Bigravity and all that

Vladimir O. Soloviev

XXXIV International Workshop on High Energy Physics
"From Quarks to Galaxies: Elucidating Dark Sides"

November, 22, 2022

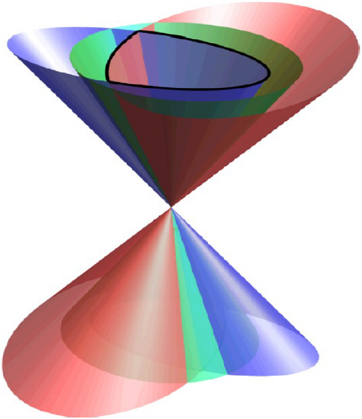
— O, My God, they have found one more elementary particle. What we shall do?

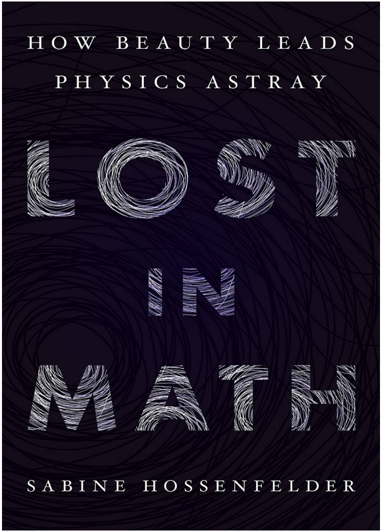
— Let us add one more term to the TRUE UNIFIED FIELD THEORY LAGRANGIAN.

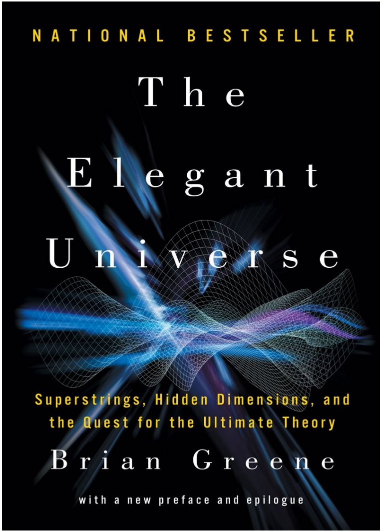
General Relativity (Einstein, Grossmann, Hilbert, 1913 – 1915) has not introduced new physical constants, maybe only one: Λ – cosmological constant.

Bigravity (Hassan, Rosen, de Rham, Gabadadze, Tolley, 2010 – 2011) has 6 new constants: ratio of two gravitational constants G_f/G_g, and five constants of the gravitational potential $\beta_0, \ldots, \beta_4$.

Bigravity: the two lightcones (Fig. by Kocic)




HOW BEAUTY LEADS PHYSICS ASTRAY
LOST IN MATH
SABINE HOSSENFELDER


NATIONAL BESTSELLER
The Elegant Universe
Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory
Brian Greene
with a new preface and epilogue

Motivation

Phenomenological reasons for inventing bigravity: the problems of dark energy and dark matter.
Theoretical reasons: curiosity to find the massive spin 2 field theory.

On the history of bimetric and bigravity theories

Nathan Rosen (1940, and later works) – the problem of energy-momentum of the gravitational field.
Kraichnan, Gupta, Feynman, Weinberg, Deser . . .,
(1950-1970) – the field theory approach to gravitation theory.
Isham, Salam, Strathdee, (1971) – the fifth force problem (on interaction of spin-2 mesons with gravity)
Wess, Zumino (1980) – the massive gravity
Damour, Kogan (2002) – gravity in higher dimensions and its consequences
Hassan, Rosen (2010 – 2011) – no-ghost dRGT potential for interaction of two metric tensors

