Unfree gauge symmetry

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Outline

1. Examples of unfree gauge symmetry in specific field theories
2. Identification of main constituents of the unfree gauge symmetry algebra: operators of gauge parameter constraints and completion functions
3. Modification of Noether identities for unfree gauge symmetry
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6. Faddeev-Popov ansatz for unfree gauge symmetry
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9. Hamiltonian BFV-BRST formalism for unfree gauge symmetry
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Examples of unfree gauge symmetry

Unimodular Gravity (UG), $\det g = -1$. *Consequences:*

- Gauge symmetry reduces to the volume preserving diffeomorphisms

$$\delta_\epsilon \det g = 0\,, \quad \delta_\epsilon g_{\mu\nu} = \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu \quad \Rightarrow \quad \nabla_\mu \epsilon^\mu = 0\,. \tag{1}$$

- Einstein's equations become traceless, hence they are not transverse. This makes Λ "integration constant", not pre-defined parameter:

$$S = \int d^d x\, R\,, \quad \frac{\delta S}{\delta g^{\mu\nu}} \equiv R_{\mu\nu} - \frac{1}{d} g_{\mu\nu} R \approx 0\,; \tag{2}$$

$$\nabla^\nu \frac{\delta S}{\delta g^{\mu\nu}} \equiv \frac{d-2}{d} \partial_\mu R \approx 0 \quad \Rightarrow \quad R \approx \Lambda = const\,. \tag{3}$$

On-shell relation $R - \Lambda \approx 0$ is *not a differential consequence* of EoM (2), nor Λ is a charge of any local conserved current.
Volume preserving diffeomorphisms form the subalgebra

$$\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1} = \delta_{[\epsilon_1,\epsilon_2]}\,, \quad \nabla \cdot \epsilon_{1,2} = 0 \quad \Rightarrow \quad \nabla \cdot [\epsilon_1, \epsilon_2] = 0\,. \tag{4}$$

The subalgebra is singled out by imposing PDE onto the gauge parameters ϵ^μ rather than by explicitly separating subset of generators.

Further examples of unfree gauge symmetry

Generalizations of the UG – *Barvinsky, Kamenshchik, et al, 2017–2022.* *Starting point*: unimodularity condition is replaced by a more general relation, $N = N(\overset{*}{g})$. *Consequences:* Λ is still an integration constant + new options to describe "k-essence".

Higher Spin (HS) Linearised Gravities

- Irreducible HS, traceless tensors, *E.Skvortsov, M.Vasiliev, 2008*:

$$\operatorname{Tr}\widetilde{h} = 0\,, \quad \delta_{\widetilde{\epsilon}}\,\widetilde{h}_{\mu_1\dots\mu_s} = \partial_{(\mu_1}\widetilde{\epsilon}_{\mu_2\dots\mu_s)}\,, \quad \operatorname{Tr}\widetilde{\epsilon} = 0\,, \quad \partial\cdot\widetilde{\epsilon} = 0\,. \qquad (5)$$

- "Maxwell-like" HS, tracefull tensors, *A.Campoleoni, D.Francia, 2013*:

$$\delta_\epsilon h_{\mu_1\dots\mu_s} = \partial_{(\mu_1}\epsilon_{\mu_2\dots\mu_s)}\,, \quad \partial\cdot\epsilon = 0\,. \qquad (6)$$

Both models don't involve auxiliary fields, unlike Fronsdal's action.

HS gravity models with unfree gauge symmetry admit "global conserved quantities" being HS analogs of cosmological constant (see *V.Abakumova, SL, PRD, 2020*).
Number of these "HS cosmological constants" is growing with spin.

