CHIRAL EFFECTS: an UPDATE

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Part I Introduction. Quantum hydrodynamics

Discovery of quark-gluon plasma at RHIC (2005) triggered U-turn of interests in high-energy physics: back
to condensed matter (in relativistic and quantum set up)

–Discoveries at RHIC: fast thermalization, nearly ideal fluid.... Why?

–Rather intense Experiment-Theory interaction
(e.g., imotivation to consider rotated, accelerated matter in magn.field)

-discipline“Quantum fluids” becomes driving force of FT

However, models describing experiment and “theoretical models"are quite far apart from each other as a rule

Son-Surowka paper (2009)

A crucial point: hydrodynamics utilizes, essentially, only conservation laws and gradient expansion, and $\partial_\alpha s^\alpha \geq 0$

In case of anomalous chiral symmetry

$$\partial_\alpha J_5^\alpha = C_5 \vec{E} \cdot \vec{B} \qquad (\textit{gauge anomaly})$$

$$\partial_\alpha J_5^\alpha = C_{gr} R\tilde{R} \quad (\textit{gravitational anomaly})$$

Non-vanishing r.h.s.–being a quantum one-loop effect– inevitably penetrates physical observables, e.g.:

$$\vec{J}^{el}_{phen} = C_5 \mu_5 \vec{B} \qquad \textit{chiral magnetic effect} \quad (CME)$$

$$\vec{J}_{5,phen} = C_5 \mu_V^2 \vec{\Omega} \qquad \textit{chiral vortical effect} \quad (CVE)$$

$\vec{E}, \vec{B}$ and $\vec{\Omega}$ are e-m fields and angular velocity of the fluid

μ_V, μ_5 are chemical potentials (some constants omitted)

Outline of the talk

Part II Most actual issue
(interpretation of results of the seach for CME at RHIC)
2a. Revision of models
2b Revision of theory of CME

Part III Strong claims in theory
(physics in non-inertial frames)
3a Nonconservation of electric current
3a Gravitational analog of Son-Surowka
3c Checks of Generalized Equivalence Principle

Part IV Most interesting
4a Matching theory of quantum phase transitions

Part II Isobar run at RHIC

To elucidate the role of CME, collide heavy ions with same atomic number but different electric charge
(Zi, Q=40 or Ru Q=44)

Basic idea: Magnetic field is sensitive to charge of projectiles and different for Q=40 and Q=44
the background is eliminated by equality of atomic numbers

Within this interpretation of results
the results of experiment are certainly negative

Revision of the model

Prompted by:

- The story of background is more complicated. Shapes of the nuclei are somewhat different. Need further phenomenological, experimental parameters
- Make theory as phenomenological as possible. No input from theory of the QCD vacuum.

The scheme adopted: Son-Surowka at short distances/times and then expansion, thermalization of the plasma

A few reference on significance of isobar run

"Implications of the isobar run results for chiral magnetic effect in heavy ion collisions". Dmitri E. Kharzeev , Jinfeng Liao , Shuzhe Shi 2205.00120 [nucl-th].

"Utilization of event shape in search of the chiral magnetic effect in heavy-ion collisions", Ryan Milton, Gang Wang , Maria Sergeeva, Shuzhe Shi, Jinfeng Liao et al. 2110.01435 .

"Investigation of experimental observables in search of the chiral magnetic effect in heavy-ion collisions in the STAR experiment", Subikash Choudhury , Xin Dong , Jim Drachenberg, James Dunlop , Shinichi Esum , 2105.06044 [nucl-ex] .

"Shuzhe Shi , Hui Zhang , Defu Hou , Jinfeng Liao " Phys.Rev.Lett. 125 (2020) 242301 e-Print:1910.14010.

"Another" Chiral Magnetic effect

"Fresh look at CME":
think about CME as of example of "indifferent equilibrium":

$$\vec{J}_{el} \sim \vec{v} \sim \vec{B}$$

Indeed, no force acting on charged particle if $\vec{v} \parallel \vec{B}$.
Moreover, if there is no friction, any velocity is allowed and its actual value is determined by amount of energy stored

However, it is not the CME we accustomed to (see above):

- the strength of the current is not predictible
- it can be perfectly classical, not quantum
- it is not necessarily relativistic
- the main problem seems stability of the motion

Helicity conservation in ideal fluid

Actually classical CVE (CME) is the eldest version of effects. As is known since long, in case of ideal fluid there is conserved axial charge

$$Q^A_{non-rel} \sim \int d^3x(\vec{v} \cdot rot\vec{v})$$

Conservation of $Q^A_{non-rel}$ is manifestation of diffeomorphism not of chiral symmetry

$$\vec{J}^A \sim \mu^2_{5,non-rel}\vec{\Omega}$$

is an alternative, non-relativistic description of CVE in terms of a different symmetry and of different chemical potential $\mu_{5,non-rel}$

