XXXIV International (ONLINE) Workshop on High Energy Physics
"From Quarks to Galaxies: Elucidating Dark Sides"

Phases of Strong Interactions: the Lattice approach

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Based on an upcoming Strong-2020 report, with thanks to all the Authors

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$$\mathcal{L} = \sum_{a=1}^{n} \bar{q}_{La}\not{\partial} q_{La} + \bar{q}_{Ra}\not{\partial} q_{Ra} - m(\bar{q}_{La}q_{La} + \bar{q}_{Ra}q_{Ra}) + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + \mathcal{L}_{gauge}$$

With $m = 0$, invariant under

$q_L \to V_L q_L q_R \to V_R q_R$, with $V \in U(n)$

Global symmetry:

$$U(n)_L \times U(n)_R \cong SU(n) \times \; SU(n) \times U(1)_V \times U(1)_A$$

.

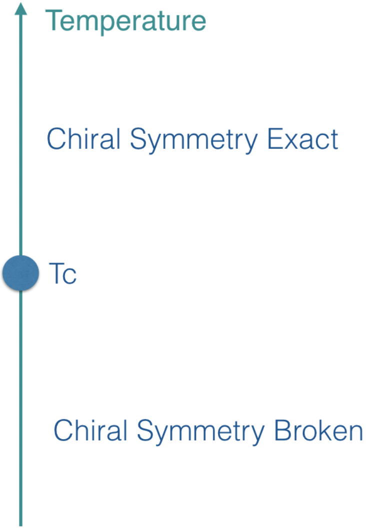
Spontaneously Broken, (n^2 - 1) GB

baryon number

Explicitely broken

Experimental Evidence

Phases of massless QCD

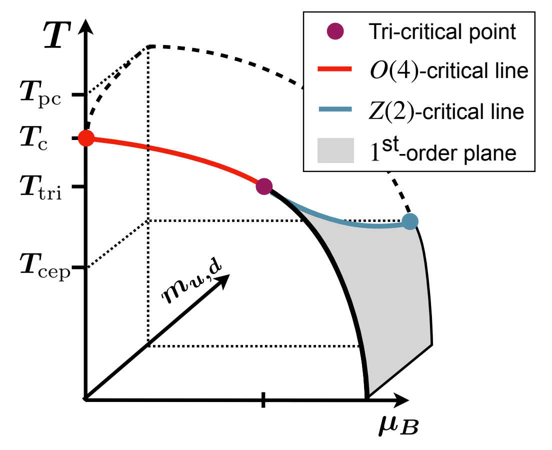


Outline

- Thermal Phase Transitions and Critical points
- Nature and phenomenology of the Quark Gluon Plasma
- Methodological challenges: spectral functions and high baryon density
- Cold and dense matter
- Topology and axions
- Parting comments

Thermal Phase Transitions and Critical Points

Standard expectation [*from Ch. Schmidt &HotQCD*]



Less standard expectations/observations

Universality class of the high T transition: theoretical (*lack of*)guidance

Parisen Toldin, Pelissetto, Vicari 2003

N_f	$\mathrm{U}(1)_A$ broken	$\mathrm{U}(1)_A$ restored
1	crossover or 1^{st}ord	$\mathrm{O}(2) \to \mathbb{Z}_2$ or 1^{st}ord
2	$\mathrm{O}(4) \to \mathrm{O}(3)$ or 1^{st}ord Pisarski, Wilczek 1984	$\mathrm{U}(2)_L \otimes \mathrm{U}(2)_R \to \mathrm{U}(2)_V$ or 1^{st}ord
≥ 3	1^{st}ord ?	1^{st}ord

Challenged by Cuteri, Philipsen, Sciarra 2020-2022

Where do we stand

Two (plus one) flavor QCD

$T_{pc} \simeq 156.5$ MeV (WB, HotQCd)
$T_c = 132^{+3}_{-6}$ MeV (HotQCD); 134^{+6}_{-4} (Kotov, MpL,Trunin)

Scaling Window

Signatures of O(4) scaling have been observed for pion masses up to 140 MeV, and temperatures up to 300 MeV (Kotov, MpL, Trunin)

