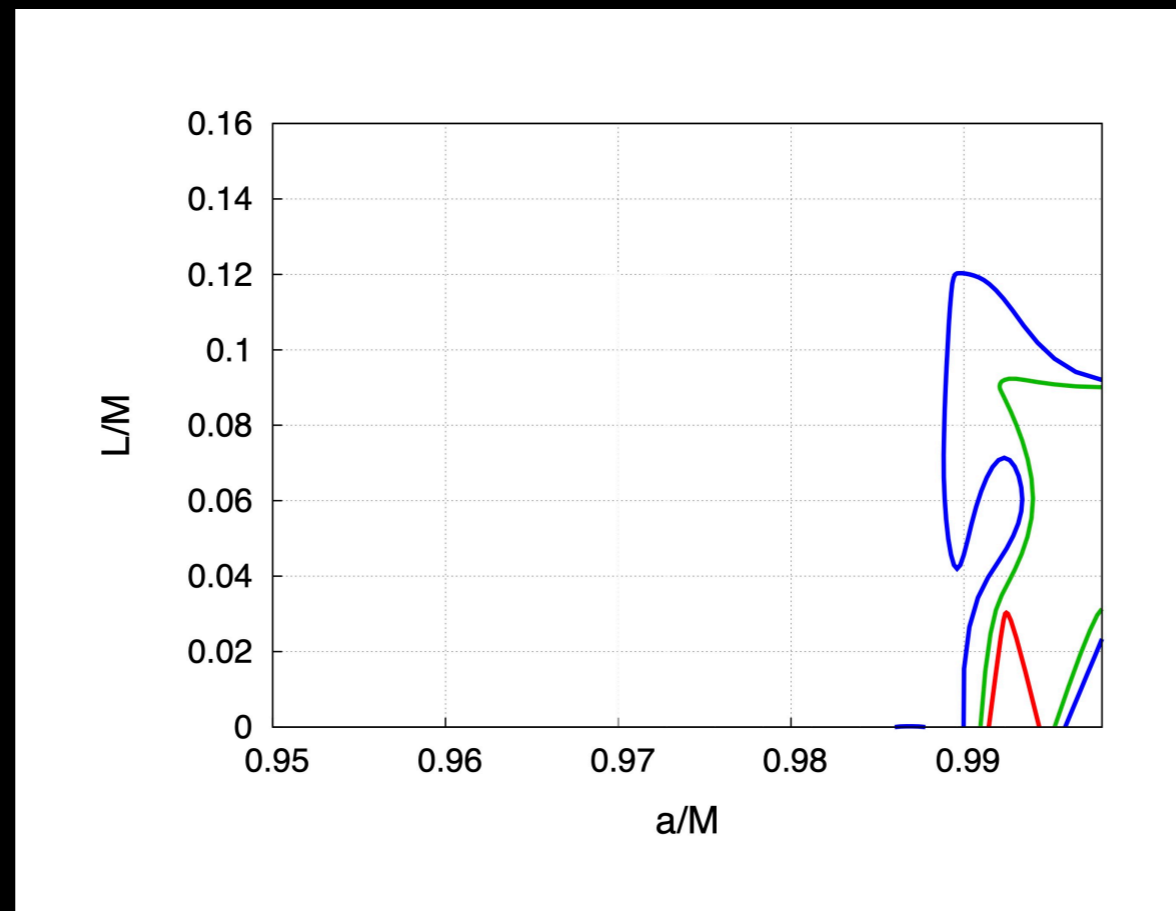
# Einstein-Maxwell-Dilaton-Axion Gravity

- Tripathi et al. 2021
- EXO 1846-031 (NuSTAR)
- Constraint:  $r_2 < 0.011$  (90% CL)



