Higgs-scalaron inflation

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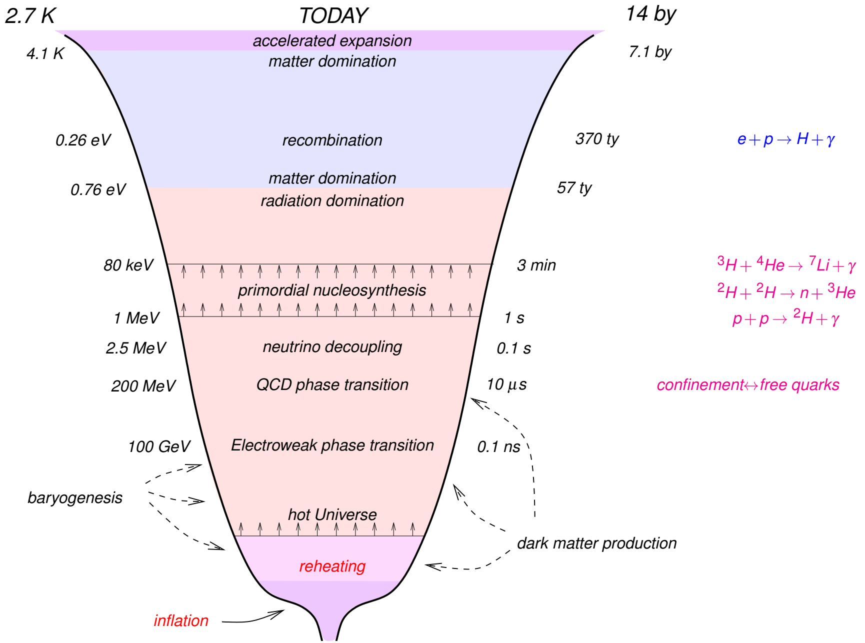
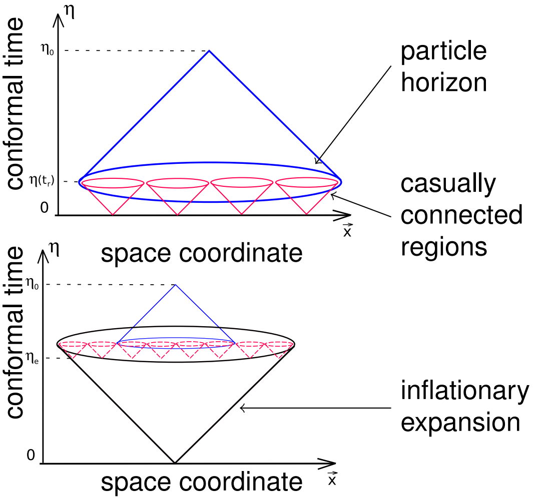
Hot Big Bang within GR and SM: problems

- Dark Matter
- Baryogenesis
- Horizon, Enthropy, Flatness, ... problems
  $l_{H_0}/l_{\mathrm{H},r}(t_0) \sim \sqrt{1+z_r} \simeq 30$
- Singularity at the beginning
- Heavy relics
- Initial perturbations $\delta T/T \sim \delta\rho/\rho \sim 10^{-4}$, scale-invariant
- Dark Energy $0 \neq \Lambda \ll M_{Pl}^4\ M_W^4\ \Lambda_{QCD}^4$ etc ?
- Coincidence problems:
  $$\Omega_B \sim \Omega_{DM} \sim \Omega_\Lambda\ ,$$
  $$\eta_B = n_B/n_\gamma \sim (\delta T/T)^2\ ,$$
  $$T_d^n \sim (m_n - m_p)\ ,$$
  $$\dots$$
- ΛCDM tensions: H_0?, σ_8? dwarfs? cusps? ... (reionization @ $z \simeq 10$, etc)


2.7 K
TODAY
14 by
accelerated expansion
4.1 K
matter domination
7.1 by
0.26 eV
recombination
370 ty
$e + p \to H + \gamma$
matter domination
0.76 eV
radiation domination
57 ty
80 keV
3 min
$^3H + {}^4He \to {}^7Li + \gamma$
primordial nucleosynthesis
$^2H + {}^2H \to n + {}^3He$
1 MeV
1 s
$p + p \to {}^2H + \gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD phase transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Inflationary solution of Hot Big Bang problems

- no initial singularity in dS space
- all scales grow exponentially, including the radius of the 3-sphere
  the Universe becomes exponentially flat
- any two particles are at exponentially large distances
  no heavy relics
  no traces of previous epochs!
- no particles in post-inflationary Universe
  to solve entropy problem we need post-inflationary reheating



Chaotic inflation at large fields: graceful entrance

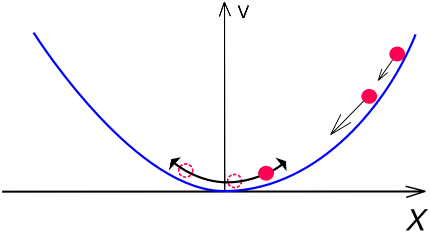
Chaotic inflation, A.Linde (1983), A.Linde (1984)

in all domains of Planck size

each of the form of inflaton energy fluctuates similarly

$$\frac{1}{2}\dot{\phi}^2 \sim \frac{1}{2}(\partial_i\phi)^2 \sim V(\phi) \lesssim M_{\rm Pl}^4$$

If $V(\phi)$ dominates by chance



$$\ddot{\phi} - \Delta\phi/a^2 + 3H\dot{\phi} + V'(\phi) = 0$$

for power-law potential at $\phi > M_{Pl}$

$$V \simeq \text{const}$$

“slow roll” solution

$$H^2 = \frac{8\pi}{3M_P^2}V(\phi)\ , \quad a(t) \propto e^{Ht}$$

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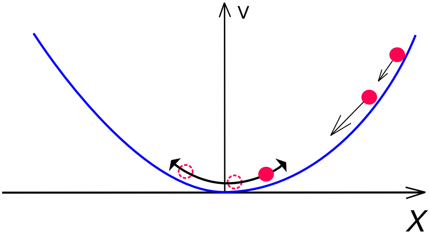
valid while slow roll conditions

$$M_P^2 \frac{V''}{V} \ll 1\,, \quad M_P^2 \frac{V'^2}{V^2} \ll 1$$



Inflaton must couple to Standard Model fields to reheat the Universe after inflation

The idea is great,

but is not verifiable
except for the 3d-flatness

Does it make the idea wrong...?

Unexpected bonus: generation of perturbations

- Quantum fluctuations of wavelength λ of a free massless field φ have an amplitude of $\delta\varphi_\lambda \simeq 1/\lambda$
- In the expanding Universe: $\lambda \propto a$

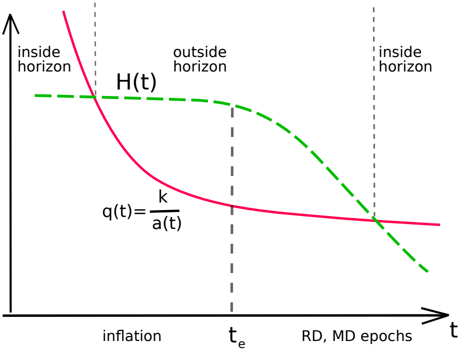
  inflation: $l_H \sim 1/H = \text{const}$, so modes "exit horizon"
  Ordinary stage: $l_H \sim 1/H \propto t$, $l_H/\lambda \nearrow$, modes "enter horizon"

Evolution at inflation

- inside horizon: $\lambda < l_H$
  $\lambda \propto a \Rightarrow$ $\delta\varphi_\lambda \propto 1/\lambda \propto 1/a$

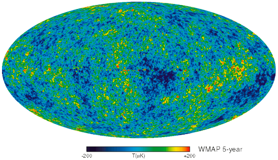
- outside horison: $\lambda > l_H$
  $\lambda \propto a \Rightarrow$ $\delta\varphi_\lambda = \text{const} = H_{infl}$!!!

