

# Galaxy Rotation Curves: Dark Matter or MOND?

Federico Lelli

INAF – Arcetri Astrophysical Observatory

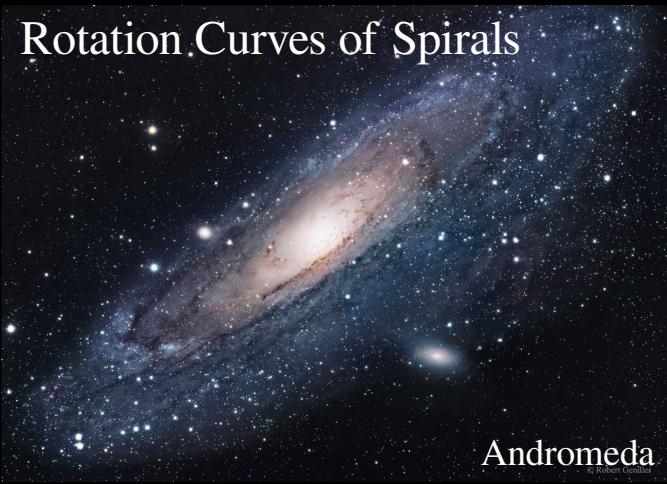


SPARC team: Stacy McGaugh, James Schombert, Pengfei Li,  
Marcel Pawlowski, Harry Desmond, Kyu-Hyun Chae

# Evidence of Dark Matter at Various Scales

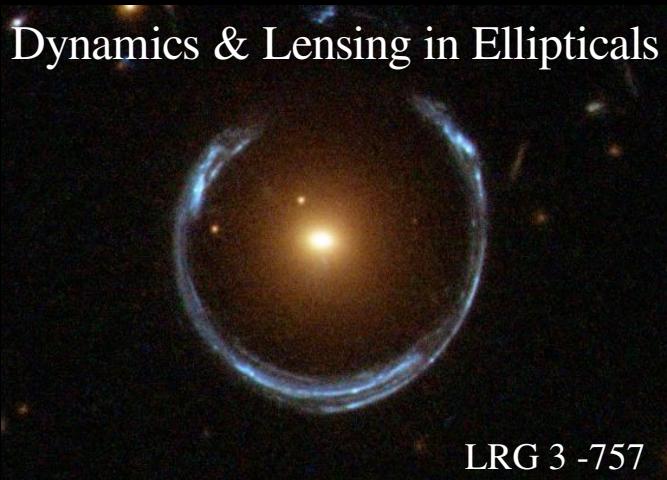
## Galaxy Scales (~1-100 kpc)

Rotation Curves of Spirals



Andromeda

Dynamics & Lensing in Ellipticals



LRG 3 -757

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Rotation Curves of Spirals



Andromeda  
© Robert Gendler

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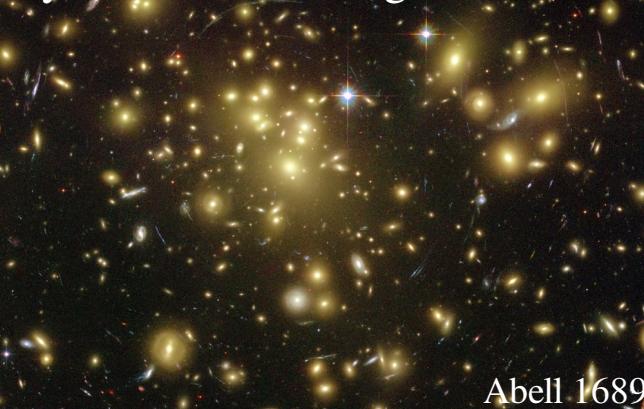
## Groups & Clusters (~1-5 Mpc)

Interactions in Galaxy Groups



Stephan's Quartet

Dynamics & Lensing in Clusters



Abell 1689

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Andromeda  
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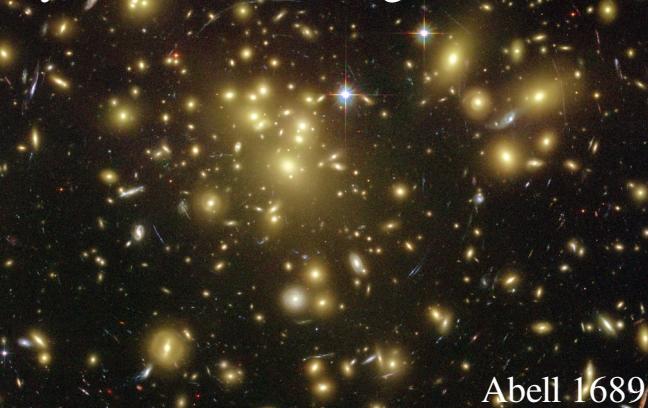
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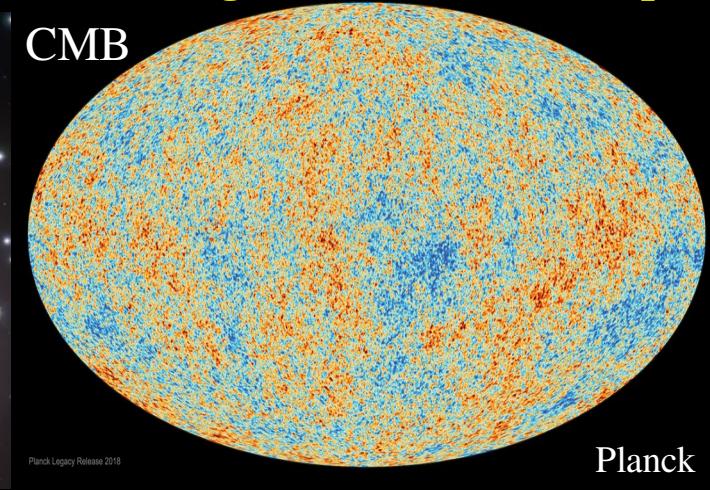
Dynamics & Lensing in Clusters



Abell 1689

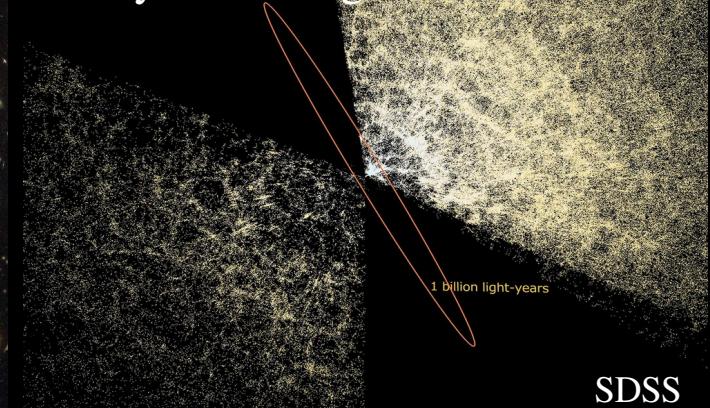
## Cosmological Scales (>100 Mpc)

CMB



Planck

Galaxy Clustering



SDSS

# ~~Evidence of Dark Matter at Various Scales~~

## Galaxy Scales (~1-100 kpc)

Rotation Curves of Spirals



Andromeda  
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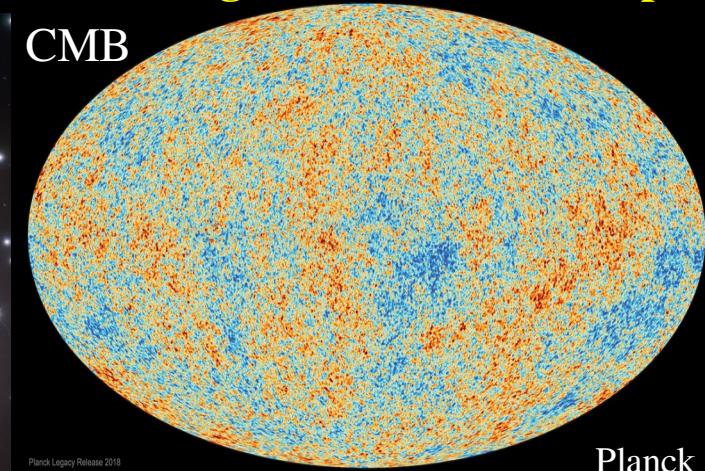
Interactions in Galaxy Groups



Stephan's Quartet

## Cosmological Scales (>100 Mpc)

CMB



Planck

This is not direct evidence for particle dark matter!

Standard Laws of Gravity (Einstein & Newton) +

Standard Model of Particle Physics = Do not work

# Outline of the Talk:

I. Empirical Evidence on Galaxy Scales

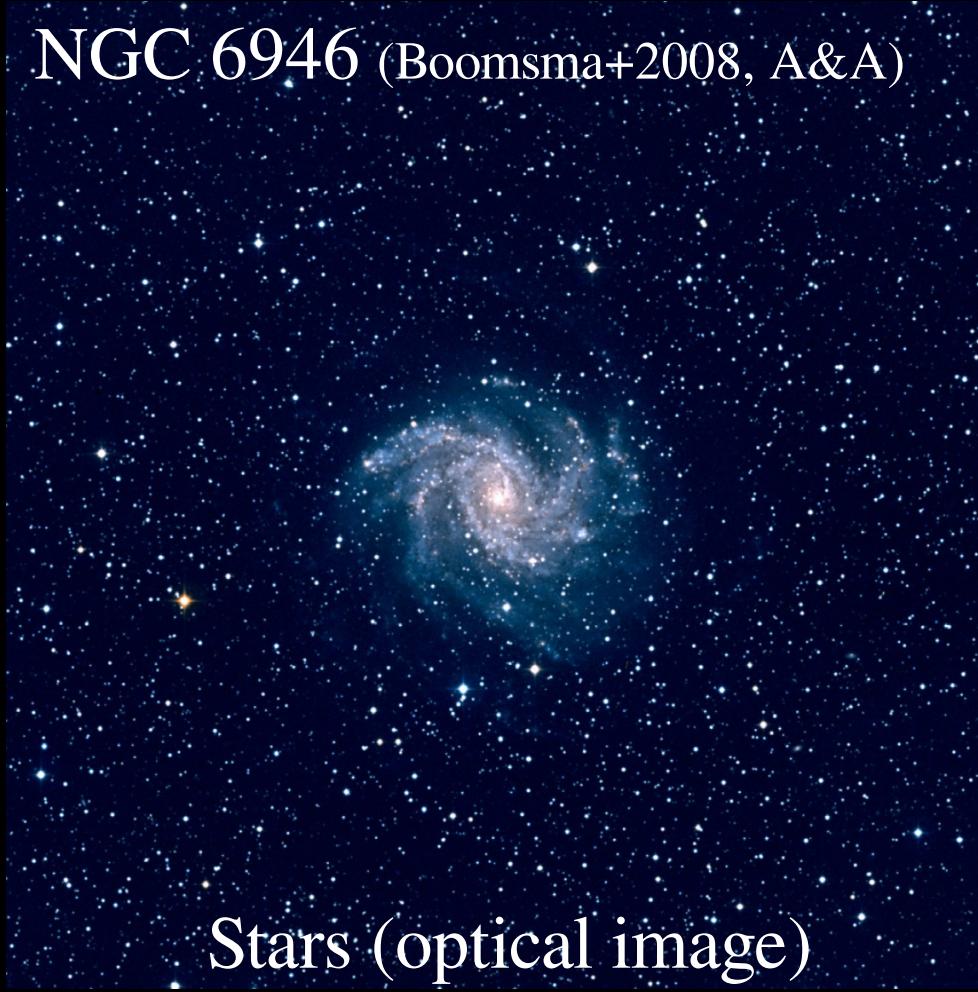
→ Rotation Curves & Dynamical Laws

II. Theoretical Interpretation

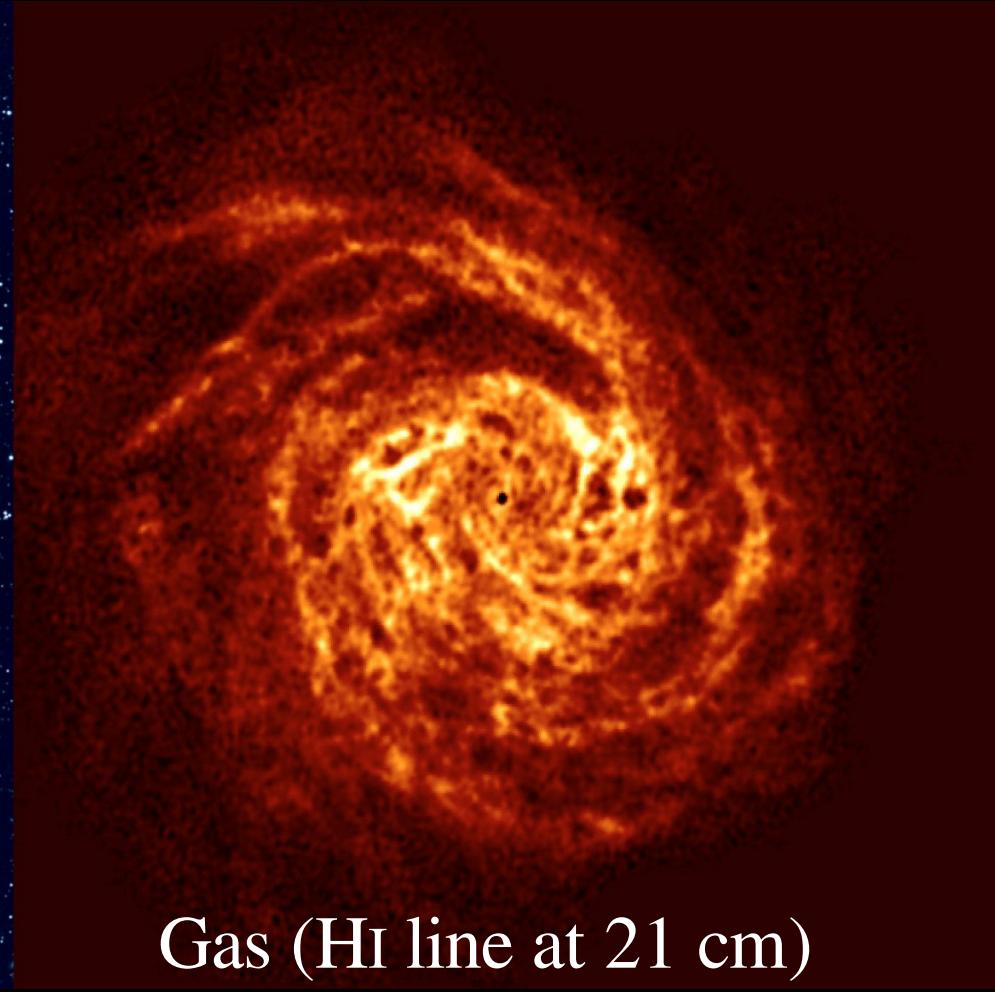
→  $\Lambda$ CDM models vs MOND theories

# Rotation Curves of Disk Galaxies

NGC 6946 (Boomsma+2008, A&A)



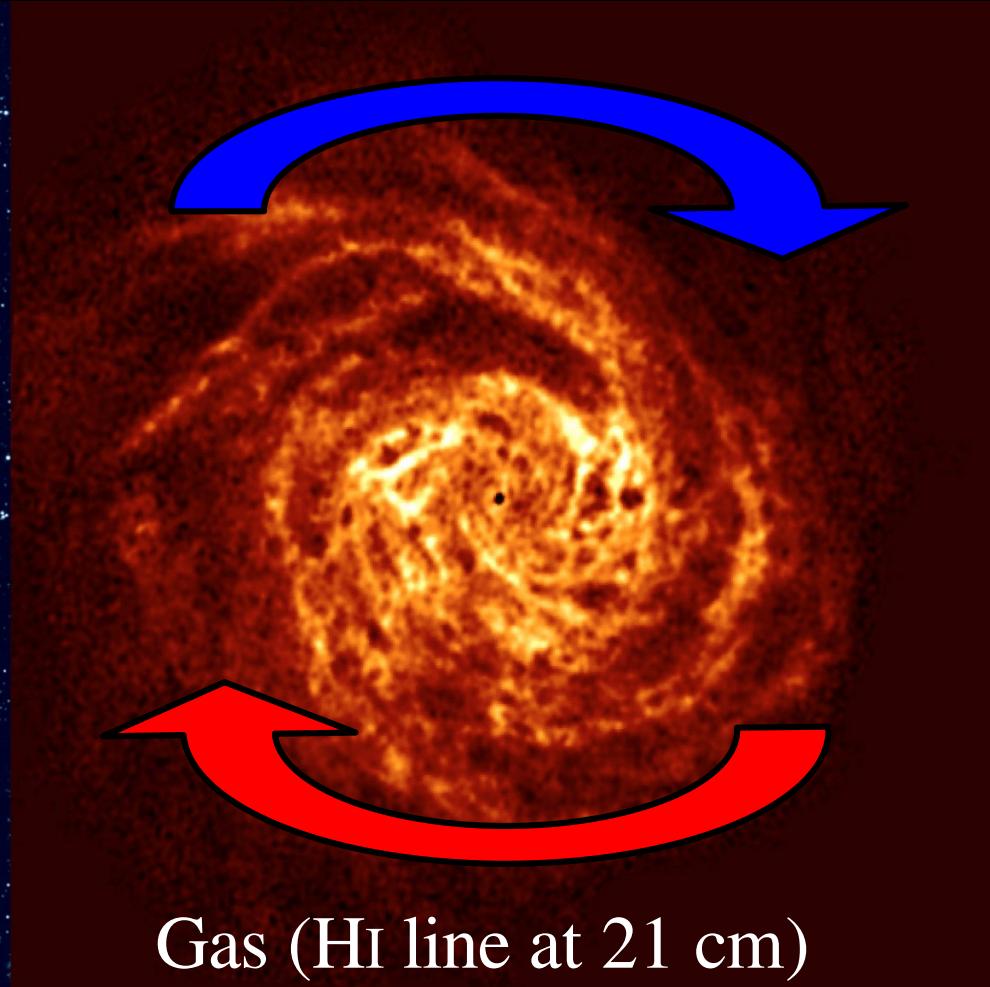
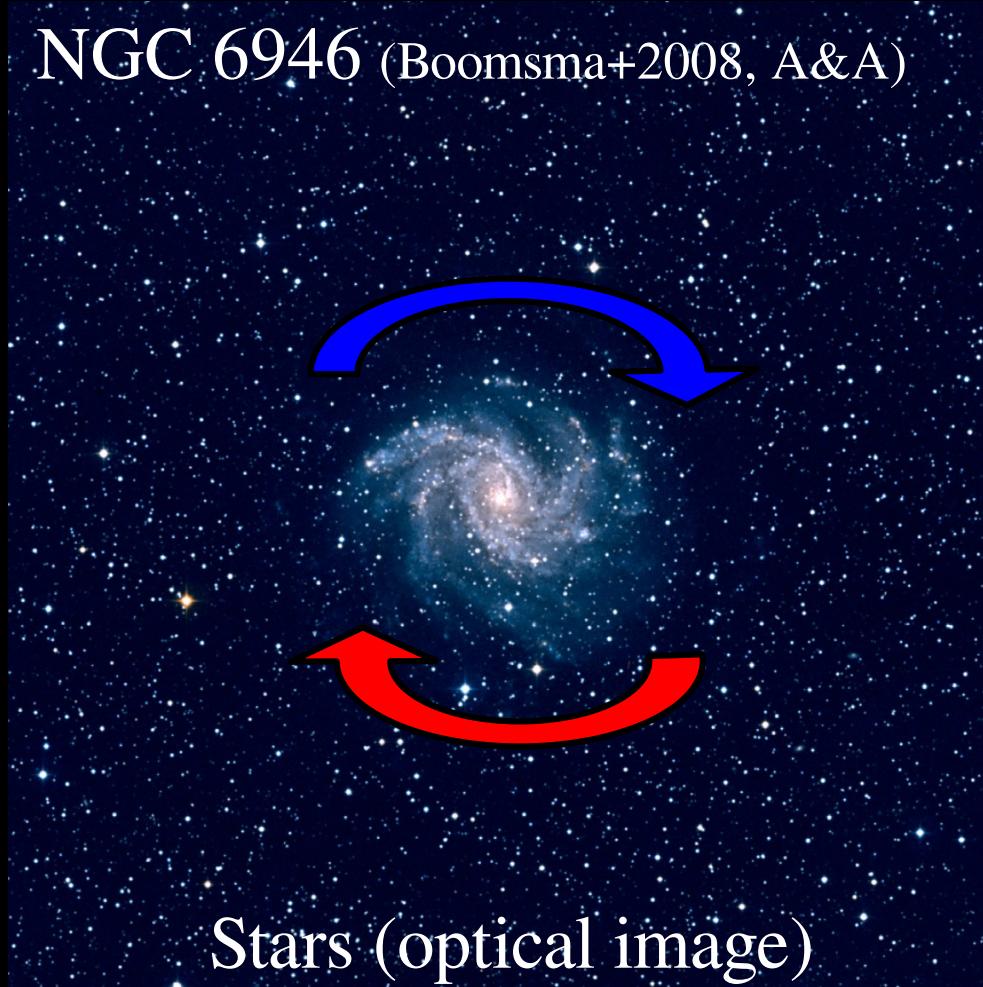
Stars (optical image)



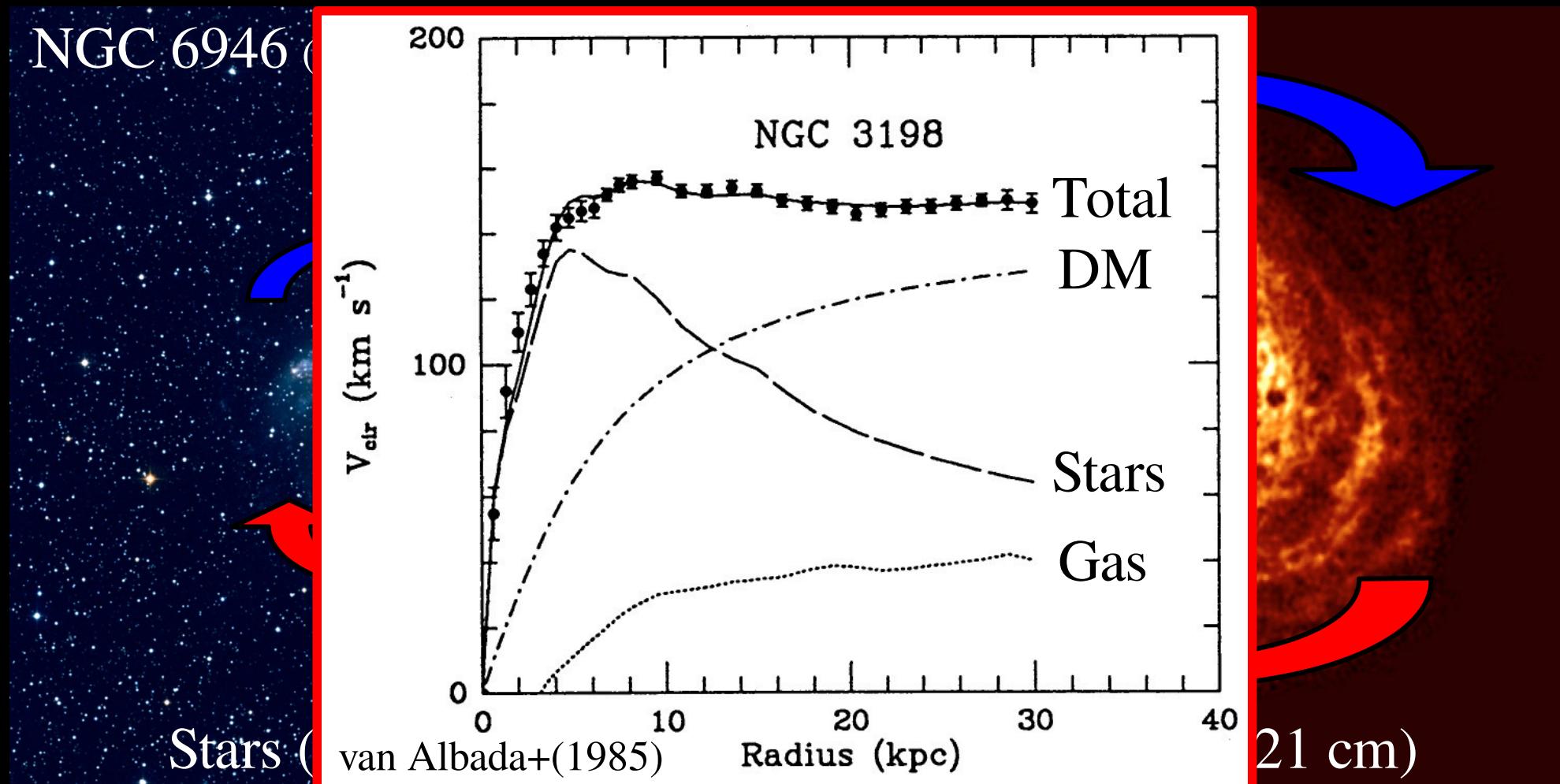
Gas (HI line at 21 cm)