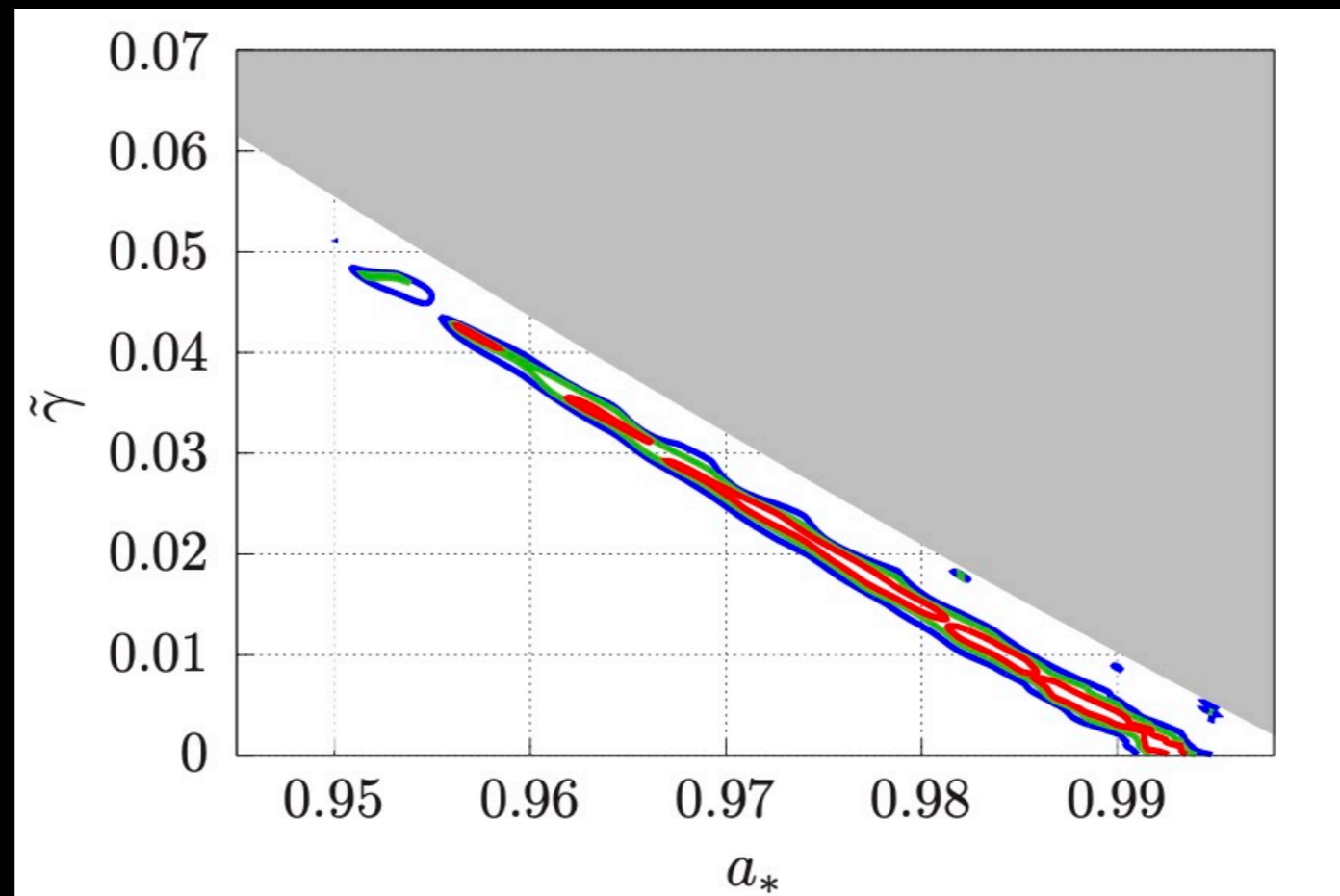
# Conformal Gravity

- Zhou et al. 2018, 2019
- GS 1354-645 (NuSTAR)
- Constraint:  $L/M < 0.12$  (99% CL)



# Asymptotically Safe Quantum Gravity

- Zhou et al. 2021
- GRS 1915+105 (Suzaku)
- Constraint:  $\gamma < 0.047$  (90% CL)