- got "classical" fluctuations:
  $\delta\varphi_\lambda = \delta\varphi_\lambda^{quantum} \times e^{N_e}$

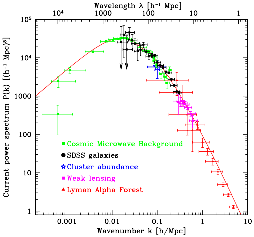


Inflationary solution of Hot Big Bang problems

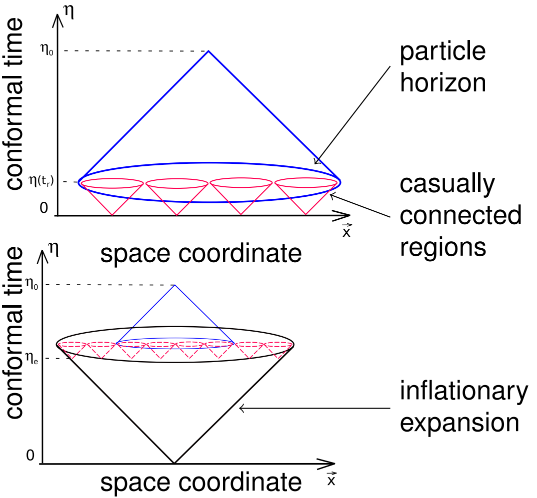
Temperature fluctuations $\delta T/T \sim 10^{-5}$



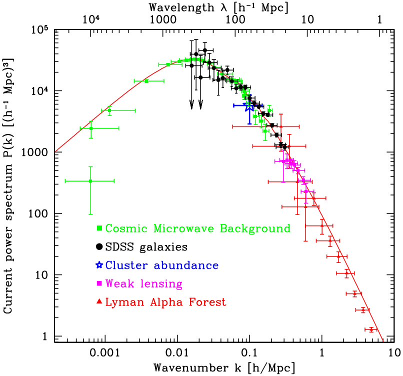
Universe is uniform!

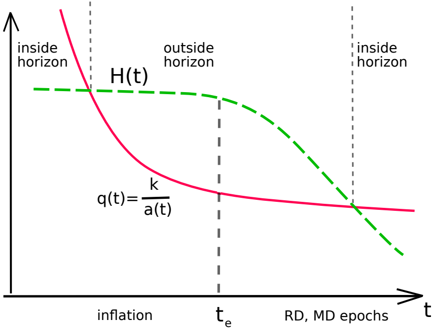


$\delta\rho/\rho \sim 10^{-5}$



Probing the matter power spectrum





$$\delta\phi \to \frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot\phi} \propto \frac{V^{3/2}}{V'}\,, \qquad h \sim \frac{H}{M_{Pl}} \propto V^{1/2}$$

$$A_S \to \frac{V^{3/2}}{V'}\,, \qquad n_S \to \frac{V''}{V}\,, \left(\frac{V'}{V}\right)^2\,, \qquad r \equiv \frac{A_T}{A_S} \to \left(\frac{V'}{V}\right)^2$$

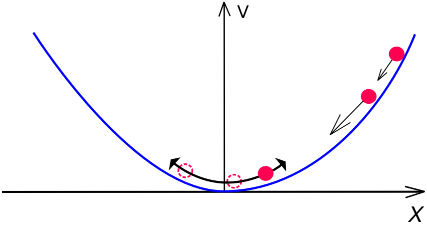
Chaotic inflation: simple realization

$$S = \int d^4x \sqrt{-g}\left(-\frac{M_P^2}{2}R + \frac{(\partial_\mu X)^2}{2} - \beta X^4\right)$$

$$\ddot{X} + 3H\dot{X} + V'(X) = 0$$

$$H^2 = \frac{1}{M_P^2}V(X)\,, \qquad a(t) \propto e^{Ht}$$

Chaotic inflation, A.Linde (1983)



slow roll conditions get satisfied at

$X_e > M_{Pl}$ $\qquad M_P^2 = M_{Pl}^2/(8\pi)$

generation of scale-invariant scalar (and tensor) perturbations from exponentially stretched quantum fluctuations of X $\Longrightarrow$ $\delta\rho/\rho \sim 10^{-5}$ requires $V = \beta X^4 : \beta \sim 10^{-13}$

We have scalar in the SM! The Higgs field!

In a unitary gauge $H^T = \left(0, (h+v)/\sqrt{2}\right)$ (and neglecting $v = 246\,\text{GeV}$) $\lambda \sim 0.01 - 1$

$$S = \int d^4x \sqrt{-g}\left(-\frac{M_P^2}{2}R + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda h^4}{4}\right)$$

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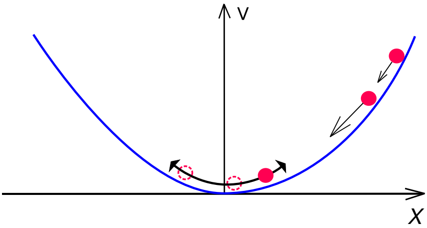

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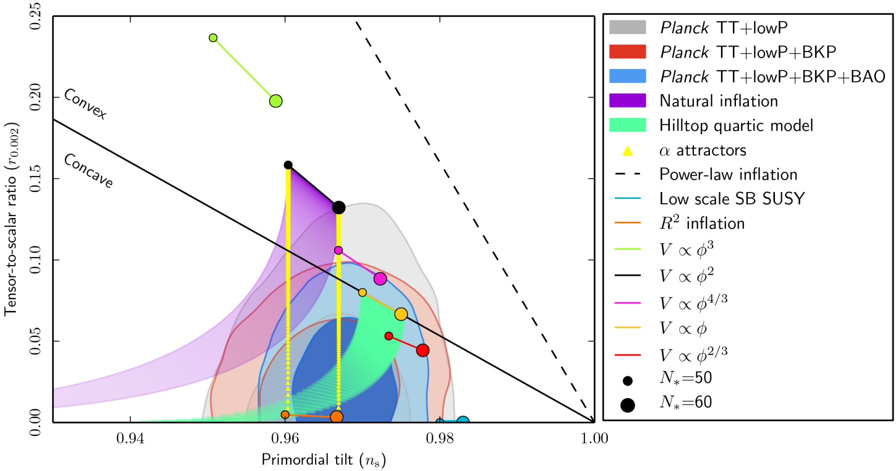
Planck 2015 favors flat inflaton potentials



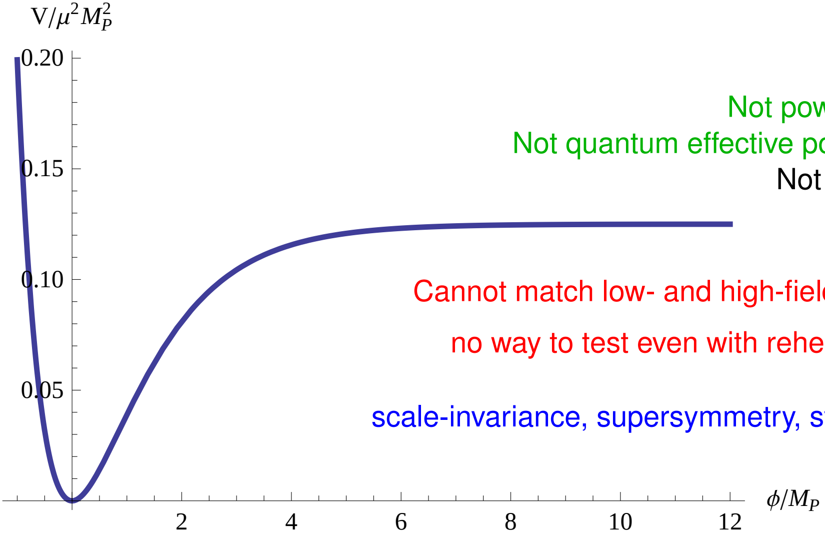
$$r = \frac{A_T}{A_S} \propto \frac{\dot{\phi}^2}{H^2 M_{Pl}^2} \propto \left(\frac{V'}{V}\right)^2 \ll 1$$