Cosmological constant analogs for HS gravities

- For $s = 3$, the field equations admit differential consequences:

$$\widetilde{R}^{\mu} = \partial_\nu \partial_\lambda \widetilde{h}^{\mu\nu\lambda} , \quad \partial_\mu \widetilde{R}_\nu + \partial_\nu \widetilde{R}_\mu - \frac{2}{d} \eta_{\mu\nu} \partial \cdot \widetilde{R} \approx 0 ; \qquad (7)$$

$$R^{\mu} = \partial_\nu \partial_\lambda h^{\mu\nu\lambda} , \quad \partial_\mu R_\nu + \partial_\nu R_\mu \approx 0 . \qquad (8)$$

Instead of $\partial_\mu R \approx 0$ for UG, for $s = 3$ we arrive at (conformal) Killing eqs. The general solution to eqs (7), (8) reads

$$R_\mu = \Lambda_\mu + \Lambda_{\mu\nu} x^\nu , \quad \widetilde{R}_\mu = \Lambda_\mu + \Lambda_{\mu\nu} x^\nu + \Lambda x^\mu + \Lambda'_\nu (2x^\nu x_\mu - \delta^\nu_\mu x_\lambda x^\lambda) , \qquad (9)$$

where $\Lambda_{\mu\nu} = -\Lambda_{\nu\mu}, \Lambda'_\nu, \Lambda_\nu, \Lambda$ are arbitrary "integration constants".

- For $s > 3$, the higher Ricci's $R_{\mu_1 \dots \mu_{s-2}}$ of the rank $s - 2$, or traceless $\widetilde{R}_{\mu_1 \dots \mu_{s-2}}$ obey (conformal) Killing tensor eqs, as the differential consequences of EoM's. The rank $s-2$ (conformal) Killing tensor is decomposed into the product of $s - 2$ (conformal) Killing vectors. Therefore, the number of the "cosmological constants" is $10 \times (s - 2)$ for the "Maxwell-like" HS theory, and $15 \times (s - 2)$ for the UG-like HS gravity in $d = 4$.

Examples of alternative parameterization

Massless spin 2 ($d = 4$):

$$\partial_\mu \epsilon^\mu = 0 \quad <\approx> \quad \epsilon^\mu = \partial_\nu \epsilon^{\mu\nu} \,, \quad \epsilon^{\mu\nu} = -\,\epsilon^{\nu\mu} \,. \tag{10}$$

Equivalence is modulo (Hodge dualised) De Rham cohomology.
This form of the volume preserving diffeomorphism is a reducible gauge symmetry. Gauge transformations of gauge parameters read

$$\delta_\omega \epsilon^{\mu\nu} = \varepsilon^{\mu\nu\lambda\rho} \partial_\lambda \omega_\rho \,, \quad \delta_\eta \omega^\mu = \partial^\mu \eta \,. \tag{11}$$

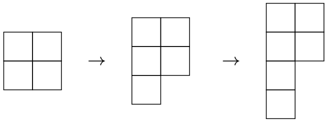
"Maxwell-like" $s = 3$ ($d = 4$):

$$\partial_\nu \epsilon^{\mu\nu} = 0 \,, \quad \epsilon^{\mu\nu} = \epsilon^{\nu\mu} \quad <\approx> \quad \epsilon^{\mu\nu} = \partial_\lambda \partial_\rho \epsilon^{\mu\nu\lambda\rho} \,, \tag{12}$$

where $\epsilon^{\mu\nu\lambda\rho} = \epsilon^{\nu\mu\lambda\rho}$, $\epsilon^{\mu\nu\lambda\rho} = \epsilon^{\mu\nu\rho\lambda}$.
Gauge symmetry is reducible,

$$\delta_\omega \epsilon^{\mu\nu\lambda\rho} = \partial_\sigma \omega^{\mu\nu\lambda\rho\sigma} \,, \quad \delta_\eta \omega^{\mu\nu\lambda\rho\sigma} = \partial_\tau \eta^{\mu\nu\lambda\rho\sigma\tau} \,, \tag{13}$$

with gauge parameters of the following symmetry type:



General setup for unfree gauge symmetry: modified Noether identities and global conserved quantities