# Rotation Curves of Disk Galaxies

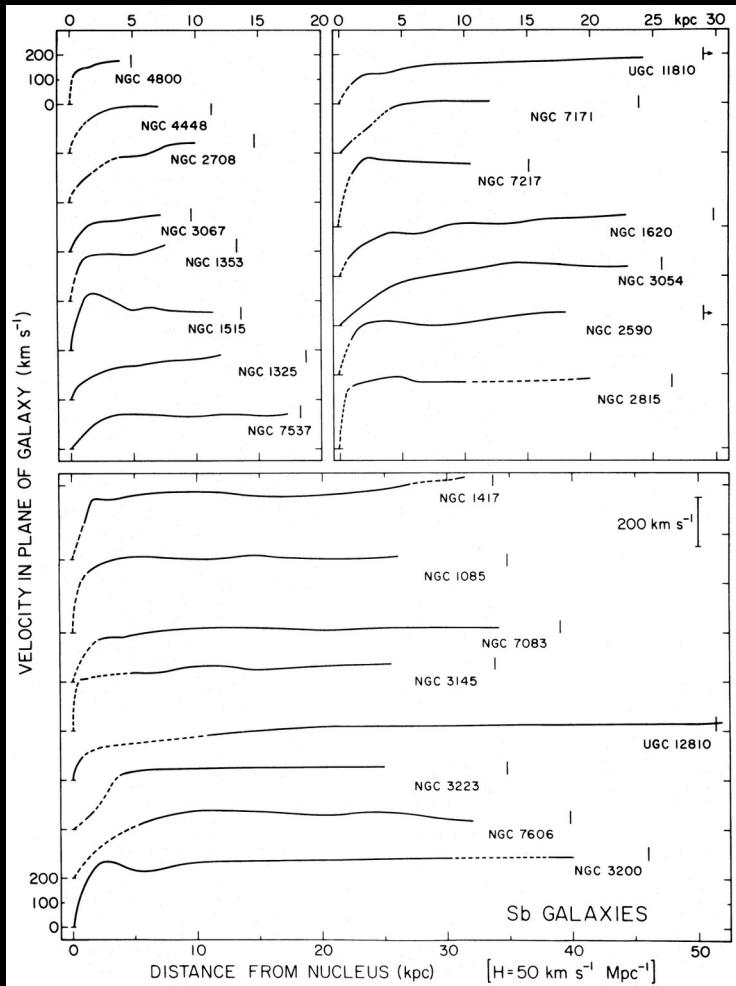
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# Rotation Curves of Disk Galaxies



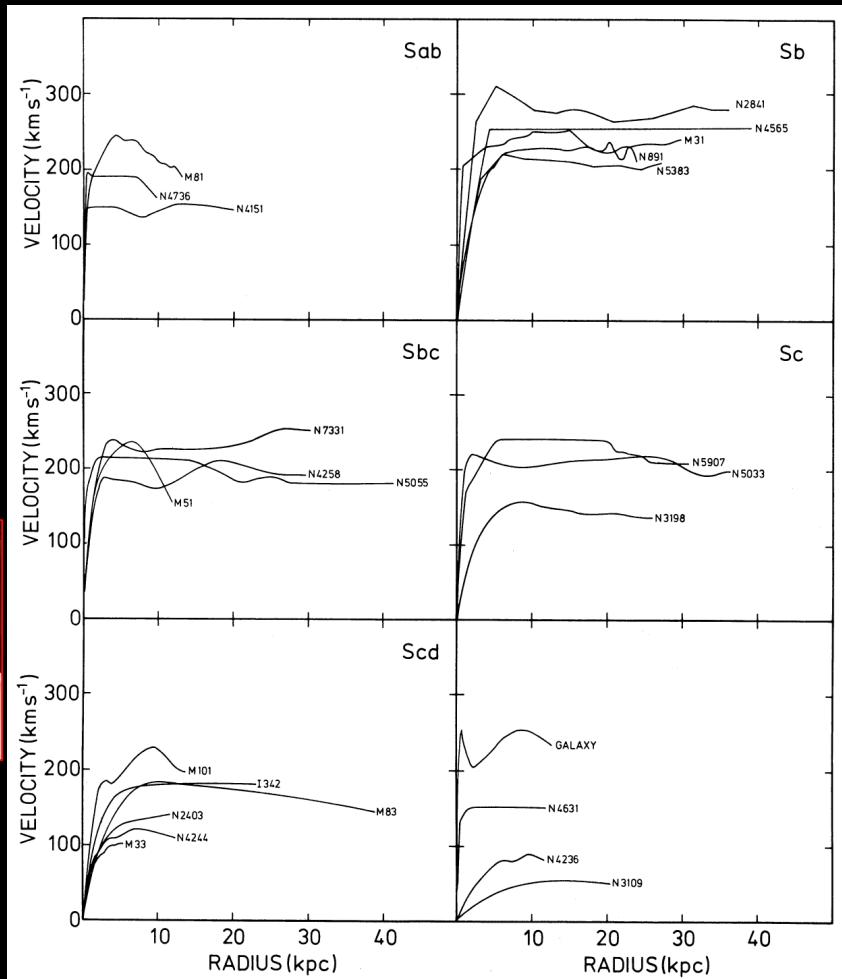
# All Rotation Curves become Flat at Large Radii



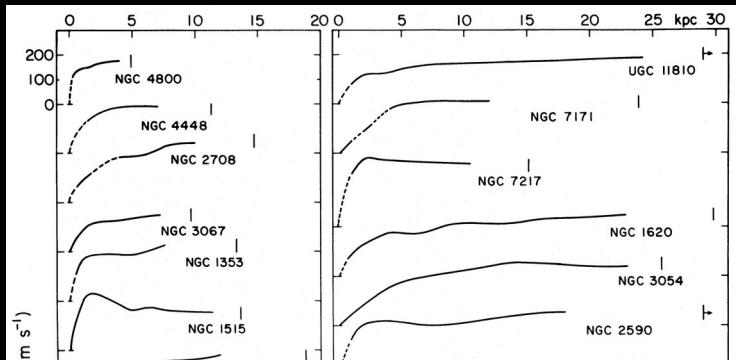
Vera Rubin  
et al. (1982, ApJ)  
Optical Spectroscopy  
of Ionized Gas (Hα)



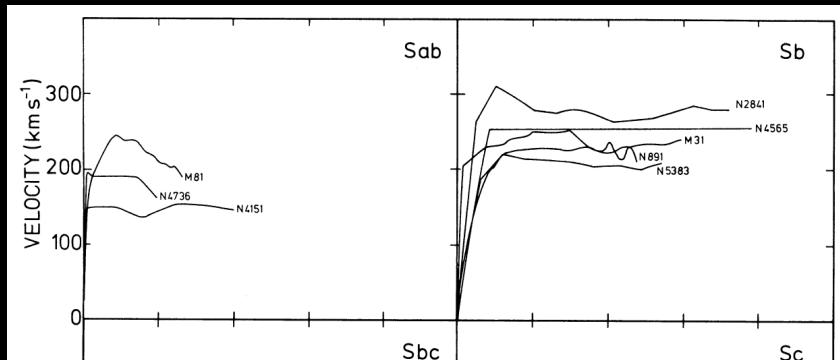
Albert Bosma  
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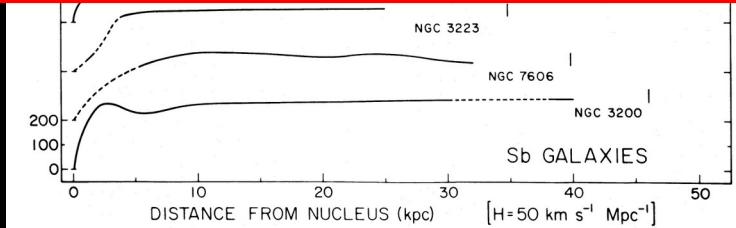


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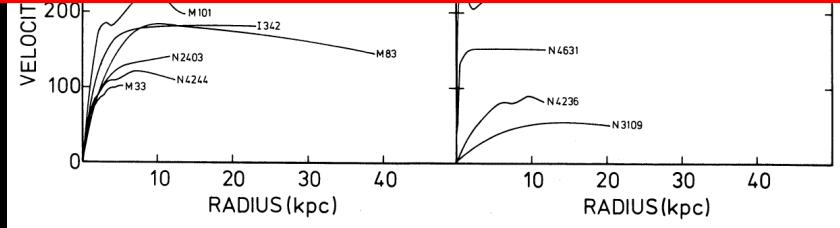


Flat rotation curves are only the beginning of the story...

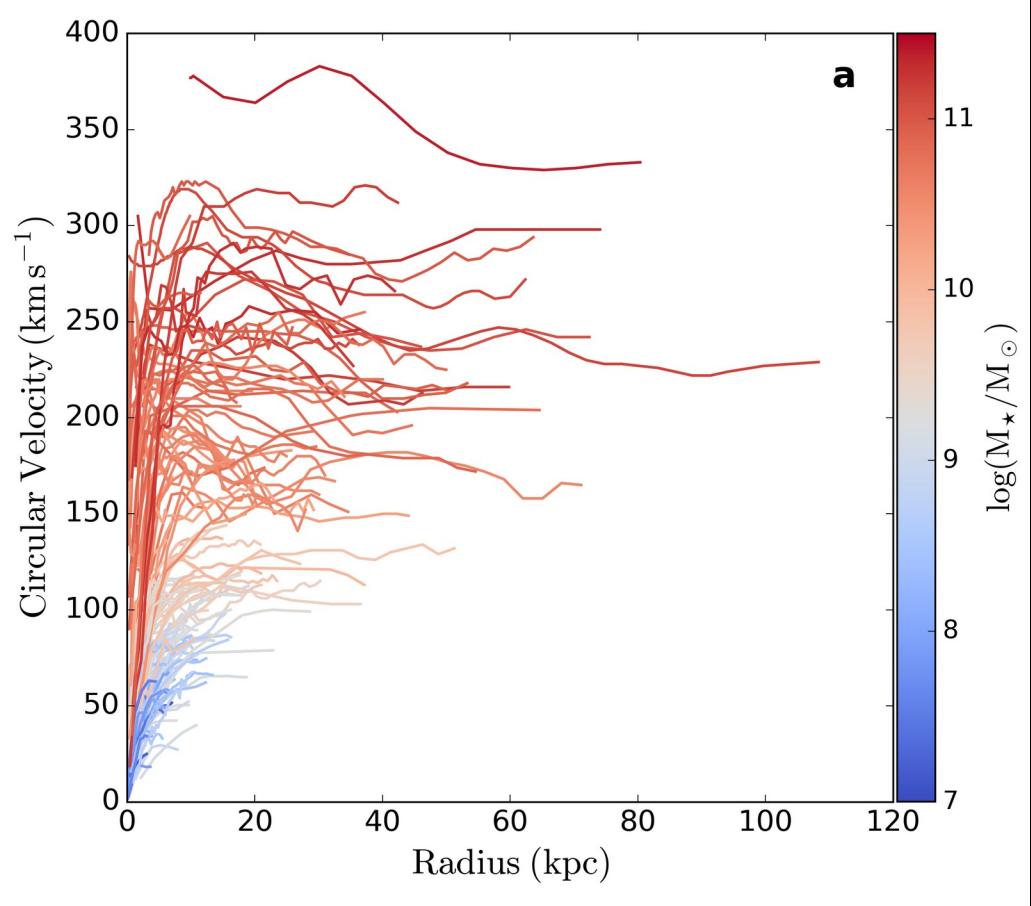
There's much more to learn from the relation  
between rotation curves and baryon distribution!



Albert Bosma  
(1981, AJ)  
Radio Interferometry  
of Atomic Gas (HI)



# Rotation Curve Shapes $\leftrightarrow$ Baryon Distribution

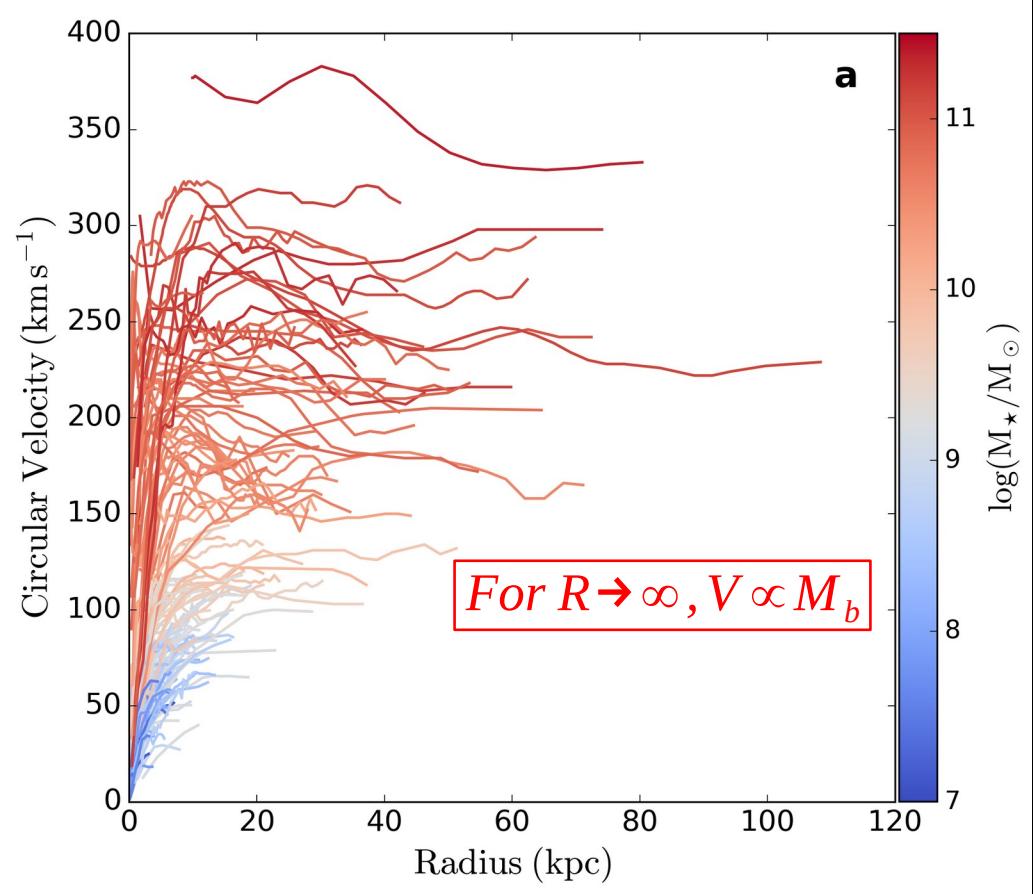


Lelli (2022, Nature Astronomy)



Database of 175 galaxies (Lelli et al. 2016, AJ)  
Public data: <http://astroweb.cwru.edu/SPARC/>

