Theories with massive gravitons

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Lectures at the ITPM, Moscow University /YouTube/

Motivations for massive gravity

- Cosmic acceleration $\Rightarrow$ either Λ-term or modification of gravity

$$\text{Newton} \quad \frac{1}{r} \quad \rightarrow \quad \text{Yukawa} \quad \frac{1}{r}\, e^{-mr}$$

  $\Rightarrow$ gravity is weaker at large distance $=$ cosmic acceleration, $m \sim 1/(\text{Hubble radius}) \sim 10^{-33}$ eV.

- Small m is more natural than small Λ.

- GW observations $\Rightarrow$ $m < 10^{-22}$ eV

Fierz-Pauli massive gravity

Linear massless gravitons – linearized GR

$$\boxed{G_{\mu\nu} = \kappa T_{\mu\nu}}$$

If $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $\tilde{h}_{\mu\nu} = h_{\mu\nu} - (h/2)\eta_{\mu\nu}$ then

$$\Box\tilde{h}_{\mu\nu} - \partial_\mu\partial^\alpha\tilde{h}_{\alpha\nu} - \partial_\nu\partial^\alpha\tilde{h}_{\alpha\mu} + \eta_{\mu\nu}\partial^\alpha\partial^\beta\tilde{h}_{\alpha\beta} = -2\kappa T_{\mu\nu}$$

or

$$\boxed{\Box h_{\mu\nu} + \ldots = -2\kappa T_{\mu\nu}}$$

Gauge invariance $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu \Rightarrow$ allows one to impose 4 gauge conditions

$$\partial^\mu\tilde{h}_{\mu\nu} = 0 \qquad \Rightarrow \qquad \Box\tilde{h}_{\mu\nu} = -2\kappa T_{\mu\nu}$$

and 4 more conditions can be imposed with the residual $\Box\xi_\mu = 0$, there remains $\Rightarrow$ $10 - 4 - 4 = 2$ DoF.

Linear massive gravitons – Fierz and Pauli /1939/

$\Box\phi = 0 \Rightarrow \Box\phi = m^2\phi$.   Similarly for gravitons

$$\Box h_{\mu\nu} + \ldots = m^2(h_{\mu\nu} - \alpha\, h\, \eta_{\mu\nu}) - 2\kappa T_{\mu\nu}$$

no gauge invariance. Taking the divergence gives 4 constraints

$$m^2(\partial^\mu h_{\mu\nu} - \alpha\partial_\nu h) = 0$$

Taking the trace gives

$$2(\alpha - 1)\,\Box h = m^2(1 - 4\alpha)\, h - 2\kappa T$$

$\Rightarrow$ for $\boxed{\alpha = 1}$ one gets the fifth constraint

$$h = -\frac{2\kappa}{3m^2}\, T$$

$\Rightarrow$ $10 - 5 = 5$ DoF=graviton polarizations.
If $\alpha \neq 1 \Rightarrow$ sixth DoF with negative kinetic energy =ghost.

Summary

The Fierz-Pauli equations

$$\begin{aligned} \Box h_{\mu\nu} + \ldots &= m^2(h_{\mu\nu} - h\,\eta_{\mu\nu}) - 2\kappa T_{\mu\nu} \\ \partial^\mu h_{\mu\nu} &= \partial_\nu h \\ h &= -\frac{2\kappa}{3m^2}\,T \end{aligned}$$

describe massive gravitons with 5=2 tensor+2 vector + 1 scalar polarizations.

One can think that GR will be recovered if $m \to 0$. However, only the 2 vector polarizations decouple for $m \to 0$ and there remain 2 tensor+ 1 scalar polarizations. The extra scalar polarization changes the gravity force.