Other ways of testing inflation

- Curvature: the World is flat
  not convincing for many
- Relic tensor modes (gravitational waves)
  low-l *B*-mode: well below Galactic foreground
- preheating: $T_{reh} \to N_e$, GW ?
  tiny effects, $n_s, r = f(log(N_e))$, GW from clumps
- Direct tests: inflaton potential
  only in specific models with light inflaton
- Generic for many-field inflation are
  isocurvature modes, non-Gaussianity
- Exotic signatures
  primordial black holes, GW from oscillons, etc

and the calculations must be reliable

Inflaton potential is apparently nonrenormalizable



Not power-law

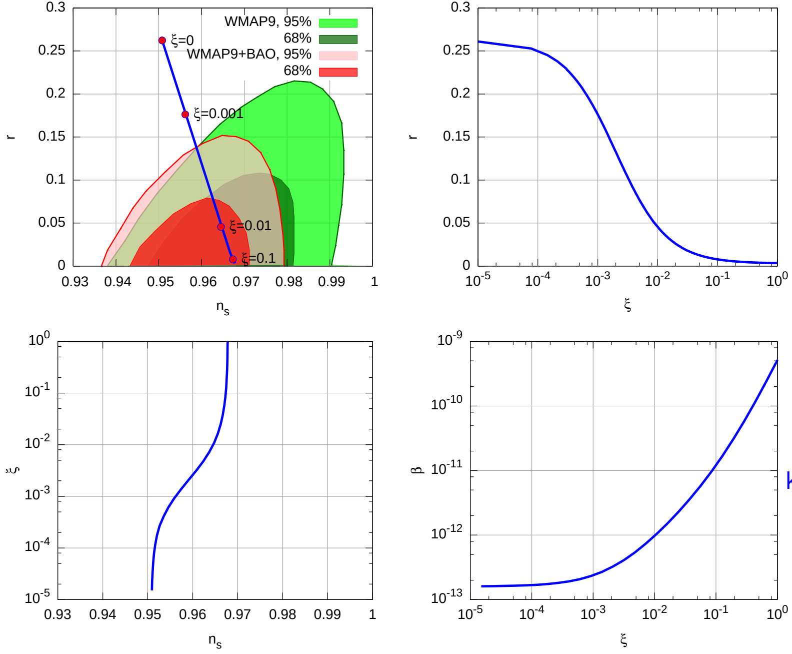
Not quantum effective potential

Not $\log\phi$

Cannot match low- and high-field parts

no way to test even with reheating !!

scale-invariance, supersymmetry, strings?



F.Bezrukov, D.G. (2013)

How it works…

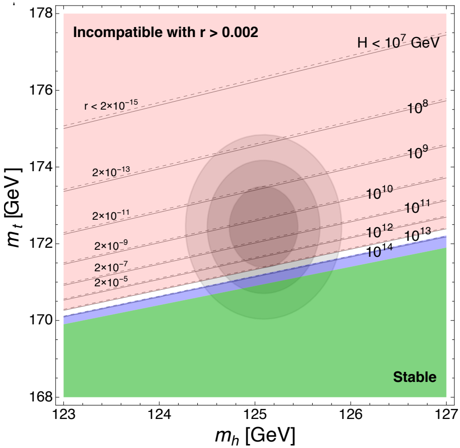
$$V = \beta\phi^4$$

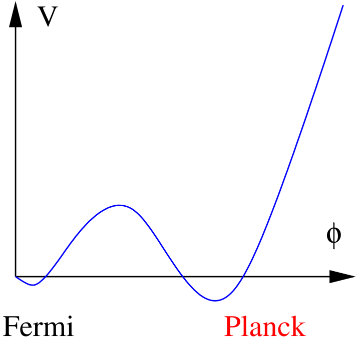
$$\xi\phi^2 R$$

it changes GR-scale

it changes kinetic term hence changes scalar and tensor spectra

Wrong EW vacuum: $\Phi \sim H/(2\pi)$





1607.00381

Thus we either constrain inflation, $H \lesssim \ldots 10^{10}\,\text{GeV}\ldots$ and hence GW, that is r or just assume we are 2σ-off and $\lambda > 0$

Higgs-driven inflation

F.Bezrukov, M.Shaposhnikov (2007)

$$S^{JF} = \int d^4x\sqrt{-g}\left(-\frac{M_P^2}{2}R - \xi H^\dagger H R + \mathscr{L}_{SM}\right)$$

In a unitary gauge $H^T = \left(0, (h+v)/\sqrt{2}\right)$ (and neglecting $v = 246\,\text{GeV}$)

$$S = \int d^4x\sqrt{-g}\left(-\frac{M_P^2 + \xi h^2}{2}R + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda h^4}{4}\right)$$

slow roll behavior due to modified kinetic term even for $\lambda \sim 1$

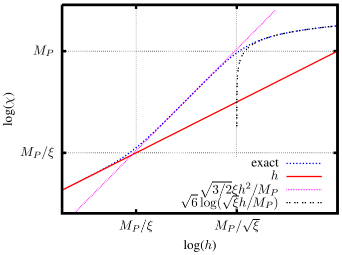
Go to the Einstein frame: $(M_P^2 + \xi h^2)\, R^{JR} \to M_P^2\, R^{EF}$

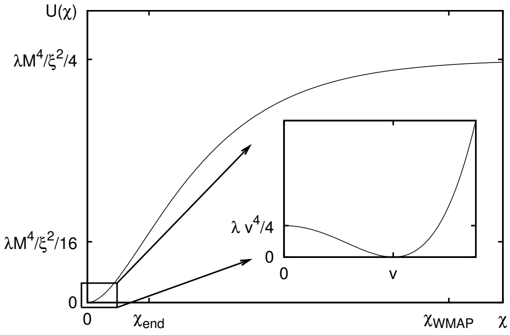
$$g_{\mu\nu}^{JF} = \Omega^{-2}\tilde{g}_{\mu\nu}^{EF}\,, \qquad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

with canonically normalized χ: interval ds^2 changes !

$$\frac{d\chi}{dh} = \frac{M_P\sqrt{M_P^2 + (6\xi+1)\xi h^2}}{M_P^2 + \xi h^2}\,,\ \ U(\chi) = \frac{\lambda M_P^4 h^4(\chi)}{4(M_P^2 + \xi h^2(\chi))^2}\,.$$

we have a flat potential at large fields: $U(\chi) \to \text{const}$ @ $h \gg M_P/\sqrt{\xi}$





Reheating by Higgs field

after inflation: $M_P/\xi < h < M_P/\sqrt{\xi}$

exponentially flat potential! @ $h \gg M_P/\sqrt{\xi}$:

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - \exp\left(-\frac{\sqrt{2}\chi}{\sqrt{3}M_P}\right)\right)^2$$

effective dynamics : $h^2 \to \chi$

$$\mathscr{L} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{\lambda}{6}\frac{M_P^2}{\xi^2}\chi^2$$

Advantage: NO NEW interactions to reheat the Universe

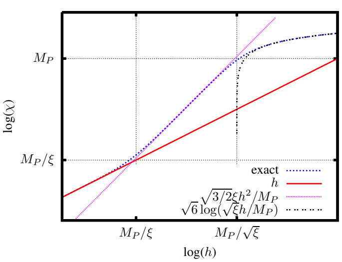
inflaton couples to all SM fields!