Consider set of fields ϕ^i (DeWitt condensed labels assumed). EoM's read

$$\partial_i S(\phi) \approx 0 \,. \tag{14}$$

Proceeding from the observations noticed in the examples, we assume the action $S(\phi)$ to obey *modified Noether identities*:

$$\Gamma^i_\alpha \partial_i S + \Gamma^a_\alpha \tau_a \equiv 0 \,, \tag{15}$$

where Γ's are matrices of differential operators, τ are local quantities. Operator Γ^a_α has *finite kernel*,

$$\Gamma^a_\alpha u_a = 0 \quad \Rightarrow \quad u \in K \,, \quad \dim K = k \in \mathbb{N} \,. \tag{16}$$

Elements of K are parameterised by k independent constants Λ_I,

$$\forall\, U \in K \quad \Rightarrow \quad U = u^I \Lambda_I, \quad I = 1 \dots k \,. \tag{17}$$

The quantities τ_a are assumed off-shell independent, while on-shell they reduce to elements of K, because of (15):

$$\mathcal{T}_a(\phi, \Lambda) \equiv \tau_a(\phi) - u_a(\Lambda) \approx 0, \quad u_a(\Lambda) \in K \,. \tag{18}$$

These relations can be resolved w.r.t. the constants:

$$J_I(\phi) \approx \Lambda_I \,, \tag{19}$$

that means J_I are the *global conserved quantities*. The constants Λ are understood as *modular parameters* of the fields.

Unfree gauge symmetry specifics: completion functions and constraints on gauge parameters

The local Λ-dependent quantities $\mathcal{T}_a(\phi, \Lambda)$ vanish on shell, while they do not reduce to the linear combinations of EoM's:

$$\mathcal{T}_a(\phi, \Lambda) = \tau_a(\phi) - u_a(\Lambda) \approx 0 \,, \quad \mathcal{T}_a \neq \Theta^i_a \partial_i S \,. \tag{20}$$

These quantities are termed *completion functions*.

The modified Noether identity (15), $\Gamma^i_\alpha \partial_i S + \Gamma^a_\alpha \tau_a \equiv 0$, means $S(\phi)$ is invariant under gauge transformations

$$\delta_\epsilon \phi^i = \Gamma^i_\alpha \epsilon^\alpha \,, \tag{21}$$

provided for the gauge parameters ϵ^α are restricted by equations

$$\Gamma^a_\alpha \epsilon^\alpha = 0 \,. \tag{22}$$

With this regard, Γ^a_α are termed *gauge parameter constraint operators*.

Unfree gauge symmetry algebra distinctions

Gauge symmetry with unrestricted parameters:

- Any on-shell trivial quantity reduces to linear combination of EoM's;
- The gauge parameters are unrestricted.

Commutation relations between gauge transformations, and the higher structure relations of gauge algebra, are deduced from Noether identities.

Unfree gauge symmetry:

- Any on-shell trivial quantity reduces to linear combination of EoM's ***and*** *completion functions* $\mathcal{T}_a(\phi, \Lambda)$;
- The gauge parameters ϵ^α are *restricted by the equations* $\Gamma^a_\alpha \epsilon^\alpha = 0$.

Structure relations of unfree gauge symmetry algebra follow from modified Noether identities, and they involve, besides gauge generators and EoM's, also completion functions and gauge parameter operators.

Unfree gauge algebra: structure relations

Proceeding from modified Noether identities, with appropriate regularity assumptions for the generators and completion functions, we arrive at the structure relations involving gauge generators completion functions:

$$\Gamma^i_\alpha \partial_i \tau_a = R^i_{\alpha a} \partial_i S + R^b_{\alpha a} \tau_b + W_{ab} \Gamma^b_\alpha \, ; \qquad (23)$$

$$\Gamma^i_\alpha \partial_i \Gamma^j_\beta - \Gamma^i_\beta \partial_i \Gamma^j_\alpha = U^\gamma_{\alpha\beta} \Gamma^j_\gamma + E^{aj}_{\alpha\beta} \tau_a + E^{ij}_{\alpha\beta} \partial_i S + R^j_{\alpha a} \Gamma^a_\beta - R^j_{\beta a} \Gamma^a_\alpha \, ; \qquad (24)$$