vDVZ problem

No smooth $m \to 0$ limit

vDVZ discontinuity /1970/

Different tensor structure of the propagator:

$$\text{GR}: \quad \frac{1}{k^2}\, T^{\mu\nu} \left(\frac{1}{2}\, \eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2}\, \eta_{\nu\alpha}\eta_{\mu\beta} - \frac{1}{2}\, \eta_{\alpha\beta}\eta_{\mu\nu} \right) T^{\alpha\beta}$$

$$\text{FP}: \quad \frac{1}{k^2 + m^2}\, T^{\mu\nu} \left(\frac{1}{2}\, \eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2}\, \eta_{\nu\alpha}\eta_{\mu\beta} - \frac{1}{3}\, \eta_{\alpha\beta}\eta_{\mu\nu} \right) T^{\alpha\beta}$$

different force between massive particles for $m \to 0$

$$V_{GR} = -\frac{GM_1 M_2}{r}, \qquad V_{FP} = -\frac{4}{3}\frac{GM_1 M_2}{r}\, e^{-mr},$$

but the same effect on masseless particles – same light deflection.
Newton's law is wrong.

/van Dam, Veltman 1970/, /Zakharov 1970/

vDVZ solution

$$ds^2 = -e^{\nu(r)}dt^2 + \left(1 + \frac{r\mu'}{2}\right)^2 e^{\lambda(r)+\mu(r)}dr^2 + r^2 e^{\mu(r)}d\Omega^2$$

In GR, $\mu(r)$ is a gauge parameter, setting $\mu = 0$ yields

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2 d\Omega^2$$

In FP, μ describes the scalar polarization. The FP equations

$$\begin{aligned} \frac{1}{r}\lambda' + \frac{1}{r^2}\lambda &= -\frac{m^2}{2}(\lambda + 3\mu + r\mu') \\ -\frac{1}{r}\nu' + \frac{1}{r^2}\lambda &= -m^2(\mu + \frac{\nu}{2}) \\ m^2(\frac{\nu'}{2} - \frac{\lambda}{r}) &= 0 \end{aligned}$$

For $m = 0$ one gets the GR solution: μ is arbitrary,

$$\lambda = -\nu = \frac{2\kappa M}{r} \qquad \boxed{\nu + \lambda = 0}$$

For $m \neq 0$ one obtains

vDVZ potential

$$\nu = -\frac{2C}{r}\, e^{-mr}, \quad \lambda = \frac{C}{r}\,(1 + mr)\, e^{-mr}$$

$$\mu = C\, \frac{1 + mr + (mr)^2}{m^2 r^3}\, e^{-mr}$$

In the near zone, for $r \ll 1/m$, this reduces to

$$\nu = -\frac{2C}{r}, \qquad \lambda = \frac{C}{r}, \qquad \mu = \frac{C}{r(mr)^2} \sim \frac{1}{r^3},$$

one has $\boxed{\nu + \lambda \neq 0}$. Light bending should be same as in GR,

$$\delta\phi_{FP} = \frac{3C}{\rho} = \delta\phi_{GR} = \frac{4GM}{\rho} \quad \Rightarrow \quad C = \frac{4}{3}\, GM$$

Newton potential

$$V_{FP} = -\frac{C}{r} = -\frac{4}{3}\frac{GM}{r}$$

Validity of VdVZ solution

Defining the Schwarzschild radius $2GM = r_g$, solution applies if

$$\mu \sim \frac{r_g}{m^2 r^3} \ll 1 \quad \Rightarrow \quad r \gg \frac{(m r_g)^{1/3}}{m}$$

If $r_g = 3$ km=Sun and $m = 1/\text{Hubble radius} = 10^{-10}$Ly then

$$r \gg \frac{(m r_g)^{1/3}}{m} \approx \; 400 \text{ L}y$$

Vainshtein improved this bound by showing that non-linear corrections to the VdVZ are small if only

$$r \gg r_V = \frac{(m r_g)^{1/5}}{m} \approx \; 400\,000 \text{ L}y$$

Vainshtein conjectured that for $r < r_V$ the GR is restored because the scalar polarization is strongly coupled by non-linear effects.