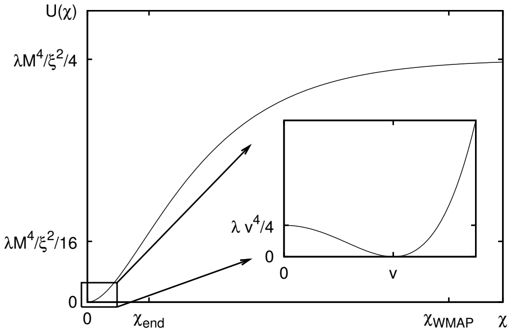
NO NEW d.o.f.

Different reheating temperature. . .

0812.3622, 1111.4397

from WMAP-normalization: $\xi \approx 47000 \times \sqrt{\lambda}$





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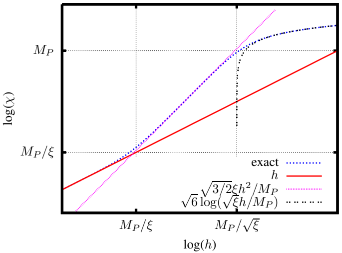
F.Bezrukov, D.G., M.Shaposhnikov, 0812.3622



$$m_W^2(\chi) = \frac{g^2}{2\sqrt{6}} \frac{M_P|\chi(t)|}{\xi}$$

$$m_t(\chi) = y_t \sqrt{\frac{M_P|\chi(t)|}{\sqrt{6}\xi}} \,\mathrm{sign}\,\chi(t)$$

Reheating by Higgs field

after inflation: $M_P/\xi < h < M_P/\sqrt{\xi}$

reheating via W^+W^-, ZZ production at zero crossings

then nonrelativistic gauge bosons scatter to light fermions

$$\chi \to W^+W^- \to f\bar{f}$$

effective dynamics : $h^2 \to \chi$

$$\mathscr{L} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{\lambda}{6}\frac{M_P^2}{\xi^2}\chi^2$$

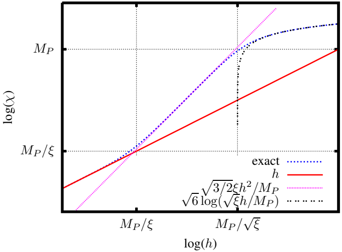
Hot stage starts almost from $T = M_P/\xi \sim 10^{14}$ GeV:

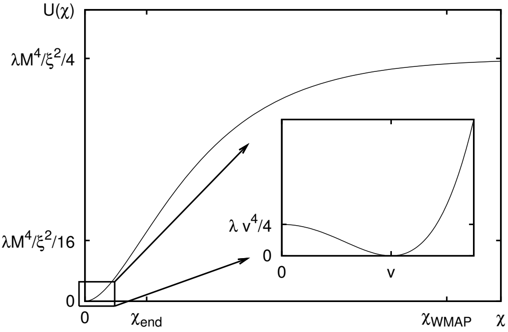
$$3.4 \times 10^{13}\,\text{GeV} < T_r < 9.2 \times 10^{13} \left(\frac{\lambda}{0.125}\right)^{1/4} \text{GeV}$$

Advantage: NO NEW interactions to reheat the Universe

inflaton couples to all SM fields!

$n_s = 0.967$, $r = 0.0032$ F.Bezrukov, D.G.,





$$\frac{d\chi}{dh} = \frac{M_P\sqrt{M_P^2+(6\xi+1)\xi\, h^2}}{M_P^2+\xi\, h^2}\,,$$

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exponentially flat potential! @ $h \gg M_P/\sqrt{\xi}$:

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1-\exp\left(-\frac{\sqrt{2}\chi}{\sqrt{3}M_P}\right)\right)^2$$

- renormalizable except gravity

  frame-dependent renormalization scale

- strong coupling (ϕ-dependent)

  save for inflation

  but reheating is questionable

F.Bezrukov et al (2008)

$V_0 \simeq 10^{-12}\,M_{Pl}^4$

from WMAP-normalization: $\xi \approx 47000\times\sqrt{\lambda}$

F.Bezrukov, D.G., M.Shaposhnikov (2008,2011)

And the reheating is almost instant

1609.05209, 1810.01304

We made an error, solving

$$\left(m_W^2(h(t)) + \Box\right) W_\nu^\pm = 0$$

while the true equation is

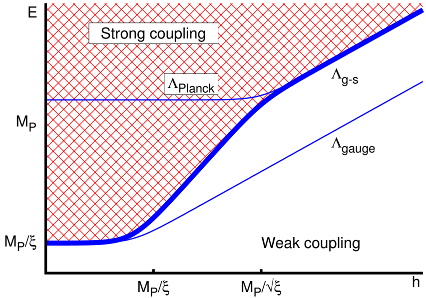
$$W_\nu^\pm m_W^2(h(t)) + \partial_\mu W_{\mu\nu}^\pm = 0$$

the $\partial_\mu W_\mu^\pm = 0$ gauge does not go through the equation...

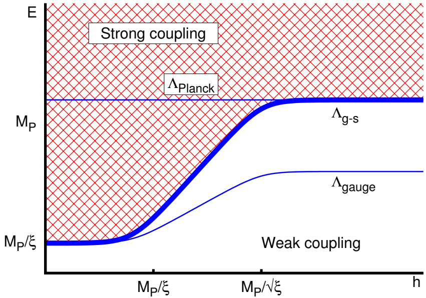
So, the longitudinal components of vector boson get contributions $\omega_L^2 \propto \dot{m}_W, \ddot{m}_W$
instant reheating well inside the strong coupling domain...

Strong coupling in Higgs-inflation

Jordan frame



Einstein frame



gravity-scalar sector:

$$\Lambda_{g-s}(h) \simeq \begin{cases} \frac{M_P}{\xi}\,, & \text{for } h \lesssim \frac{M_P}{\xi}\,, \\ \frac{\xi h^2}{M_P}\,, & \text{for } \frac{M_P}{\xi} \lesssim h \lesssim \frac{M_P}{\sqrt{\xi}}\,, \\ \sqrt{\xi} h\,, & \text{for } h \gtrsim \frac{M_P}{\sqrt{\xi}}\,. \end{cases}$$

gravitons: $\Lambda^2_{\rm Planck} \simeq M_P^2 + \xi h^2$

gauge interactions:

$$\Lambda_{\rm gauge}(h) \simeq \begin{cases} \frac{M_P}{\xi}\,, & \text{for } h \lesssim \frac{M_P}{\xi}\,, \\ h\,, & \text{for } \frac{M_P}{\xi} \lesssim h\,, \end{cases}$$

1008.5157

We must modify the model to restore the unitarity

Natural completion with R^2

Y.Ema (2017), D.G., A.Tokareva (2018)

$\xi h^2 R$ induces R^2-term

hep-th/9510140

$$S_0 = \int d^4x \sqrt{-g}\left(-\frac{M_P^2 + \xi h^2}{2} R + \frac{\beta}{4} R^2 + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4}(h^2 - v^2)^2\right).$$

introduce a Lagrange multiplier L and auxiliary scalar $\mathscr{R}$

$$S = \int d^4x \sqrt{-g}\left(\frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4}(h^2 - v^2)^2 - \frac{M_P^2 + \xi h^2}{2}\mathscr{R} + \frac{\beta}{4}\mathscr{R}^2 - L\mathscr{R} + LR\right).$$

integrate out $\mathscr{R}$

$$S = \int d^4x \sqrt{-g}\left(\frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4}(h^2 - v^2)^2 + LR - \frac{1}{\beta}(L + \frac{1}{2}\xi h^2 + \frac{1}{2}M_P^2)^2\right)$$

$$\xi \to \xi^2/\beta$$

with

$$\beta \gtrsim \frac{\xi^2}{4\pi}$$

everything here look healthy

Further transformations...