# **Accuracy of GR Tests with X-ray Data**

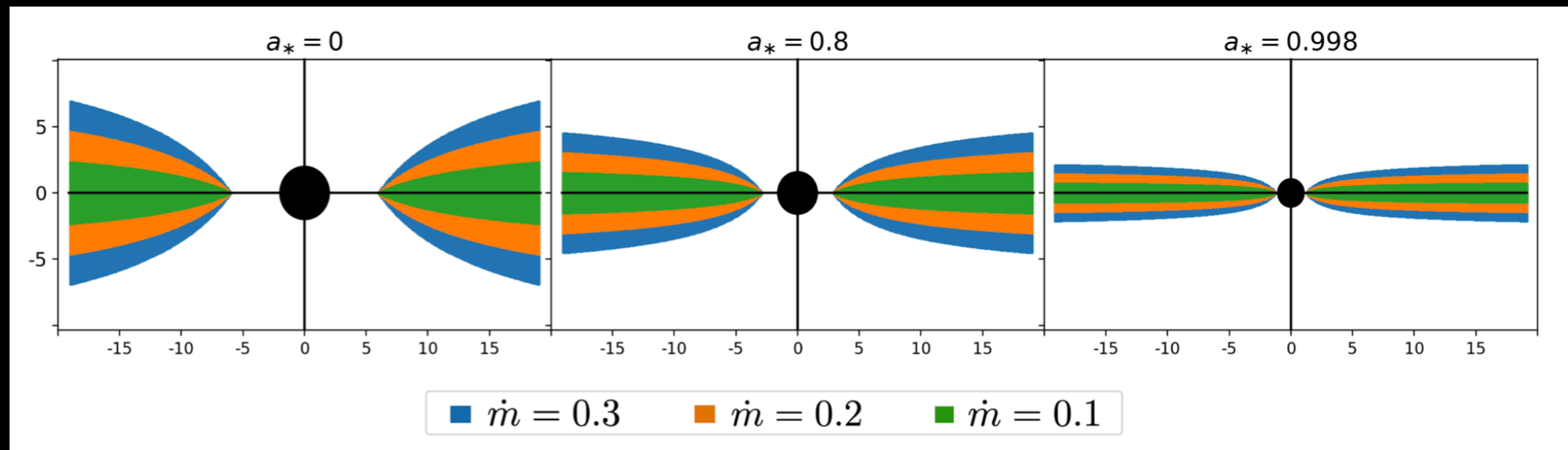
# Source Selection

Source selection for X-ray reflection spectroscopy:

- High spin ( $a > 0.9$ )
- Compact corona close to the black hole
- Prominent broadened iron line
- $L \sim 0.05-0.30 L_{\text{Edd}}$
- Bright source
- Constant flux
- High resolution at the iron line + hard X-ray band (e.g. XMM+NuSTAR)