$$\Gamma^i_\alpha \partial_i \Gamma^a_\beta - \Gamma^i_\beta \partial_i \Gamma^a_\alpha = U^\gamma_{\alpha\beta} \Gamma^a_\gamma + R^a_{\alpha b} \Gamma^b_\beta - R^a_{\beta b} \Gamma^b_\alpha + E^{ab}_{\alpha\beta} \tau_b + E^{ai}_{\alpha\beta} \partial_i S \, , \qquad (25)$$

where the structure coefficient W_{ab} is on-shell symmetric, and the structure functions E are antisymmetric, $E^{ij}_{\alpha\beta} = - E^{ji}_{\alpha\beta}$, $E^{ab}_{\alpha\beta} = - E^{ba}_{\alpha\beta}$.

Relation (23) means the completion functions are on-shell invariant under unfree gauge variation; (24) demonstrates possible off-shell disclosure of the composition of gauge transformations, including deviation of the parameters from the equations restricting them; and relation (25) demonstrates that equations imposed on gauge parameters are gauge invariant under unfree gauge variation.

Faddev-Popov (FP) action for unfree gauge symmetry

Impose independent gauges $\chi^I(\phi)$, the FP matrix is rectangular,

$$\frac{\delta_\varepsilon \chi^I}{\delta \varepsilon^\alpha} = \Gamma^i_\alpha(\phi)\partial_i \chi^I(\phi)\,. \tag{26}$$

The number of gauges plus the number of equations restricting gauge parameters equals to the number of gauge parameters. The unfree gauge variation has to be transverse to the gauge condition surface.

FP ghosts are introduced being restricted by the equations

$$\Gamma^a_\alpha(\phi)C^\alpha = 0\,, \quad \mathrm{gh}(C^\alpha) = 1\,, \quad \epsilon(C^\alpha) = 1\,, \tag{27}$$

where $\Gamma^a_\alpha(\phi)$ are operators of gauge parameter constraints.

Anti-ghosts are introduced for gauges and equations imposed on ghosts:

$$\mathrm{gh}(\bar{C}_I) = \mathrm{gh}(\bar{C}_a) = -1\,,\ \epsilon(\bar{C}_I) = \epsilon(\bar{C}_a) = 1\,,\ \mathrm{gh}(\pi_I) = \epsilon(\pi_I) = 0\,. \tag{28}$$

The FP path integral is adjusted to the case of unfree gauge symmetry:

$$Z = \int [d\Phi] \exp\left\{\frac{i}{\hbar} S_{FP}(\phi)\right\}, \quad \Phi = \{\phi, \pi_i, C^\alpha, \bar{C}_I, \bar{C}_a\}\,, \tag{29}$$

where the FP action reads

$$S_{\mathrm{FP}} = S(\phi) + \pi_I \chi^I(\phi) + \bar{C}_I \Gamma^i_\alpha(\phi)\partial_i \chi^I(\phi) C^\alpha + \bar{C}_a \Gamma^a_\alpha(\phi) C^\alpha\,. \tag{30}$$

Path integral (29) remains unchanged under variation of gauge χ.

BV-BRST formalism for unfree gauge symmetry

Starting point of the formalism extension to the unfree gauge symmetry is the idea that ghosts are constrained

$$\Gamma^a_\alpha C^\alpha = 0\,. \tag{31}$$

This equation is considered on equal footing with the original EoM's. The equation is non-Lagrangian, so it has to be assigned with the antifield ξ^a.

(Non-Lagrangian BV-BRST – *P.Kazinski, SL, A.Sharapov, JHEP, 2005*.)