Y.Ema (2017)

introducing scalaron ϕ with $m = M_P/\sqrt{3\beta}$

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu}\,, \quad \Omega^2 \equiv \frac{2L}{M_P^2}\,, \; L \to \phi \equiv M_P\sqrt{\frac{2}{3}}\log\Omega^2\,.$$

and setting $M_P = 1/\sqrt{6}$

$$S = \int d^4x\sqrt{-g}\left(-\frac{R}{12} + \frac{1}{2}e^{-2\phi}(\partial h)^2 + \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-4\phi}\left(\lambda h^4 + \frac{1}{36\beta}(e^{2\phi} - 1 - 6\xi h^2)^2\right)\right)$$

both gravity and scalar sector are weakly coupled up to M_P with $\beta \gtrsim \xi^2/(4\pi)$

And one more...

D.G., A.Tokareva (2018)

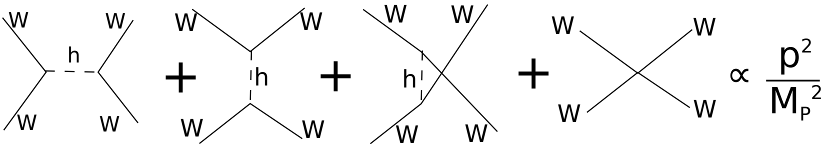
$$h = e^{\Phi}\tanh H,\ \ \phi = e^{\Phi}/\cosh H,$$

The scalar sector becomes

$$L = \frac{1}{2}\cosh^2 H(\partial\Phi)^2 + \frac{1}{2}(\partial H)^2 - \frac{\lambda}{4}\sinh^4 H - \frac{\lambda}{144\beta}\left(1 - e^{-2\Phi}\cosh^2 H - 6\xi\sinh^2 H\right)^2.$$

and the Higgs coupling to gauge bosons, e.g.,

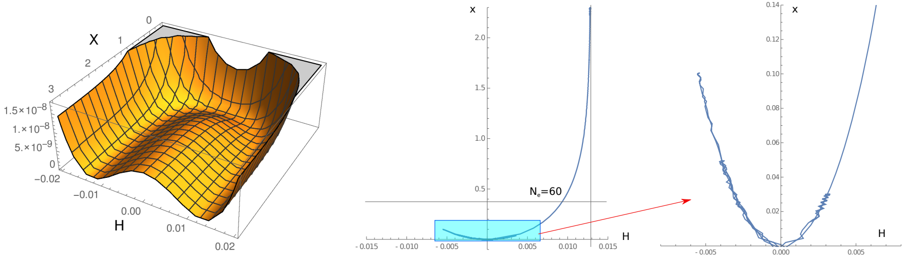
$$L_{gauge} = \frac{g^2h^2}{4}e^{-2\phi}\,W_\mu^+W_\mu^- = \frac{g^2}{4}\sinh^2 H\ W_\mu^+W_\mu^-.$$



$$\mathscr{A} \sim \frac{g^2p^2}{m_W^2}\left(\frac{4}{g^2}\left(\frac{dm_W(H)}{dH}\right)^2 - 1\right) \ \rightarrow \mathscr{A} \propto \frac{p^2}{M_P^2}$$

Cosmological spectra

D.G., A.Tokareva 1807.02392



Scalar perturbations: adiabatic

1701.07665

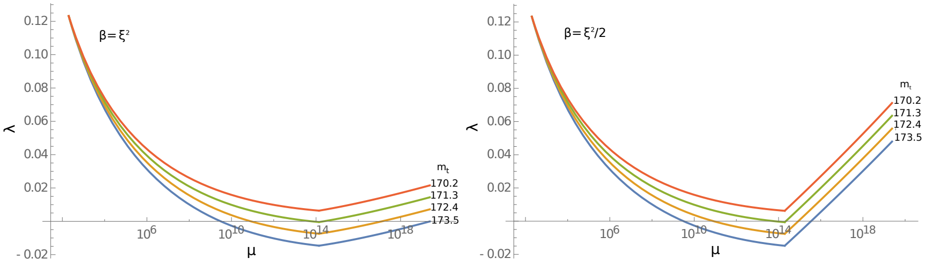
$$\beta + \frac{\xi^2}{\lambda} \simeq 2 \times 10^9$$

At small β like in the Higgs-inflation

heavy scalaron is integrated out

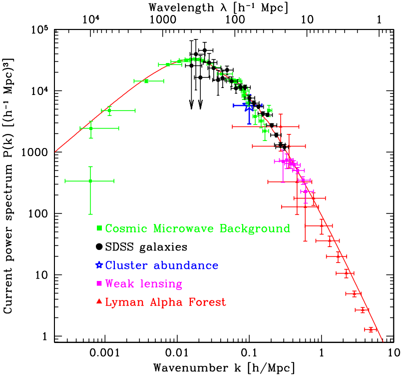
$$\frac{\xi^2}{4\pi} < \beta < \frac{\xi^2}{\lambda} \quad \rightarrow \quad 5 \times 10^{13}\,\text{GeV} < m < 1.5 \times 10^{15}\,\text{GeV}$$

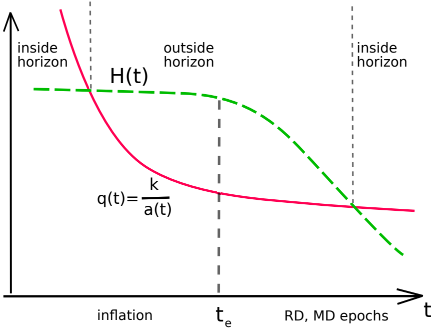
Bonus: stable for a bit heavier top-quark



We can calculate observables at any energy scale up to Planck

Probing the spectrum at energy scale $\mu = \mu(T_{reh})$





$$\delta\phi \to \frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot\phi} \propto \frac{V^{3/2}}{V'}\,, \qquad h \sim \frac{H}{M_{Pl}} \propto V^{1/2}$$

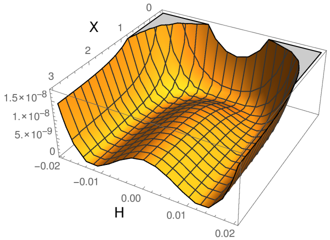
$$A_S \to \frac{V^{3/2}}{V'}\,, \qquad n_S \to \frac{V''}{V}\,, \left(\frac{V'}{V}\right)^2, \qquad r \equiv \frac{A_T}{A_S} \to \left(\frac{V'}{V}\right)^2$$

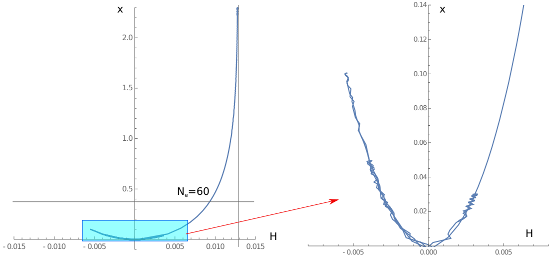
Reheating. . . all masses depend on oscillating Higgs