Three flavor QCD, and beyond

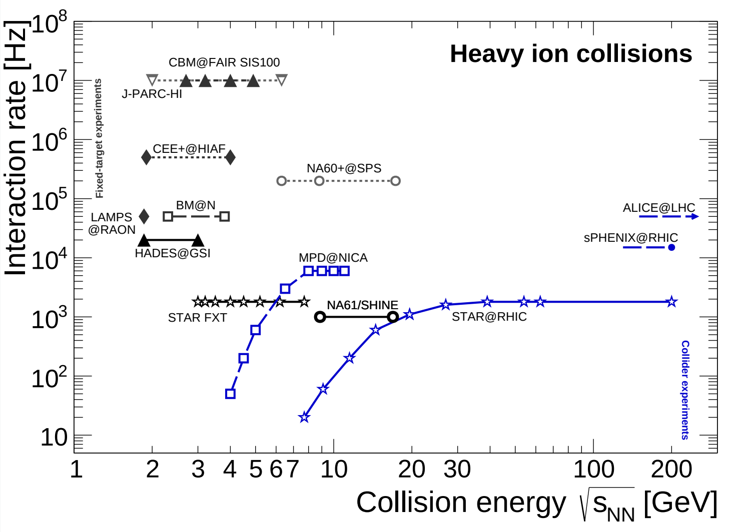
$T_c = 98^{+3}_{-6}$ MeV (Sipaz Sharma & HotQCD)
No evidence for the first order phase transition was found in the pion mass range explored (Cuteri Philipsen; Sipaz Sharma & HotQCD)
Cuteri Philipsen's results suggest that the transition remains of second order up to the onset of the conformal window at about $N_f = 12$ [Later]

The road ahead

The order of the transition as a function of N_f, and the nature of scaling needs a complete clarification.
Reasons of interest:

- The chiral behaviour in QCD may well constrain the phase diagram, in particular the location of the critical point [Next]
- Interplay of chiral and axial symmetry, and fate of the anomaly.
- Approach to the conformal window
- Apparent discrepancies with FRG results (Pawlowski et al.)

Overview over future and past experiments; Credit: Tetyana Galatyuk



Nature and Phenomenology of the QGP (Cont'd)
Credits: Jana Guenther

Fluctuations

Fluctuations are computed as the derivatives of the pressure with respect to various chemical potentials. The allow the computation of pressure and density.

$$\chi_{i,j,k}^{B,Q,S} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k}\,,\ \hat{\mu} = \frac{\mu}{T} \tag{1}$$

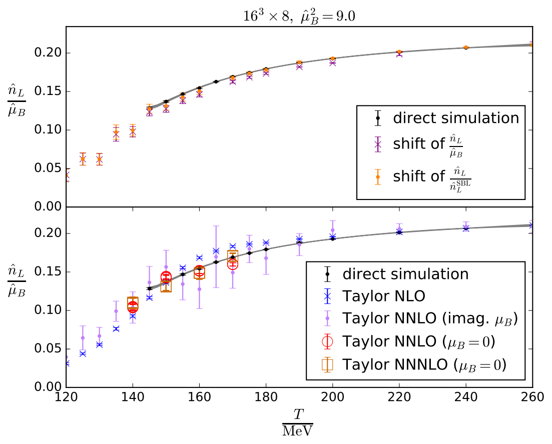
Phenomenological parametrization

The lattice input is especially well suited for the temperature range around the crossover between the hadronic phase that can often be described by the Hadron Resonance Gas and the QGP-phase.

Equation of State

Fluctuations allow the computation of pressure and density as a function of chemical potential. This has posed a significant challenge for several years.