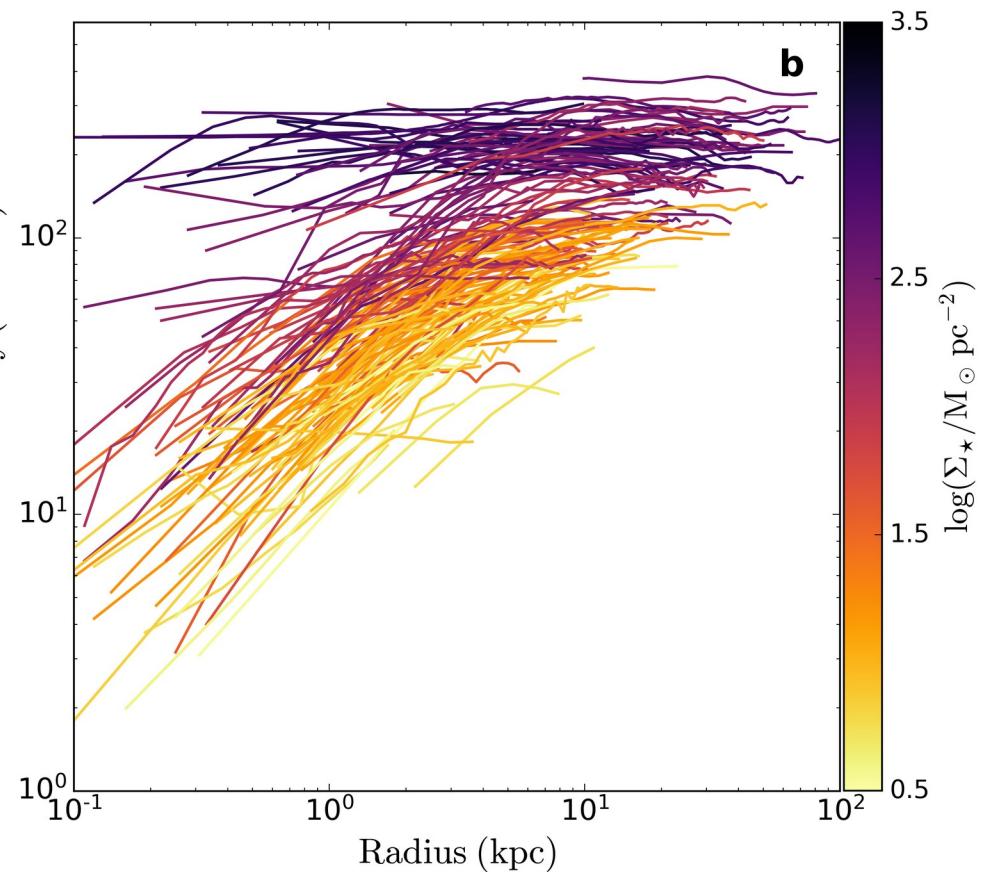
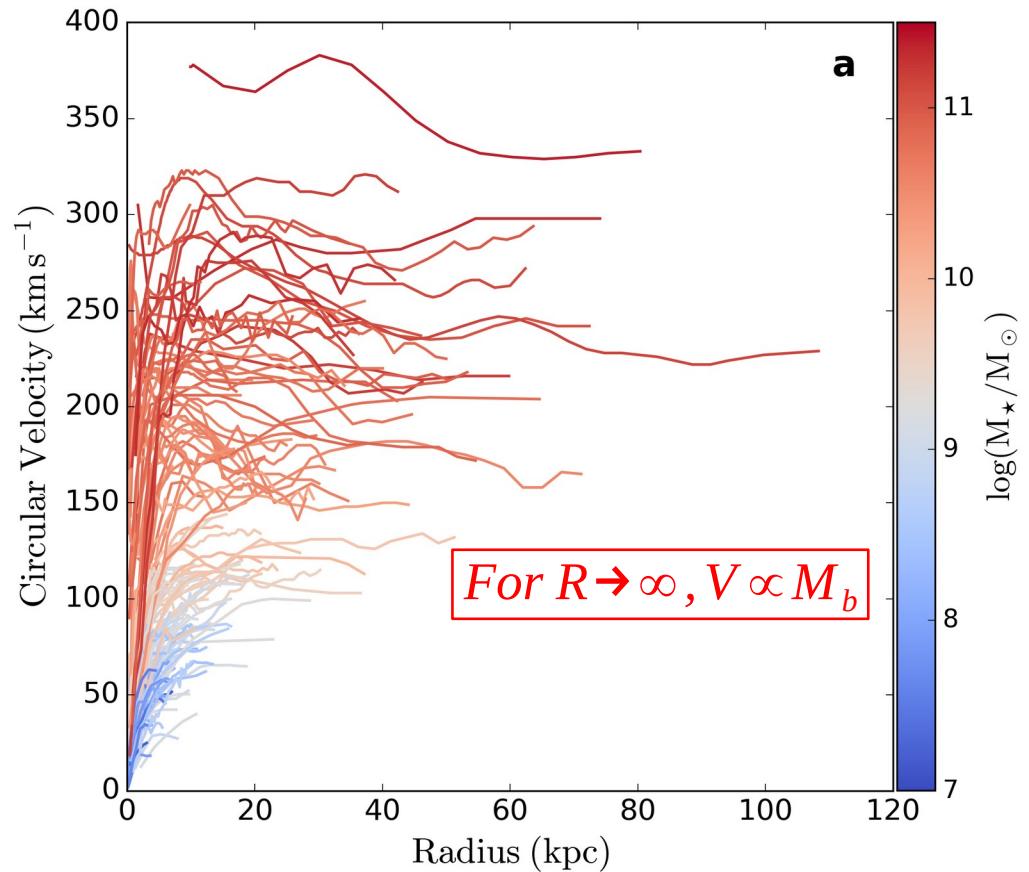
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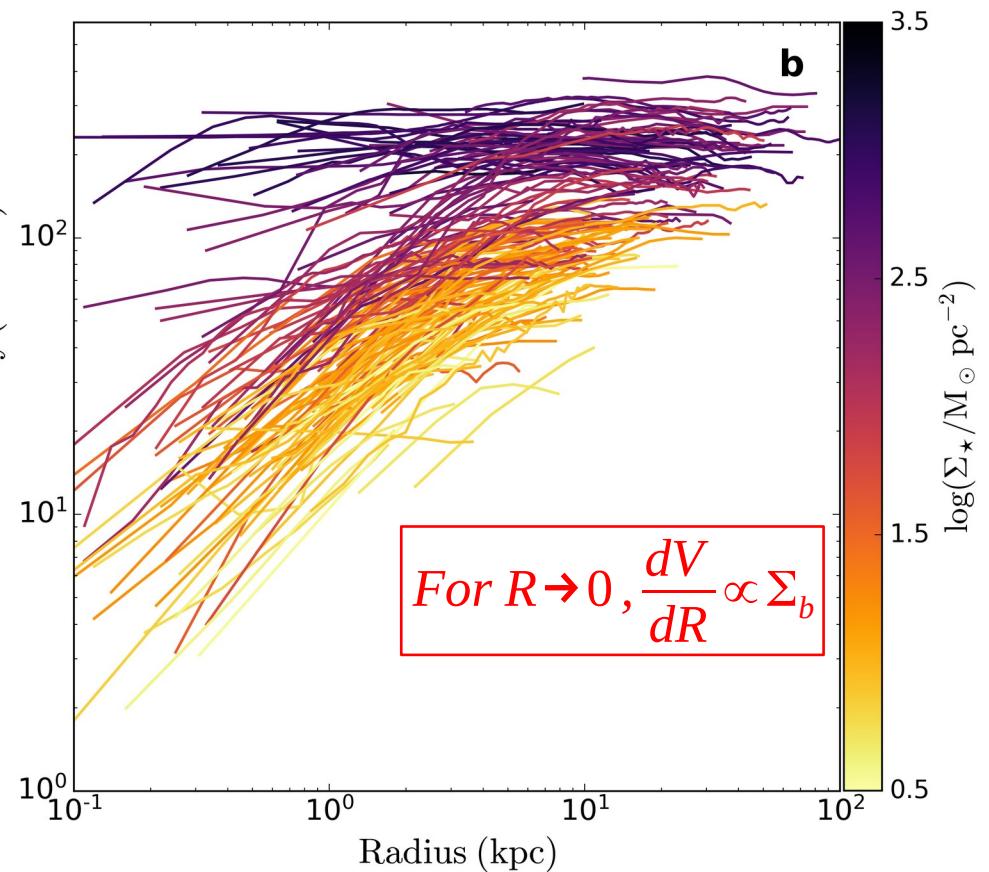
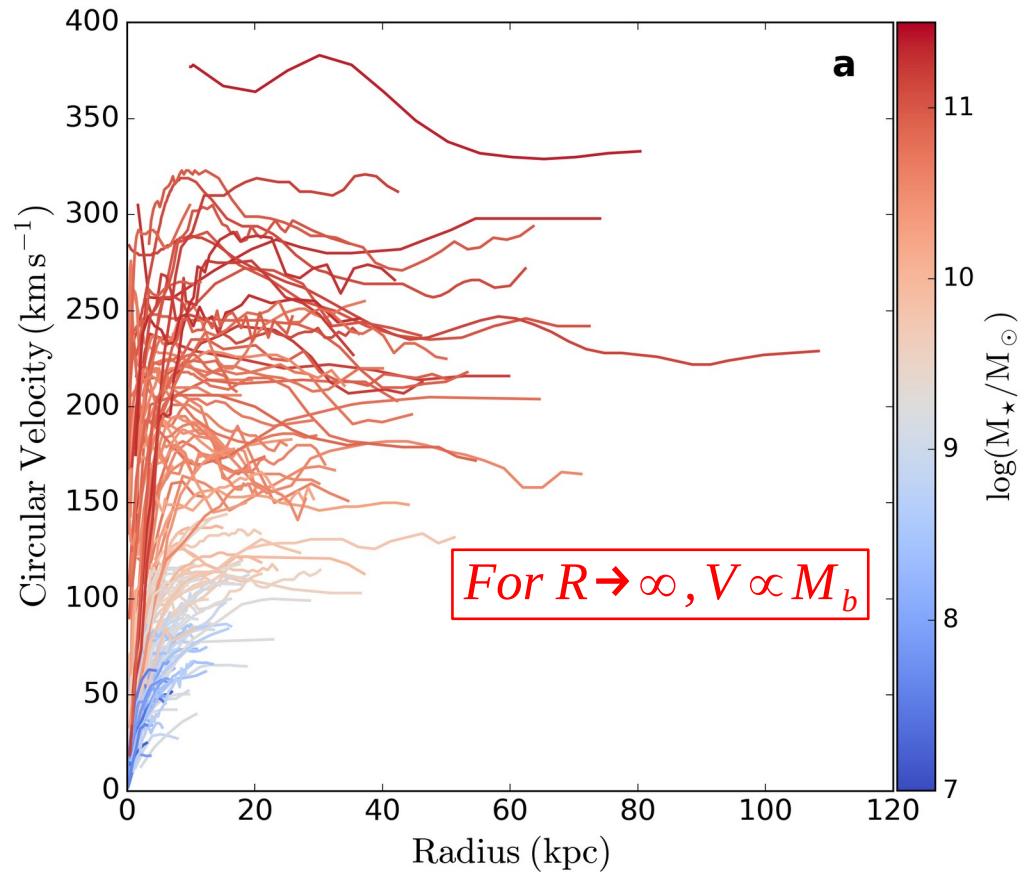
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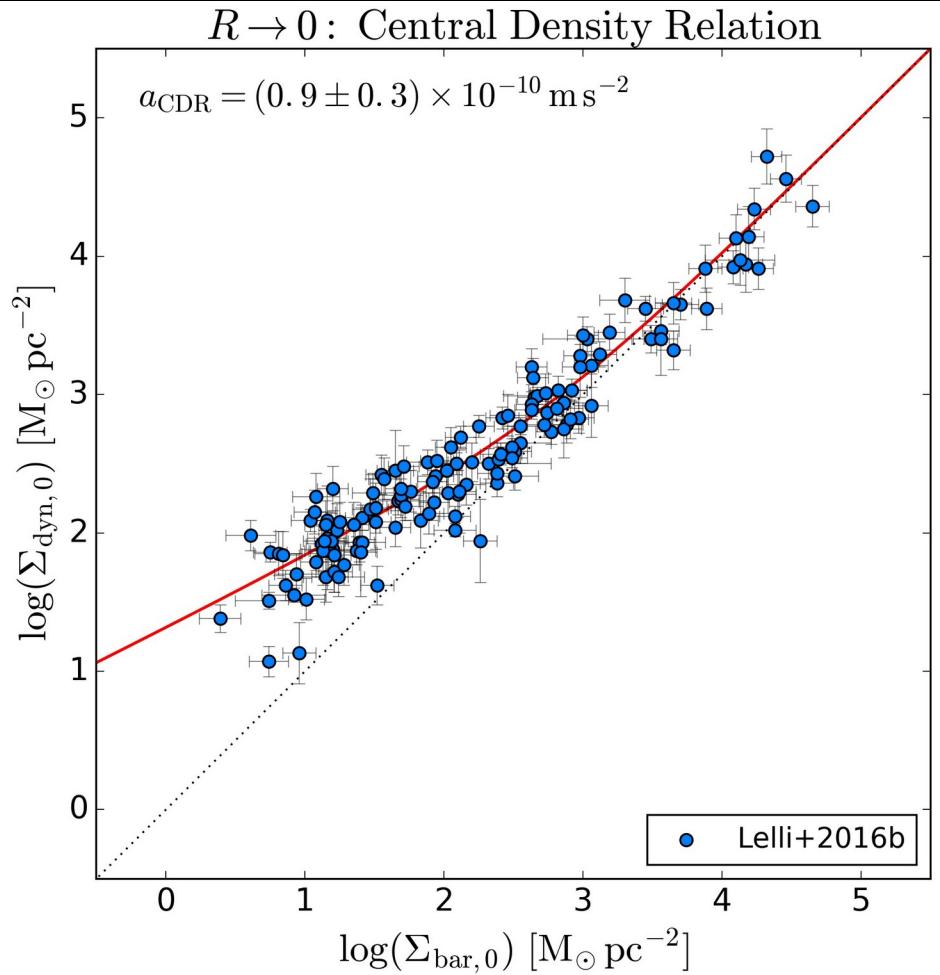
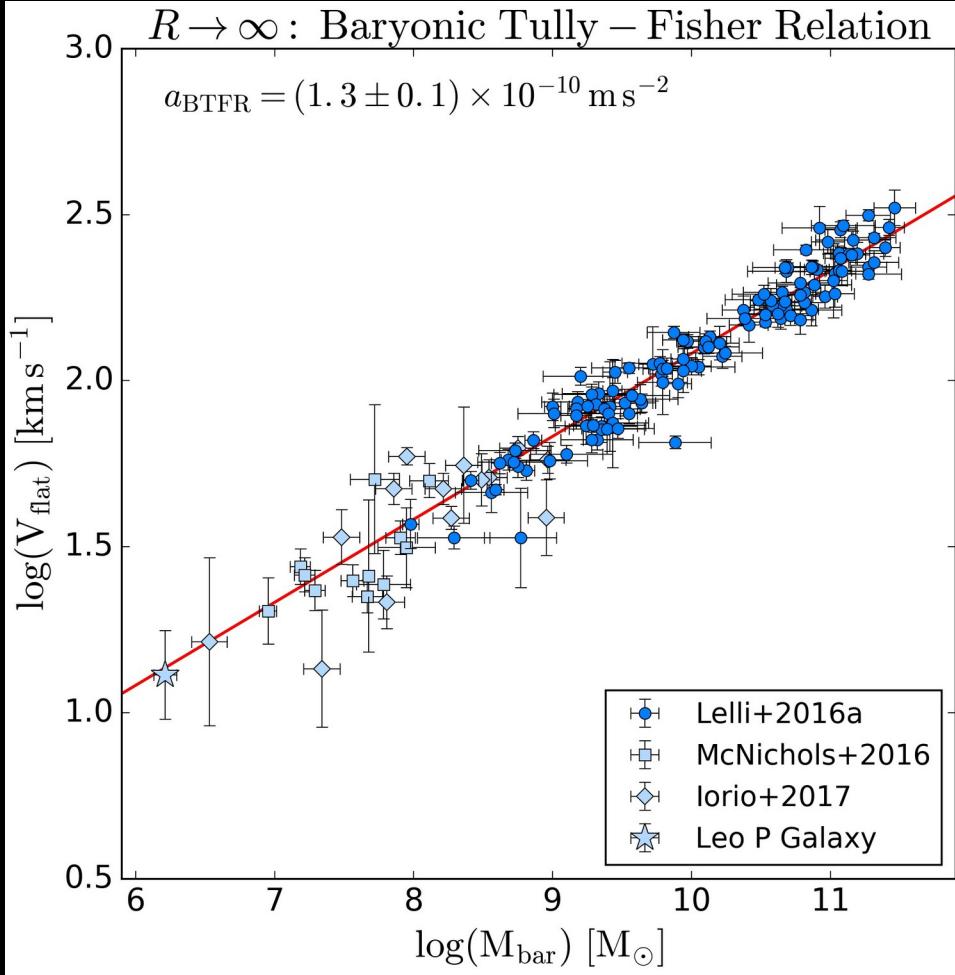
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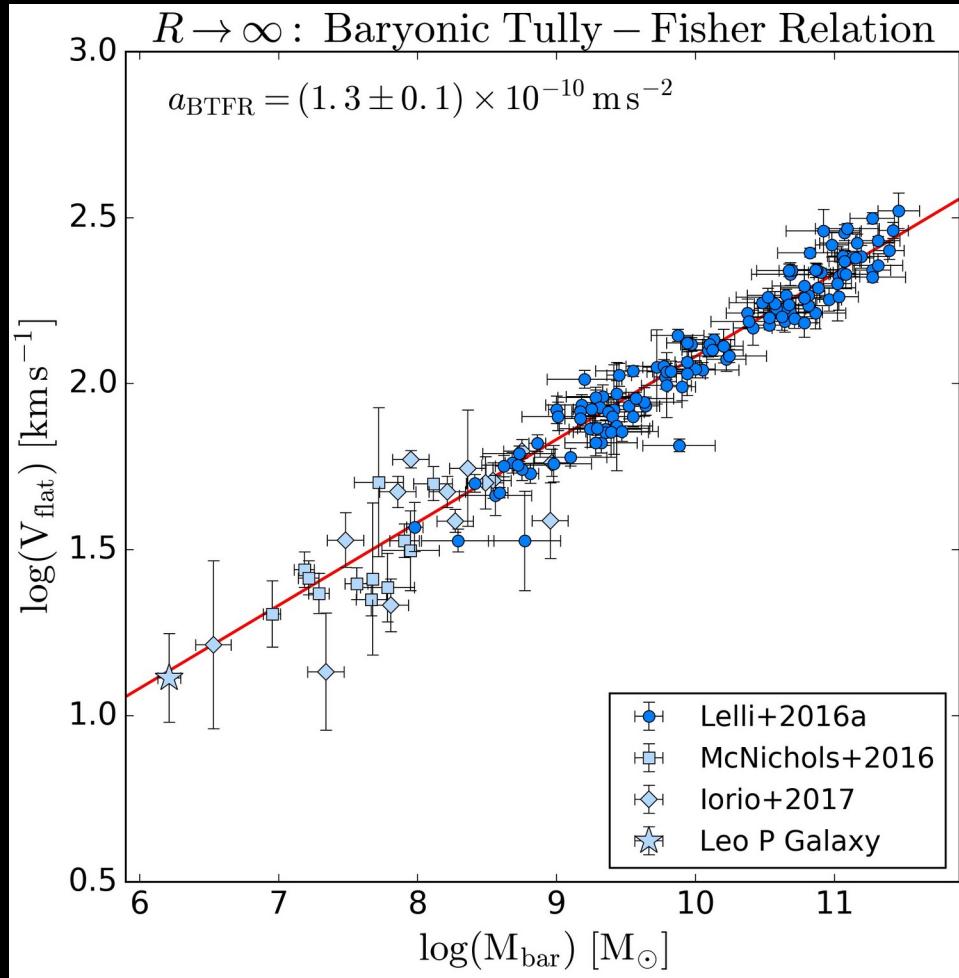


Lelli (2022, Nature Astronomy)

# Dynamical Laws for Disk Galaxies



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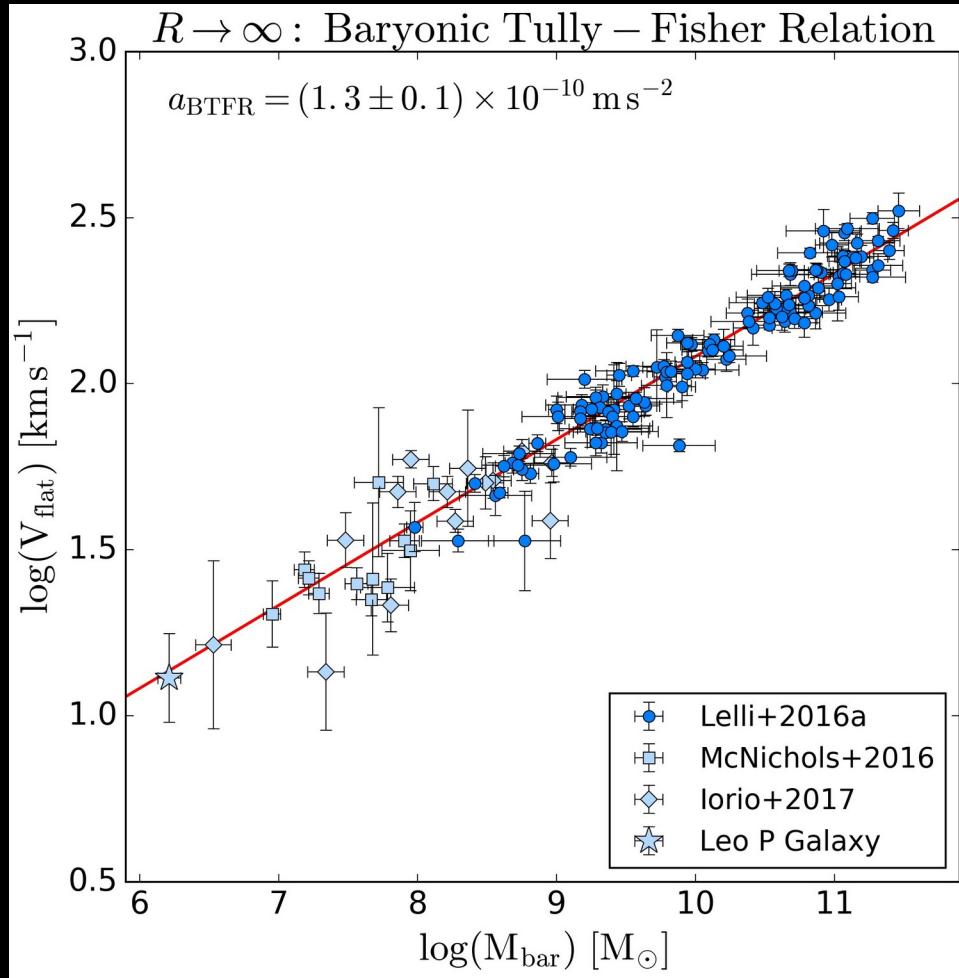
Take-home points on BTFR:

- Key Quantities (e.g. Lelli+2019):

$$M_b = M_{\text{star}} + M_{\text{gas}}$$

$$V_{\text{flat}} = \langle V_{\text{rot}} \rangle \text{ along flat part}$$

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$$V_{\text{flat}} = \langle V_{\text{rot}} \rangle \text{ along flat part}$$

- Linear with Slope  $\sim 4.0$

$$M_b = N V_{\text{flat}}^4 \quad \longrightarrow \quad N = \frac{1}{G_N a_{BTFR}}$$

$$a_{BTFR} = \frac{1}{N G_N} \simeq 10^{-10} \text{ m/s}^2$$

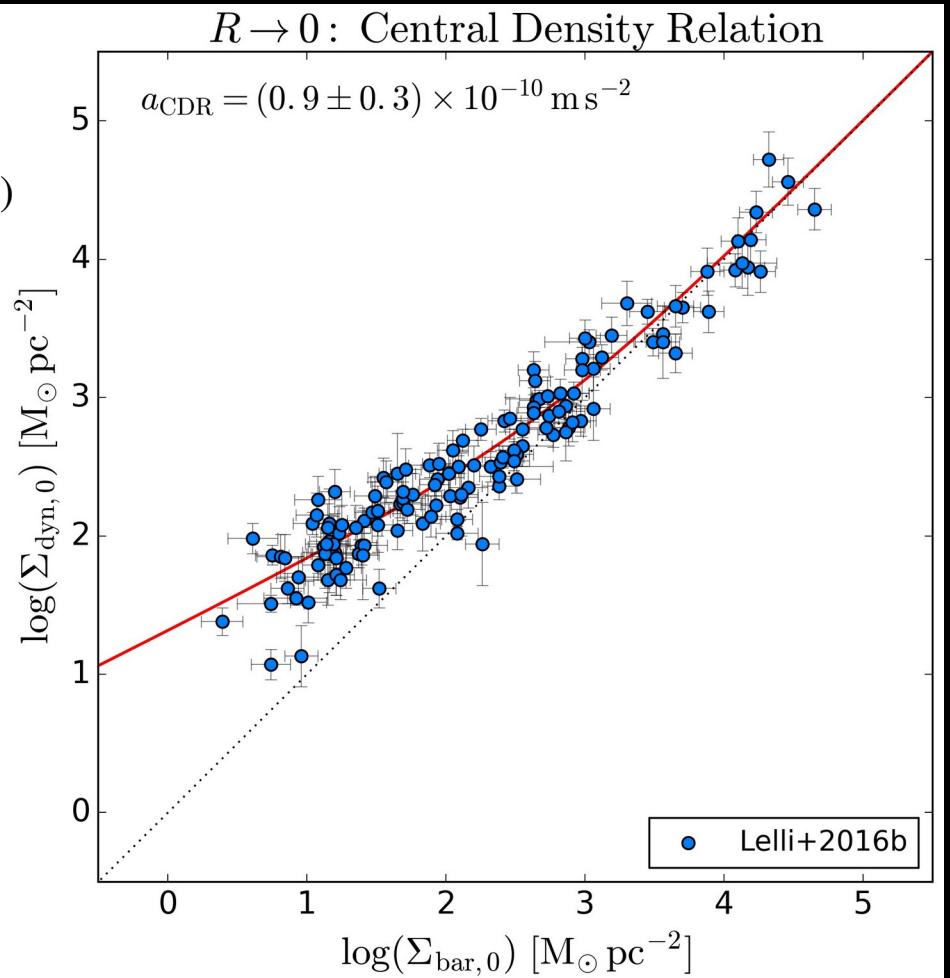
# Dynamical Laws for Disk Galaxies

## Take-home points on CDR:

- **Dynamical Surf. Density** (Toomre 1963)

$$\Sigma_{dyn}(0) = \frac{1}{2\pi G} \int_0^\infty \frac{V^2}{R^2} dR$$

→ Inner Steepness of Rot. Curve



# Dynamical Laws for Disk Galaxies

## Take-home points on CDR:

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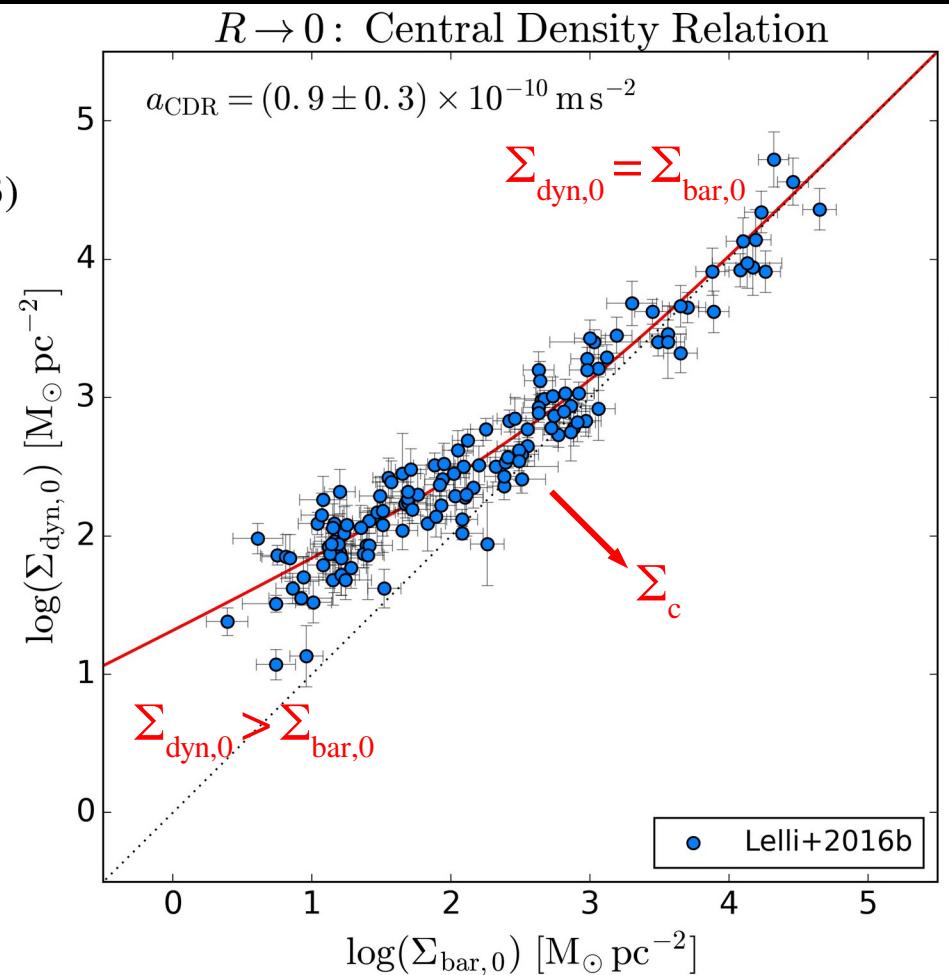
→ Inner Steepness of Rot. Curve