- Huge spikes do not reheat !! 1812.10099
- it is a highly nonlinear system
- ω^2 for W_L and Z_L rapidly oscillates and becomes negative for some time
- similar for one of the scalars (a mixture of Higgs and scalaron)
- we expect instant preheating, at least for a region in model parameter space F.Bezrukov, D.G., Ch.Shepherd, A.Tokareva (2019)
- but for precise number the backreaction must be taken into account

Cosmological spectra

D.G., A.Tokareva 1807.02392





Scalar perturbations:

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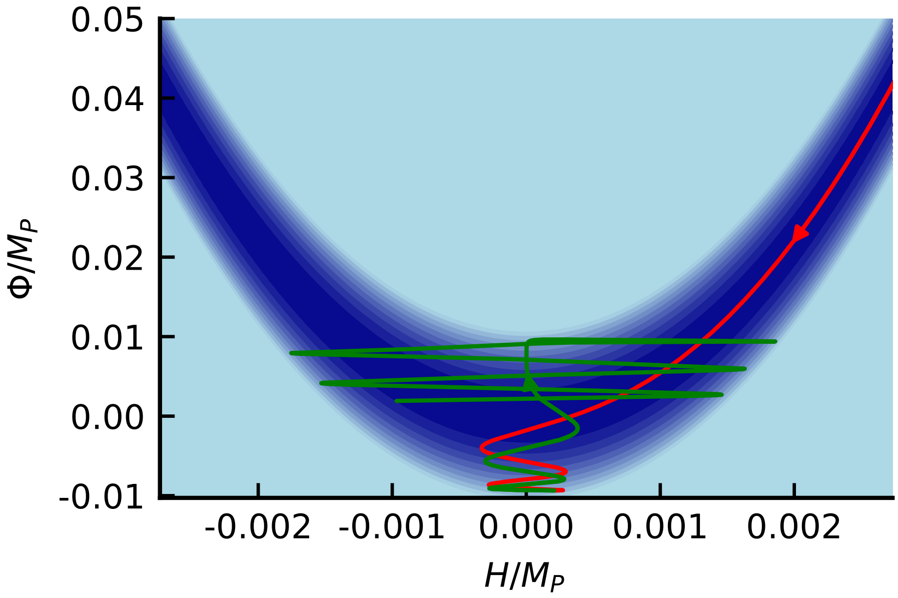
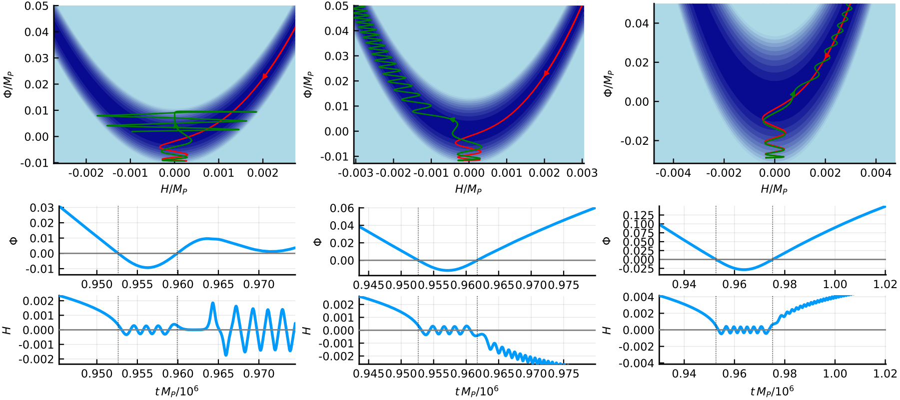

0.05
0.04
0.03
0.02
0.01
0.00
-0.01
Φ/M_P
-0.002
-0.001
0.000
0.001
0.002
H/M_P

Scalaron Φ and Higgs H evolution after inflation

Homogeneous modes (mixing in kinetic sector), $\dot{\Phi} < 0$, $\dot{\Phi} > 0$



$\beta = 1.869 \times 10^6$ $\beta = 2.9 \times 10^6$ $\beta = 18 \times 10^6$

$$V(H,\Phi) = \frac{1}{4}\left(\lambda + \frac{\xi^2}{\beta}\right)H^4 + \frac{M_P^2}{6\beta}\Phi^2 - \frac{\xi M_P}{\sqrt{6}\beta}\Phi H^2 + \frac{7}{108\beta}\Phi^4 + \frac{\xi}{6\beta}\Phi^2 H^2 - \frac{M_P}{3\sqrt{6}\beta}\Phi^3$$

Linear equations for gauge bosons

Gauge bosons (e.g. $W^{\pm}$)

$$L_g^{(2)} = -\frac{1}{2}\left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+\right)\left(\partial_\lambda W_\rho^- - \partial_\rho W_\lambda^-\right) g^{\mu\lambda} g^{\nu\rho} + \frac{g^2 H_0^2}{4} W_\mu^+ W_\nu^- g^{\mu\nu},$$

transverse modes

$$\ddot{W}_k^T + 3\mathscr{H}\dot{W}_k^T + \frac{k^2}{a^2} W_k^T + m_T^2 W_k^T = 0, \qquad m_T \equiv \frac{g}{2} H_0$$

longitudinal modes

$$\ddot{W}_k^L + 3\mathscr{H}\dot{W}_k^L + \omega_W^2(\mathbf{k}) W_k^L = 0.$$

$$\omega_W^2(\mathbf{k}) = \frac{k^2}{a^2} + m_T^2 - \frac{k^2}{k^2 + a^2 m_T^2}\left(\dot{\mathscr{H}} + 2\mathscr{H}^2 + 3\mathscr{H}\frac{\dot{m}_T}{m_T} + \frac{\ddot{m}_T}{m_T} - \frac{3(\dot{m}_T + \mathscr{H} m_T)^2}{k^2/a^2 + m_T^2}\right).$$

for $k/a \gg m_T$ after inflation

$$\omega_W^2 = \frac{k^2}{a^2} + \frac{g^2}{4} H_0^2 + \frac{\xi}{3\beta}\Phi_0^2 + \left(\lambda + \frac{\xi^2}{\beta}\right) H_0^2 - \frac{\xi\sqrt{2}}{\beta\sqrt{3}} M_P \Phi_0,$$

Linear equations for scalaron and Higgs

A mixutre of the two scalars

$$m^2_{L,H} = \frac{1}{2}\left(V_{H_0H_0} + V_{\Phi_0\Phi_0}\right) \times \left(1 \pm \sqrt{1 - 4\frac{V_{\Phi_0\Phi_0}V_{H_0H_0} - V^2_{\Phi_0H_0}}{\left(V_{H_0H_0} + V_{\Phi_0\Phi_0}\right)^2}}\right).$$

after inflation can be approximated as

$$m^2_{H,L} \approx V_{H_0H_0} \approx 2\left(\lambda + \frac{\xi^2}{\beta}\right)H_0^2 + \left(\lambda + \frac{\xi^2}{\beta}\right)H_0^2 - \frac{\sqrt{2}\xi}{\sqrt{3}\beta}M_P\Phi_0$$