Equation of State



The light quark density computed at fixed chemical potential by the WB collaboration)

Nature and Phenomenology of the QGP (Cont'd)

Influence of a magnetic field *Credits: Lorenzo Maio*

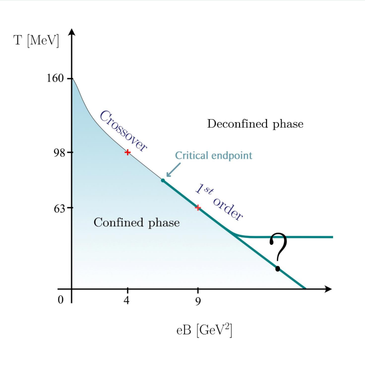


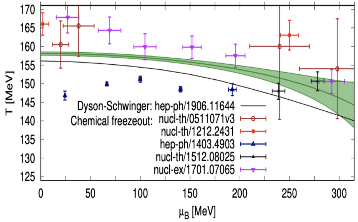
Figure: Updated QCD phase diagram for $N_f = 2 + 1$ at the physical point in an external magnetic field from the Pisa group

BEST efforts

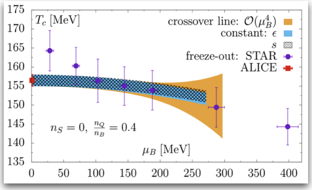
- The BEST-collaboration combines first-principles lattice QCD calculations and phenomenological approaches, to create a framework for the analysis of experimental data at low collision energies.
- One of the main goal is the search for the QCD critical point.
- To definitively claim or rule out the presence of a QCD critical point or anomalous transport requires a comprehensive framework for modeling the salient features of heavy ion collisions at BES energies, which allows for a quantitative description of the data.
- A quantitative understanding of fluctuations near the critical point needs to be developed as well.

QCD crossover temperature

WB:PRL (2020)



HotQCD: PLB (2019)



$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$$

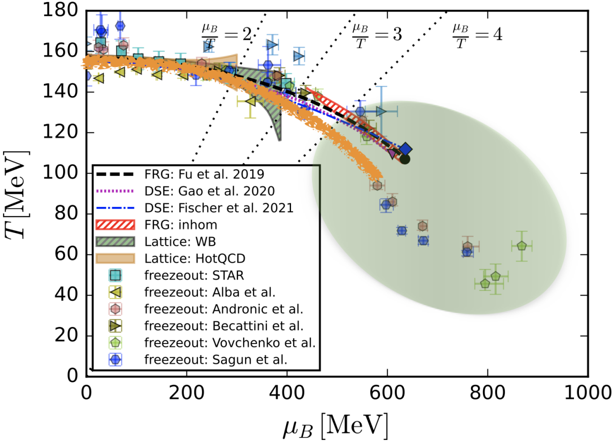
T_0=158.0±0.6 MeV
κ_2=0.0153±0.0018
κ_4=0.00032±0.00067

T_0=156.5±1.5 MeV
κ_2=0.012±0.004
κ_4=0.000±0.004

Slide from Claudia Ratti

Nature and Phenomenology of the QGP)(Cont'd)
Credits: Jan Pawlowski

Chiral phase structure (theory) & freeze out data (Exp. data+Pheno)



Green area: candidate region for the critical point. No clear signature yet.

Nature and Phenomenology of the QGP (Cont'd)
Credits: Olga Soloveva

Transport Properties

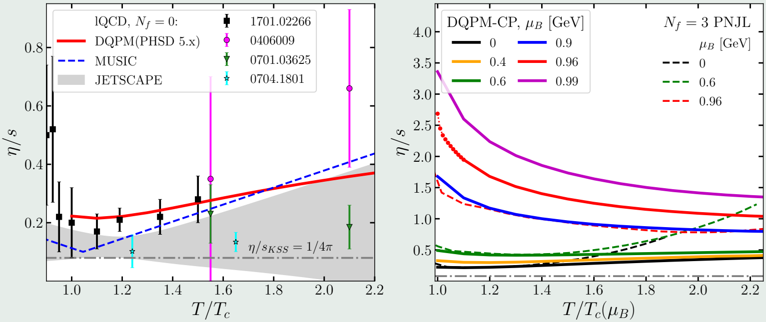


Figure: Specific shear viscosity as a function of the scaled temperature T/T_c at $\mu_B = 0$ (left) and at finite μ_B (right). The symbols corresponds to the lQCD results for pure SU(3) gauge theory (black squares) (Astrakhantsev et al), (green triangles and magenta circles) (Nakamura et al), (cyan stars) (Meyer et al.). At finite density only model calculations are possible so far.