# Thickness of the Disk

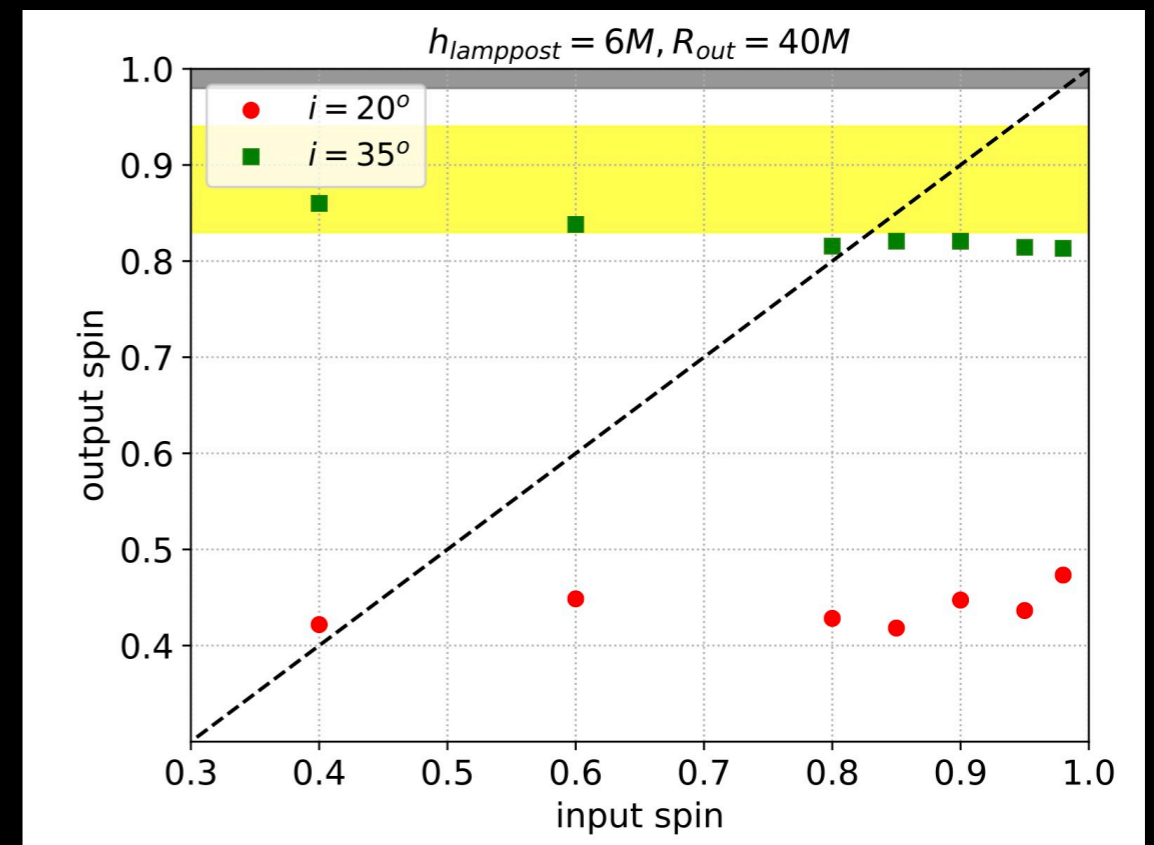
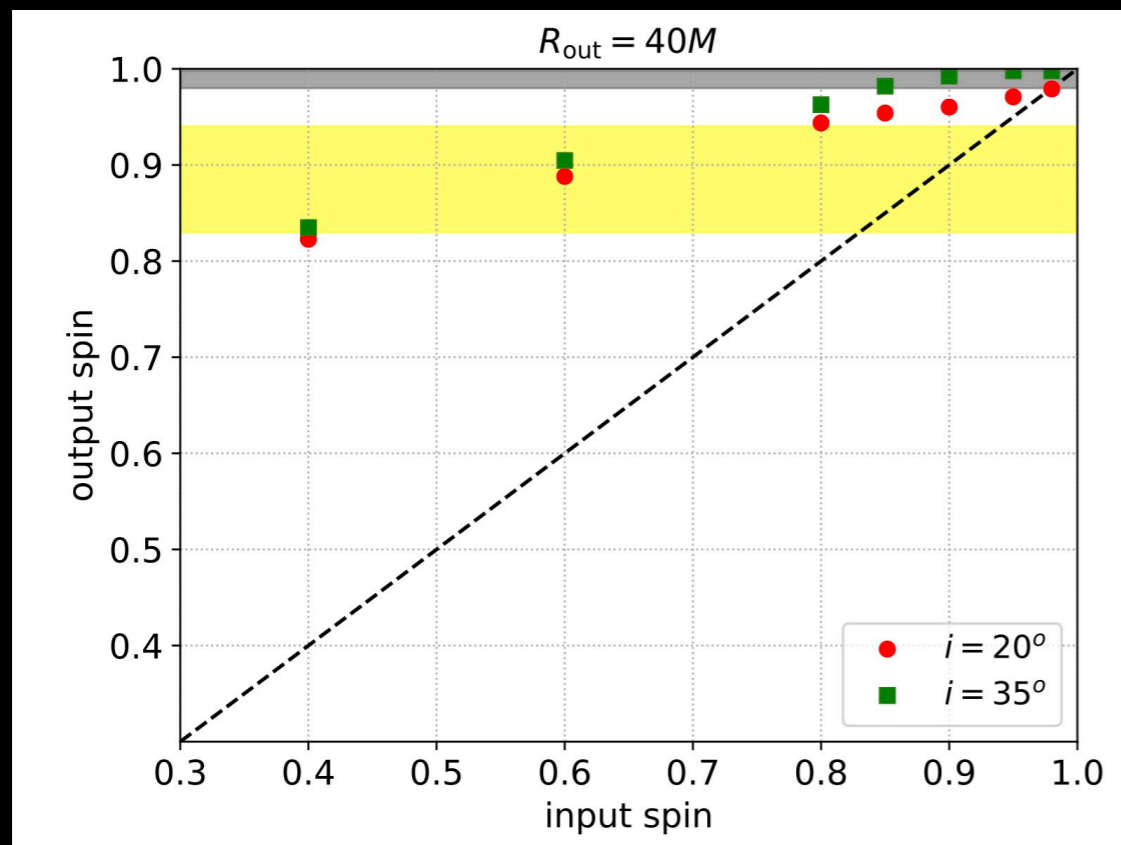
- Thin disks of finite thickness
- Abdikamalov et al. 2020 (model, GRS 1915+105)
- Tripathi et al. 2021 (MCG-6-30-15, EXO 1846-031)
- Jiang et al. 2022 (lampost corona, MCG-6-30-15)