Vainshtein mechanism

Vainshtein solution /1972/

For $r \ll r_V$ one finds the VdVZ with non-linear corrections

$$\nu = -\frac{2r_g}{r}\left(1 + c_1\frac{r_g}{m^4 r^5} + \ldots\right), \quad \lambda = \frac{r_g}{r}\left(1 + c_2\frac{r_g}{m^4 r^5} + \ldots\right)$$
$$\mu = \frac{r_g}{m^2 r^3}\left(1 + c_3\frac{r_g}{m^4 r^5} + \ldots\right)$$

For $r_g \ll r_V$ one finds the GR in the leading order

$$\nu = -\frac{r_g}{r}\left(1 + a_1 (mr)^2\sqrt{r/r_g} + \ldots\right)$$
$$\lambda = \frac{r_g}{r}\left(1 + a_2 (mr)^2\sqrt{r/r_g} + \ldots\right)$$
$$\mu = \sqrt{\frac{a r_g}{r}}\left(1 + a_3 (mr)^2\sqrt{r/r_g} + \ldots\right)$$

The two must agree for $r \sim r_V$ /checked in 2009/.
The linear VdVZ applies only for $r \gg r_V$. For $r \ll r_V$ the non-linear effects confine the extra polarization and restore GR.

Vainshtein scenario

- The VdVZ discontinuity is only visible in the linear regime, for

$$r \gg r_V = \left(\frac{r_g}{m^4}\right)^{1/5}$$

.

- For $r \ll r_V$ the scalar graviton is bound by non-linear effects and does not propagate $\Rightarrow$ GR is recovered.
- For $r \sim r_V$ there is a transition between the two regimes.

The VdVZ problem is cured by the non-linear effects.
This restores GR.

Non-linear Fierz-Pauli

Non-linear Fierz-Pauli

One uses two metrics, the physical one and a flat reference metric, $g_{\mu\nu}$, $f_{\mu\nu} = \eta_{AB}\partial_\mu \Phi^A \partial_\nu \Phi^B$ to build a matrix $\boxed{\hat{S} = \hat{g}^{-1}\hat{f}}$ to determine the potential for the physical metric,

$$U(g,f) = U\left([\hat{S}], [\hat{S}^2], [\hat{S}^3], \det(\hat{S})\right),$$

which enters the action

$$S = \frac{1}{\kappa}\int \sqrt{-g}\left(\frac{1}{2}R - m^2\, U(g,f)\right) d^4x$$

Theory is not unique. If

$$U = \frac{1}{8}\left([\hat{H}^2] - [\hat{H}]^2\right) + \mathcal{O}(H^3) \qquad \text{with} \qquad \hat{H} = \hat{S} - \hat{1}$$

then for weak fields $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ one has $H^\mu{}_\nu = h^\mu{}_\nu$ and everything reduces to Fierz-Pauli.

Equations

One can define two energy-momentum tensors

$$T_{\mu\nu} = 2\frac{\partial U}{\partial g_{\mu\nu}} - U g_{\mu\nu}, \qquad \mathcal{T}_{\mu\nu} = 2\frac{\partial U}{\partial f_{\mu\nu}} - U f_{\mu\nu},$$

Diff. invariance of U implies the identity

$$\sqrt{-g}\, \nabla^{\mu} T_{\mu\nu} - \sqrt{-\eta}\, \partial^{\mu} \mathcal{T}_{\mu\nu} \equiv 0$$

The equations are

$$G_{\mu\nu} = m^2 T_{\mu\nu} \quad \Rightarrow \quad \nabla^{\mu} T_{\mu\nu} = 0 \quad \Rightarrow \quad \partial^{\mu} \mathcal{T}_{\mu\nu} = 0$$

there are two conserved tensors.

Theory of Ogievetsky-Polubarinov /1965/

$$S = \frac{1}{\kappa} \int \sqrt{-g} \left(\frac{1}{2} R(g) - m^2\, U(g,\eta) \right) d^4x$$

$$\text{with} \qquad U = \frac{1}{4n^2} \left(\det(\hat{S}) \right)^{-s/2} [\hat{S}^n]$$

Defining the graviton field $h^{\mu\nu}$ via

$$\left(\frac{\sqrt{-g}}{\sqrt{-\eta}} \right)^{s+1} ((\hat{g}^{-1})^n)^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}.$$

the equations, with $\lambda = -s/(2n)$,

$$G_{\mu\nu} = m^2\, T_{\mu\nu} \quad \rightarrow \quad \Box h_{\mu\nu} = m^2\, h_{\mu\nu} + \text{non-linear terms}$$

$$\partial^\mu T_{\mu\nu} = 0 \quad \rightarrow \quad \partial^\mu h_{\mu\nu} = \lambda \partial_\nu h$$

look like a non-linear completion of theory of free gravitons, first field-theory derivation of the Einstein-Hilbert term.