- Successful studies of the critical line and fluctuations in the working region of LHC, and up to densities where the critical point is not observed.
- The study of fluctuations to identify a possible first order phase transition and a possible QCD critical point should be further pursued. This requires better control over large densities [Next]
- Transport coefficients, hence spectral functions, need to be improved [Next]
- Joint work with phenomenologists is essential in order to make the most of lattice results

The main limitation of the lattice approach: The sign problem

- Lattice QCD calculations crucially rely on the interpretation of the Boltzmann factor as a probability density for the numerical sampling of the path integral.
- This is possible once the time is rotated to imaginary values, and chemical potential is zero.
- Once the Boltzmann factor is no longer strictly positive, or even becomes genuine complex, this interpretation is lost and standard Monte Carlo methods for the calculation of the path integral cease working.
- This is known as 'Sign problem'

Methodological Challenges: Spectral Functions (Cont'd)

Spectral functions as an (ill-defined) inverse problem

The computation of spectral functions begins with lattice correlators.

Bottomonium has been used as an important case study: first, it has a great physical interest, due to the rich production at the LHC. And secondly, the inversion required to compute spectral functions is a "simple" inverse Laplace transform, for which a wealth of methods has been designed.

Despite a qualitative coherence among the results, a quantitative agreement has not been reached yet.

The alliance of EFTs and lattice, with lattice correlators defined inside the EFT appears to be a novel and promising avenue.

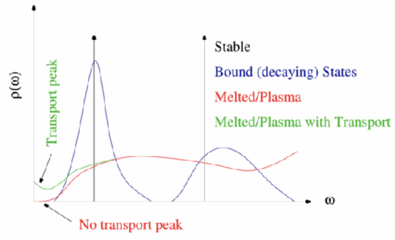
Methodological Challenges: Spectral Functions (Cont'd)

$$G(\tau) = \int \frac{d\omega}{2\pi} K(\omega,\tau)\,\rho(\omega)\,d\omega$$

$\uparrow$ Euclidean Correlator

$\nearrow$ (Lattice) Kernel

$\downarrow$ Spectral Function



$K(\omega,\tau) \sim e^{-\omega\tau}$ for NRQCD

Slide from Chris Allton

Transport coefficients from Kubo's formulæ:

$$D = \frac{1}{3\chi_q} \lim_{\omega \to 0^+} \sum_{i=1}^{3} \frac{\rho^{ii}(\omega,\vec{0})}{\omega}$$

The critical endpoint from the Taylor/Pade' series

- One strategy to locate the critical point: the radius of convergence of the Taylor expansion.
- The radius would be limited by any genuine singularity, including the critical point in the QCD phase diagram.
- Alternatively, one may use a Pade' representation and locate the CEP from its poles
- The limiting singularity may also be located in the complex $\hat{\mu}_B$ plane.
- Estimating the radius of convergence from the lattice results of the Taylor coefficients $\chi_{i,j,k}^{B,Q,S} \equiv c_n$ is very challenging, due to the limited number of coefficients, usually $(i + j + k = n) \leq 8$, and the increasing statistical error.

Different strategies at a glance

Compute $\mathcal{Z} = <\mathrm{Det}>$. This is a polynomial in complex $e^{3\mu/T}$ whose zeros can be computed.

GCFP
on the lattice
Glasgow-style

I.M. Barbour, N. Behilil, E. Dagotto, F. Karsch, A.Moreo, M.Stone, H. Wyld, Nuclear Physics **B275** (FS17)(1986) 296.

Compute c_n at $\mu = 0$
Study the radius of convergence of the Taylor series in μ/T

Gavai-Gupta et al. 2003—

Pressure
on the lattice Bielefeld-Swansea -style

C. R. Allton, S. Ejiri, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, and C. Schmidt, Phys. Rev. **D66**, 074507 (2002); **D68**, 014507 (2003); C. R. Allton, M. Doering, S. Ejiri, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, and K. Redlich, **D71**, 054508 (2005).