Once eq. (31) is ghost number one, the anti-field is ghost number zero! We equip all the fields, including original ones, ghosts, and antifields ξ with anti-canonical conjugate. The grading is arranged in the table:

	ϕ^i	ξ^a	C^α	ϕ^*_i	ξ^*_a	C^*_α
ε	0	0	1	1	1	0
gh	0	0	1	-1	-1	-2
deg	0	1	0	1	1	2

Given the anti-canonical pairs, the anti-bracket reads

$$(A,B) = \frac{\partial^R A}{\partial\varphi^I}\frac{\partial^L B}{\partial\varphi^*_I} - \frac{\partial^R A}{\partial\varphi^*_I}\frac{\partial^L B}{\partial\varphi^I}\,, \tag{32}$$

where $\varphi^I = (\phi^i, \xi^a, C^\alpha)$, $\varphi^*_I = (\phi^*_i, \xi^*_a, C^*_\alpha)$, and

$$\mathrm{gh}((A,B)) = \mathrm{gh}(A) + \mathrm{gh}(B) + 1\,, \quad \varepsilon((A,B)) = \varepsilon(A) + \varepsilon(B) + 1\,. \tag{33}$$

BV Master equation

The BV action is defined by the equation

$$(S, S) = 0\,. \tag{34}$$

The solution is sought for as the expansion w.r.t. resolution degree

$$S = \sum_{k=0} S_k\,, \quad \mathrm{gh}(S_k) = \varepsilon(S_k) = 0\,, \quad \deg S_k = k\,. \tag{35}$$

The boundary condition is defined by the first two orders

$$S_0 = S(\phi), \quad S_1 = \tau_a \xi^a + (\phi^*_i \Gamma^i_\alpha + \xi^*_a \Gamma^a_\alpha) C^\alpha\,, \tag{36}$$

where S is the original action, while S_1 includes the basic constituents of unfree gauge symmetry: completion functions τ_a, gauge generators Γ^i_α, and operators of gauge parameter constraints Γ^a_α. The second order reads

$$\begin{aligned} S_2 = \frac{1}{2}(C^*_\gamma U^\gamma_{\alpha\beta} + \phi^*_j \phi^*_i E^{ij}_{\alpha\beta} + 2\xi^*_a \phi^*_i E^{ia}_{\alpha\beta} + \xi^*_b \xi^*_a E^{ab}_{\alpha\beta}) C^\alpha C^\beta \\ - \xi^b (\phi^*_i R^i_{b\alpha} + \xi^*_a R^a_{b\alpha}) C^\alpha - \frac{1}{2} \xi^b \xi^a W_{ab}\,. \end{aligned} \tag{37}$$

Master equation (34) identifies all the coefficients in S_2 with structure functions in structure relations (23)-(25) of unfree gauge symmetry algebra.

Existence theorem and homological perturbation theory

BRST differential s is anti-Hamiltonian vector field for the master action:

$$sA = (A, S)\,, \quad s^2 = 0\,, \quad \mathrm{gh}(s) = 1\,, \quad \varepsilon(s) = 1\,. \tag{38}$$

It can be decomposed w.r.t. resolution degree

$$s = \delta + \gamma + \overset{(1)}{s} + \dots\,,\ \deg\delta = -1\,,\ \deg\gamma = 0\,,\ \deg\overset{(1)}{s} = 1\,. \tag{39}$$

Because of master equation, the first orders are connected by the relations

$$s^2 = 0 \ \Rightarrow\ \delta^2 = 0\,,\ \delta\gamma + \gamma\delta = 0\,,\ \gamma^2 + (\delta\overset{(1)}{s} + \overset{(1)}{s}\delta) = 0\,, \tag{40}$$

where Kozul-Tate differntial δ is defined as

$$\delta A = -\frac{\partial^R A}{\partial\phi^*_i}\partial_i S - \frac{\partial^R A}{\partial C^*_\alpha}(\phi^*_i\Gamma^i_\alpha + \xi^*_a\Gamma^a_\alpha) + \frac{\partial^R A}{\partial\xi^a}\Gamma^a_\alpha C^\alpha\,. \tag{41}$$

By virtue of Noether identity for unfree gauge symmetry, δ squares to zero

$$\delta^2 A = -\frac{\partial^R A}{\partial C^*_a}(\Gamma^j_\alpha\partial_j S + \Gamma^a_\alpha\tau_a) \equiv 0\,. \tag{42}$$

One can verify that δ is acyclic in strictly positive resolution degrees, that insures existence of solution for s in the deg > 0, Q.E.D.