/S.Mukohyama, M.S.V. 2018/

Boulware-Deser problem

Boulware-Deser ghost

There are infinitely many ways to choose the potential for the theory

$$S = \frac{1}{\kappa} \int \sqrt{-g} \left(\frac{1}{2} R - m^2 \, U(g, f) \right) d^4x$$

such that

$$U = \frac{1}{8} \left([\hat{H}^2] - [\hat{H}]^2 \right) + \mathcal{O}(H^3) \qquad \text{with} \qquad \hat{H} = \hat{S} - \hat{1}$$

All these theories propagate 5 degrees of freedom in the weak field limit and show the Vainshtein mechanist in the strong field limit, so that they do not have the VdVZ problem.

However, away from the weak field limit, they show an additional 6-th dynamical mode which is ghost.

Hamiltonian formulation

The Lagrangian

$$\mathcal{L} = \left(\frac{1}{2} R - m^2 \mathcal{U}\right) \sqrt{-g}$$

after the ADM decomposition

$$\begin{aligned} ds_g^2 &= -N^2 dt^2 + \gamma_{ik}(dx^i + N^i dt)(dx^k + N^k dt) \\ ds_f^2 &= -dt^2 + \delta_{ik} dx^i dx^k \end{aligned}$$

determines the Hamiltonian

$$\boxed{H = \pi^{ik}\dot{\gamma}_{ik} - \mathcal{L} = N^\mu \mathcal{H}_\mu(\pi^{ik}, \gamma_{ik}) + m^2 \mathcal{V}(N^\mu, \gamma_{ik})}$$

with $\mathcal{V} = \sqrt{\gamma}\, N\, U$ and

$$\mathcal{H}_0 = \frac{1}{\sqrt{\gamma}} (2\pi_{ik}\pi^{ik} - (\pi^k_k)^2) - \frac{1}{2}\sqrt{\gamma} R^{(3)}, \quad \mathcal{H}_k = -2\nabla^{(3)}_i \pi^i_k$$

π^{ik}, γ_{ik} span the phase space of dimension 12. Secondary constrints

$$-\dot{p}_{N_\mu} = \frac{\partial \mathcal{H}}{\partial N^\mu} = \mathcal{H}_\mu(\pi^{ik}, \gamma_{ik}) + m^2 \frac{\partial \mathcal{V}(N^\mu, \gamma_{ik})}{\partial N^\mu} = 0$$

Degrees of freedom

$$\frac{\partial \mathcal{H}}{\partial N^\mu} = \mathcal{H}_\mu(\pi^{ik}, \gamma_{ik}) + m^2 \frac{\partial \mathcal{V}(N^\mu, \gamma_{ik})}{\partial N^\mu} = 0$$

- If $m = 0$ this gives 4 first class constraints $\mathcal{H}_\mu(\pi^{ik}, \gamma_{ik}) = 0$, $\{\mathcal{H}_\mu, \mathcal{H}_\nu\} \sim \mathcal{H}_\alpha$, which generate gauge symmetries, one can impose 4 gauge conditions, there remain

$$12 - 4 - 4 = 4 = 2 \times (2 \text{ DoF}) \quad \Rightarrow \quad 2 \text{ graviton polarizations}$$

- If $m \neq 0$ this gives 4 equations for laps and shifts whose solution is $N^\mu(\pi^{ik}, \gamma_{ik})$. No constraints arise $\Rightarrow$ there are

$$12 = 2 \times (5 + 1 \text{ degrees of freedom})$$

The energy $H = N^\mu \mathcal{H}_\mu + m^2 \mathcal{V}(N^\mu, \gamma_{ik})$ is unbounded below because the ghost comes back. This stopped all progress for $\sim$40 years.