The bigravity Lagrangian

Let us take two copies of the GR Lagrangian

$$
\begin{aligned}
S_0 &= \int d^4x \frac{M_f^2}{2}\sqrt{-f}\, f^{\mu\nu} R^{(f)}_{\mu\nu} + \mathcal{L}^{(f)}_M(\psi^A, f_{\mu\nu}) && (1)\\
&+ \int d^4y \frac{M_g^2}{2}\sqrt{-g}\, g^{\mu\nu} R^{(g)}_{\mu\nu} + \mathcal{L}^{(g)}_M(\phi^A, g_{\mu\nu}), && (2)
\end{aligned}
$$

they have the two independent diffeomorphysm invariances

$$
x^\mu \to x'^\mu(x^\alpha), \qquad y^\mu \to y'^\mu(y^\alpha). \qquad (3)
$$

But introducing the potential we loose one of them, now only diagonal diffeomorhysms are allowed

$$
S = S_0 - m^2 M_g^2 \int d^4z \sqrt{-g}\, U(g_{\mu\nu}, f_{\mu\nu}). \qquad (4)
$$

Invariants for the potential

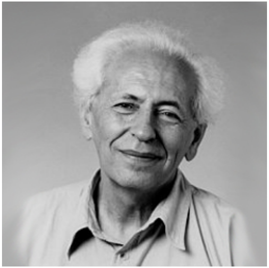
Take matrix $\mathsf{Y} = ||g^{\mu\alpha} f_{\alpha\nu}||$, and consider its invariants:

$$
\begin{aligned}
e_0 &= 1, \\
e_1 &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\
&\equiv \mathrm{Tr}\mathsf{Y}, \\
e_2 &= \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_4 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_4 \\
&\equiv \frac{1}{2}\left((\mathrm{Tr}\mathsf{Y})^2 - \mathrm{Tr}\mathsf{Y}^2\right), \\
e_3 &= \lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_1\lambda_2\lambda_4 \\
&\equiv \frac{1}{6}\left((\mathrm{Tr}\mathsf{Y})^3 - 3\mathrm{Tr}\mathsf{Y}\mathrm{Tr}\mathsf{Y}^2 + 2\mathrm{Tr}\mathsf{Y}^3\right), \\
e_4 &= \lambda_1\lambda_2\lambda_3\lambda_4 \\
&\equiv \det \mathsf{Y},
\end{aligned}
$$

An example of the potential

The RTG (Relativistic Theory of Gravitation) by A.A. Logunov and his collaborators is a theory of massive gravity with the potential

$$\boxed{\sqrt{-g}U = \left(\sqrt{-g}\left(\frac{1}{2}\mathrm{Tr}\mathsf{Y} - 1\right) - \sqrt{-f}\right)} \qquad (5)$$



Boulware-Deser ghost

In general case there are 8 degrees of freedom, one more than required for one massless and one massive field of spin-2 in 4-dimensional spacetime.

Also it has been shown that there is a negative kinetic energy for one degree of freedom, so it is a ghost. For a long period of time (1972 – 2010) it was believed that nonlinear massive gravity (and the bigravity) was impossible to construct.

The dRGT (de Rham, Gabadadze, Tolley) potential



$$\boxed{U_{\mathrm{dRGT}} = \sum_{i=0}^{i=4} \beta_i e_i(\mathsf{X}) \qquad \mathsf{X} = \sqrt{\mathsf{Y}}}.$$

Fawad Hassan and Rachel Rosen

Proportional metrics as backgrounds

Let metrics

$$\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}, \tag{6}$$

be solutions of the vacuum (no matter) bigravity equations

$$G_{\mu\nu}(\bar{g}) + V^g_{\mu\nu} = 0, \quad G_{\mu\nu}(\bar{f}) + V^f_{\mu\nu} = 0. \tag{7}$$

As usual, we have Bianchi identities

$$\nabla^\mu_g G^g_{\mu\nu} \equiv 0, \qquad \nabla^\mu_f G^f_{\mu\nu} \equiv 0, \tag{8}$$

and therefore on-shell equations

$$\nabla^\mu_g V^g_{\mu\nu} \equiv 0, \qquad \nabla^\mu_f V^f_{\mu\nu} \equiv 0. \tag{9}$$

It follows from here that $c = \mathrm{const}$.

