Heisenberg's Uncertainty Principle and Particle Trajectories

XXXIV International Workshop on High Energy Physics "From Quarks to Galaxies: Elucidating Dark Sides"

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Detlef Dürr

Fundamental significance of the uncertainty principle

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It was through this paper [Heisenberg 1927] that the revolutionary character of the new conception became clear. It showed that not only the determinism of classical physics must be abandoned, but also the naive concept of reality which looked upon the particles of atomic physics as if they were very small grains of sand. At every instant a grain of sand has a definite position and velocity. This is not the case with an electron.

M.Born Nobel lecture (1954)

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Thus the mechanics which governs atomic phenomena – quantum mechanics or wave mechanics – must be based on ideas of motion which are fundamentally different from those of classical mechanics. In quantum mechanics there is no such concept as the path of a particle. This forms the content of what is called the uncertainty principle, one of the fundamental principles of quantum mechanics, discovered by W. Heisenberg in 1927.

[L.D. Landau, E.M. Lifshitz, Course of Theoretical Physics Vol. 3: Quantum Mechanics, Pergamon Press, Oxford, 1977, p. 2]

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1. Bohmian mechanics [D. Dürr, S. Teufel, Springer, 2009.]
2. Nelson's stochastic mechanics [E. Nelson, J. Phys. Conf. Ser., 361 (2012) 012011]

A (very) brief introduction to Bohmian mechanics

- Also called: *de Broglie-Bohm* theory or *pilot wave* theory
- Discovered by de Broglie in 1927, presented at the Solvay conference [G. Bacciagaluppi, A. Valentini, Cambridge University Press 2009, p. 57-84.]
- Rediscovered by D. Bohm in 1952 [D. Bohm, Phys. Rev. 85(2) (1952)]

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- Wave equation (Schrödinger, Pauli, Dirac, Proca)

$$i\hbar\,\partial_t \Psi(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N, t) = H\Psi(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N, t)$$

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- Guiding equation:

$$\frac{d\mathbf{Q}_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\left[\frac{\nabla_i \Psi}{\Psi}\right]\left(\bar{\mathbf{Q}}, t\right) = \frac{\mathbf{j}\left(\bar{\mathbf{Q}}, t\right)}{\rho\left(\bar{\mathbf{Q}}, t\right)},$$

  where $\bar{\mathbf{Q}} = (\mathbf{Q}_1, \mathbf{Q}_2, ..., \mathbf{Q}_N)$, $\rho = |\Psi|^2$, $\mathbf{j} = \frac{\hbar}{2mi}\left[\Psi^* \nabla \Psi - \Psi \nabla \Psi^*\right]$.

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For non-relativistic spin-1/2 particles see e.g. [Bohm & Hiley (1987), Ch. 10]
For Dirac particles see e.g. [P.R. Holland Phys. Rev. A 60(6) (1999) 4326]
For spin-1 bosons see [W. Struyve, W. De Baere, J. De Neve, S. De Weirdt, Phys. Let. A 322(1-2) (2004) 8495.]

A (very) brief introduction to Bohmian mechanics

Born's rule holds in Bohmian mechanics!

- Apply BM to subsystems, assume the Born's rule for $t = 0$. By virtue of the velocity field $\mathbf{v}^{\psi}$ it is going to hold for later times.

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{v}^{\psi}) = 0, \quad \rho = |\psi|^2.$$

- Apply BM to the whole universe, derive the Born's rule for subsystems [D. Dürr, S. Goldstein, & N. Zanghì, J. Stat. Phys. 67, (1992).] (the result holds for spin-0 case).

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BM gives correct predictions for:

- All known non-relativistic experiments: measurements of position, momentum, spin; scattering... [D. Dürr, S. Teufel, Springer, 2009.]
- Experiments with single relativistic particles

As of today there is no BM for the systems of many interacting relativistic particles.

Bohmian mechanics for double slit experiment

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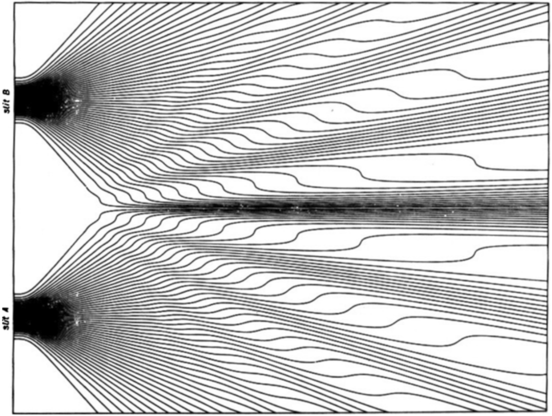

[C. Philippidis, C. Dewdney, and B. Hiley, IL. Nuov. Cim. B 52, 15 (1979)].

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Therefore it has to be something wrong with the arguments of Heisenberg against particles trajectories. We would like to find out what.

Heisenberg's arguments in 1927

Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. Z. Phys. 43 (1927) 172–198.

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Heisenberg redefines the familiar physical notions of position, velocity, and trajectory:

> *When one wants to be clear about what is to be understood by the words position of the object, for example of the electron <...>, then one must specify definite experiments with whose help one plans to measure the position of the electron; otherwise this word has no meaning.*

[W. Heisenberg, The Physical Content of Quantum Kinematics and Mechanics, In: J.A. Wheeler, W.H. Zurek, Princeton University Press, Princeton, 1984, 62–84.]