Methodological Challenges: Baryon density(Cont'd)

Searching for the poles

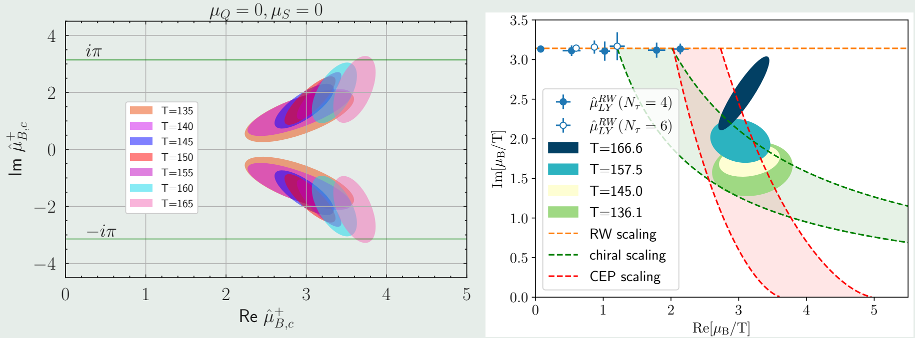


Figure: Poles in the complex $\hat{\mu}_B$ plane from the [4, 4]-Padé re-summation of the Taylor series about $\mu_B = 0$ (left) and from the multi-point Padé approach applied to lattice QCD data at imaginary μ_B (right). Also shown in the right panel is the expected scaling behaviour of the Lee-Yange edge singularities for different critical points, indicated by dashed lines/bands. (Di Renzo, Karsch, Schmidt)

Methodological Challenges: Baryon density(Cont'd)

The Canonical approach

Calculation in pure μ_I regions, where no sign problem.

$$Z_n = \int \frac{d\theta}{2\pi} e^{in\theta} \boxed{Z(T, \theta \equiv \frac{Im\,\mu}{T}) \quad = e^{-\int_0^{\mu_I/T} n^n(x)dx}}$$

Then calculate Z in real μ regions.

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

$$\xi \equiv e^{\mu/T}$$

elopments:

The results reveal the Roberge-Weiss transition above T_c.

Lee-Yang zeros in lattice QCD for searching phase transition points

M. Wakayama[a], V. G. Bornyakov[b,c,d], D. L. Boyda[b,d,e], V. A. Goy[b], H. Iida[b,a,f], A. V. Molochkov[b,d], A. Nakamura[b,g,a], V. I. Zakharov[b,d,h]

The road ahead

- The motivation for going beyond simple importance sampling is very strong and comes from collider experiments and astrophysics.
- New methods have been developed and are currently vigorously pursued, and oldish methods are continuously improved.
- Continual interactions with colleagues pursuing analytic approaches such as EFT on one side, and mathematicians developing advanced methods on the other are beneficial and should be further pursued.

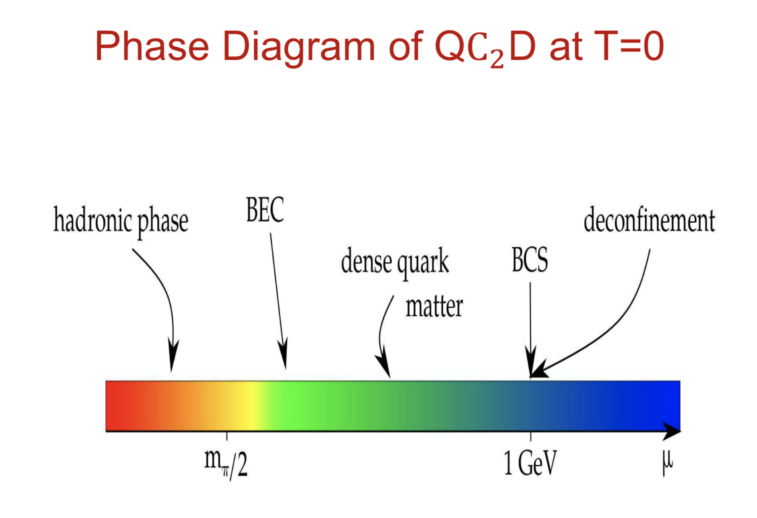
Cold and Dense Matter

- 'Cold' and dense matter include neutron stars interiors and is particularly topical also in view of a possible generation of gravitational waves in neutron stars collisions.
- As discussed in the previous Section, the region of high baryon density is not accessible at the moment to lattice simulations of QCD.
- In this region we have to rely on model calculations, or study models without the sign problem.
- There are important cases which do not suffer from sign problem: isospin dense matter, and QCD with two colors.