Details

All formulas are simple for proportional metrics

$$f_{\mu\nu} = c^2 g_{\mu\nu}, \rightarrow \mathsf{Y} = ||g^{-1}f|| = c^2\mathsf{I}, \quad \mathsf{X} = \sqrt{\mathsf{Y}} = c\mathsf{I}, \quad (10)$$

Then

$$V^g_{\mu\nu} = g_{\mu\nu}\Lambda_g, \quad \Lambda_g = m^2 B_0(c), \quad (11)$$

$$V^f_{\mu\nu} = f_{\mu\nu}\Lambda_f, \quad \Lambda_f = \frac{m^2}{\alpha^2}\frac{B_1(c)}{c^3}, \quad (12)$$

where notations are the following

$$\alpha = \frac{M_f}{M_g}, \quad B_i(c) = \beta_i + 3\beta_{i+1}c + 3\beta_{i+2}c^2 + \beta_{i+3}c^3. \quad (13)$$

Spacetimes of constant curvature

In order to discuss the graviton mass we are to consider small fields on maximally symmetric space-times. Then for background proportional metrics we have equations

$$G_{\mu\nu}(\bar{g}) + \bar{g}_{\mu\nu}\Lambda_g = 0, \quad G_{\mu\nu}(\bar{f}) + \bar{f}_{\mu\nu}\Lambda_f = 0, \tag{14}$$

and as the Einstein tensor does not change when metric is multiplied by constant we must have

$$\Lambda_g = c^2\Lambda_f. \tag{15}$$

This provides the following equation for c:

$$\alpha^2\beta_3 c^4 + (3\alpha^2\beta_2 - \beta_4)c^3 + 3(\alpha^2\beta_1 - \beta_3)c^2 + (\alpha^2\beta_0 - 3\beta_2)c - \beta_1 = 0. \tag{16}$$

Minkowski background (1)

We come to the simplest case when both tensors $g_{\mu\nu}$ and $f_{\mu\nu}$ have the common flat background

$$\Lambda_f = 0 = \Lambda_g, \qquad c = 1, \qquad \bar{g}_{\mu\nu} = \eta_{\mu\nu} = \bar{f}_{\mu\nu}. \tag{17}$$

It is achieved, for example, if

$$\beta_1 = \beta_3 = 0, \qquad \beta_0 = \beta_4 = -3\beta_2. \tag{18}$$

Minkowski background (2)

Linear perturbations $h_{\mu\nu}$ and $\ell_{\mu\nu}$ are as follows

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_g} h_{\mu\nu}, \qquad f_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_f} \ell_{\mu\nu}, \tag{19}$$

and for simplest case ($\beta_0 = 3$, $\beta_1 = -1$, $\beta_2 = 0 = \beta_3$, $\beta_4 = 1$) we get the following Fierz-Pauli term

$$-\frac{m^2 M_{\text{eff}}^2}{4} \left[\left(\frac{h^\mu_\nu}{M_g} - \frac{\ell^\mu_\nu}{M_f} \right)^2 - \left(\frac{h^\mu_\mu}{M_g} - \frac{\ell^\mu_\mu}{M_f} \right)^2 \right], \tag{20}$$

corresponding to variable

$$v_{\mu\nu} = M_{\text{eff}} \left(\frac{h_{\mu\nu}}{M_g} - \frac{\ell_{\mu\nu}}{M_f} \right), \qquad M_{\text{eff}} = \sqrt{M_g^2 + M_f^2}. \tag{21}$$

More general case

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_g} h_{\mu\nu}, \qquad f_{\mu\nu} = c^2 \bar{g}_{\mu\nu} + \frac{c}{M_f} \ell_{\mu\nu}, \tag{22}$$

Then we get massive $M_{\mu\nu}$ and massless $G_{\mu\nu}$ fluctuations (here $\alpha = M_f / M_g$)

$$G_{\mu\nu} = \frac{1}{1 + c^2\alpha^2} \left(h_{\mu\nu} + c\alpha \ell_{\mu\nu} \right), \tag{23}$$

$$M_{\mu\nu} = \frac{1}{1 + c^2\alpha^2} \left(\ell_{\mu\nu} - c\alpha h_{\mu\nu} \right), \tag{24}$$

and the Fierz-Pauli mass appears as

$$m^2_{\mathrm{FP}} = m^4 \left(\frac{1}{c^2 M_f^2} + \frac{1}{M_g^2} \right) (\beta_1 c + 2\beta_2 c^2 + \beta_3 c^3). \tag{25}$$