Unfree gauge symmetry in Hamiltonian formalism

Hamiltonian action for the theory with primary constraints T_α:

$$S = \int dt\big(p_i\dot{q}^i - H_T\big)\,,\ H_T = H + \lambda^\alpha T_\alpha\,, \tag{43}$$

where the role of fields is played by canonical variables q^i, p_i, and Lagrange multipliers λ^α. Assume that there are no second-class constraints. Conservation of T_α leads to secondary constraints τ_a,

$$\dot{T}_\alpha \equiv \{T_\alpha\,, H_T\} = W_\alpha^\beta T_\beta(q,p) + \Gamma_\alpha^a \tau_a(q,p) \approx 0\,, \tag{44}$$

where W, Γ are local differential operators, Γ has *finite kernel*. Secondary constraints τ are considered as *completion functions*, and gauge symmetry should be unfree. Once the kernel of Γ is finite, completion functions can be redefined by adding modular parameters Λ to make τ vanishing on-shell

$$\Gamma_\alpha^a \tau_a = 0 \ \Leftrightarrow\ \tau_a = \Lambda_a\,,\ \Lambda_a \in \operatorname{Ker}\Gamma_\alpha^a\ :\ \tau_a \mapsto \tau_a - \Lambda_a\,. \tag{45}$$

Assume no tertiary constraints appear,

$$\dot{\tau}_a \equiv \{\tau_a\,, H_T\} = W_a^\alpha T_\alpha(q,p) + W_a^b \tau_b(q,p) \approx 0\,. \tag{46}$$

(For more general case, see *V. Abakumova and SL, PRD, 2020.*)

Termination of the Dirac-Bergmann algorithm means the modified gauge identities as the EoM's turn out dependent with their differential consequences *and completion functions*:

$$\{T_\alpha\,,q^i\}\frac{\delta S}{\delta q^i}+\{T_\alpha\,,p_i\}\frac{\delta S}{\delta p_i}+\big(\delta^\beta_\alpha\frac{d}{dt}-W^\beta_\alpha\big)\frac{\delta S}{\delta\lambda^\beta}+\Gamma^a_\alpha\tau_a\equiv 0\,;$$
$$\{\tau_a\,,q^i\}\frac{\delta S}{\delta q^i}+\{\tau_a\,,p_i\}\frac{\delta S}{\delta p_i}-W^\alpha_a\frac{\delta S}{\delta\lambda^\alpha}+\big(-\delta^b_a\frac{d}{dt}+W^b_a\big)\tau_b\equiv 0\,. \qquad (47)$$

Corresponding unfree gauge symmetry transformations:

$$\delta_\varepsilon O(q,p)=\{O\,,T_\alpha\}\varepsilon^\alpha+\{O\,,\tau_a\}\varepsilon^a\,,$$
$$\delta_\varepsilon\lambda^\alpha=\dot\varepsilon^\alpha+W^\alpha_\beta\varepsilon^\beta+W^\alpha_a\varepsilon^a\,. \qquad (48)$$

Constraints on gauge parameters:

$$\big(\delta^a_b\frac{d}{dt}+W^a_b\big)\varepsilon^b+\Gamma^a_\alpha\varepsilon^\alpha=0\,. \qquad (49)$$

Direct computation confirms that action (43) is invariant under transformations (48), (49),

$$\delta_\varepsilon S_H\equiv\int dt\big[\big((\delta^a_b\frac{d}{dt}+W^a_b)\varepsilon^b+\Gamma^a_\alpha\varepsilon^\alpha\big)\tau_a-\frac{1}{2}\frac{d}{dt}\big(T_\alpha\varepsilon^\alpha+\tau_a\varepsilon^a\big)\big]=0\,. \qquad (50)$$