Heisenberg's arguments in 1927

Regarding the possibility of measuring the trajectories of the electrons, Heisenberg wrote:

> *By path we understand a series of points in space (in a given reference system) which the electron takes as positions one after the other. As we already know what is to be understood by position at a definite time, no new difficulties occur here. Nevertheless, it is easy to recognize that, for example, the often used expression, the 1s orbit of the electron in the hydrogen atom, from our point of view has no sense. In order to measure this 1s path we have to illuminate the atom with light whose wavelength is considerably shorter than 10^{-8} cm. However, a single photon of such light is enough to eject the electron completely from its path* <...>.

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$$X := P(X)$$

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$$\Delta q \Delta p \geq \hbar,$$

$(\Delta q)^2 = 2\int (q' - \bar{q})^2 |\psi_q(q')|^2 \mathrm{d}q'$, $\bar{q} = \int q' |\psi_q(q')|^2 \mathrm{d}q'$,
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It is a mathematical fact proven by Kennard in 1927. [E.H. Kennard, Z. Physik 44 (1927) 326352.]

What is its empirical input?

Heisenberg's arguments in 1929

<...> if the velocity of the electron is at first known and the position then exactly measured, the position for times previous to the measurement may be calculated. Then for these past times $\Delta p \Delta x$ *is smaller than the usual limiting value, but this knowledge of the past is of a purely speculative character, since it can never (because of the unknown change in momentum caused by the position measurement) be used as an initial condition in any calculation of the future progress of the electron and thus cannot be subject to experimental verification. It is a matter of personal belief whether such a calculation concerning the past history of the electron can be ascribed any physical reality or not.*

[W. Heisenberg, Dover Pub., New York, 1949, p.20]

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Because of the UR the 'experimental verification' is impossible so it is a *matter of personal belief* whether the trajectories exist or not.

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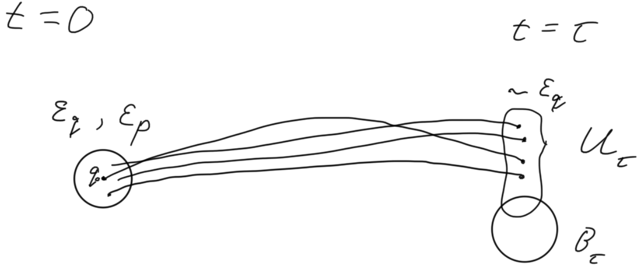
Assume the particles move along trajectories according to a certain law:



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- The wave function has clearly epistemological character.

> **Any knowledge** *of the coordinate* q *of the electron can be expressed by a probability amplitude* $S(q')$, $|S(q')|^2 \mathrm{d}q'$ *being the probability of finding the numerical value of the coordinate of the electron between* q' *and* $q' + \mathrm{d}q'$.

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- How accurately can we know the momentum at the same time? If we know p_{prior} and δp then $p_{prior} + \delta p = p$. Thus our knowledge of p is dictated by $\epsilon_{\delta p}$ – the accuracy of the determination of the disturbance of momentum introduced by the position measurement.

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Let us come back to the picture of 'experimental verification'.

Heisenberg's arguments in 1929

The summary of the argument: Heisenberg assumes a TCQT, in which the experimental verification (as we defined it above) is impossible because of the (Kennard) UR.

According to Heisenberg, if such an experimental verification is impossible it is 'a matter of personal belief' to accept or to reject particle trajectories.

γ-microscope and an old question

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Not a surprise, but simply an illustration of

$$\epsilon_q \epsilon_{\delta p} \equiv \Delta q \Delta p \geq \hbar.$$

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- In Bohmian mechanics the answer is yes. [see the discussion in S. Aristarhov arxiv.org/abs/2208.12735]

TCQT or a ψ-complete theory: a matter of personal belief?

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   - Trajectories in quantum theory do not allow for accurate future prediction of particles' positions.
   - Calculation of the trajectories requires the solution of an additional equation.
   - Why bother? The wave function is enough. Unless...

Time of arrival measurements

A big class of experiments for which standard quantum formalism does not give predictions.

- A particle(s) is first trapped in a certain region of space and then released at a known time, which is set to zero. A detector of given geometry is placed at a certain distance from the region of initial confinement. At time $\tau > 0$, it clicks. This experiment is repeated many times and the distribution of the arrival times is acquired.

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- There is no canonical time-operator. [W. Pauli, Springer, Berlin, 1980]
- Many add-ons to standard formalism have been suggested [J.G. Muga, C.R. Leavens, Phys. Rep. 338(4) (2000) 353–438.]
- The adequacy of many of them is questionable. The range of applicability is limited [see the ref. in S.Aristarhov arXiv:2208.12735]

Time of arrival measurements

In a TCQT, in particular, in Bohmian mechanics, the prediction of the arrival time distribution is a problem with an almost obvious solution: if particles follow trajectories, the time when a given trajectory crosses a certain surface can be easily calculated. The arrival time distribution can thus be obtained if the distribution of the initial positions is known. [see e.g. S. Das, D. Dürr, Sci. Rep. 9 (2019) 2242.]

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1. A physical theory is a set of rules allowing us to obtain numerical predictions for experiments
   - TCQT is preferable.
2. A physical theory has to tell us what the world consists of, how the constituents behave and how it results into our observation.
   - The theory has to state *what there is*.
   - The theory has to furnish a dynamical equation describing the behaviour of what there is.
   - TCQTs (in particular Bohmian mechanics) are like that. ψ-complete theories are not. The former are obviously preferable.

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- Nevertheless the analysis in Bohmian mechanics shows that, at least in this TCQT, Heisenberg's conclusion was correct.
- Despite that TCQTs and, in particular, Bohmian mechanics, are favourable in comparison to the ψ-complete theories: The set of experiments for which TCQTs give unambiguous predictions is strictly larger than that of ψ-complete theories.