- **Relation is Non-Linear**

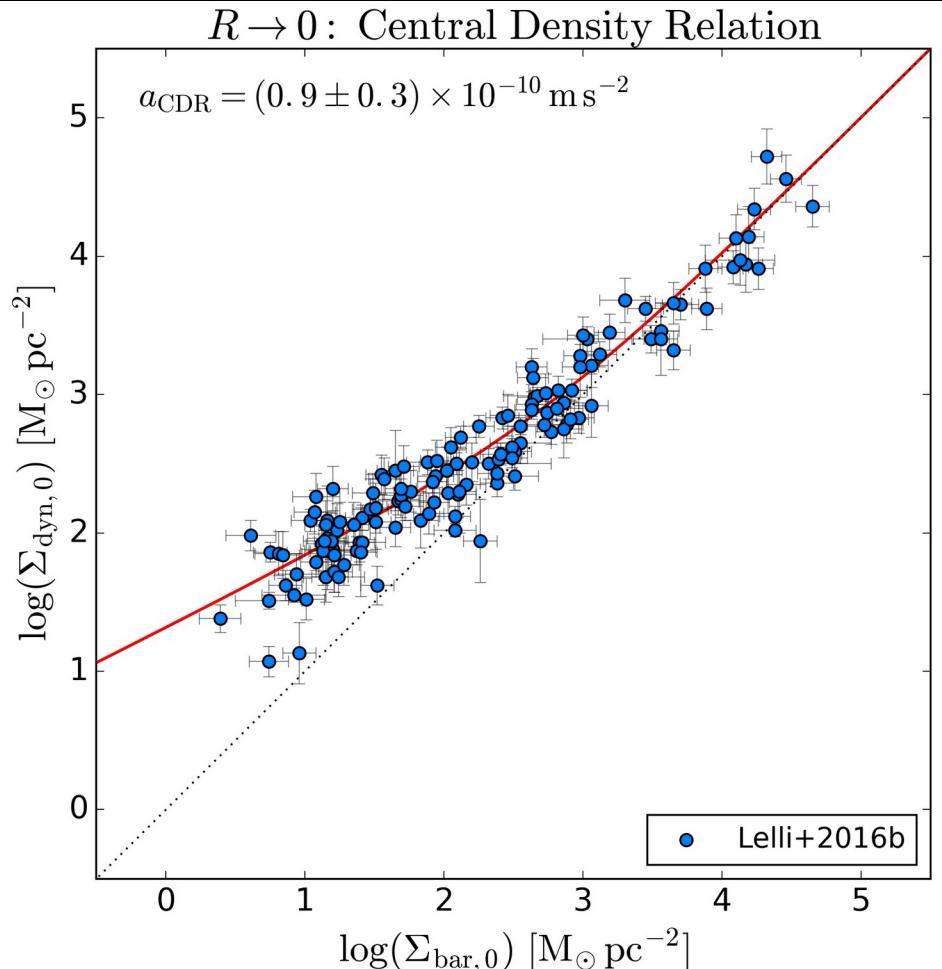
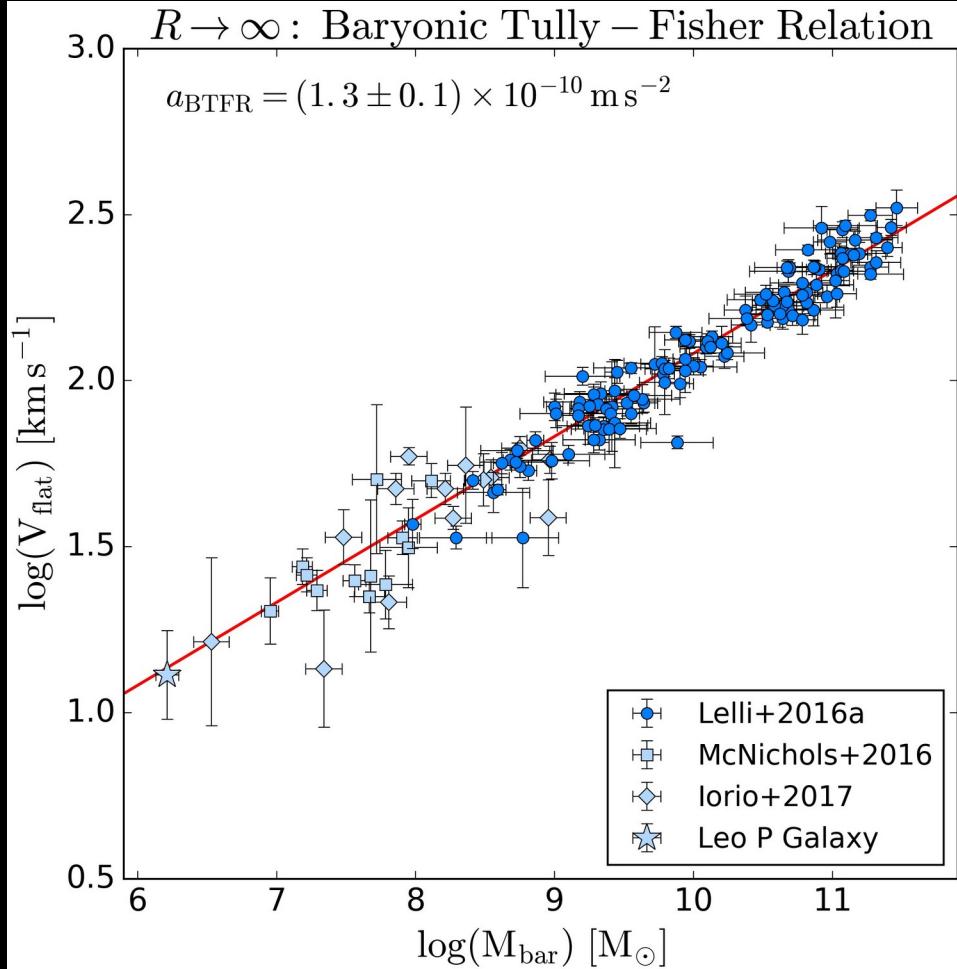
$\Sigma_{bar,0} > \Sigma_c \rightarrow$  baryons dominates

$\Sigma_{bar,0} < \Sigma_c \rightarrow$  DM dominates

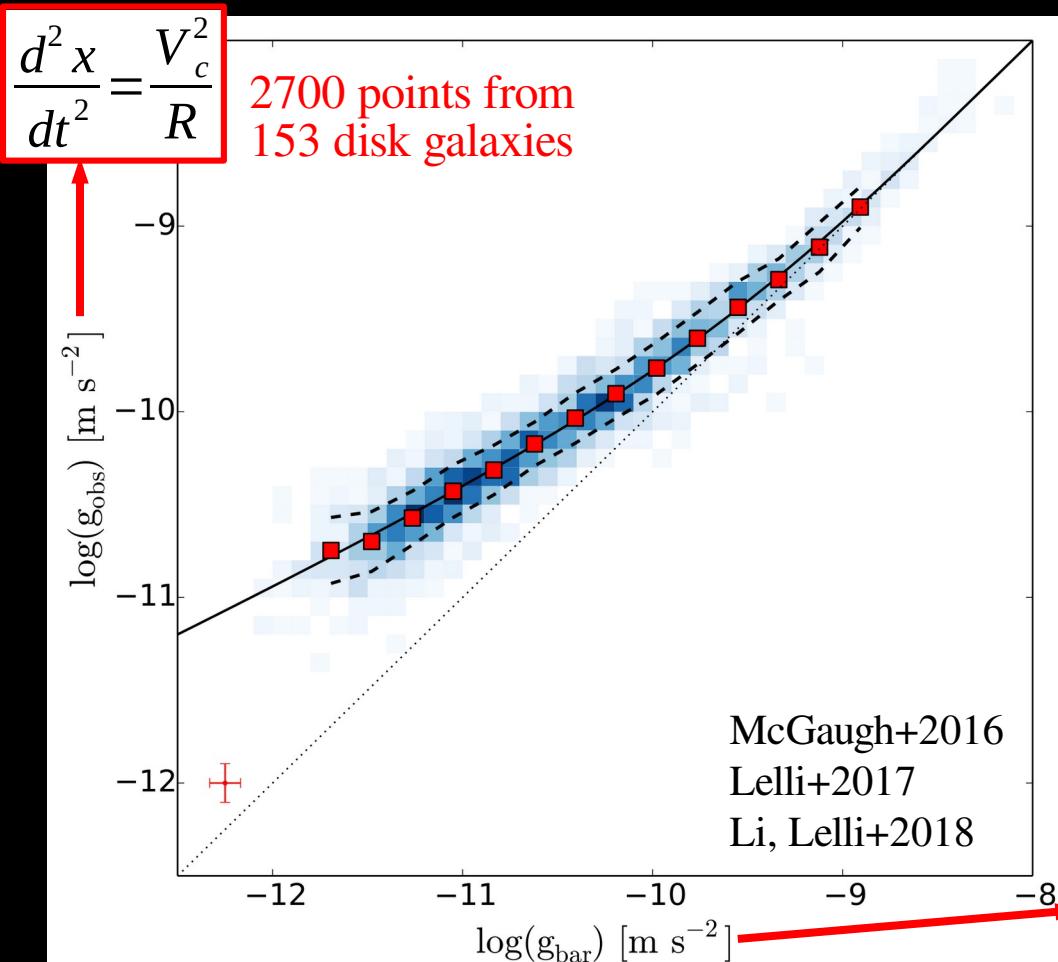
$$a_{CDR} = 2\pi G_N \Sigma_c \simeq 10^{-10} \text{ m/s}^2$$



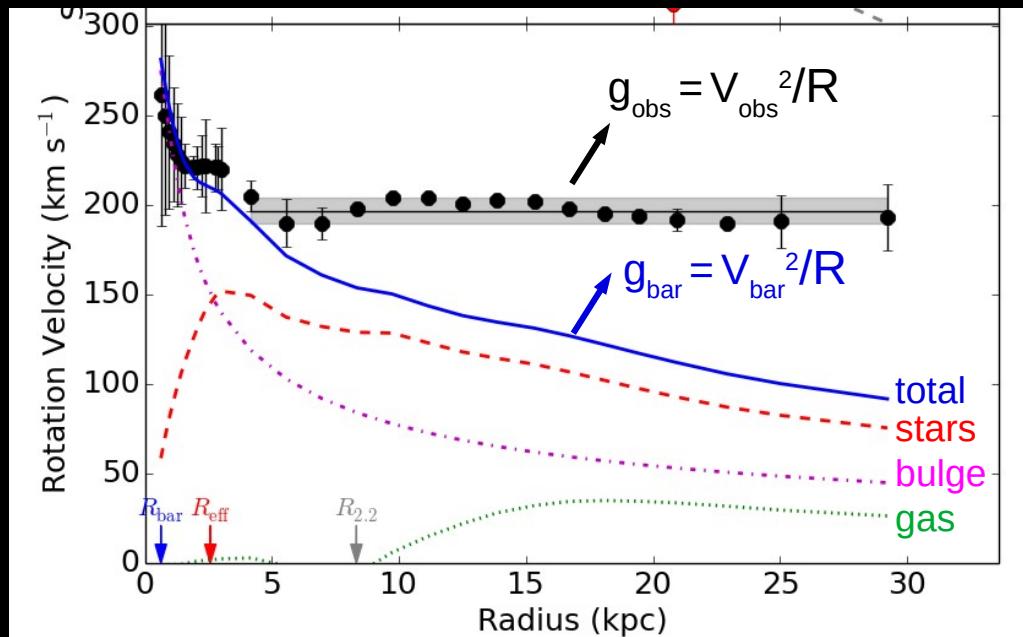
# “Asymptotic” Laws $\rightarrow$ One Point = One Galaxy



# Radial Acceleration Relation (RAR)



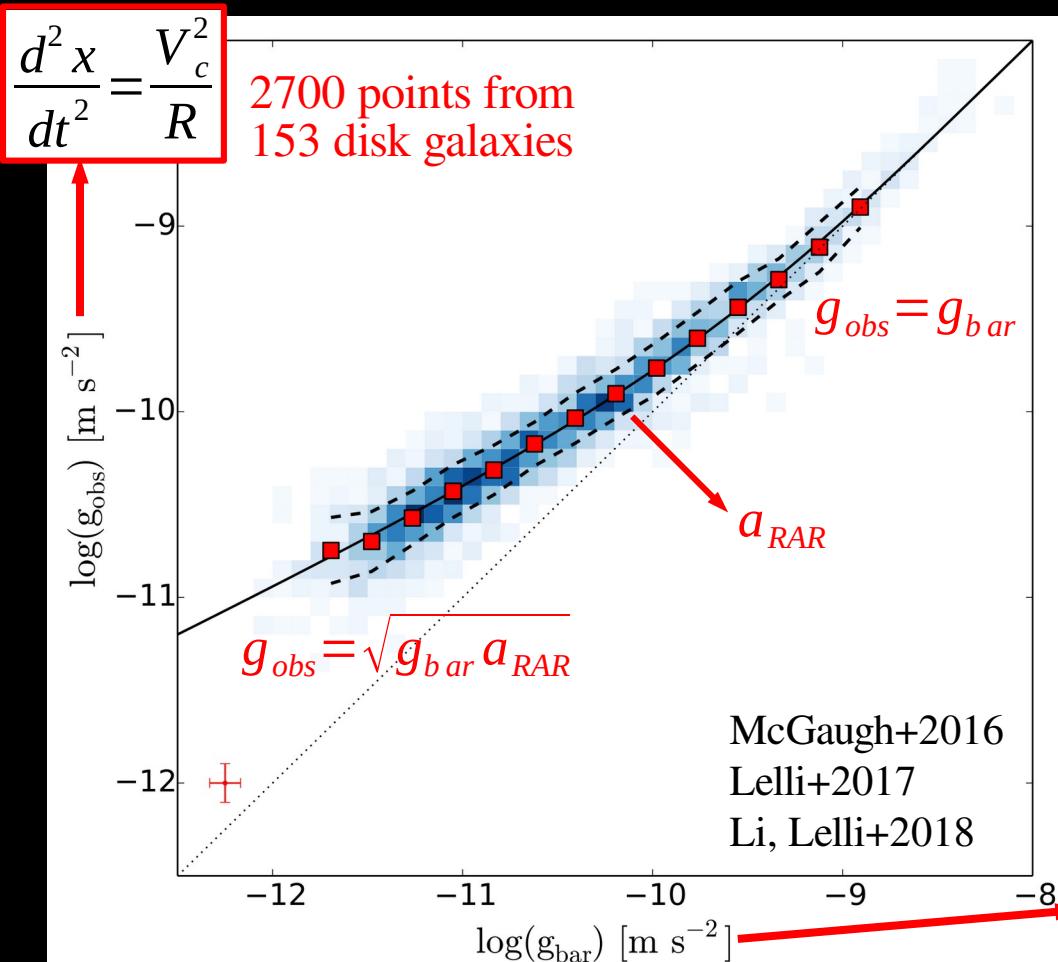
Spatially resolved quantities for each galaxy:



$$g_b = -\nabla \Phi_b$$

$$\nabla^2 \Phi_b = 4\pi G \rho_b$$

# Radial Acceleration Relation (RAR)

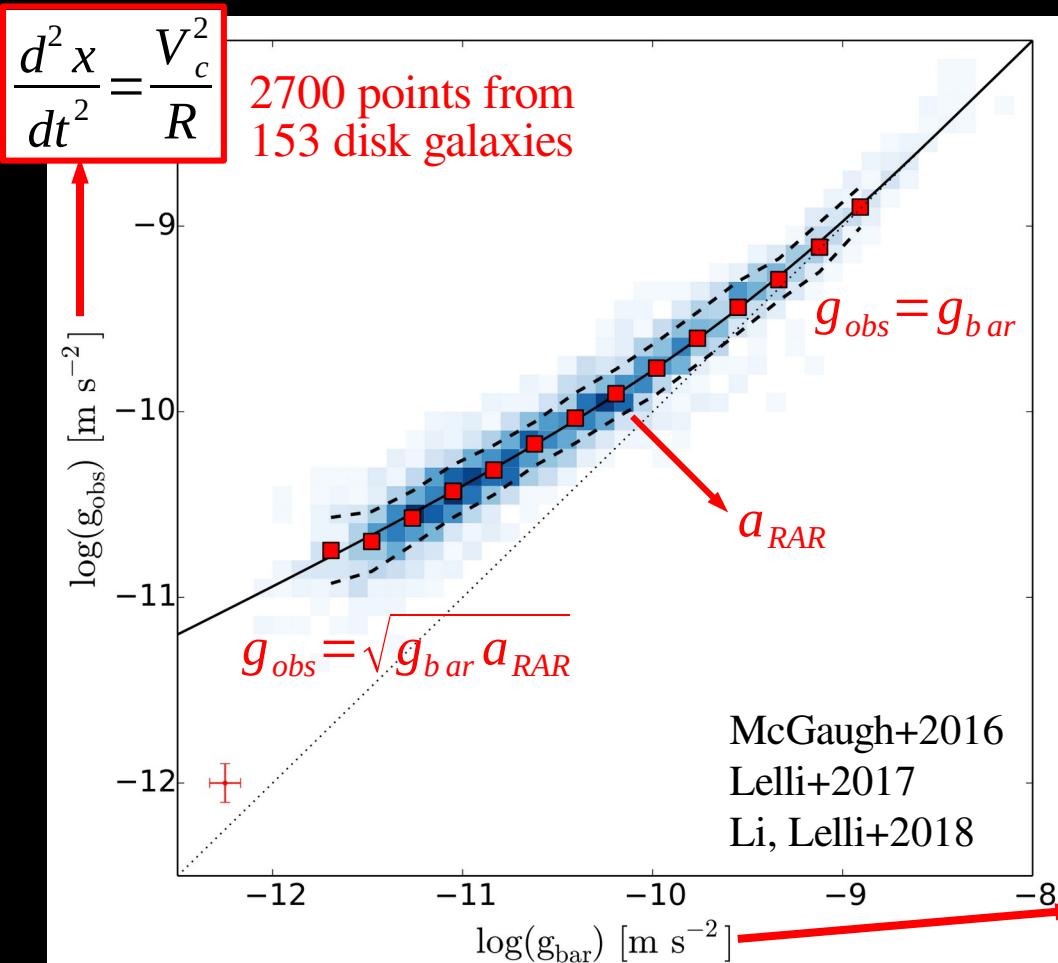


Take-home points on RAR:

- Acceleration scale ( $\sim 10^{-10} \text{ m/s}^2$ ) below which DM effect kicks in

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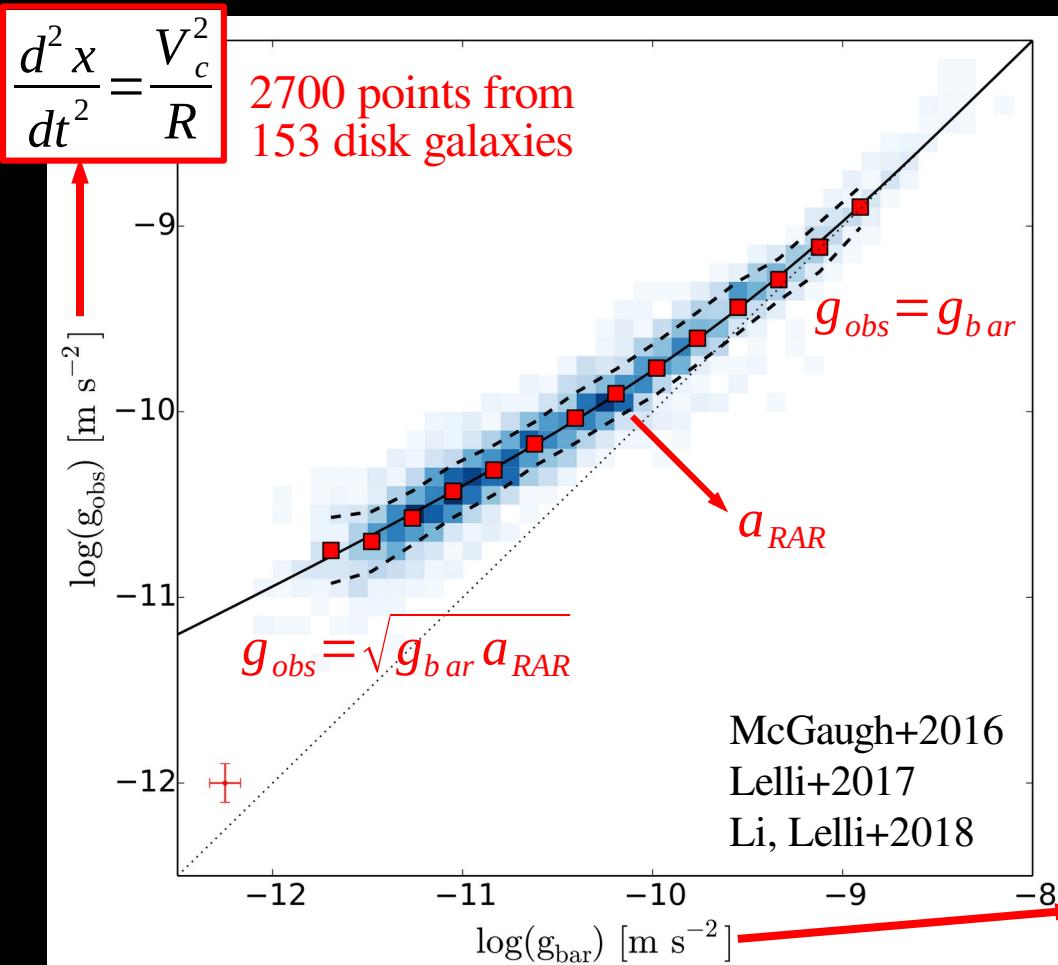


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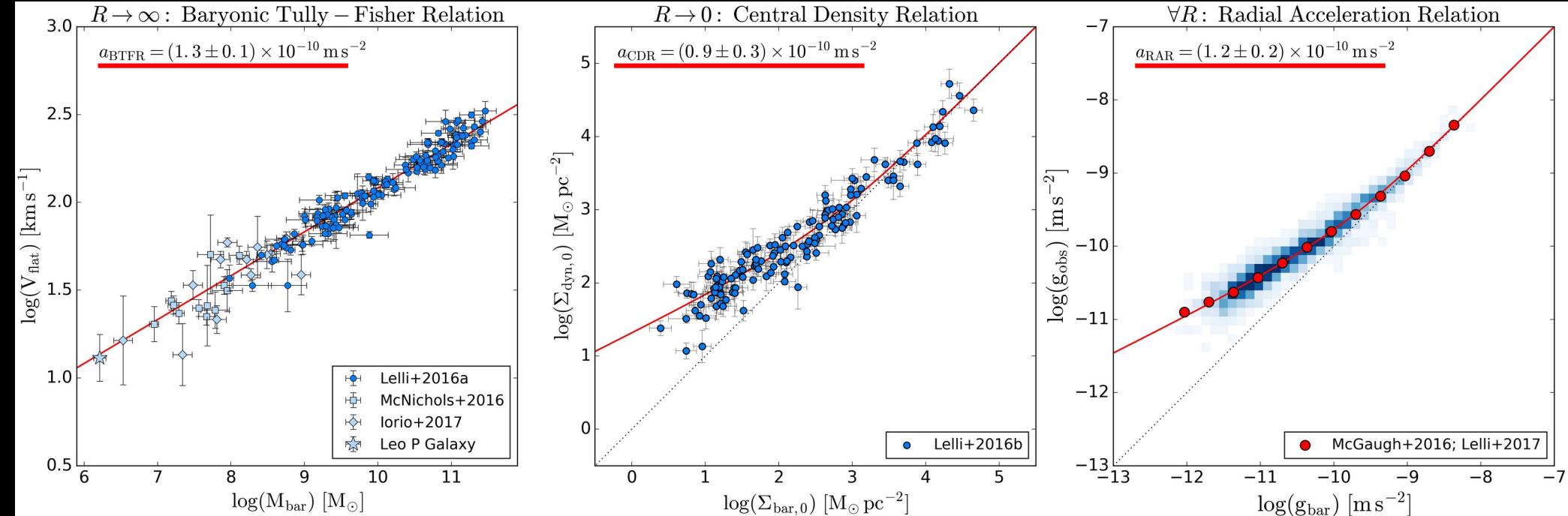


Take-home points on RAR:

- Acceleration scale ( $\sim 10^{-10} \text{ m/s}^2$ ) below which DM effect kicks in
  - Small observed scatter (0.12 dex) consistent with obs. errors
  - Local baryon-DM coupling
- Baryon distribution  $\leftrightarrow$  Rot. curve

$$g_b = -\nabla \Phi_b$$
$$\nabla^2 \Phi_b = 4\pi G \rho_b$$

# Kepler-like Empirical Laws for Rotating Galaxies



$a_{\text{BTFR}}$  → Normalization BTFR  
→ Global baryon-to-DM ratio across galaxies

$a_{\text{CDR}}$  → Critical Surface Density  
→ Transition baryon to DM dominated galaxies at R=0

$a_{\text{RAR}}$  → Acceleration Scale  
→ Transition baryon to DM domination inside galaxies

# Outline of the Talk:

I. Empirical Evidence on Galaxy Scales

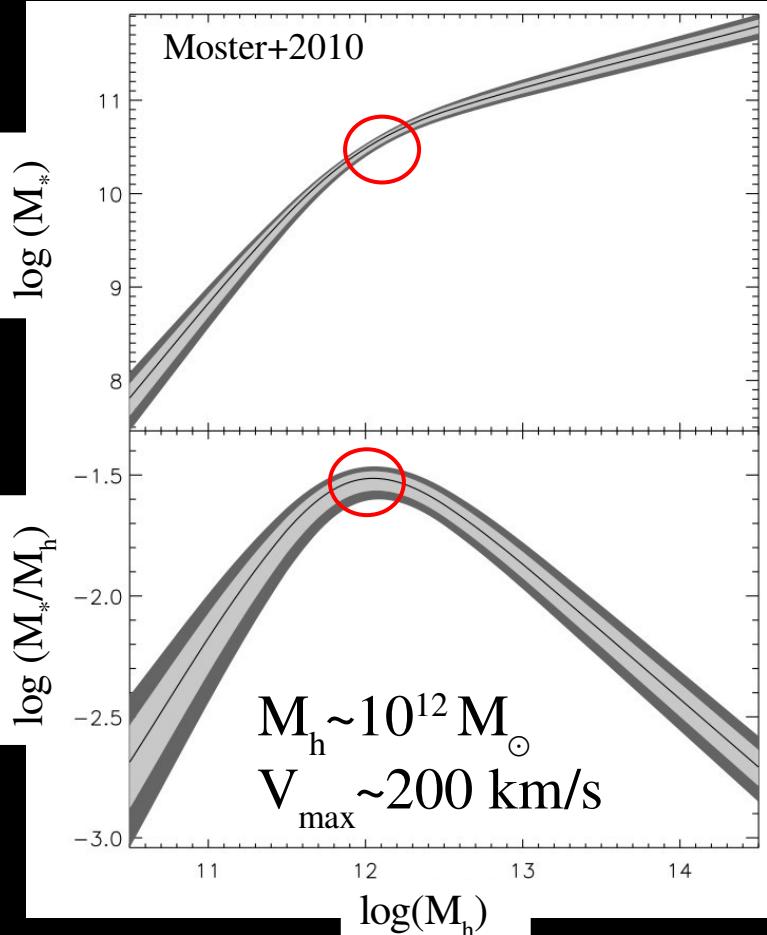
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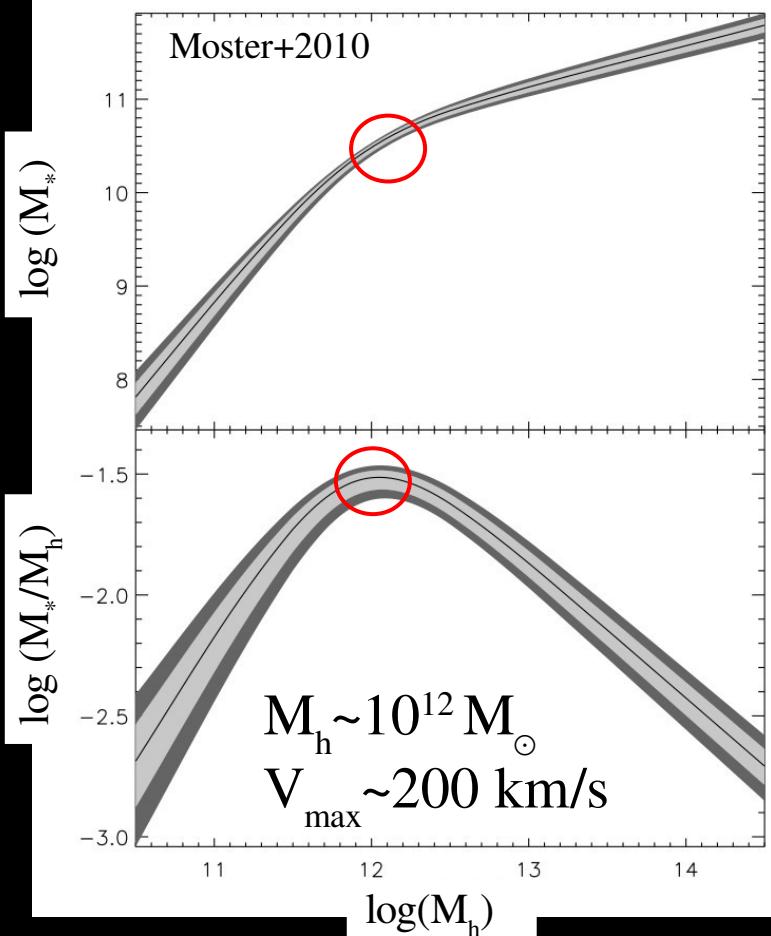
# The BTFR in a $\Lambda$ CDM Cosmology

Theory: Stellar Mass – Halo Mass Relation

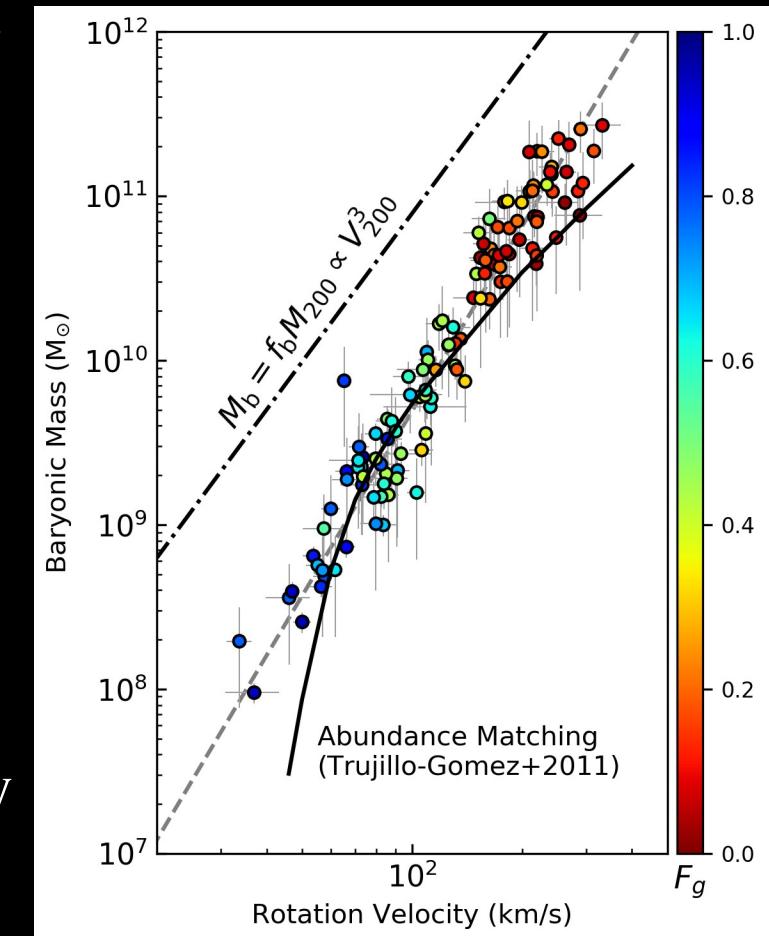


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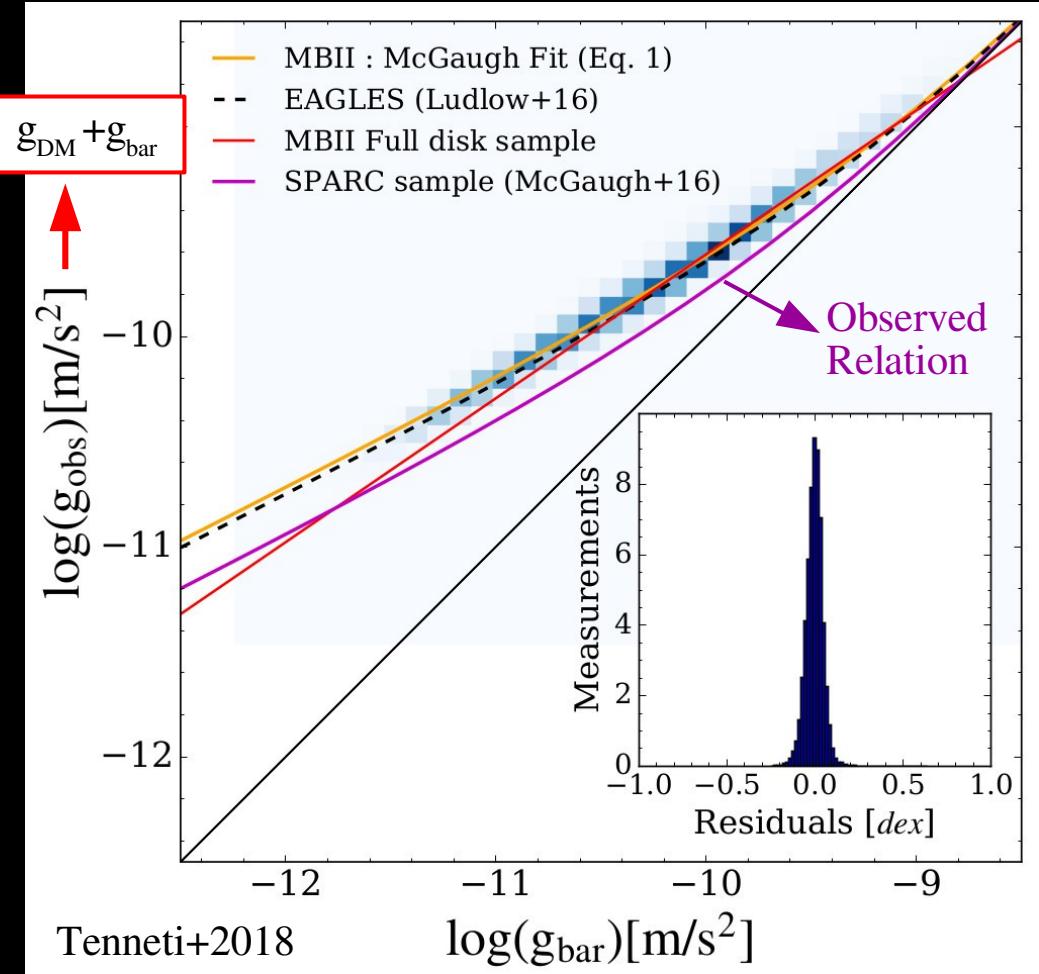


Observations: Mass – Velocity Relation



General Issues:  
BTFR shape:  
mass scale *vs*  
acceleration scale  
BTFR scatter:  
stochasticity *vs*  
observed regularity  
(scatter  $\sim 6\%$ )

# The RAR in a $\Lambda$ CDM Cosmology



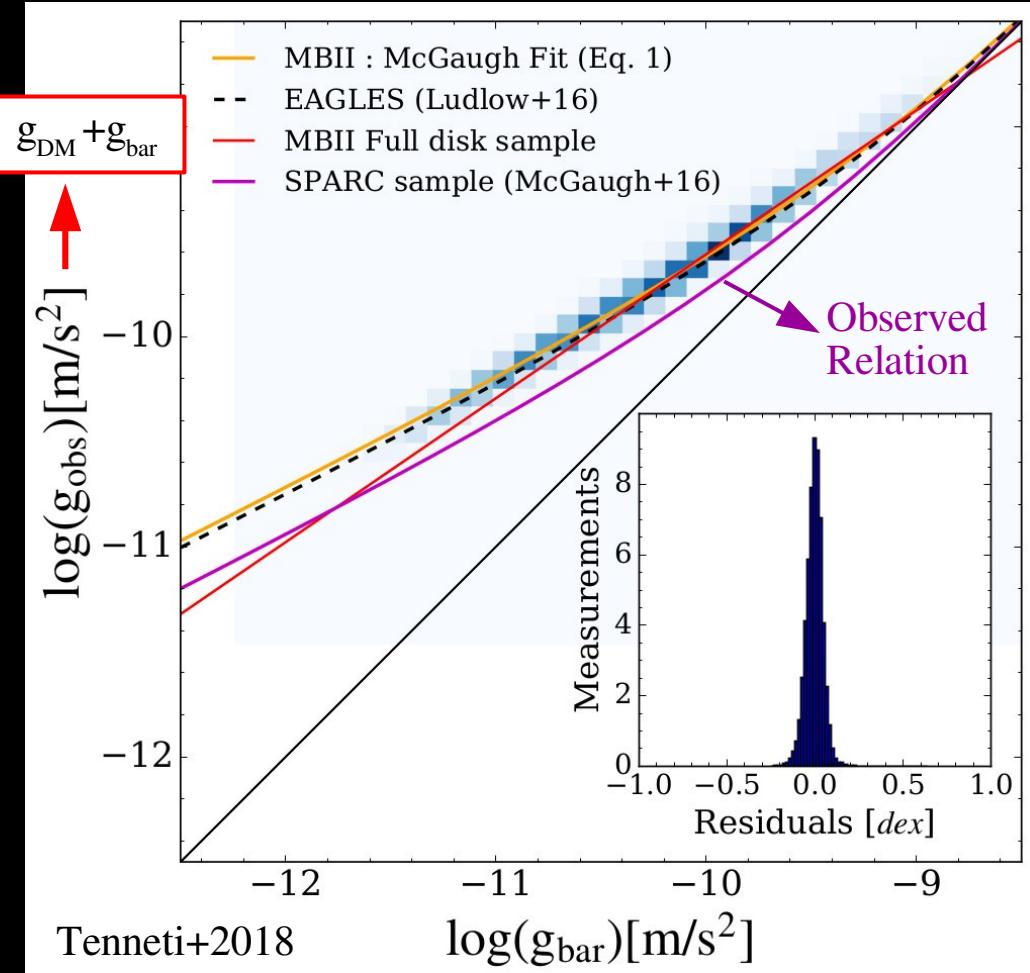
Semi-analytic AM models:

(Di Cintio & Lelli 2016; Desmond 2017; Navarro+2017;  
Paranjape & Sheth 2021; Li, McGaugh, Lelli+2022)

Hydrodynamic Simulations:

(Santos Santos 2016; Keller & Wadsley 2017;  
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*General Issues:*

**Acc. Scale:** Existence and value

**Shape:** Too much DM at all radii

**Scatter:** Galaxy formation is stochastic  
→ Observed tightness of ~25% hard to  
explain in galaxy formation models

# Dynamical Laws of Galaxies predicted by MOND

MOND = NO DM

*or* Modified Newtonian Dynamics *or* MilgrOmiaN Dynamics

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- Proposed by Mordehai Milgrom (1983a,b,c).
- MOND modifies laws of gravity and/or inertia when  $a \ll a_0$ .
- Scale Invariance (Milgrom 2009):  $(\vec{x}', t') \rightarrow (\lambda \vec{x}, \lambda t)$

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$$a = \sqrt{g_N a_0}$$

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Circular orbit at large R

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$$a = \sqrt{g_N a_0} \quad \xrightarrow{\text{Circular orbit at large R}} \quad \frac{V^2}{R} = \sqrt{\frac{a_0 G M_b}{R^2}} \quad \xrightarrow{\text{Baryonic TF relation}} \quad V^4 = a_0 G M_b$$

Circular orbit at large R

Baryonic TF relation

# A-priori Predictions by Milgrom 40 years ago

A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXIES<sup>1</sup>

M. MILGROM

Department of Physics, Weizmann Institute, Rehovot, Israel; and The Institute for Advanced Study

Received 1982 February 4; accepted 1982 December 28

## ABSTRACT

I use a modified form of the Newtonian dynamics (inertia and/or gravity) to describe the motion of bodies in the gravitational fields of galaxies, *assuming that galaxies contain no hidden mass*, with the following main results.

1. The Keplerian, circular velocity around a finite galaxy becomes independent of  $r$  at large radii, thus resulting in asymptotically flat velocity curves.

2. The asymptotic circular velocity ( $V_\infty$ ) is determined only by the total mass of the galaxy ( $M$ ):  $V_\infty^4 = a_0 GM$ , where  $a_0$  is an acceleration constant appearing in the modified dynamics. This relation is consistent with the observed Tully-Fisher relation if one uses a luminosity parameter which is proportional to the observable mass.

3. The discrepancy between the dynamically determined Oort density in the solar neighborhood and the density of observed matter disappears.

4. The rotation curve of a galaxy can remain flat down to very small radii, as observed, only if the galaxy's average surface density  $\Sigma$  falls in some narrow range of values which agrees with the Fish and Freeman laws. For smaller values of  $\Sigma$ , the velocity rises more slowly to the asymptotic value.

5. The value of the acceleration constant,  $a_0$ , determined in a few independent ways is approximately  $2 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1})^2 \text{ cm s}^{-2}$ , which is of the order of  $CH_0 = 5 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1}) \text{ cm s}^{-2}$ .

The main predictions are:

1. Rotation curves calculated on the basis of the *observed* mass distribution and the modified dynamics should agree with the observed velocity curves.

2. The  $V_\infty^4 = a_0 GM$  relation should hold exactly.

3. An analog of the Oort discrepancy should exist in all galaxies and become more severe with increasing  $r$  in a predictable way.



$$a_0 = a_{\text{BTFR}} = a_{\text{RAR}} = a_{\text{CDR}}$$

→ Baryonic TF relation

→ Central density relation

→ Radial acceleration relation

# Linking Newtonian & Milgromian Regimes

Interpolation function  $\mu(x)$  with  $x = a/a_0$ :

$$a\mu(x) = g_N \left\{ \begin{array}{l} \lim_{x \rightarrow \infty} \mu \rightarrow 1 \rightarrow a = g_N \quad \text{Newtonian regime} \\ \lim_{x \rightarrow 0} \mu \rightarrow x \rightarrow \frac{a^2}{a_0} = g_N \rightarrow a = \sqrt{a_0 g_N} \quad \text{MOND regime} \end{array} \right.$$

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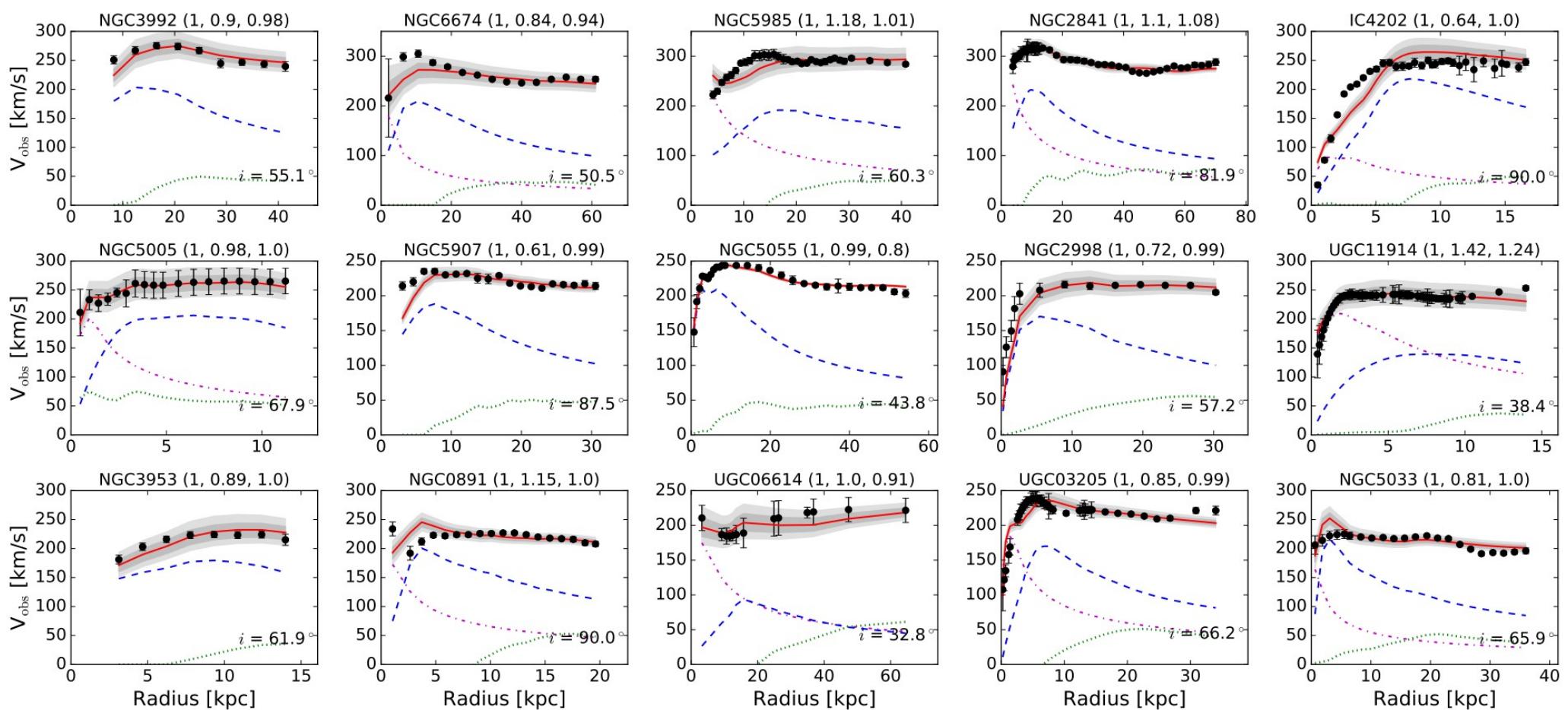
Examples of interpolation functions in Physics:

- Lorentz Factor  $\gamma$  (via  $c$ ): Newton's second law  $\leftrightarrow$  special relativity
- Planck's Law for the Blackbody (via  $\hbar$ ): Rayleigh-Jeans  $\leftrightarrow$  Wein regimes