Effective Planck mass is

$$M_p = M_g\sqrt{1 + c^2\alpha^2} \equiv \sqrt{M_g^2 + c^2 M_f^2}. \tag{26}$$

The linearized equations are

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} G_{\rho\sigma} + \Lambda_g G_{\mu\nu} = \frac{1}{M_p}\left(\delta T_{\mu\nu}^{(g)} + c^2 \delta T_{\mu\nu}^{(f)}\right), \tag{27}$$

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} M_{\rho\sigma} + \Lambda_g M_{\mu\nu} = \frac{c}{M_p\alpha}\left(\delta T_{\mu\nu}^{(f)} - \alpha^2 \delta T_{\mu\nu}^{(g)}\right) - $$

$$- \frac{m_{\mathrm{FP}}^2}{2}(M_{\mu\nu} - \bar{g}_{\mu\nu}\bar{g}^{\rho\sigma} M_{\rho\sigma}) \tag{28}$$

Einstein was the first who tried to find equation for the Universe. But he was not brave enough to consider Universe as a dynamical system.

Alexander Friedmann has predicted the expanding Universe

(1922–2022)

Also: *How Friedmann shod Einstein.* arXiv:2204.10650

Friedmann's ansatz

Homogeneous and isotropic Universe may be described as a spacetime with metric

$$ds^2 = -dt^2 + R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \quad (29)$$

where one of the opportunities $k = -1, 0, +1$ must be chozen. The matter, i.e. a source of gravitation, is also supposed to be homogeneous isotropic and at rest. Usually, the ideal fluid is playing this role

$$T^\mu_\nu = \mathrm{diag}(\rho(t), p(t), p(t), p(t)). \quad (30)$$

Friedmann's equations

The main equation is a constraint from the Hamiltonian view, because it is of the 1st order in time derivatives

$$3\left(\frac{\dot{R}}{R}\right)^2 + \frac{3k}{R^2} = \frac{\rho}{M_g^2} + \Lambda \equiv \frac{1}{M_g^2}\left(\rho + M_g^2\Lambda\right), \qquad (31)$$

therefore the initial data $H(t_0)$, $\rho(t_0)$, $R(t_0)$ must fulfil it. l.h.s contains the Hubble constant $H = \frac{\dot{R}}{R}$ and the curvature parameter k. For simplicity we consider only flat space case when $k = 0$.

The 2nd order in time derivative equation is as follows

$$\dot{H} = -\frac{1}{M_g^2}(\rho + p), \tag{32}$$

and there is a consequence of the above equations:

$$\dot{\rho} = -3H(\rho + p), \tag{33}$$

which is an energy-momentum conservation law.

УДК 519.62

Кузнецов А.Е., Коньков В.В.

МОДЕЛИРОВАНИЕ РАЗЛИЧНЫХ ЭТАПОВ РАЗВИТИЯ ВСЕЛЕННОЙ СОГЛАСНО ЧАСТНЫМ РЕШЕНИЯМ УРАВНЕНИЯ ФРИДМАНА

MODELING DIFFERENT STAGES OF DEVELOPMENT OF THE UNIVERSE ACCORDING TO PARTICULAR SOLUTIONS OF THE FRIEDMANN EQUATION

Филиал «Протвино» государственного университета «Дубна»
Секция «Естественные и инженерные науки»

Авторы: Кузнецов Алексей Евгеньевич, Коньков Владислав Владимирович, студенты 4 курса кафедры «Информатика и вычислительная техника» Государственного университета «Дубна» (филиал Протвино).

Научный руководитель: Зюзько Татьяна Николаевна, кандидат технических наук, доцент кафедры Общеобразовательных дисциплин Государственного университета «Дубна» (филиал Протвино).