**Conclusion: no significant impact on the parameter estimate for disks with high radiative efficiency**

# Thickness of the Disk

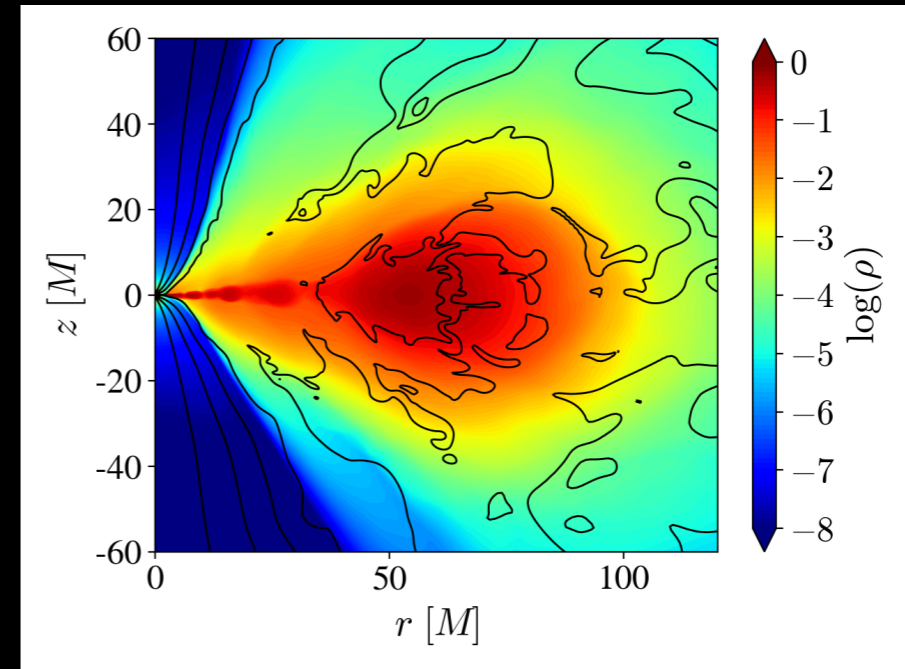
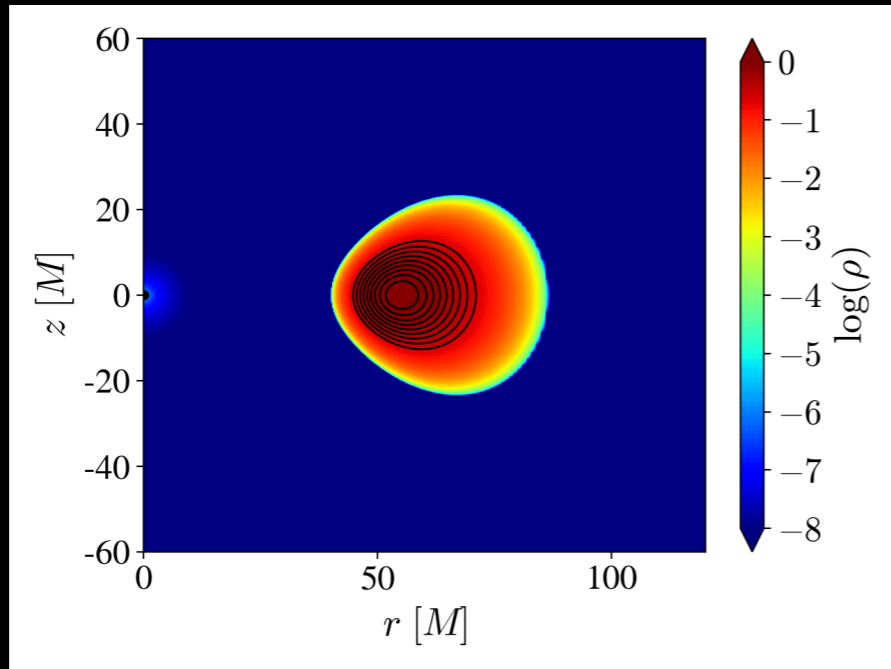
- Thick disks
- Riaz et al. 2020a, 2020b



