Dynamical Tension Strings Braneworlds and Lifetime of the homogeneous and isotropic cosmology

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Talk at XXXIV International Workshop on High Energy Physics, FROM QUARKS TO GALAXIES, ELUCIDATING DARK SIDES, NOVEMBER 22, 2022

Instead of, or alongside with, $\sqrt{-g}$ we can employ one or several different alternative *non-Riemannian* volume elements as in (1) given by non-singular *exact* D-forms $\omega^{(j)} = dB^{(j)}$ where:

$$B^{(j)} = \frac{1}{(D-1)!} B^{(j)}_{\mu_1 \ldots \mu_{D-1}} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_{-1}}$$

$$\longrightarrow \quad \Omega^{(j)} \equiv \Phi(B^{(j)}) = \frac{1}{(D-1)!} \varepsilon^{\mu_1 \ldots \mu_D} \partial_{\mu_1} B^{(j)}_{\mu_2 \ldots \mu_D}$$

One way to define a metric independent measure (as opposed to $\sqrt{-g}$) , a density, is

by means of four scalar fields φ_a $(a = 1, 2, 3, 4)$

$$\Phi = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_\mu\varphi_a\partial_\nu\varphi_b\partial_\alpha\varphi_c\partial_\beta\varphi_d.$$

- There is an invariant volume element, $\Phi\, d^D x$
- Φ is a density, transforms according to the inverse of the jacobian of a coordinate transformation, while $d^D x$ transforms according to the jacobian

In this contex it is interesting to mention the work of Pirogov, where the use of 4 scalars in gravity theories have been studied from a more general point of view, not just for use in the measure-

- See, Quartet metric gravity and Dark components of the Univrse, arXiv 1712.00612

We use these fields only in the measure however, both in Gravity and for String or Branes

- We have used this these measure in the past to construct modified theories of gravity

The generic form of modified gravity actions involving (one or more) non-Riemannian volume-elements, called for short *NRVF (Non-Riemannian Volume-Form)* actions, read (henceforth $D = 4$, and we will use units

2 The Modified Measure Theory String Theory

The standard world sheet string sigma-model action using a world sheet metric is [21], [22], [23]

$$S_{sigma-model} = -T \int d^2\sigma \frac{1}{2} \sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \tag{1}$$

Here γ^{ab} is the intrinsic Riemannian metric on the 2-dimensional string worldsheet and $\gamma = det(\gamma_{ab})$; $g_{\mu\nu}$ denotes the Riemannian metric on the embedding spacetime. T is a string tension, a dimension full scale introduced into the theory by hand.

From the variations of the action with respect to γ^{ab} and X^μ we get the following equations of motion:

$$T_{ab} = (\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu) g_{\mu\nu} = 0, \tag{2}$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0, \tag{3}$$

where $\Gamma^\mu_{\nu\lambda}$ is the affine connection for the external metric.

There are no limitations on employing any other measure of integration different than $\sqrt{-\gamma}$. The only restriction is that it must be a density under arbitrary diffeomorphisms (reparametrizations) on the underlying spacetime manifold. The modified-measure theory is an example of such a theory.

In the framework of this theory two additional worldsheet scalar fields $\varphi^i(i=1,2)$ are introduced. A new measure density is

$$\Phi(\varphi)=\frac{1}{2}\epsilon_{ij}\epsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j. \tag{4}$$

Then the modified bosonic string action is (as formulated first in [10] and latter discussed and generalized also in [11])

$$S=-\int d^2\sigma\Phi(\varphi)(\frac{1}{2}\gamma^{ab}\partial_aX^\mu\partial_bX^\nu g_{\mu\nu}-\frac{\epsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)), \tag{5}$$

where F_{ab} is the field-strength of an auxiliary Abelian gauge field A_a: $F_{ab}=\partial_aA_b-\partial_bA_a$.

It is important to notice that the action (5) is invariant under conformal transformations of the intrinsic measure combined with a diffeomorphism of the measure fields,

$$\gamma_{ab}\to J\gamma_{ab}, \tag{6}$$

$$\varphi^i\to\varphi'^i=\varphi'^i(\varphi^i) \tag{7}$$

such that

$$\Phi\to\Phi'=J\Phi \tag{8}$$

Here J is the jacobian of the diffeomorphim in the internal measure fields which can be an arbitrary function of the world sheet space time coordinates, so this can called indeed a local conformal symmetry.

To check that the new action is consistent with the sigma-model one, let us derive the equations of motion of the action (5).