Аннотация

С открытием реликтового излучения и космологического красного смещения в моделировании состояния Вселенной стала преобладать теория нестационарного расширения. Вселенная Фридмана была первой моделью расширяющейся Вселенной с положительной, нулевой и отрицательной постоянной кривизной, она являлась первым основным развитием Общей Теории Относительности после Эйнштейна, и по сей день на ней основана современная космология.

В данной статье изучаются этапы и ключевые переходные моменты развития Вселенной,

Для построения каждой из кривых было получено частное решение по Ω_1 и Ω_2. Вклад Ω_1 отображен кривой синего цвета, а Ω_2- зеленого. Полученный результат на Рис. 1:

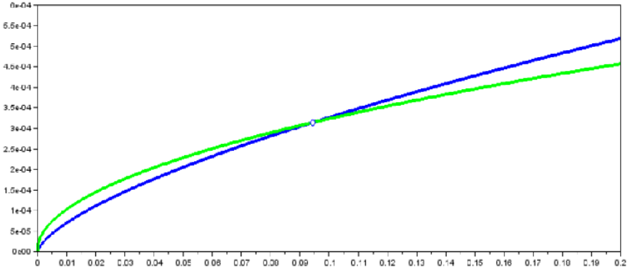


Рис.1 Переход от эпохи доминирования излучения к эпохе доминирования вещества

По оси OX откладывается время в млн лет. По оси OY откладывается радиус Вселенной в относительных единицах (где 0 – Большой Взрыв, 1 — настоящее время). Также, на Рис.1 отображена точка пересечения кривых, в которой вклады сравнялись на моменте времени около 90 000 лет (t = 94300 лет). Момент перехода означает равенство плотностей энергии нерелятивистской и релятивисткой материи. Положение данной точки соответствует пересечению кривых. Важно отметить, что кривые существенно отличаются - кривая излучения (Ω_2) круче идет к нулю, что характеризует преобладание ультрарелятивистских частиц в первые минуты после Большого Взрыва.

На Рис. 2 изображен полученный график радиуса Вселенной от Большого взрыва до 20 млрд лет. Кривая масштабного фактора - синим цветом. Для наглядности была проведена касательная красного цвета и черная вертикальная линия – наше время (13.7 млрд лет).

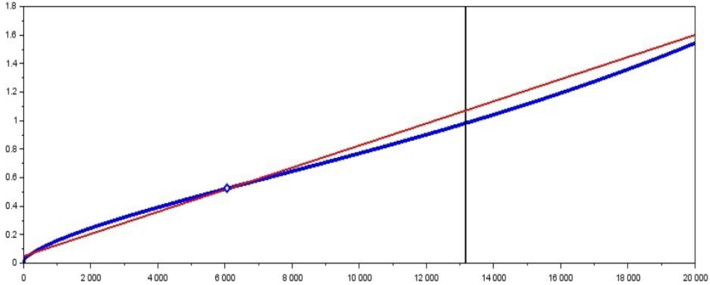


Рис. 2 - График радиуса Вселенной от Большого взрыва до 20 млрд лет.

В момент времени примерно 6 млрд лет отображена точка перегиба – момент перехода от интенсивного, но замедляющегося расширения к расширению с ускорением. Отрицательное ускорение соответствует отрицательной второй производной и выпуклому графику, а положительное ускорение – положительной второй производной и вогнутому графику.

Известно, что точное решение уравнения (1) может быть получено только для частного случая, когда вклады Ω_2 и Ω_3 равны нулю. [1, с. 86]. Исходя из этого, соответствующими вкладами можно пренебречь. Тогда точное решение уравнения Фридмана примет вид:

$$R' = \left(\frac{\Omega_1}{\Omega_4}\right)^{\frac{1}{3}} * sh\left(\left(\frac{3}{2}\right) * \sqrt{\Omega_4} * H_0 * t\right)^{\frac{2}{3}} \qquad (7)$$

Рассматривая точное решение уравнения Фридмана (7) при постоянно растущем R, было замечено, что вклад темной энергии Ω_4, как неизвестного перелятивистского вещества, начинает очень быстро возрастать относительно любой формы материи, что характеризуется практически экспоненциальным ростом масштабного фактора Вселенной в будущем - Рис 3.