Ghost-free massive gravity and bigravity

Ghost-free massive gravity /2010/

One has

$$\frac{\partial \mathcal{H}}{\partial N^{\mu}} = \mathcal{H}_{\mu}(\pi^{ik}, \gamma_{ik}) + m^2 \frac{\partial \mathcal{V}(N^{\mu}, \gamma_{ik})}{\partial N^{\mu}} = 0 \qquad (\star)$$

with

$$\mathcal{V}(N^{\mu}, \gamma_{ik}) = \frac{1}{8}\sqrt{-g}\left([H^2] - [H]^2\right) + \text{higher order terms}$$

One can choose the higher order terms such that

$$\text{rank}\left(\frac{\partial^2 \mathcal{V}(N^{\mu}, \gamma_{ik})}{\partial N^{\nu} \partial N^{\mu}}\right) = 3$$

$\Rightarrow$ the 4 equations $(\star)$ determine only 3 shifts $N^k = N^k(\pi^{ik}, \gamma_{ik})$, the lapse N remains undetermined, the 4-th equation reduces to a constraint

$$\mathcal{C}(\pi^{ik}, \gamma_{ik}) = 0 \qquad \Rightarrow \qquad \dot{\mathcal{C}} = \{\mathcal{C}, H\} \equiv S = 0.$$

The two constraints $\mathcal{C}, \mathcal{S}$ remove one DoF, there remain 5.

dRGT theory

Explicitely

$$S = M_{\mathrm{Pl}}^2 \int \left(\frac{1}{2} R - m^2 \, U \right) \sqrt{-g} d^4 x$$

$$U = b_0 + b_1 \sum_a \lambda_a + b_2 \sum_{a<b} \lambda_a \lambda_b + b_3 \sum_{a<b<c} \lambda_a \lambda_b \lambda_c + b_4 \lambda_0 \lambda_1 \lambda_2 \lambda_3$$

where b_k are parameters and λ_a are eigenvalues of the matrix

$$\boxed{\gamma^\mu_{\ \nu} = \sqrt{g^{\mu\alpha} f_{\alpha\nu}}}$$

Theory propagates 5 and not 5+1 degrees of freedom, therefore called "ghost-free". /de Rham, Gabadadze, Tolley 2010 (1500 citations)/ However, it is unclear if the 5 degrees of freedom are all "healthy". Characteristic surfaces can be locally timelike ⇒ superluminal signals, but it is unclear if this implies aucausality.

Old non Lorenz-invariant models with 5 DoF /Rubakov, Tynakov/

Bigravity

$$S = \frac{M_{\rm Pl}^2}{2\kappa_1}\int R(g)\sqrt{-g}\,d^4x + \frac{M_{\rm Pl}^2}{2\kappa_2}\int R(f)\sqrt{-f}\,d^4x$$
$$- m^2\int \mathcal{U}\sqrt{-g}\,d^4x + S_{\rm mat}[g,\Psi_g] + S_{\rm mat}[f,\Psi_f]$$

with $\kappa_1 + \kappa_2 = 1$ and the same potential as before

$$\mathcal{U} = b_0 + b_1\sum_a \lambda_a + b_2\sum_{a<b}\lambda_a\lambda_b + b_3\sum_{a<b<c}\lambda_a\lambda_b\lambda_c + b_4\lambda_0\lambda_1\lambda_2\lambda_3$$

Both $g_{\mu\nu}$ and $g_{\mu\nu}$ are dynamical, there are two gravitons, one massive and one massless. There are 7 =5+2 degrees of freedom. /Hassan and Rosen 2012/ There are various ways to show that the 8-th mode is excluded /talk of V.Soloviev/. The theory has a better behaviour as compared to the dRGT massive gravity.

Some applications

Massive gravitons in curved space

C.Mazuet, M.S.V. JCAP (2018)

Massive fields in curved space

How to generalize wave equations to curved space ? Spin-0: Klein-Gordon equation

$$(\eta^{\mu\nu}\partial_\mu\partial_\nu - m^2)\Phi = 0$$

generalizes via simply $\eta_{\mu\nu} \Rightarrow g_{\mu\nu}$, $\partial_\mu \Rightarrow \nabla_\mu$ which yields

$$(g^{\mu\nu}\nabla_\mu\nabla_\nu - m^2)\Phi = 0$$

Similarly for spins $1/2, 1$. The procedure fails for 2: the Fierz-Pauli generalizes to curved space only if $R_{\mu\nu} = \Lambda g_{\mu\nu}$, otherwise it shows the 6-th polarization. A long standing problem.