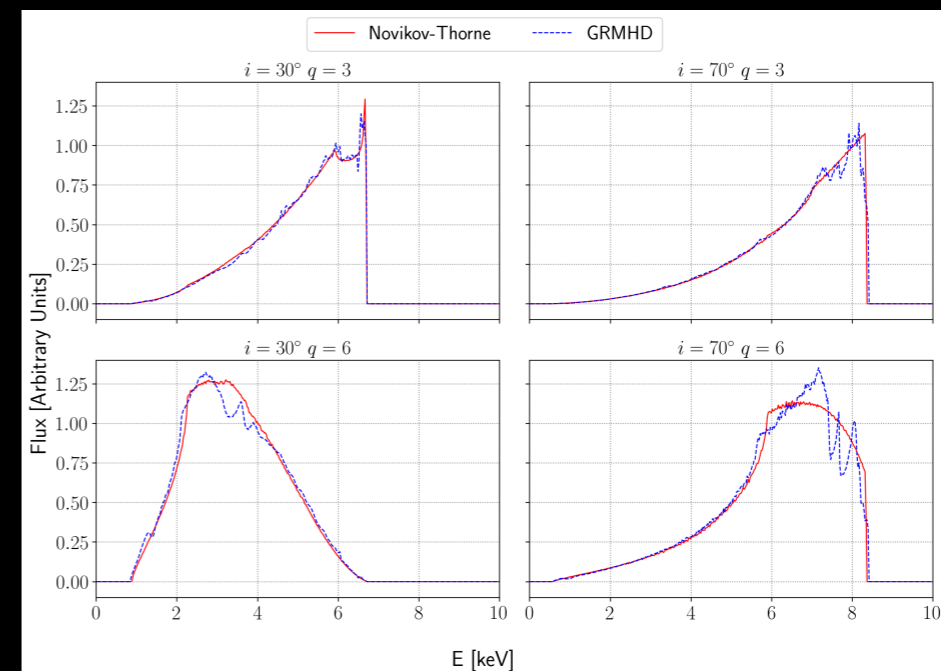
**Conclusion: large systematic uncertainties,  
wrong measurements**

# GRMHD-Simulated Thin Disks

- Shashank et al. 2022



**Conclusion: we recover well the input parameters**





# Concluding Remarks

# Past/Present Work

- Models: `relxill_nk`, `nkbb`
- Public on GitHub: <https://github.com/ABHModels>
- Agnostic tests
- Tests on specific theories of gravity

# Future Work

- More sophisticated astrophysical model (necessary to analyze the data of the next generation of X-ray missions)
- Tests of atomic physics and values of fundamental constants in the strong gravitational fields of black holes

**Thank You!**