Insight from models without the sign problem

- Phase diagram and thermodynamics of isospin asymmetric QCD, with a focus on (1) signatures of the superconducting BCS phase expected on perturbation theory grounds, and (2) the role of pion condensation in the early universe evolution at non vanishing lepton flavour asymmetries. *F. Cuteri*
- Two colors lattice QCD with two flavors of staggered fermions at nonzero quark chemical potential μ_q. The line of the deconfinement transition in the $\mu_q - T$ plane was found for the ranges 900 MeV $\lesssim \mu_q \lesssim$ 1700 MeV, 100 MeV $\lesssim T \lesssim$ 150 MeV. *V. Bornyakov*

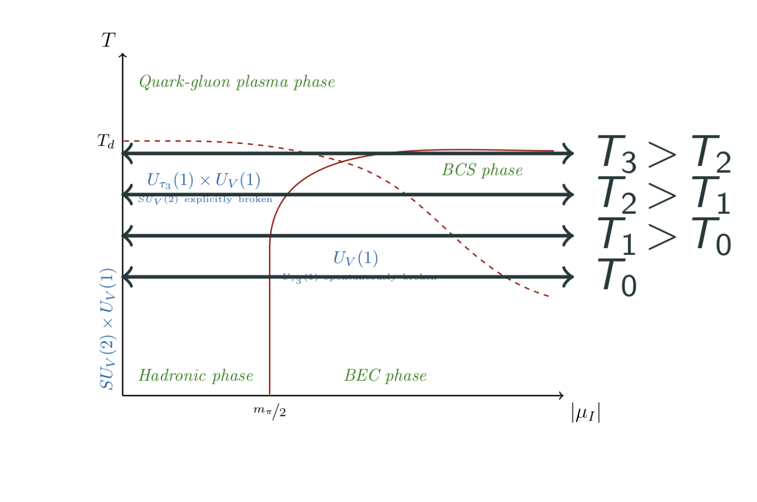
Two color QCD



Slide from V. Bornyakov

Cold and Dense Matter (Cont'd)

Isospin dense matter



Slide from F.Cuteri

Cold and Dense Matter -The road ahead

- It will probably take a while before we can have true ab-initio studies of the cold sector of the phase diagram.
- Clearly the effort described in Section should be further pursued hoping to access this region.
- At the same time, it is important to continue explore what we can: finite isospin density, two color QCD.
- Having a line of communication with model studies and astrophysics is most important

The strong CP problem

- QCD allows for an explicit breaking of CP due to the coupling the the CP-odd topological charge Q, $\mathcal{L}_\theta \propto \theta Q$.
- $$Q = \frac{1}{16\pi^2} \int \left\{ F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x) \right\} d^4 x$$
- Experimental measures of the neutron electric dipole moment put the extremely stringent upper bound $|\theta| \lesssim 10^{-9} - 10^{-10}$ on this parameter
- A particularly interesting solution proposed by Peccei, Quinn, Weinberg and Wilczek to solve this issue is the *axion*, an hypothetical pseudo-scalar particle introduced as the pseudo Nambu–Goldstone Boson (NGB) of the spontaneous breaking of a new global $\mathrm{U}(1)_{\mathrm{PQ}}$ axial symmetry, the Peccei–Quinn (PQ) symmetry.

Axion as a Dark Matter candidate

The axion's couplings with SM particles is suppressed by the axion scale f_a, which is expected to be extremely large from astrophysical and cosmological bounds: 10^8 GeV $\lesssim f_a \lesssim$ 10^{12}GeV This makes the axion a natural Dark Matter candidate.