Hamiltonian formalism for Linearised UG (LUG)

$$S_H[h,\Pi,\lambda] = \int d^4x\big(\Pi^{ij}\dot{h}_{ij} - H - \lambda^i T_i\big)\,, \quad T_i = -\,2\partial^j\Pi_{ij}\,,$$
$$H = \Pi^{ij}\Pi_{ij} - \frac{1}{2}\Pi^2 + \frac{1}{4}\big(2\partial^i h_{ij}\partial_k h^{kj} - \partial_i h\partial^i h - \partial_i h_{jk}\partial^i h^{jk}\big)\,, \tag{51}$$

where $i,j,k = 1,2,3$, $h = \eta^{ij}h_{ij}$, $\Pi = \eta_{ij}\Pi^{ij}$, $\lambda^i = h^{0i}$.
Conservation of primary constraints T_i leads to the secondary ones,

$$\dot{T}_i = \{T_i, H\} = -\,\partial_i\tau_0 = 0\,, \quad \tau_0 \equiv \partial^i\partial^j h_{ij} - \partial_i\partial^i h - \Lambda = 0\,. \tag{52}$$

The secondary constraints are conserved by virtue of the primary ones:

$$\dot{\tau}_0 = \{\tau_0, H\} = -\,\partial^i T_i\,. \tag{53}$$

Unfree gauge symmetry transformations read

$$\delta_\varepsilon h_{ij} = \partial_i\varepsilon_j + \partial_j\varepsilon_i\,,\ \delta_\varepsilon\Pi^{ij} = -\,\partial^i\partial^j\varepsilon^0 + \eta^{ij}\partial_k\partial^k\varepsilon^0\,,\ \delta_\varepsilon\lambda^i = \dot{\varepsilon}^i + \partial^i\varepsilon^0\,. \tag{54}$$

Gauge variation of the action:

$$\delta_\varepsilon S_H \equiv \int d^4x\big((\dot{\varepsilon}^0 + \partial_i\varepsilon^i)\tau_0 - \partial_0(T_i\varepsilon^i + \tau_0\varepsilon^0)\big)\,. \tag{55}$$

So, gauge parameters have to obey equation

$$\dot{\varepsilon}^0 + \partial_i\varepsilon^i = 0\,. \tag{56}$$

(For analogue in the non-linear UG, see *I.Karataeva and SL, PRD, 2022.*)

Hamiltonian BFV-BRST formalism

To avoid technical complexities, we restrict consideration by simplified involution relations

$$\{T_\alpha, H\} = V_\alpha^a \tau_a\,,\ \{\tau_a, H\} = V_a^\alpha T_\alpha\,,$$
$$\{T_\alpha, T_\beta\} = \{T_\alpha, \tau_a\} = \{\tau_a, \tau_b\} = 0. \tag{57}$$

with structure coefficients V_α^a, V_a^α being constants.

Complete BRST-charge reads

$$Q = T_\alpha C^\alpha + \tau_a C^a + \pi_\alpha P^\alpha \tag{58}$$

Given the gauge conditions,

$$\dot\lambda^\alpha - \chi^\alpha = 0\,, \tag{59}$$

the gauge fermion is introduced,

$$\Psi = \bar{C}_\alpha \chi^\alpha + \lambda^\alpha \bar{P}_\alpha\,, \tag{60}$$

and gauge-fixed Hamiltonian is defined by the usual rule,

$$H_\Psi = \mathcal{H} + \{Q, \Psi\} = H - \overline{P}_\alpha V_a^\alpha C^a - \overline{P}_a V_\alpha^a C^\alpha + T_\alpha \lambda^\alpha + \pi_\alpha \chi^\alpha$$
$$+ \overline{P}_\alpha P^\alpha + \overline{C}_\alpha \{\chi^\alpha, T_\beta\} C^\beta + \overline{C}_\alpha \{\chi^\alpha, \tau_a\} C^a\,. \tag{61}$$