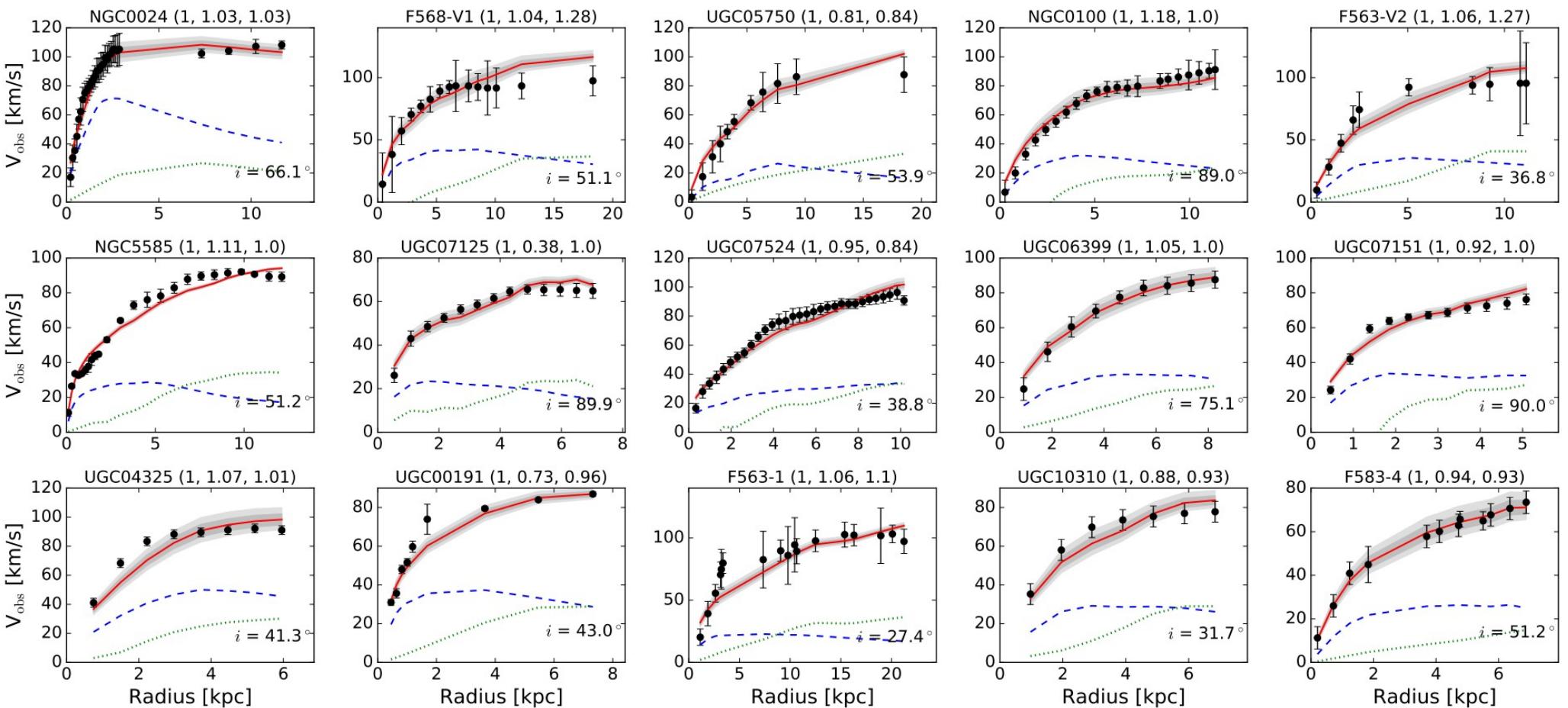
MOND postulates (scale invariance below  $a_0$ ) do NOT specify  $\mu$

$\rightarrow$  Empirical shape of the RAR  $\rightarrow a = v(g_N/a_0)g_N$  with  $v = \mu^{-1}$

# MOND Fits to Rotation Curves (Li, Lelli, McGaugh 2018)



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# AQUAL = Aquadratic Lagrangian (Bekenstein & Milgrom 1984)

$$S = \int dt L = \int dt d^3x \left( \rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right)$$

↓

$$- \frac{a_0^2}{8\pi G} F \left( \frac{|\vec{\nabla} \Phi|^2}{a_0^2} \right)$$

Newton: Quadratic in  $\nabla \Phi$

$F(z) \rightarrow z$  for  $z = |\nabla \Phi|^2/a_0^2 \gg 1$

$F(z) \rightarrow z^{3/2}$  for  $z = |\nabla \Phi|^2/a_0^2 \ll 1$

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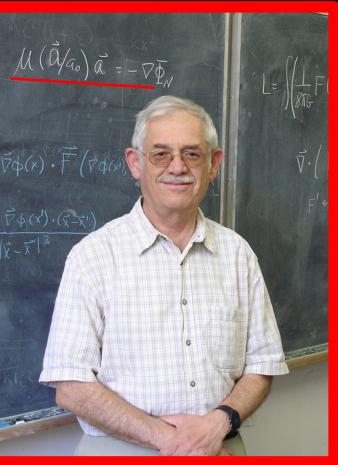
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$$\frac{\delta S}{\delta \Phi} = 0 \rightarrow \nabla \cdot \left[ \mu \left( \frac{|\vec{\nabla} \Phi|}{a_0} \right) \vec{\nabla} \Phi \right] = 4\pi G \rho$$

Modified Poisson's Equation

$$\mu(x) = \frac{dF(z)}{dz} \quad z = x^2 \quad F(z) \text{ provides the interpolation function } \mu = v^{-1}$$



# DOES THE MISSING MASS PROBLEM SIGNAL THE BREAKDOWN OF NEWTONIAN GRAVITY?

JACOB BEKENSTEIN

Department of Physics, Ben Gurion University of the Negev, Beer-Sheva

AND

MORDEHAI MILGROM<sup>1</sup>

Department of Physics, Weizmann Institute of Science, Rehovot

*Received 1984 March 28; accepted 1984 May 17*



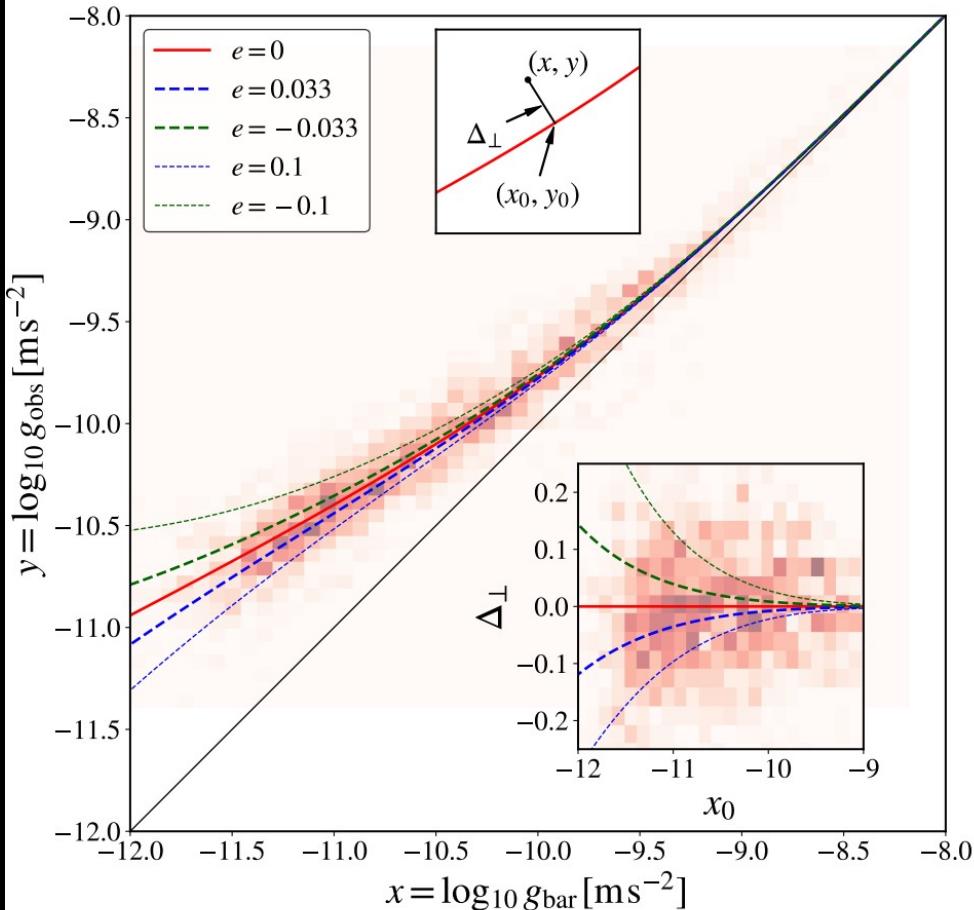
## ABSTRACT

We consider a nonrelativistic potential theory for gravity which differs from the Newtonian theory. The theory is built on the basic assumptions of the modified dynamics, which were shown earlier to reproduce dynamical properties of galaxies and galaxy aggregates without having to assume the existence of hidden mass. The theory involves a modification of the Poisson equation and can be derived from a Lagrangian. The total momentum, angular momentum, and (properly defined) energy of an isolated system are conserved. The center-of-mass acceleration of an arbitrary bound system in a constant external gravitational field is independent of any property of the system. In other words, all isolated objects fall in exactly the same way in a constant external gravitational field (the weak equivalence principle is satisfied). However, the internal dynamics of a system in a constant external field is different from that of the same system in the absence of the external field, in violation of the strong principle of equivalence. These two results are consistent with the phenomenological requirements of the modified dynamics. We sketch a toy relativistic theory which has a nonrelativistic limit satisfying the requirements of the modified dynamics.

*Subject headings:* cosmology — galaxies: internal motions — gravitation

**Violation of Local-Position Invariance  
for Gravitational Experiments**

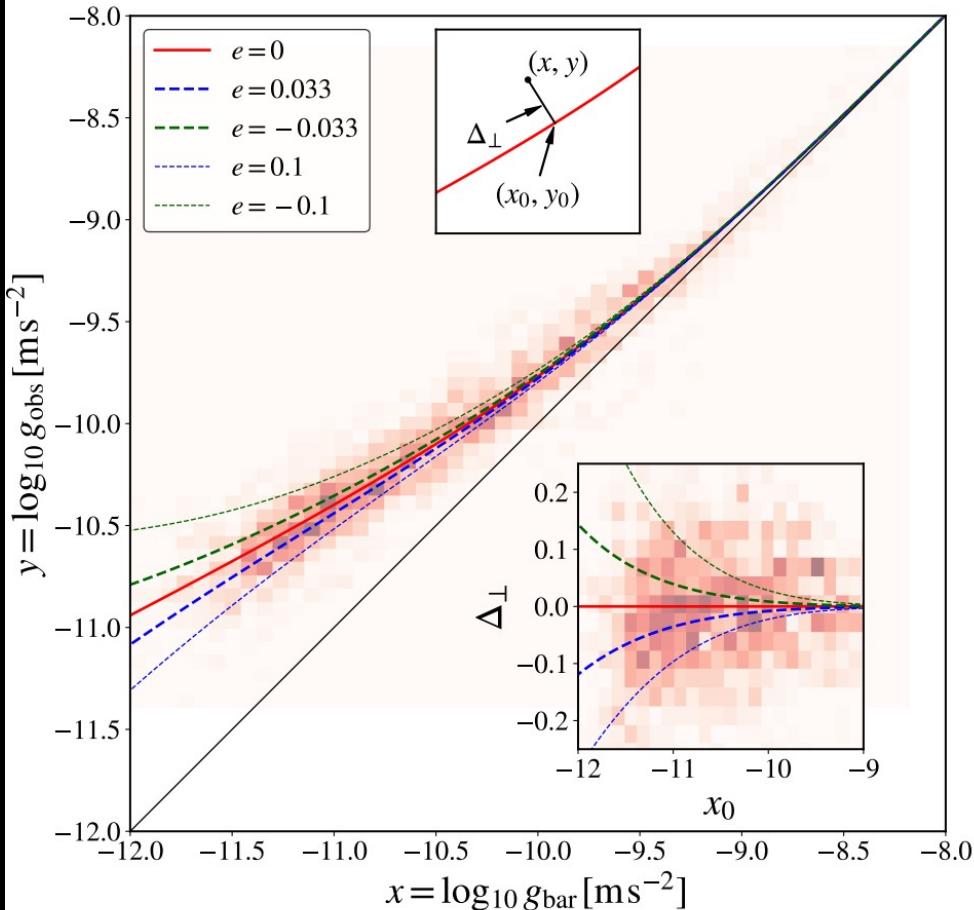
# The External Field Effect (EFE) in MOND



- For truly *isolated* galaxies:  
$$\mathbf{a} = v_0(g_N/a_0)g_N$$
- For galaxies subjected to  $e = g_{\text{ext}}/a_0$ :  
$$\mathbf{a} = v_e(g_N/a_0; e)g_N$$

Chae, Lelli, Desmond et al. (2020)

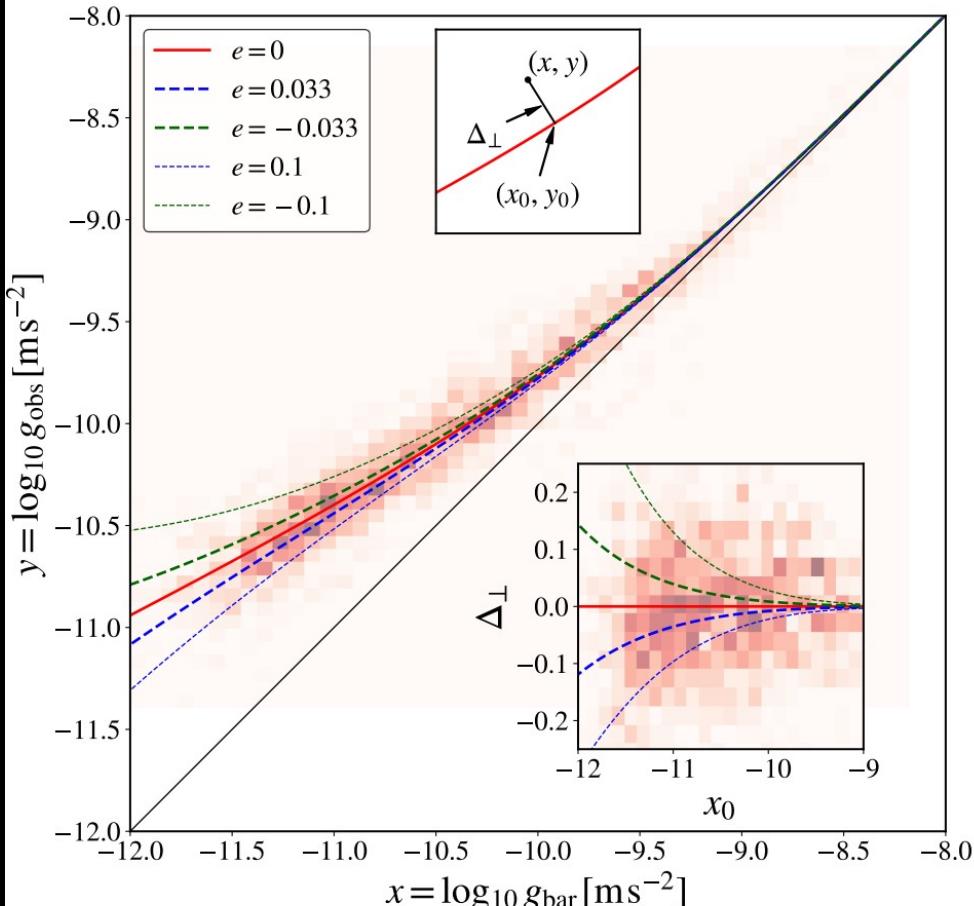
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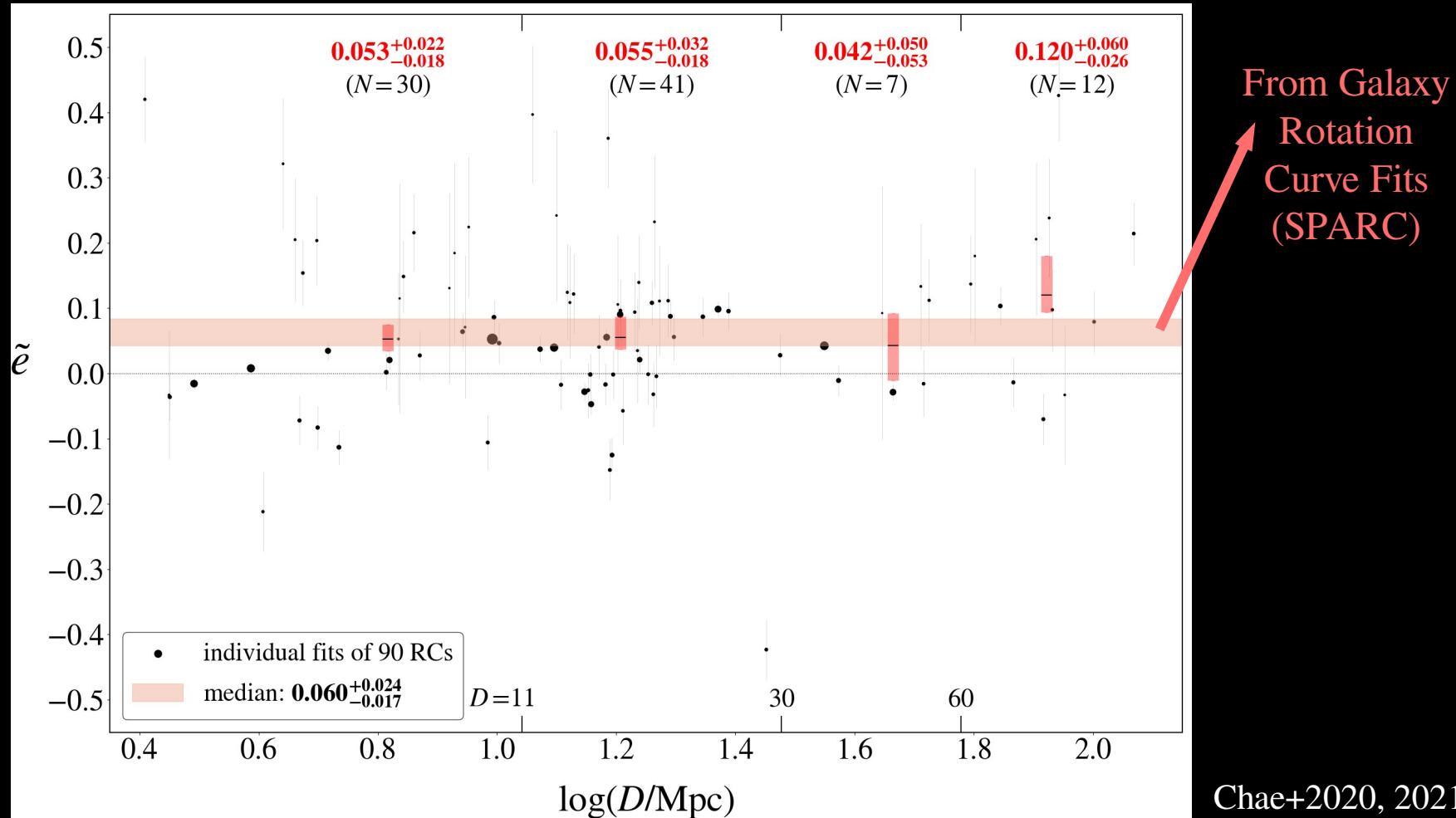
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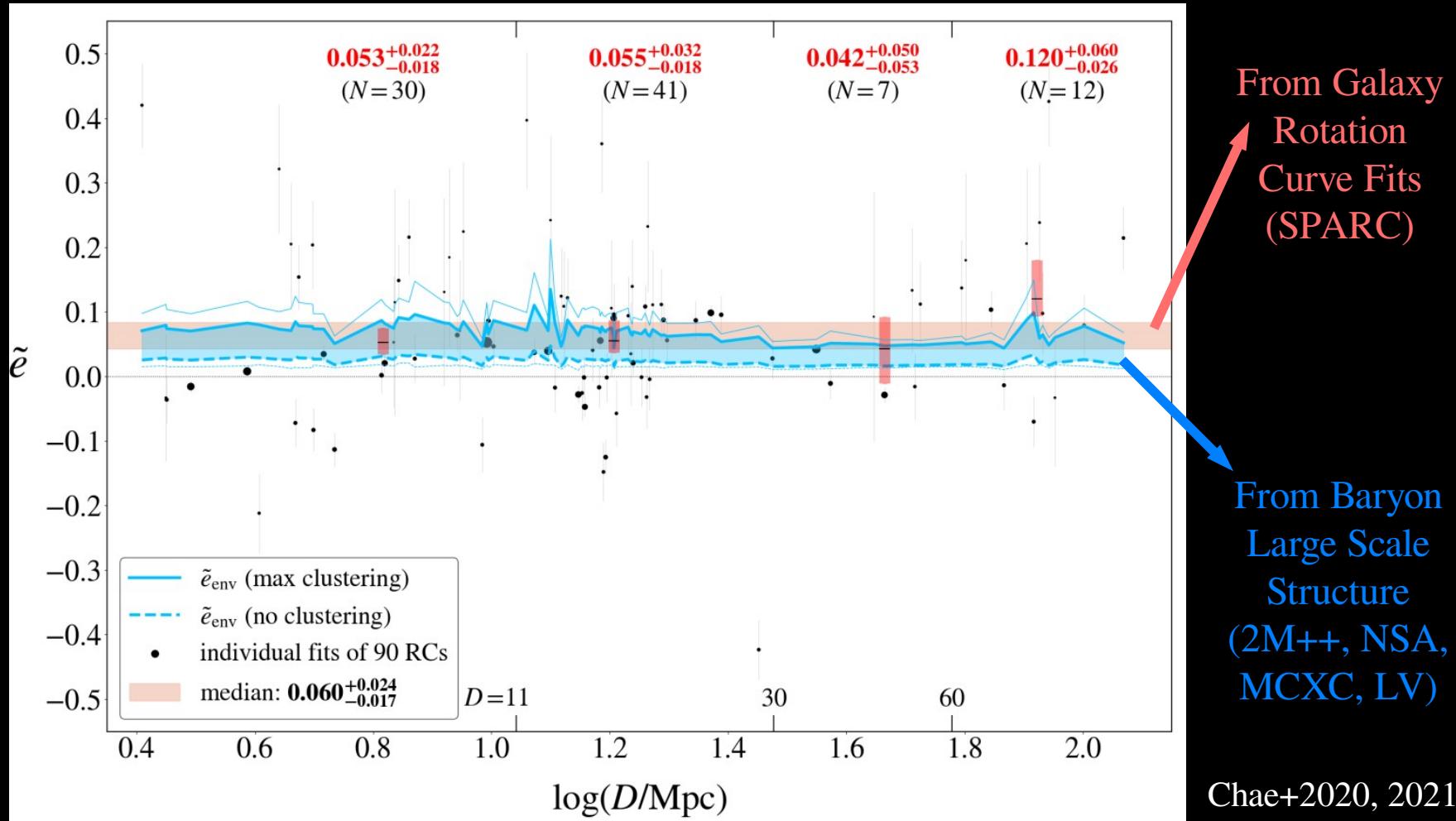
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- The RAR should be a *family of curves* depending on the galaxy environment
- We can fit RCs to infer the value of  $e$ . Independently, we estimate  $e_{\text{env}}$  from the large-scale environment of galaxies

Chae, Lelli, Desmond et al. (2020)

# EFE is detected at $>4\sigma$ and agrees with LSS!



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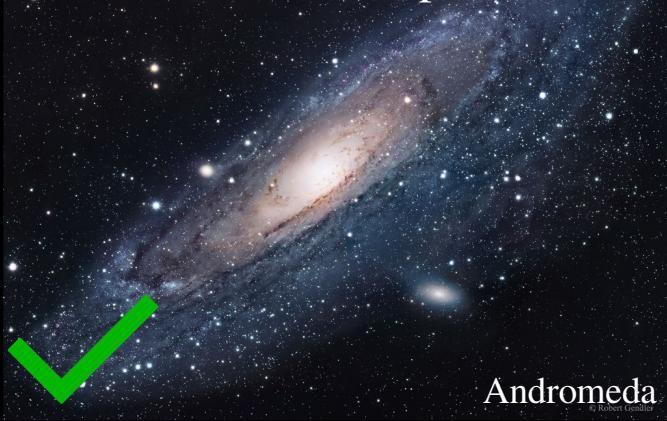
1. Empirical baryon-DM coupling in galaxies  
Three dynamical laws → three acceleration scales
2. In  $\Lambda$ CDM, dynamical laws reproduced a-posteriori  
Theory is reactive → scatter & shape are problematic
3. In MOND, dynamical laws predicted a-priori  
Theory is predictive → EFE may be a definitive test