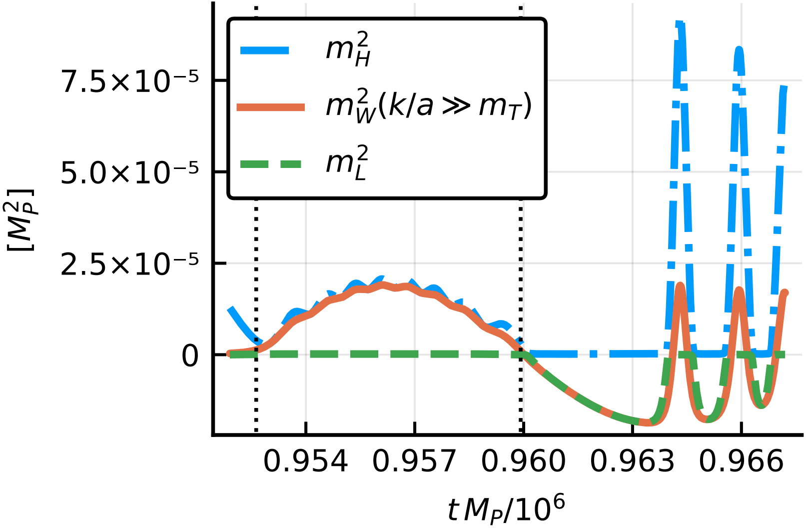
Then we calculate the Bogolubov coefficients from the field solutions $f_{\mathbf{k}}(t) = e^{-i\omega t}/\sqrt{2\omega(\mathbf{k})}$ at $t \to 0$, which gives for the number density

$$n_{\mathbf{k}} = \frac{1}{2}\left|\sqrt{\omega(\mathbf{k})}f_{\mathbf{k}} - \frac{i}{\sqrt{\omega(\mathbf{k})}}\dot{f}_{\mathbf{k}}\right|^2$$

and the physical energy

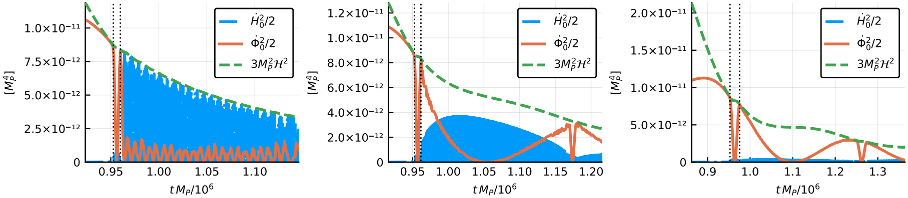
$$\rho = \int \frac{d^3\mathbf{k}}{(2\pi)^3a^3(t)}\omega(\mathbf{k})n_{\mathbf{k}}.$$

Numerical results: mass squared

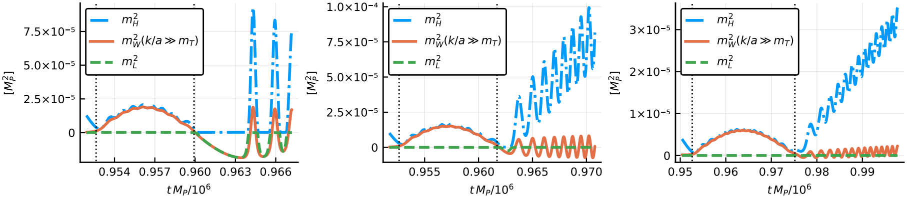


Numerical results for perturbations

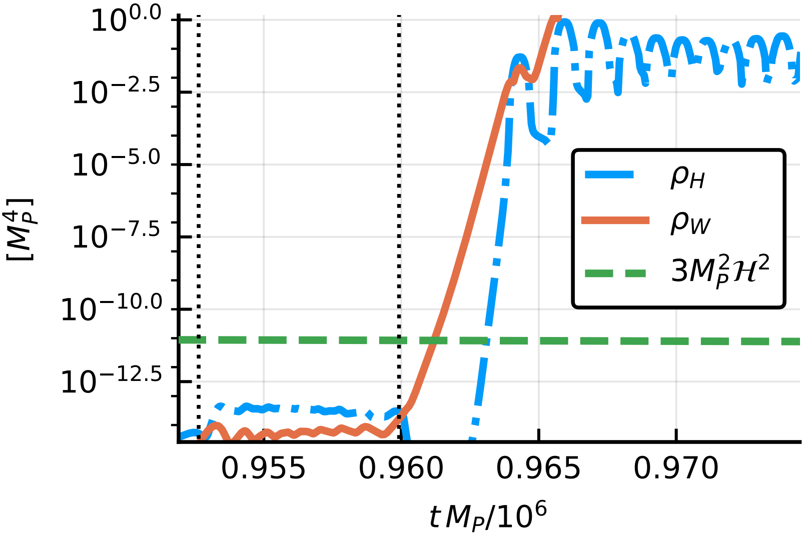
energy distribution over homogeneous modes



mass squared for the relevant perturbations

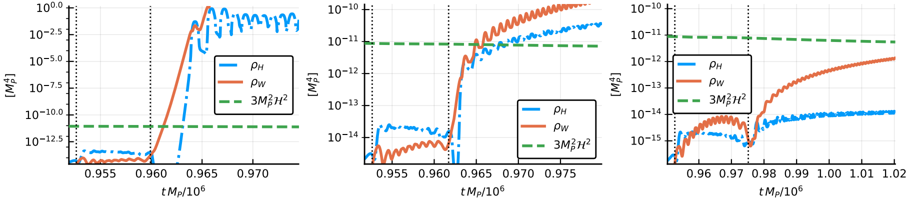


Numerical results: energy in perturbations

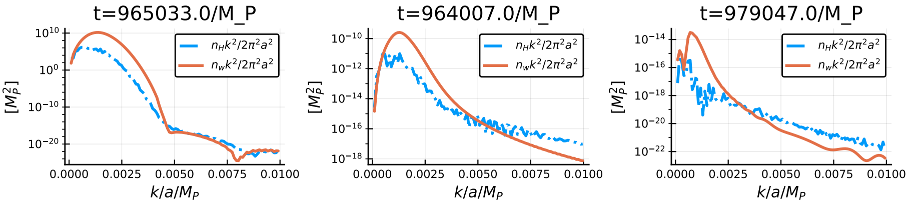


Spectra and energy density of produced particles

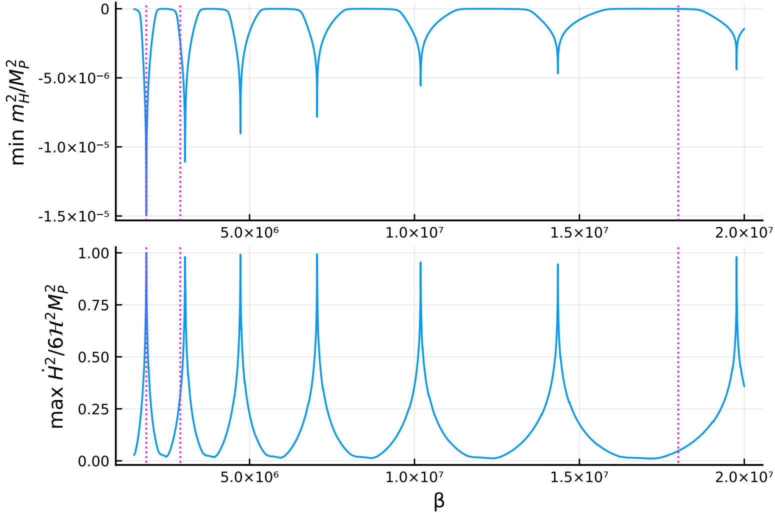
neglecting the adiabaticity conditions... and backreaction



spectra at a reference moment



The resonance positions and energy in Higgs between two zero crossings are correlated