The grading is arranged in the table:

	C^α	$\overline{P}_\alpha$	C^a	$\overline{P}_a$	λ^α	π_α	P^α	$\overline{C}_\alpha$
ε	1	1	1	1	0	0	1	1
gh	1	-1	1	-1	0	0	1	-1

Hamiltonian BFV-BRST in UG

The complete BRST charge reads

$$Q=\int d^3x\big(T_iC^i+\tau C-\overline{P}_iC^j\partial_jC^i-\overline{P}\partial_i(C^iC)$$
$$+\overline{P}_i(-\overset{*}{g})^{-1}\overset{*}{g}{}^{ij}C\partial_jC+\pi_iP^i\big)\,,\quad T_i=-2\overset{*}{g}_{ij}\big(\partial_k\Pi^{kj}+\overset{*}{\Gamma}{}^j_{kl}\Pi^{kl}\big)\,,\tag{62}$$
$$\tau=H-\Lambda=-\frac{1}{\overset{*}{g}}\mathcal{G}_{ijkl}\Pi^{ij}\Pi^{kl}+\overset{*}{R}-\Lambda\,,\quad\Lambda=const\,.$$

Introduce the gauge fermion

$$\Psi=\int d^3x\big(\overline{C}_i\chi^i+\overline{P_i}N^i\big)\,,\quad\chi^i=(-\overset{*}{g})^{-1}\partial_j\overset{*}{g}{}^{ji}+N^j\partial_jN^i\,.\tag{63}$$

The complete gauge-fixed BRST-invariant Hamiltonian reads

$$H_\Psi=\mathcal{H}+\{Q,\Psi\}=\int d^3x\big\{H+T_iN^i+\pi_i\chi^i-\overline{P}\partial_iC^i$$
$$-\overline{P}_i(-\overset{*}{g})^{-1}\overset{*}{g}{}^{ij}\partial_jC+\overline{P}\partial_i(CN^i)+\overline{P}_i(\partial_jC^iN^j-C^j\partial_jN^i)$$
$$-\overline{C}_i(-\overset{*}{g})^{-1}\big(2\partial_j\overset{*}{g}{}^{ij}\overset{*}{\nabla}_kC^k+\partial_j(\overset{*}{\nabla}{}^jC^i+\overset{*}{\nabla}{}^iC^j)\big)\tag{64}$$
$$+\overline{C}_i(-\overset{*}{g})^{-1}\big(\partial_j\overset{*}{g}{}^{ij}(-\overset{*}{g})^{-1}\Pi C-2\partial_j((-\overset{*}{g})^{-1}\Pi^{ij}C)$$
$$+\partial_j(\overset{*}{g}{}^{ij}(-\overset{*}{g})^{-1}\Pi C)\big)+\overline{C}_i(\partial_jN^iP^j+N^j\partial_jP^i)+\overline{P}_iP^i\big\}\,.$$

Summary of results

- Besides action and gauge generators, the unfree gauge symmetry algebra has two more principal constituents: operators of gauge parameter constraints and completion functions;
- Noether identities are modified involving these constituents. This results in modification of structure relations of gauge algebra;
- Modified Noether identities result in the "global conserved quantities" in any model with unfree gauge symmetry;
- The modification is found for the FP ansatz that accounts for the constraints imposed on the gauge parameters. This has consequences in the models, including UG;
- The BV-BRST field-antifield formalism is worked out that accounts for the unfree gauge symmetry;
- The unfree gauge symmetry transformations are described in terms of general constrained Hamiltonian formalism. The volume preserving diffeomorphisms are constructed in Hamiltonian form of UG;
- Hamiltonian BFV-BRST formalism is worked out for the systems with unfree gauge symmetry. Being applied to the UG, it results in previously uknown ghost vertices.

FEEL FREE WITH UNFREE GAUGE SYMMETRY!

THANK YOU FOR ATTENTION!

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