# More Slides

# Status of MOND at Various Scales

## Galaxy Scales (~1-100 kpc)

Rotation Curves of Spirals



Andromeda  
© Robert Gendler

Dynamics & Lensing in Ellipticals



LRG 3 -757

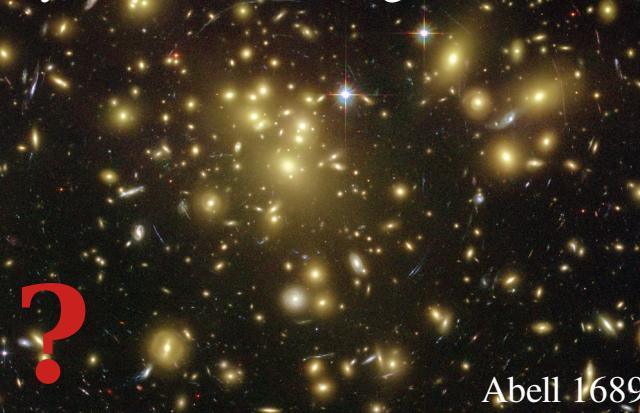
## Groups & Clusters (~1-5 Mpc)

Interactions in Galaxy Groups



Stephan's Quartet

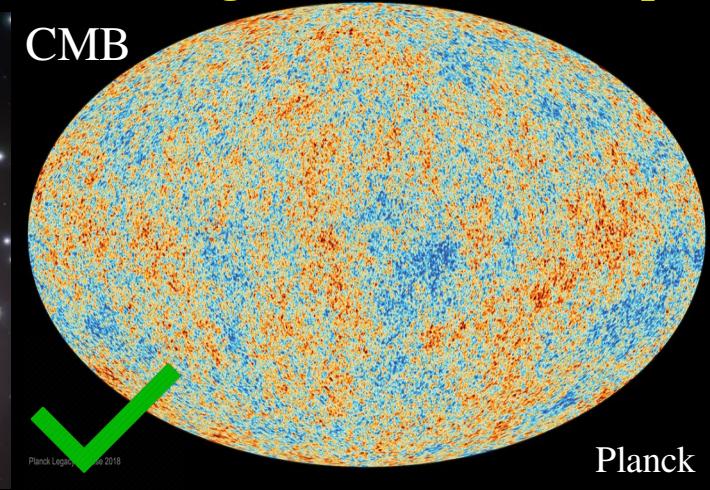
Dynamics & Lensing in Clusters



Abell 1689

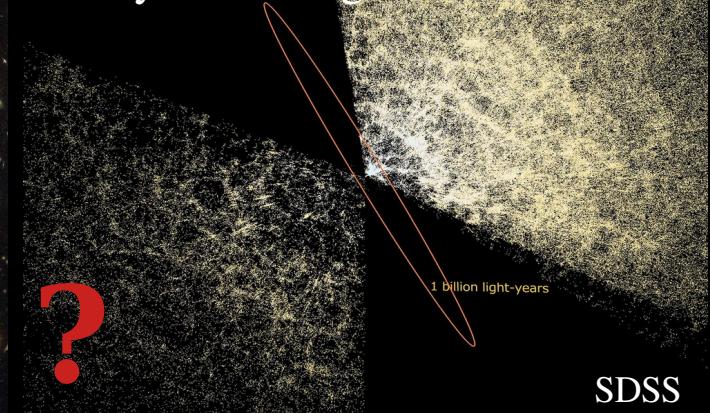
## Cosmological Scales (>100 Mpc)

CMB



Planck  
Legacy Catalog 2018

Galaxy Clustering



1 billion light-years

SDSS

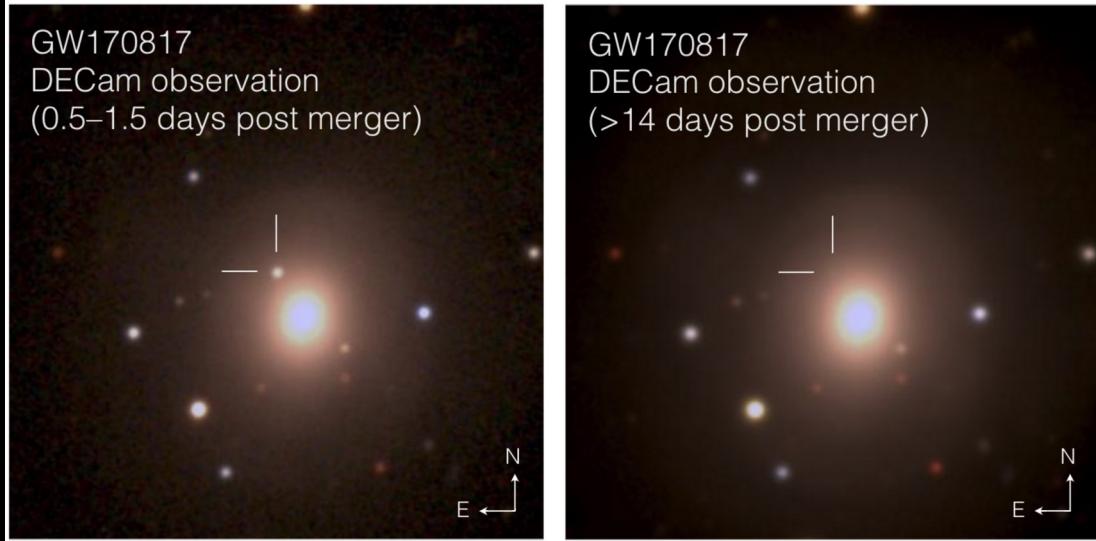
# Relativistic MOND: TeVeS (Bekenstein 2004, PhRvD)

- Tensor  $g_{\mu\nu}$  → Einstein's metric
- Vector  $A^\mu$  → For the “right” gravitational lensing (Sanders 1997, ApJ)
- Scalar  $\Phi$  → For the DM effect on matter (Bekenstein & Milgrom 1984, ApJ)
- Free Function → Interpolation Function (similar to AQUAL & QMOND)

Matter follows a “physical metric” given by a disformal transformation:

$$\tilde{g}_{\mu,\nu} = g_{\mu,\nu} e^{-2\phi} + A_\mu A_\nu e^{-2\phi} - A_\mu A_\nu e^{2\phi} = e^{-2\phi} g_{\mu,\nu} - 2 A_\mu A_\nu \sinh(2\phi)$$

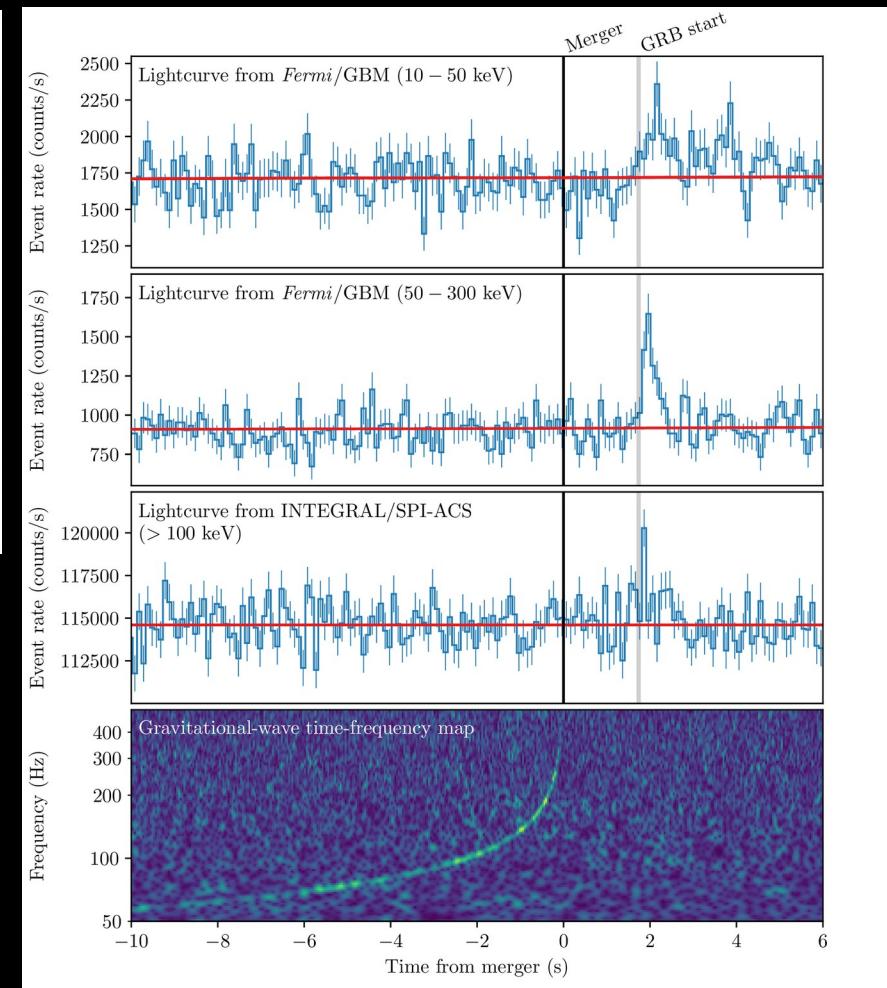
# TeVeS is ruled out by kilonova discovery (GW170817)



Gravitational wave signal immediately followed by gamma-ray signal:

$$|c_{\text{GW}} - c_{\text{EM}}| < 10^{-15} c_{\text{EM}}$$

But TeVeS predicted  $c_{\text{GW}} \neq c_{\text{EM}}$ !

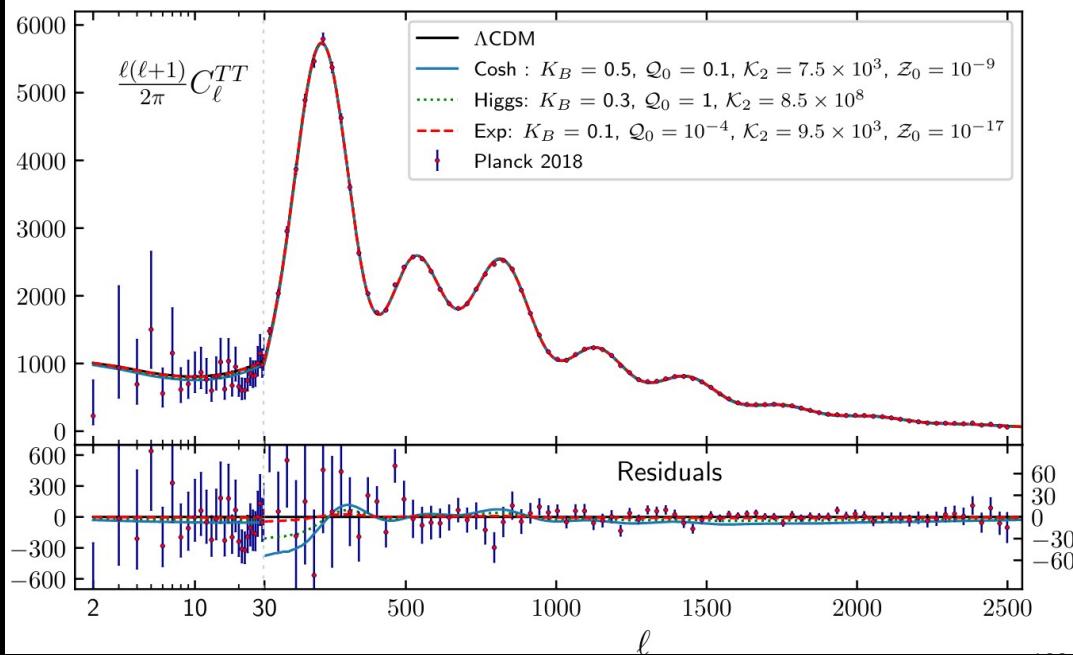


# New TeVeS-like theory (Skordis & Zlosnik 2021, PRL)

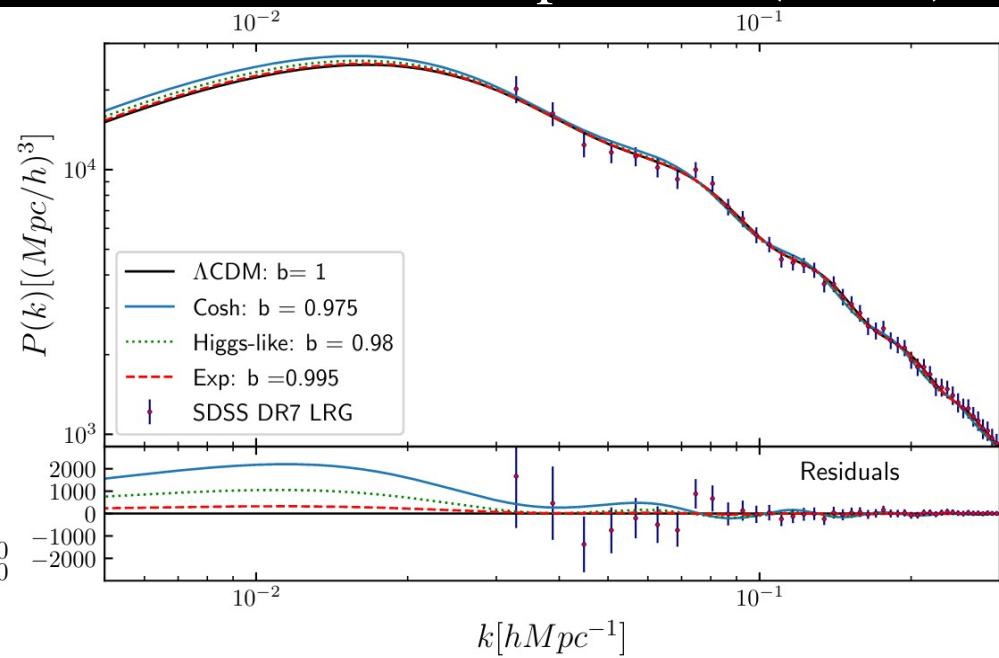
Combine scalar & vector in a time-like vector:

$$B^\mu = e^{-2\phi} A^\mu \quad \text{such that} \quad B^2 = g^{\mu\nu} B_\mu B_\nu = -e^{-2\phi} \quad \Rightarrow \quad c_{\text{GW}} = c_{\text{EM}}$$

CMB Power Spectrum (Planck)

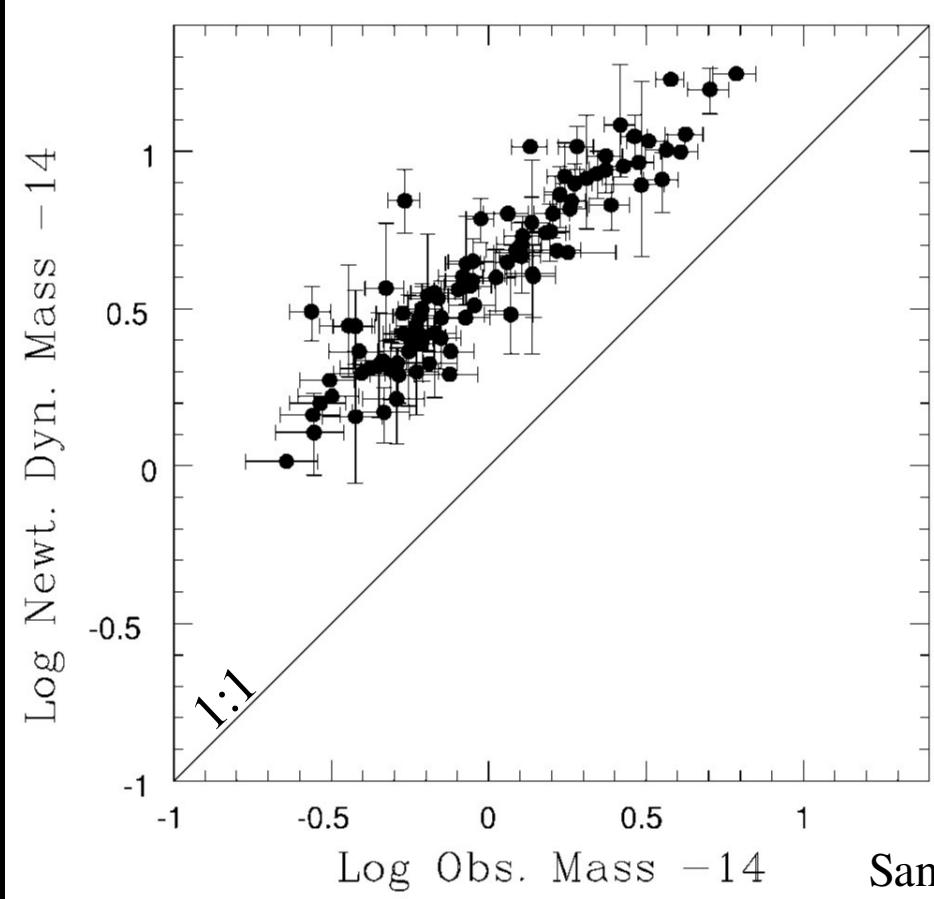


Matter Power Spectrum (SDSS)

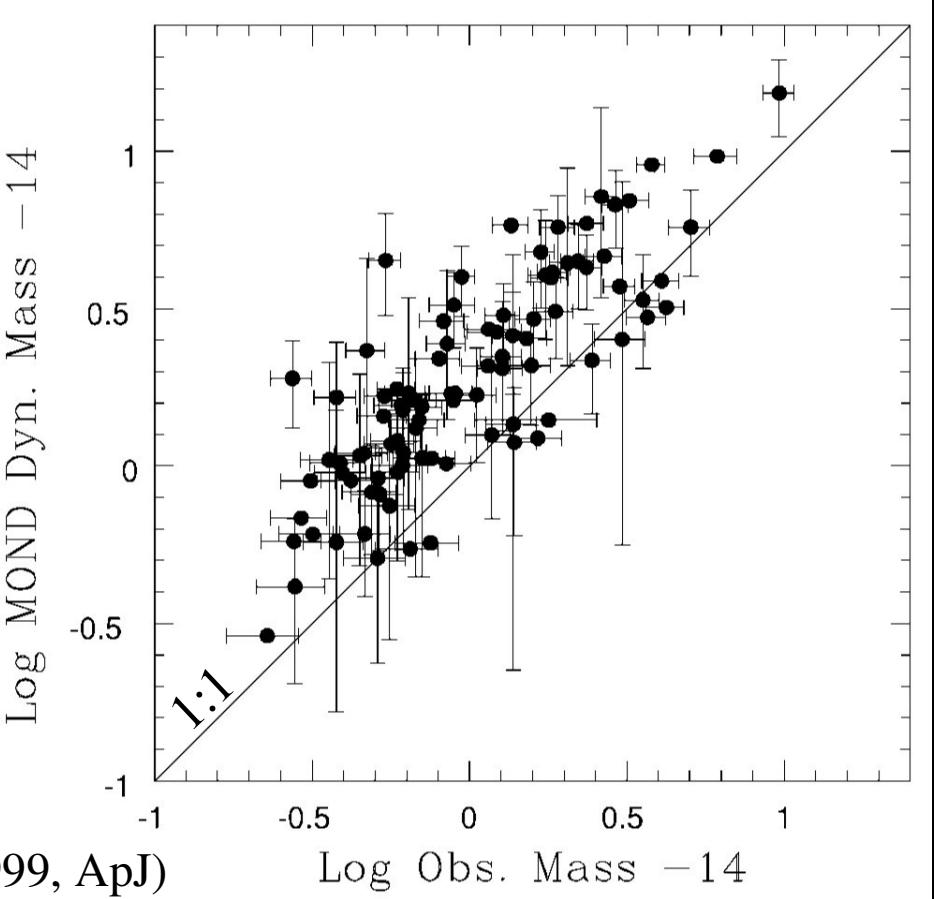


# Galaxy Clusters: Long-standing issue for MOND

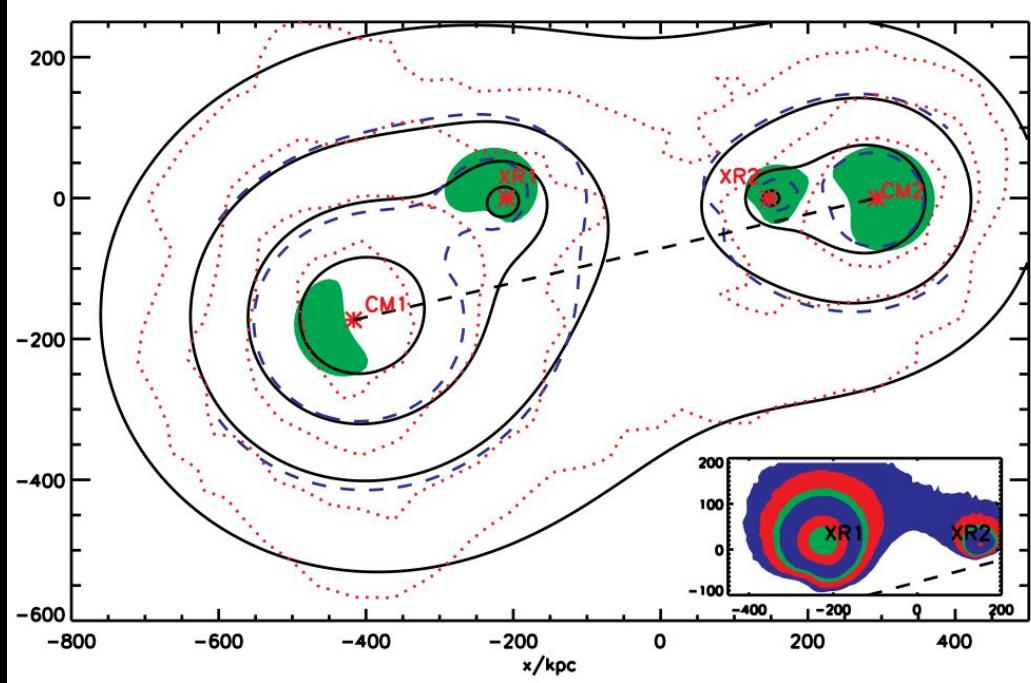
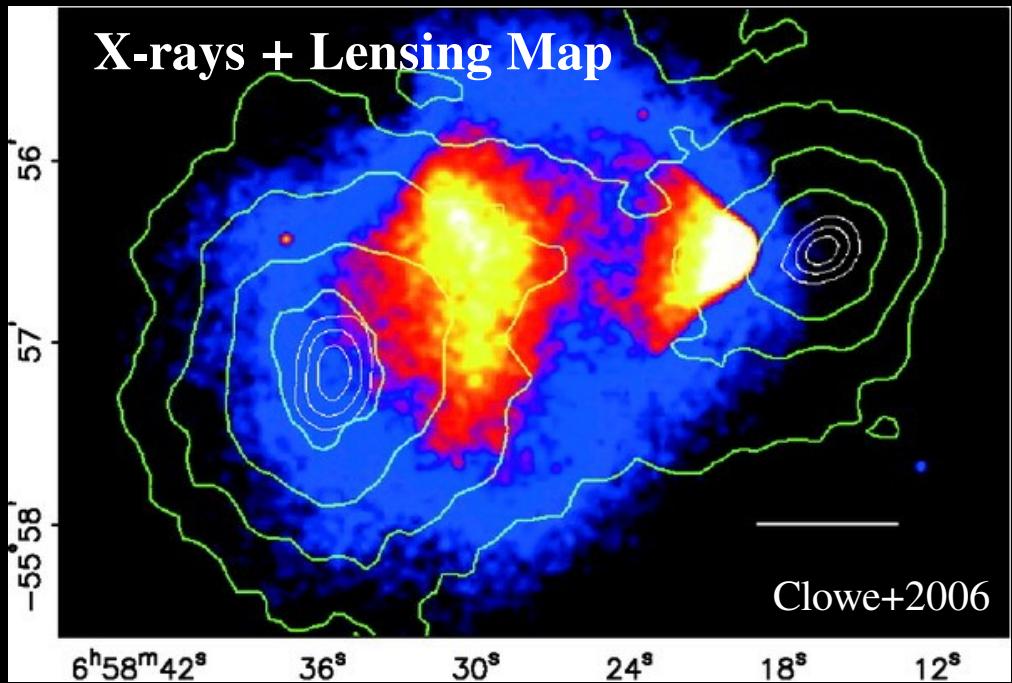
Newtonian analysis:  $M_{\text{dyn}}/M_{\text{bar}} \sim 4-5$