Axion and Gravitational Waves

A particularly well-motivated class of models assumes the existence of a confining strongly-coupled dark sector which possesses an accidental global PQ symmetry.
The spontaneous breaking of the PQ symmetry will create a GW signature amenable to be probed by future experimental interferometric Gravitational Wave (GW) observations

Topology and the QCD Axion (Cont'd)

Back to the lattice

It is possible to relate the temperature-dependent axion effective mass to the QCD topological susceptibility $\chi(T)$ via the well-known relation:

$$m_a^2(T) f_a^2 = \chi(T) = \lim_{V \to \infty} \frac{\langle Q^2 \rangle (T)}{V},$$

where V is the $4\mathrm{D}$ space-time volume and T is the temperature. Apart from the unknown constant f_a, the value of the axion mass in Eq. (2) is completely fixed by the QCD topological susceptibility χ.

High temperature and Dilute Instanton Gas

Assuming that instantons can be treated as identical non-interacting particles, it is possible to compute $\chi(T)$ at leading order in perturbation theory by performing a Gaussian integration of the fluctuations around a one-instanton configuration:

$$\chi_{\mathrm{DIGA}}(T) \sim T^{-d}$$

where $d \approx 8$ for 3 light quark flavors.

$\chi(T)$ from the lattice: current status and future challenges

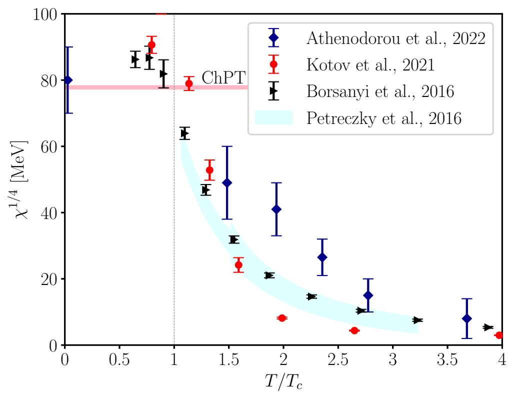


Figure: Comparison of different determinations of the fourth root of the topological susceptibility $\chi^{1/4}$ in $N_f = 2 + 1$ QCD from lattice simulations. We also report results obtained below the crossover, as well as the two-flavor $T = 0$ ChPT prediction for degenerate up-down quarks.

Figure courtesy Claudio Bonanno

Topology and the QCD axion (Cont'd)

Limits on the post-inflactionary axion mass

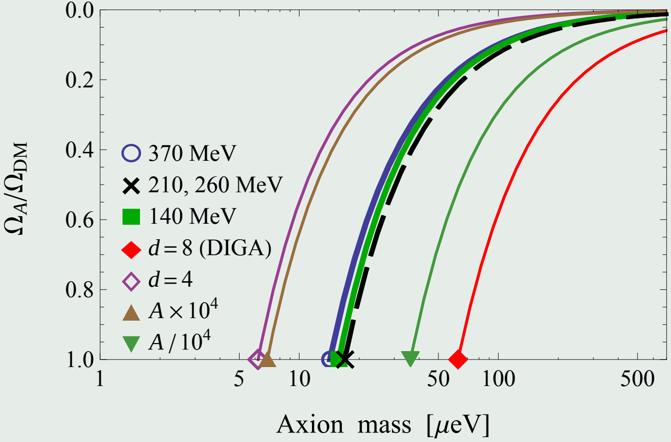


Figure: Dependence of the ratio Ω_A/Ω_{DM} on the axion mass (Kotov, MpL, Trunin). For the $m_\pi \simeq 140$ MeV ensemble, parameters A and d obtained from the best fit of data to the DIGA form are varied as written in the legend.

- Topology in QCD is a topic of central importance, which has been further revived by the quest for QCD axions
- We foresee more robust limit on the QCD axion mass and studies of the axion potential.
- On the Quark Gluon Plasma phase, we look forward to the clarification of the role of topology in the QGP threshold

Parting Remarks

- Main connections: astrophysics, high temperature experiments, high density experiments and search for the critical endpoint, axion dark matter.
- Main methodological bottleneck: Sign problem - major subject of discussion, old and new avenues investigated and pushed to the limits, very active community.
- Future developments (not discussed):
  - Machine Learning, already on the way.
  - Quantum computing maybe the future, especially for dense matter – but we are not there yet.
- Computing: very important, steady activity towards Exascale era. The Lattice community is a major user and stakeholder in the field.

Thank You !