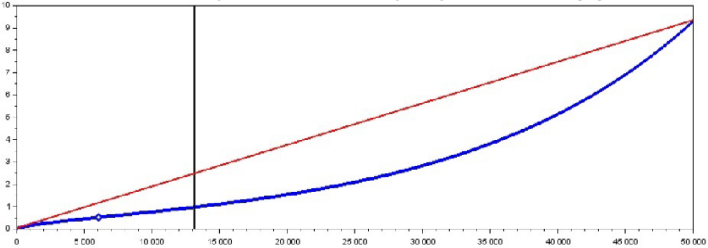


Рис.3 График эволюции Вселенной до 50 млрд лет

Friedmann's ansatz in bigravity

The starting point is two diagonal metrics with the same curvature parameter, below we will take $k = 0$

$$ds_g^2 = -dt^2 + R_g^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \quad (34)$$

$$ds_f^2 = -N(t)^2 dt^2 + R_f^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \quad (35)$$

the scale factors R_g and R_f are different, it is useful to exploit their ratio as a new variable $y = R_f/R_g$, the time components are also different.

The Friedmann equations in bigravity

The 1st order equations are as follows ($k = 0$)

$$3H_g^2 = \frac{\rho_g}{M_g^2} + \Lambda_g(y), \tag{36}$$

$$3H_f^2 = \frac{\rho_f}{M_f^2} + \Lambda_f(y), \tag{37}$$

where we can find dynamical dark energy, i.e. two different functions of y instead of cosmological constants

$$\Lambda_g(y) = m^2 \left(\beta_0 + 3\beta_1 y + 3\beta_2 y^2 + \beta_3 y^3\right), \tag{38}$$

$$\Lambda_f(y) = m^2 \frac{M_g^2}{M_f^2} \left(\frac{\beta_1}{y^3} + 3\frac{\beta_2}{y^2} + 3\frac{\beta_3}{y} + \beta_4\right) \tag{39}$$

There is an additional constraint following from dynamics

$$H_g = yH_f. \tag{40}$$

To simplify the situation let us put $\rho_f = 0 = p_f$, then

$$3H_g^2 = \frac{\rho_g}{M_g^2} + \Lambda_g(y), \tag{41}$$

$$3H_f^2 = \Lambda_f(y), \tag{42}$$

After combining the above equations we can express ρ_g or H_g as a function of y. The inverse problem has up to 4 real solutions because it is necessary to is solve the algebraic equation in order to find y:

$$\frac{M_g^2}{M_f^2}\left(\frac{\beta_1}{y} + 3\beta_2 + 3\beta_3 y + \beta_4 y^2\right) - \beta_0 - 3\beta_1 y - 3\beta_2 y^2 - \beta_3 y^3 = \frac{\rho_g}{m^2 M_g^2}. \tag{43}$$

One usually treat this as branches related to Big Bang

- Finite branch $y \to 0$ when $\rho_g \to \infty$. The ratio of scale factors y is increasing and reaches a constant value when ρ_g tends to zero.
- Infinite branch $y \to \infty$ when $\rho_g \to \infty$. This ratio is decreasing up to finite value.
- Exotic branches for the two other cases. It is possible to get the evolution equation for variable y

$$\dot{y} = (N - y)H_g. \tag{44}$$

So, we see that $y =$ const in two cases:

1. the two metrics are proportional: $N = y$,
2. cosmology is static: $H_g = 0 = H_f$,

On the proportional metrics

The bigravity cosmological solution with proportional metrics

$$ds_f^2 = \left(\frac{R_f}{R_g}\right)^2 ds_g^2, \qquad H_g = NH_f = \mathrm{const}, \tag{45}$$

provides de Sitter solutions for both metrics.
It follows also that

$$\dot{\rho}_g = -3H_g(\rho_g + p_g) = 0. \tag{46}$$

Then, matter should be dissolved completely or should have vacuum equation of state.