Comments

- CVE can be thought of as a quantum correction to $Q^A_{non-rel}$
- Why ideal fluid at all? –To ensure conservation of the CVE current
- No particle production within consistent hydrodenamic approach

Non- Conservation of Electric current

To get CME from CVE replace $\partial_\alpha \ \rightarrow \ \nabla_\alpha$

There arise amusing non-relativistic anomalies

$$\partial_\alpha J^\alpha_A \ \sim \ \vec{E} \cdot \vec{B} \quad (as\text{``}usual'')$$

$$\partial_\alpha J^\alpha_{el} \ \sim \ \vec{E}_5 \cdot \vec{B} \equiv \ \vec{\nabla}\mu_{5,non-rel} \cdot \vec{B}$$

Note, there is no short distance divergencies in non-relativistic physics – results seem reliable

Also, non-conservation of electric current is openly proclaimed, in contrast to FT where it is not allowed (remember Bardeen's regularization scheme)

A few references on ideal fluid and “anomalies”

“On consistency of hydrodynamic approximation for chiral media”, A. Avdoshkin , V.P. Kirilin , A.V. Sadofyev , V.I. Zakharov, e-Print: 1402.3587 [hep-th].

A. G. Abanov and P. B. Wiegmann,
“Axial-Current Anomaly in Euler Fluids”
Phys.Rev.Lett. 128 (2022) 5, 054501,e-Print: 2110.11480 [hep-th]) “Chiral anomaly in Euler fluid and Beltrami flow”, JHEP 06 (2022) 038. e-Print: 2202.12437 [hep-th]

“Divergence anomaly and Schwinger terms: Towards a consistent theory of anomalous classical fluids”,
Arpan Krishna Mitra and Subir Ghosh, 2111.00473 [hep-th].

Two parallel CME phenomenologies

Both approaches are intensely discussed, but no clear understanding of relation between two phenomenologies
Ideal solution :

- non-relativistic ideal fluid is IR completion of relativistic chiral theory, and vice verse
- Ultrarelativistic chiral theory is UV completion of ideal fluid

i. e., 't Hooft-consistency-condition type solution
In one direction the proof is (almost) there (Avdoshkin et al (2013)) while the other direction requires more work to unveil "secret symmetry" bridging diffeomorphismm and chrality conservation

Part III More on symmetries

"Chiral and Gravitational Anomalies on Fermi Surfaces"
G. Basar, D. E. Kharzeev , I. Zahed, 1307.2234 [hep-th]

Consider motion of levels of the Fermi sphere at finite μ caused by external grav. field, a la Nielsen&Ninomiya

$$\partial_\alpha J_5^\alpha = \frac{\mu^2}{2\pi^2}(\vec{a}_{gr} \cdot \vec{\Omega})$$

where $\vec{a}_{gr}$ is the grav. acceleration, $\vec{\Omega}$ is the angular velocity

Upon substitution of Luttinger's identity

$$\vec{a_{gr}} \rightarrow -\vec{\nabla} T / T$$

looks as a novel chiral thermal effect

Gravimagnetic“ anomaly”

The novel effect can be interpreted in terms of Gravimagnetic fields,

$$\vec{B}_{gr} = 2\epsilon\vec{\Omega}, \quad \vec{E}_{gr} = -\epsilon\vec{\nabla}\phi_{gr}$$

where ϵ is energy of test particle

Making this substitution in standard gauge anomaly reproduces the novel effect with all the coefficients correct
But: there is no place for such an anomaly in gravitational case since it is not “gauge invariant”

Resolution of the puzzle

In presence of gravity the conservation law takes on form

$$\nabla_\alpha(\mu^2\omega^\alpha) \ \sim \ \partial_\alpha(\sqrt{-g}\mu^2\omega^\alpha) \ = \ 0$$

where ∇_α is the covariant derivative.