The solution can be obtained using properties of the dRGT theory, which yields a new theory. The massive gravitons in an arbitrary background geometry $g_{\mu\nu}$ can be described by a non-symmetric tensor $X_{\mu\nu}$ which fulfills the equations

Equations: $\Delta_{\mu\nu} + \mathcal{M}_{\mu\nu} = 0$

with the kinetic term

$$\begin{aligned}\Delta_{\mu\nu} &= \frac{1}{2}\nabla^\sigma\nabla_\mu(X_{\sigma\nu}+X_{\nu\sigma})+\frac{1}{2}\nabla^\sigma\nabla_\nu(X_{\sigma\mu}+X_{\mu\sigma})\\ &- \frac{1}{2}\Box(X_{\mu\nu}+X_{\nu\mu})-\nabla_\mu\nabla_\nu X - R^\sigma_\mu X_{\sigma\nu} - R^\sigma_\nu X_{\sigma\mu}\\ &+ g_{\mu\nu}\left(\Box X - \nabla^\alpha\nabla^\beta X_{\alpha\beta} + R^{\alpha\beta}X_{\alpha\beta}\right)\end{aligned}$$

and the mass term

$$\begin{aligned}\mathcal{M}_{\mu\nu} &= \beta_1\left(\gamma^\sigma{}_\mu X_{\sigma\nu} - g_{\mu\nu}\,\gamma^{\alpha\beta}\,X_{\alpha\beta}\right)\\ &+ \beta_2\,\{-\gamma^\alpha{}_\mu\gamma^\beta{}_\nu\,X_{\alpha\beta} - (\gamma^2)^\alpha{}_\mu\,X_{\alpha\nu} + \gamma_{\mu\nu}\,\gamma_{\alpha\beta}\,X^{\alpha\beta}\\ &\quad +[\gamma]\,\gamma^\alpha{}_\beta\,X_{\alpha\nu} + ((\gamma^2)_{\alpha\beta}\,X^{\alpha\beta} - [\gamma]\,\gamma_{\alpha\beta}\,X^{\alpha\beta})\,g_{\mu\nu}\}\\ &+ \beta_3\,|\gamma|\,\left(X_{\mu\sigma}(\gamma^{-1})^\sigma{}_\nu - [X](\gamma^{-1})_{\mu\nu}\right)\end{aligned}$$

$\gamma_{\mu\nu}$ is algebraically related to the background $g_{\mu\nu}$ via

$$\begin{aligned}G_{\mu\nu}(g) &+ \beta_0\,g_{\mu\nu} + \beta_1([\gamma]\,g_{\mu\nu} - \gamma_{\mu\nu})\\ &+ \beta_2\,|\gamma|\,([\gamma]\,\gamma_{\mu\nu} - \gamma^{-2}_{\mu\nu}) + \beta_3\,|\gamma|\,\gamma_{\mu\nu} = 0\end{aligned}$$

Constraints

There are 16 equations

$$E_{\mu\nu} \equiv \Delta_{\mu\nu} + \mathcal{M}_{\mu\nu} = 0$$

for 16 components of $X_{\mu\nu}$. Their contractions imply 11 constraints, the number of DoF is $16 - 6 - 4 - 1 = 5$.

If $R_{\mu\nu} = \Lambda g_{\mu\nu}$ reduce to the Fierz-Pauli – massive gravitons in de Sitter, Higuchi bound, etc.

Massive gravitons in FRW universe form a stable condensate (Dark Matter ?).

Bigravity cosmologies

M.S.V. JHEP 01 (2012) 035

K.-i. Maeda, M.S.V. Phys.Rev. D87 (2013) 104009

FLRW ansatz

$$ds_g^2 = -dt^2 + e^{2\Omega}\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \qquad /k = 0, \pm 1/$$

$$ds_f^2 = -\mathcal{A}^2 dt^2 + e^{2\mathcal{W}}\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

This yields solutions approaching de Sitter at late times with

$$\Lambda_g = m^2 P(b_0, b_1, b_2, b_3)$$

This agrees with observations if either $1/m \sim R_H = 10^{10}$Ly or

$$\frac{1}{m} \sim 10^6\,\text{km}, \quad \frac{\kappa_1}{\kappa_2} \approx \kappa_1 \sim \left(\frac{\mathrm{M_{ew}}}{\mathrm{M_{Pl}}}\right)^2 \sim 10^{-34} \ll 1,$$

where $\mathrm{M_{ew}} \sim 100$ GeV and $\mathrm{M_{Pl}} \sim 10^{19}$ GeV. Agrees with the ΛCDM model. Can mimic the dark matter as well.