MOND analysis:  $M_{\text{dyn}}/M_{\text{bar}} \sim 2-3$



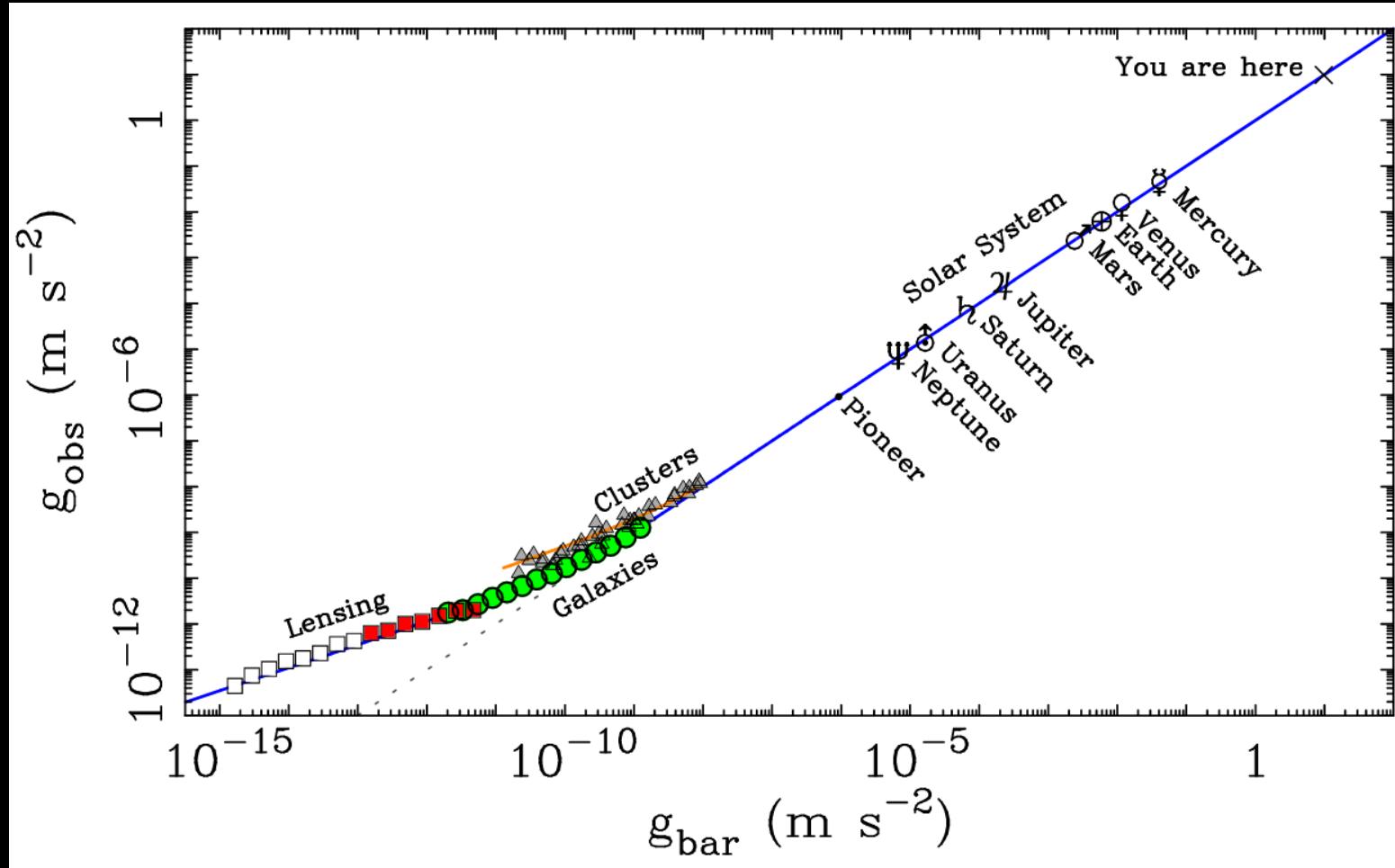
# Bullet Cluster in TeVeS (relativistic MOND)



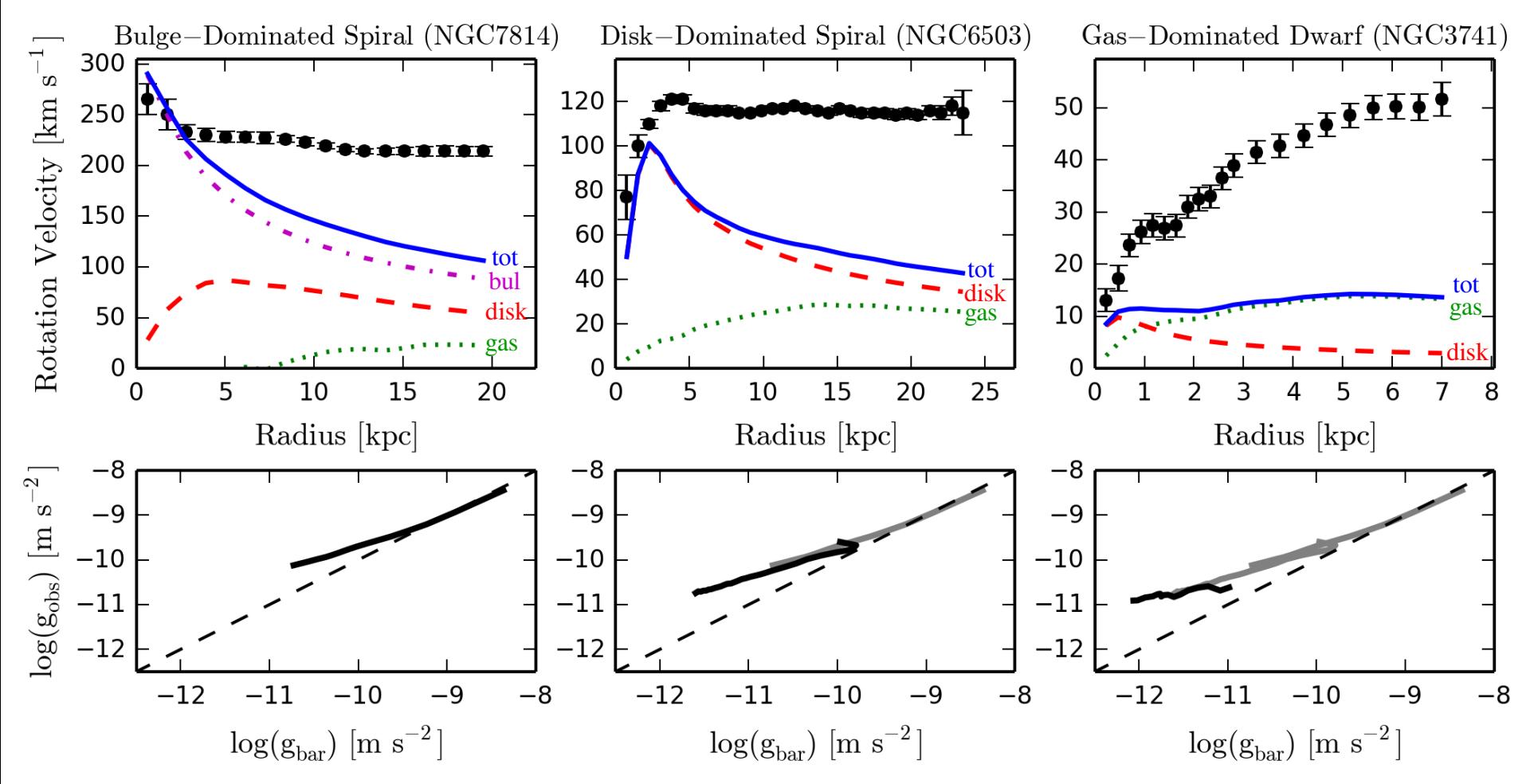
High collision speed ( $\sim 4500$  km/s) is rare in  $\Lambda$ CDM but natural in MOND.  
(Hayashi & White 2006; Farrar & Rosen 2006;  
Angus+2007; Angus & McGaugh 2008)

**MOND model with 2eV  $\nu$**  (Angus+2007):  
**Red:** Observed lensing convergence map  
**Black:** best-fit MOND+ $\nu$  convergence map  
**Blue:** total surface densities (baryons+ $\nu$ ).

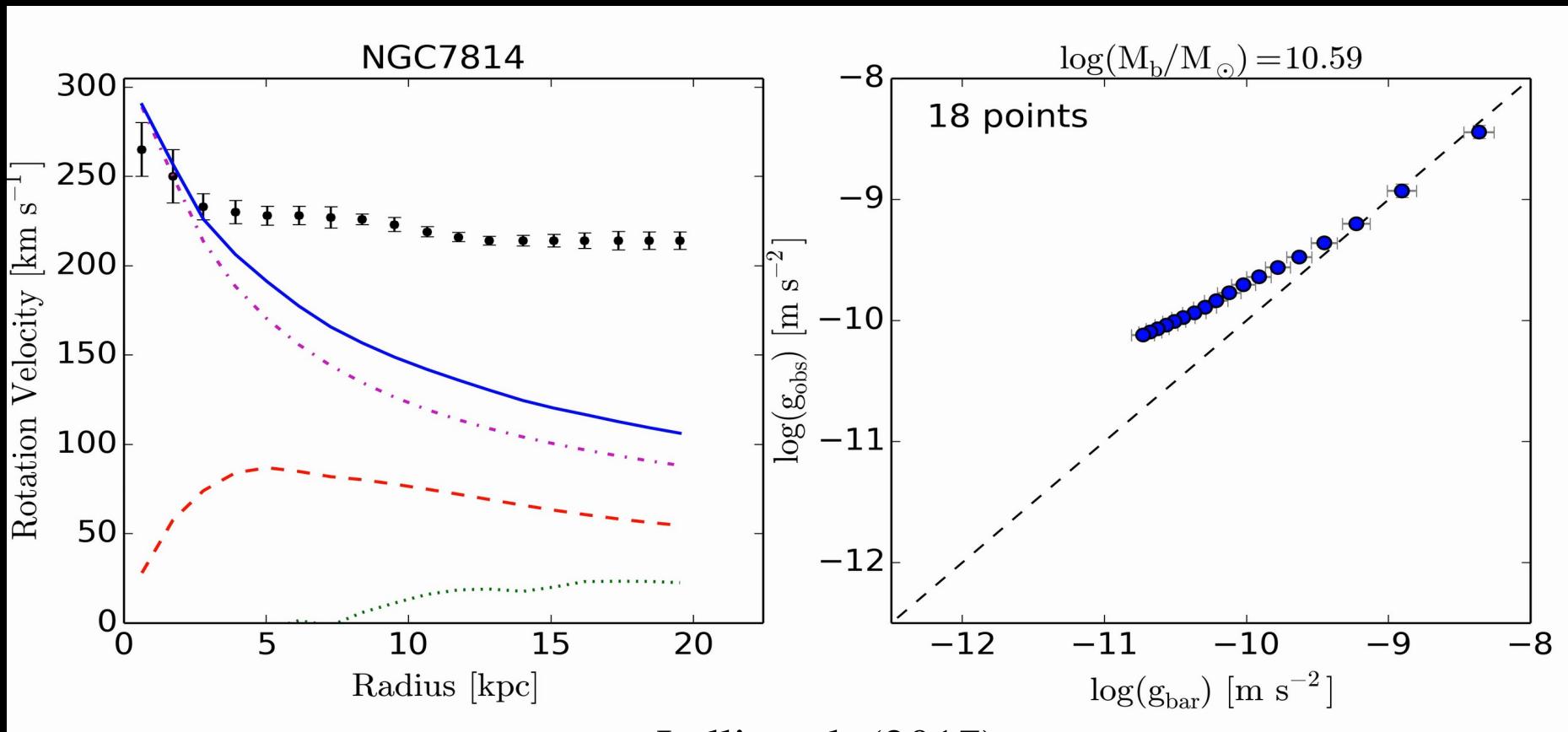
# Putting Galaxies & Galaxy Clusters in Context



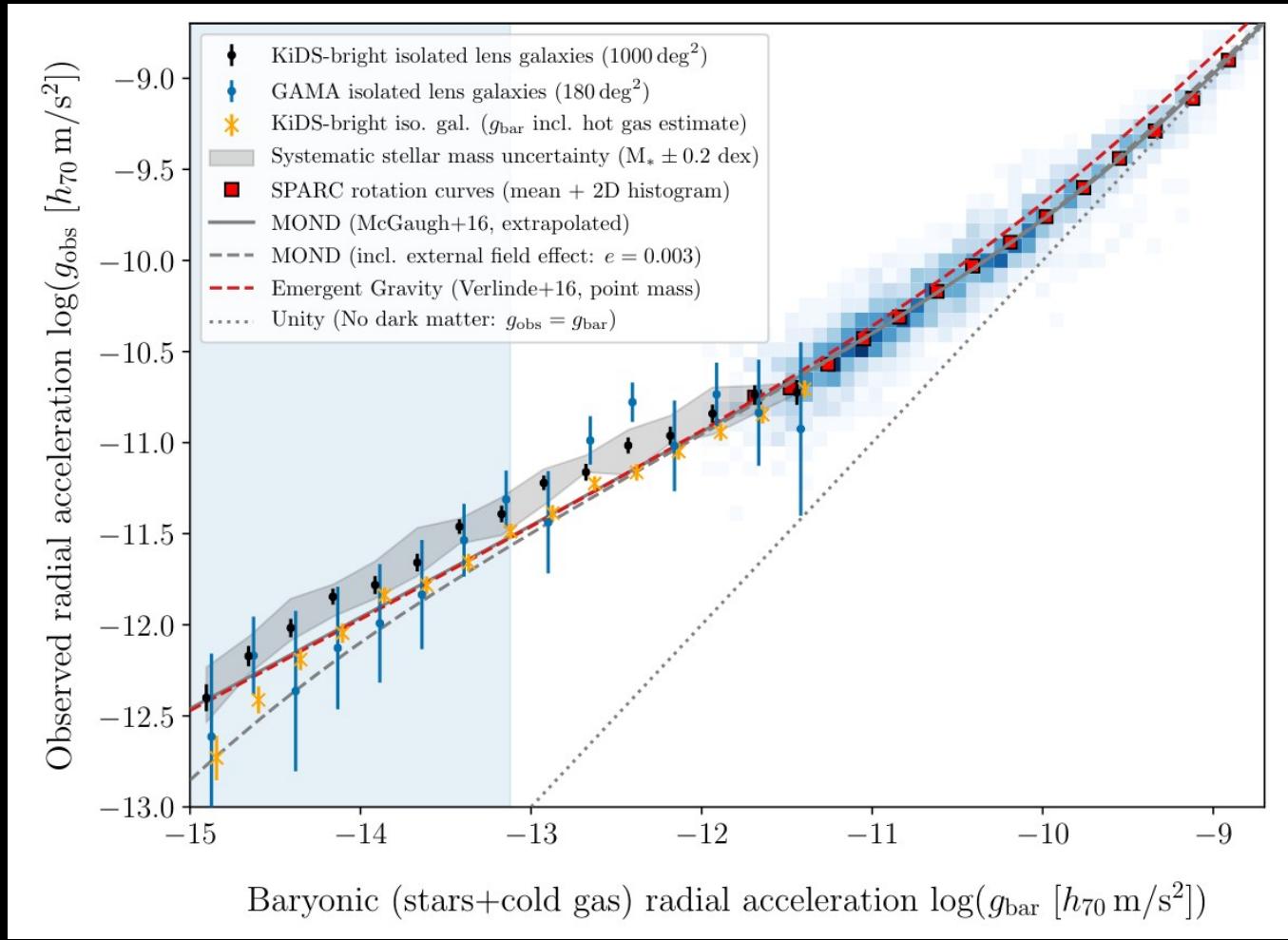
# Galaxies on the same RAR *despite* their diversity



# Building up the RAR (watch video here)



# Extending the RAR with weak lensing!



Brouwer+2021

# QUMOND = Quasi-Linear MOND (Milgrom 2010, MNRAS)

$$S = \int dt L = \int dt d^3x \left( \rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right) \quad \text{Newton: Single potential } \Phi$$
$$\downarrow$$
$$\frac{-1}{8\pi G} \left[ 2 \vec{\nabla} \Phi \cdot \vec{\nabla} \Phi_N - a_0^2 Q \left( \frac{|\vec{\nabla} \Phi_N|^2}{a_0^2} \right) \right] \quad \text{Two potentials: } \Phi \text{ and } \Phi_N !$$

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Newton: Single potential  $\Phi$

Two potentials:  $\Phi$  and  $\Phi_N$ !

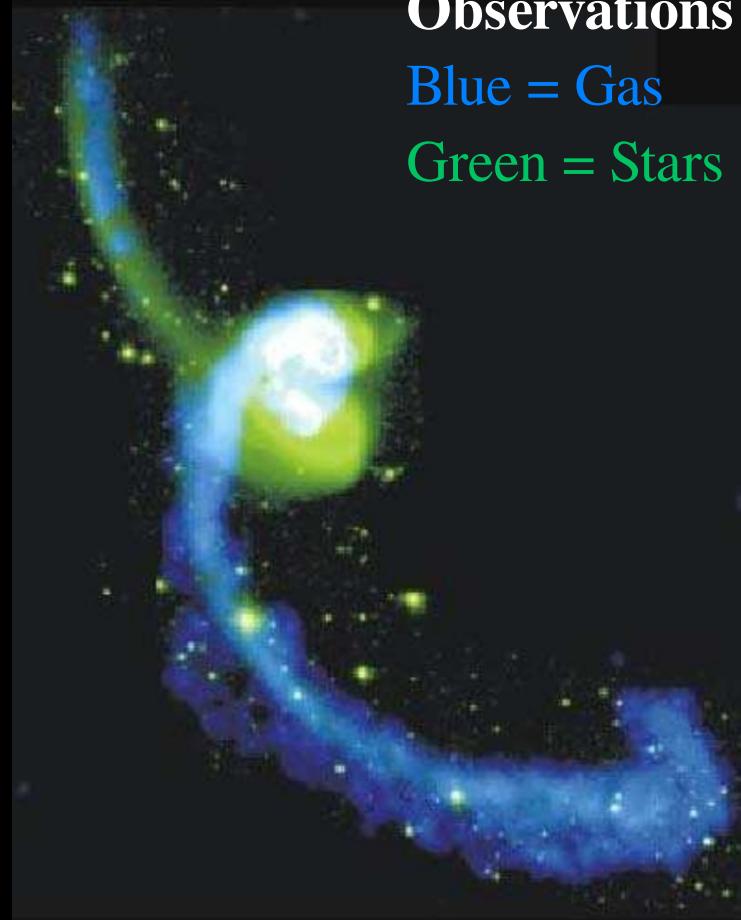
Varying the Action for  $\Phi$ ,  $\Phi_N$  and  $\bar{x}$  → set of 3 equations

$\nabla^2 \Phi_N = 4\pi G \rho$  → Standard, linear Poisson's equation for  $\Phi_N$

$\nabla^2 \Phi = \vec{\nabla} \cdot \left[ v \left( |\vec{\nabla} \Phi_N| / a_0 \right) \vec{\nabla} \Phi_N \right]$  → Non-linear step: get  $\Phi$  from  $\Phi_N$     $v(\sqrt{x}) = \frac{dQ(x)}{x}$

$\vec{a} = -\vec{\nabla} \Phi$  → Acceleration/force set by second potential  $\Phi$

# Application of AQUAL: Antennae Galaxies



Observations

Blue = Gas

Green = Stars

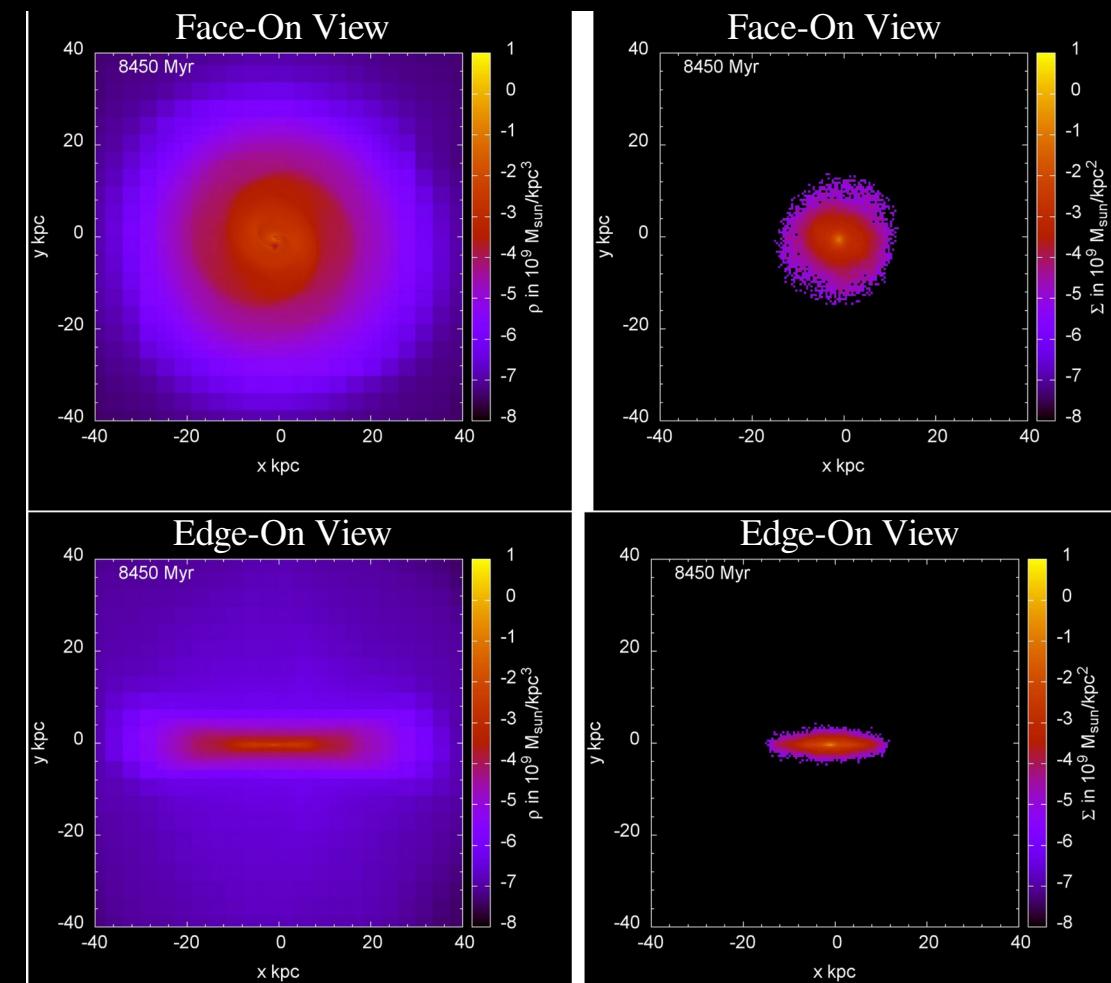
Simulations

Blue = Gas

Red = Stars

Tiret & Combes (2008, ASPC)

# Application of QUMOND: Formation of Galaxy Disks



Gas collapse → Exponential disk