Direct check of the inflation potential

– Higgs frequency is much and scalaron frequency is significantly higher than the expansion rate:
It seems that the reheating is instant (can be refined at NLO)

$$N_e = 59, \quad n_s = 0.97, \quad r = 0.0034.$$

– Higgs selfcoupling becomes canonical λ below the scalaron scale $\mu = M_P/\sqrt{3\beta}$

$$V(H,\Phi) = \frac{1}{4}\left(\lambda + \frac{\xi^2}{\beta}\right)H^4 + \frac{M_P^2}{6\beta}\Phi^2 - \frac{\xi M_P}{\sqrt{6}\beta}\Phi H^2 + \frac{7}{108\beta}\Phi^4 + \frac{\xi}{6\beta}\Phi^2 H^2 - \frac{M_P}{3\sqrt{6}\beta}\Phi^3$$

cancellation: $\xi M_P/\beta \times 1/\mu^2 \times \xi M_P/\beta \to \xi^2/\beta$

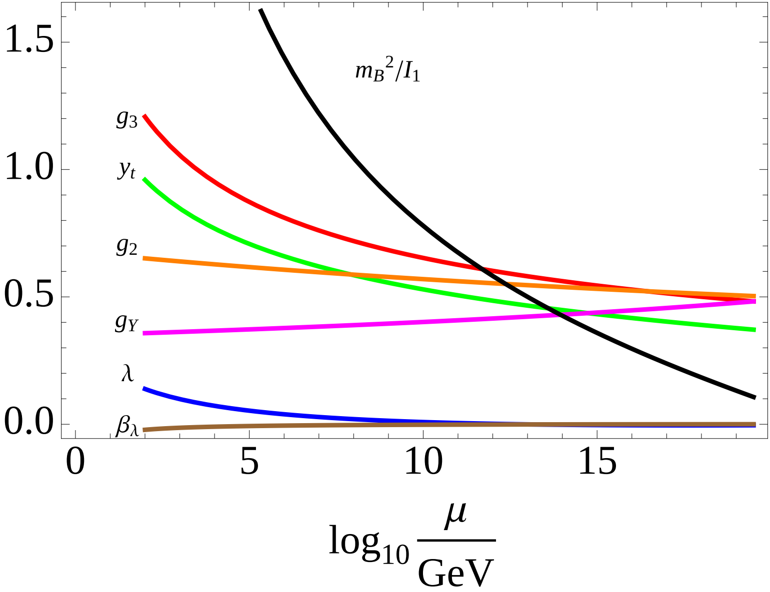
Conclusions

- Higgs inflation is a viable cosmological model unique in minimality:
  no new d.o.f., no new interactions to reheat the Universe
- however it suffers from the stong coupling problem:
  predictivity $\delta\rho/\rho \leftrightarrow \lambda$ is lost
- R^2-term with heavy scalaron cures the model:
  it seems minimal, natural, Higgs-inflation predictions remain intact
- cosmological and particle physics observables are pertrurbatively related
- to refine predictions we must study the backreaction at reheating
  and improve the accuracy of Y_t, (m_h, α_s, etc) to convince it works indeed
  ... ILC, FCC, etc
- Then one can calculate T_{reh}...
- And hopefully gravitational waves

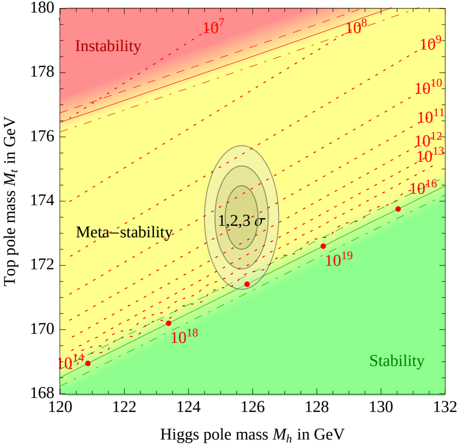
Backup slides

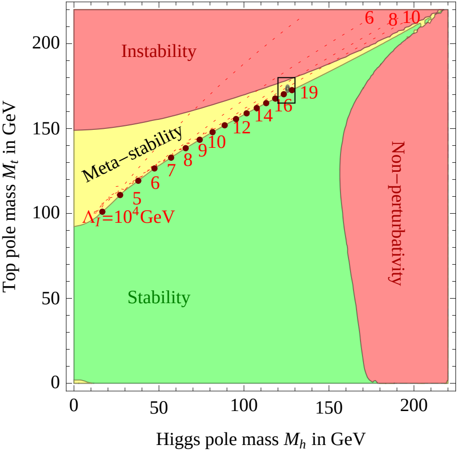
Running of the SM couplings

1305.7055



How weird to live with 125 GeV Higgs. . .





1307.7879

Power spectrum of perturbations

In the Minkowski space-time:

- fluctuations of a free quantum field φ are gaussian its power spectrum is defined as

$$\int_0^\infty \frac{dq}{q}\,\mathscr{P}_\varphi(q) \equiv \langle\varphi^2(x)\rangle = \int_0^\infty \frac{dq}{q}\,\frac{q^2}{(2\pi)^2}$$

We define amplitude as $\delta\varphi(q) \equiv \sqrt{\mathscr{P}_\varphi} = q/(2\pi)$

- In the expanding Universe momenta $q = k/a$ gets redshifted
- Cast the solution in terms $\phi(\mathbf{x},t) = \phi_c(t) + \varphi(\mathbf{x},t)\,, \quad \varphi(\mathbf{x},t) \propto e^{\pm i\mathbf{kx}}\,\varphi(\mathbf{k},t)$
  φ solves the equation

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{k^2}{a^2}\,\varphi + V''(\phi_c)\varphi = 0$$

- $q = k/a \gg H \;\Rightarrow$ as in Minkowski space-time
- $q = k/a \ll H \;\Rightarrow$ for inflaton $\varphi = \mathrm{const}$
- Matching at t_k: $q(t_k) = k/a(t_k) = H(t_k) \equiv H_k$ gives

$$\delta\varphi(q) = \frac{H_k}{2\pi} \;\Rightarrow\; \mathscr{P}_\varphi(q) = \frac{H_k^2}{(2\pi)^2}$$

amplification $H_k/q = e^{N_e(k)}$!!!

$H_k \approx \mathrm{const} = H_{infl}$ hence (almost) flat spectrum

Inflaton parameters and spectral parameters

- Observation of CMB anisotropy gives $\delta T/T$

$$\frac{\delta T}{T} \sim \frac{\delta\rho}{\rho} \quad \Rightarrow \Delta_{\mathscr{R}} \equiv \sqrt{\mathscr{P}_{\mathscr{R}}} = 5\times 10^{-5}$$

- These are so-called adiabatic perturbations!
  Other possibles (isocurvature) modes (e.g. $\delta T = 0$, but $\delta n_B/n_B \neq 0$) are not found.
- $\Delta_{\mathscr{R}} = 5\times 10^{-5} \Rightarrow$ fixes model paramaters, e.g.:

$$V(\phi) = \frac{\beta}{4}\phi^4 \to \lambda \sim 10^{-13}$$

  With such a tiny coupling perturbations are obviously gaussian
  So far confirmed by observations
- That's why Higgs boson in the SM does not help!

Inflaton parameters and spectral parameters

- In fact, spectra are a bit tilted, as H_{infl} slightly evolves

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}}\left(\frac{k}{k_*}\right)^{n_s-1}\,, \qquad \mathcal{P}_T(k) = A_T\left(\frac{k}{k_*}\right)^{n_T}\,.$$

- Measure $\Delta_{\mathcal{R}}$ at present scales $q \simeq 0.002/\mathrm{Mpc}$, it fixes the number of e-foldings left N_e
- For tensor perturbations one introduces:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = \frac{1}{\pi}\,\frac{M_{Pl}^2\,V'^2}{V} = 16\,\varepsilon \;\to\; \frac{16}{N_e} \;\text{ for }\; \beta\phi^4$$