Rewrite this equation as

$$\partial_\alpha(\mu^2\omega^\alpha) \ = \ (non - zero) \ \sim \ \mu^2(\vec{a}\cdot\vec{\Omega})$$

Covariant conservation is exactly the same as anomalous conservation in inertial frames (P. Mitkin+VZ (2021))
Pre-gravity relations. No gravitational field is needed.
Only going into non-itertial frame

Deciphering non-conservation of electric current

Exactly the same mechanism explains non-conservation of electric current discussed above

Equilibrium assumes introduction of both axial' electric field $\vec{E}_5$ and of $\vec{\nabla}\mu_5$. Absence of $\vec{E}_5$ results in acceleration and, therefore, in pure kinematic non-conservation of electric current

Chiral Kinematical Effect

Gravitational analog of Son-Surowka relations

$$J^A_\mu \;=\; \Big(-\frac{\omega^2}{24\pi^2}-\frac{a^2}{8\pi^2}\Big)\omega_\mu + O(R_{\alpha\beta\gamma\delta}) \qquad (spin\ 1/2)$$

$$J^A_\mu \;=\; \Big(-\frac{53\omega^2}{24\pi^2}+\frac{5a^2}{8\pi^2}\Big)\omega_\mu + O(R_{\alpha\beta\gamma\delta}) \qquad (spin\ 3/2)$$

$$\omega_\mu = (1/2)\epsilon_{\mu\nu\rho\sigma}u^\nu\partial^\rho u^\sigma, \qquad a_\mu = (u^\nu\partial_\nu)u_\mu$$

Input: $\partial_\alpha J^\alpha_5 = (const)R\tilde{R}, \quad \partial_\alpha s^\alpha = 0$

(G. Prokhorov, O. Teryaev, VZ (2022))

Comment

Chiral kinematical effect survives if gravity is switched off
How can it be fixed by the gravitational anomaly?

Partial answer: equilibrium survives in absence of gravity.
Equilibrium is described in tems of effective interaction,
in particular, $$\hat{H}_{int} = \vec{\Omega}\hat{\vec{M}}$$
on the other hand, metric tensor ("gravity") also knows about rotation

Isn't it possible to reduce gravity to theory of equilibrium or vice versa?
(E. Verlinde (2011))

Generalized equivalence principle

Used both FT ($\partial \cdot J = R\tilde{R}$) and thermodynamics ($\partial \cdot s = 0$)
Stronger hypothesis, reminiscent of equality of gravitational and entropic forces can also be tried
Statistically, effective, or macroscopic interaction

$$\hat{H}_{eff} = \vec{\Omega} \cdot \hat{\vec{M}} + \vec{a} \cdot \hat{\vec{K}}$$

where $\vec{M}$ is angular momentum and $\vec{K}$ is the boost
In FT

$$\hat{H}_{fund} = \frac{1}{2} \hat{\Theta}^{\alpha\beta} h_{\alpha\beta}$$

where $\Theta^{\alpha\beta}$ is the energy momentum tensor, $h_{\alpha\beta}$ is the grav. potentials accommodating the same $\vec{\Omega}$, $\vec{a}$
$* * *$ Evaluate "external probes", $< \Theta^{\alpha\beta} >, < J_5^{\alpha} >$
Expect results to be the same (duality)

More on statistical approach

- The scheme known to work in case of pure rotation. Inclusion of acceleration is recent, see "Thermodynamic equilibrium with acceleration and the Unruh effect" F. Becattini 1712.08031 [gr-qc]
- Statistical averaging involves density operator $\hat{\rho}$ where $\hat{\rho} = \frac{1}{Z} \exp\left(- b_\alpha \hat{P}^\alpha + \bar{\omega}_{\alpha\beta} \hat{J}^{\alpha\beta} \right)$ where $\hat{J}^{\alpha\beta}$ are generators of the Lorentz transformations $\bar{\omega}_{\alpha\beta} = \partial_\alpha (u_\beta / T) - \partial_\beta (u_\alpha / T)$,
- The boost operators $\hat{K}^\alpha$ are conserved but do not commute with $\hat{H}$. A novel feature!

Statistics-gravity duality at work

Evaluate energy density Θ_{00} of quantum massless spinors as function of independent a, T exploiting 'novel' density operator (G. Prokhorov, O. Teryaev, VZ+references)

$$\rho_{vac} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2}$$

This calculation entirely within statistical approach

On the gravitational side, in the Euclidean set up, temperature related to circumference in time direction acceleration is related to the distance to cone apex ,

Result for quantum correction to the vacuum energy (Casimir effect) is the same

Now the duality is extended to include grav. anomaly

“Probes” and phase transitions

At temperatures below Unruh temperature $T_U = a/(2\pi)$ our “probes ” demonstrate exotic behaviour:

vacuum energy density gets negative,
currents oscillate fast

Probably, there are phase transitions at $T = T_U$ and we enter another fast developing field of quantum phase transitions

Conclusions

Picture presented:

- Chiral effects as “indifferent equilibrium” of ideal fluids
- Equilibrium in non-inertial frames. Novel effects: non-conservation of electric current; chiral kinematical effrect; generalized equivalence principle, or duality between gravity and statistics
- Mentioned possible quantum phase transitions at Unruh temperature