Example: calculating the evolution of Universe in bigravity

We can calculate observables $\rho(t)$, $H(t)$, $\Lambda(t)$ if we find $y(t)$. First, solve (43) with parameter ρ for $y = y(\rho))$ and choose the finite branch solution. Next, solve the evolution equation

$$\dot{y} = (N - y)H_g, \tag{47}$$

taking as initial condition $y(t_0) = y(\rho(t_0))$. $N = N(y)$ can be found from the conservation of equation $H_g = yH_f$ during evolution, and $H_g(y)$ is given by the Friedmann equation. The age of Universe t_0 can be found from $y(0) = 0$.

What is the best metric for matter?

Is there any combination of $g_{\mu\nu}$ and $f_{\mu\nu}$ which is minimally coupled to matter?
One proposal was an effective metric

$$\mathcal{G}^{\mathrm{eff}}_{\mu\nu} = (E^A_\mu + \xi F^A_\mu)(E_{\nu A} + \xi F_{\nu A}), \tag{48}$$

but it was proved that this theory would not be ghost-free.
Another idea is to get a new (spatial) metric G_{ij} from the algebra of Hamiltonian constraints where there is an equation

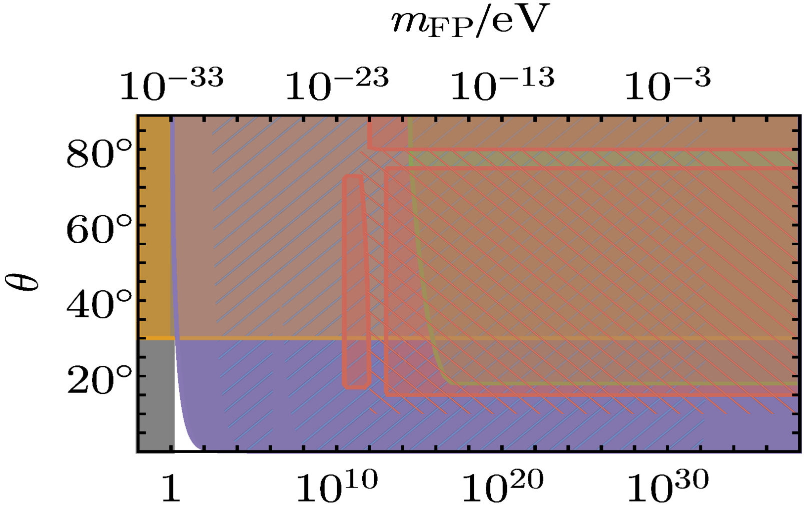
$$\{\mathcal{R}(x), \mathcal{R}(y)\} = \mathcal{G}^{ij}(\mathcal{R}_j(x)\delta_{,i}(x-y) - \mathcal{R}_j(y)\delta_{,i}(y-x)), \tag{49}$$

but it is found that both spatial metrics may appear in the above formula.

On study of observational bounds

- A. Caravano, M. Luben, J. Weller
  Combining cosmological and local bounds on bimetric theory. 2101.08791
- M. Luben, A. Schmidt-May, J. Weller
  Physical parameter space of bimetric theory and SN1a constraints. 2003.03382
- Marcus Hogas, Edvard Mortsell
  Constraints on bimetric gravity. Part I. Analytical constraints. 2101.08794
- Marcus Hogas, Edvard Mortsell
  Constraints on bimetric gravity. Part II. Observational constraints. 2101.08795

Exclusion plot (Thesis of Marcus Hogas)



Conclusions

- Bigravity is motivated theoretically and observationally
- Bigravity is not so nice as General Relativity and contains 6 arbitrary parameters
- Bigravity provides solutions for self-accelerated Universe
- Bigravity maybe has relations to dark matter
- Bigravity survives all the cosmological and local tests (as also ΛCDM model does)
- Bigravity has limits to GR and to dRGT massive gravity
- But questions prevail over answers