“Hairy” bigravity black holes

M.S.V. Phys.Rev. D85 (2012) 124043
R.Gervalle, M.S.V. Phys.Rev. D102 (2020) 124040

Schwarzschild vs. hairy black holes

$$
\begin{aligned}
g^{\mathrm{S}}_{\mu\nu} dx^\mu dx^\nu &= f^{\mathrm{S}}_{\mu\nu} dx^\mu dx^\nu \\
&= -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2 d\Omega^2
\end{aligned}
$$

with M in units of $(\mathbf{M}_{\mathrm{Pl}}/m)\,\mathbf{M}_{\mathrm{Pl}}$. This solution is stable if M is large but becomes unstable if $M < M_{\mathrm{cr}} = 0.43$.

For $M = M_{\mathrm{cr}} = 0.43$ the solution develops a zero mode, which indicates an appearance of new "hairy" solutions:

$$
\begin{aligned}
ds_g^2 &= -Q(r)^2 dt^2 + \frac{dr^2}{N(r)^2} + r^2 d\Omega^2\,, \\
ds_f^2 &= -q(r)^2 dt^2 + \frac{dU(r)^2}{Y(r)^2} + U(r)^2 d\Omega^2.
\end{aligned}
$$

Asymptotically flat hairy black holes

$$
\begin{aligned}
G^{\mu}_{\nu}(g) &= \kappa_1 \, T^{\mu}_{\nu}(g,f), \qquad \kappa_1 = 10^{-34}; \\
G^{\mu}_{\nu}(f) &= \kappa_2 \, \mathcal{T}^{\mu}_{\nu}(g,f), \qquad \kappa_2 = 1 - \kappa_1 \approx 1
\end{aligned}
$$

$\Rightarrow$ the physical g-metric is extremely close to Schwarzschild, all the hair is contained in the f-metric which is directly invisible.

Normally the hairy black holes cannot be distinguished from Schwarzschild. However, in hole collisions the source $T^{\mu}_{\nu}(g,f)$ can become strong enough to overcome the 10^{34} suppression $\Rightarrow$ the hairy features can become visible.

It is possible that "hairy signatures" are contained in GW signals from black hole mergers. These signatures should be stronger for small black holes.

Approximation for M for small κ_1

$$M(r_H) \approx \frac{r_H}{2} + \kappa_1 \, \frac{0.005}{(r_H)^{4.6}}$$

Assuming $\kappa_1 = 10^{-34}$, the minimum is

$$(\mathbf{r}_H)_{\rm min} \approx 0.52 \ \ {\rm km}, \qquad \mathbf{M}_{\rm min} \approx 0.2 \times {\rm M}_\odot$$

$\Rightarrow$ size and mass of the lightest hairy black hole. As for the heaviest,

$$(\mathbf{r}_H)_{\rm max} \sim 10^6 \ {\rm km}, \quad \mathbf{M}_{\rm max} \sim 3 \times 10^6 {\rm M}_\odot$$

Summary

If the bigravity theory indeed describes physics, the astrophysical black holes cannot be Schwarzschild because it is unstable.

They should be hairy, but the hair is contained in the f-metric not coupled to matter and not directly seen, while their g-metric is extremely close to Schwarzschild. Therefore, normally these hairy black holes should be undistinguishable from the usual GR black holes. Their mass ranges from $\sim 0.2\mathrm{M}_{\odot}$ to $\sim 0.3 \times 10^{6}\mathrm{M}_{\odot}$.

Then the deviation from GR may become visible in black hole collisions, which should be visible in signals detected by LIGO/VIRGO. The effect should be larger for small black holes.