The variation with respect to φ^i leads to the following equations of motion:

$$\epsilon^{ab}\partial_b\varphi^i\partial_a(\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}) = 0. \tag{9}$$

It implies

$$\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = M = const. \tag{10}$$

The equations of motion with respect to γ^{ab} are

$$T_{ab} = \partial_a X^\mu\partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = 0. \tag{11}$$

We see that these equations are the same as in the sigma-model formulation (2), (3). Namely, taking the trace of (11) we get that $M = 0$. By solving $\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}$ from (24) (with $M = 0$) we obtain (2).

A most significant result is obtained by varying the action with respect to A_a:

$$\epsilon^{ab}\partial_b(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}) = 0. \tag{12}$$

Then by integrating and comparing it with the standard action it is seen that

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T. \tag{13}$$

That is how the string tension T is derived as a world sheet constant of integration opposite to the standard equation (1) where the tension is put ad hoc.The variation with respect to X^μ leads to the second sigma-model-type equation (3). The idea of modifying the measure of integration proved itself effective and profitable. This can be generalized to incorporate super symmetry, see for example [11], [13], [12] , [14]. For other mechanisms for dynamical string tension generation from added string world sheet fields, see for example [17] and [18]. However the fact that this string tension generation is a world sheet effect and not a universal uniform string tension generation effect for all strings has not been sufficiently emphasized before. Now we go and review the Modified Measure Brane Theory

3 The Modified Measure Brane Theory

4 Each String and Each Brane in its own world sheet determines its own tension. Therefore the tension is not universal for all strings or branes

If we look at a single string, the dynamical string tension theories and the standard string theories appear indeed indistinguishable, there are however more than one string and/or one brane in the universe then, let us now observe indeed that it does not appear that the string tension or the brane tension derived in the sections above correspond to "the" string or brane tensions of the theory. The derivation of the string or brane tensions in the previous sections holds for a given string or brane, there is no obstacle that for another string or brane these could acquire a different string or brane tension. In other words, the string or brane tension is a world sheet constant, but it does not appear to be a universal constant same for all strings and for all branes. Similar situation takes place in the dynamical string generation proposed by Townsend for example [17], in that paper worldsheet fields include an electromagnetic gauge potential. Its equations of motion are those of the Green-Schwarz superstring but with the string tension given by the circulation of the worldsheet electric field around the string. So again ,in [17] also a string will determine a given tension, but another string may determine another tension. If the tension is a universal constant valid for all strings, that would require an explanation in the context of these dynamical tension string theories, for example some kind of interactions that tend to equalize string tensions, or that all strings in the universe originated from the splittings of one primordial string or some other mechanism.

In any case, if one believes for example in strings , on the light of the dynamical string tension mechanism being a process that takes place at each string independently, we must ask whether all strings have the same string tension.

5 Equations for the Background fields and a new background field

However, in addition to the traditional background fields usually considered in conventional string theory, one may consider as well an additional scalar field that induces currents in the string world sheet and since the current couples to the world sheet gauge fields, this produces a dynamical tension controlled by the external scalar field as shown at the classical level in [25]. In the next two subsections we will study how this comes about in two steps, first we introduce world sheet currents that couple to the internal gauge fields in Strings and Branes and second we define a coupling to an external scalar field by defining a world sheet currents that couple to the internal gauge fields in Strings and Branes that is induced by such external scalar field. This is very much in accordance to the philosophy of Schwinger [28] that proposed long time ago that a field theory must be understood by probing it with external sources.

As we will see however, there will be a fundamental difference between this background field and the more conventional ones (the metric, the dilaton field and the two index anti symmetric tensor field) which are identified with some string excitations as well. Instead, here we will see that a single string does not provide dynamics for this field, but rather when the condition for world sheet conformal invariance is implemented for two strings which sample the same region of space time, so it represents a collective effect instead.

3.1 Introducing world sheet currents that couple to the internal gauge fields

If to the action of the string we add a coupling to a world-sheet current j^a, i.e. a term

$$S_{\text{current}} = \int d^{p+1}\sigma A_a j^a, \tag{12}$$

then the variation of the total action with respect to A_a gives

$$\epsilon^{ab}\partial_a\left(\frac{\Phi}{\sqrt{-\gamma}}\right) = j^b. \tag{13}$$

We thus see indeed that, in this case, the dynamical character of the brane is crucial here.

3.2 How a world sheet current can naturally be induced by a bulk scalar field, the Tension Field

Suppose that we have an external scalar field $\phi(x^\mu)$ defined in the bulk. From this field we can define the induced conserved world-sheet current

$$j^b = e\partial_\mu\phi\frac{\partial X^\mu}{\partial\sigma^a}\epsilon^{ab} \equiv e\partial_a\phi\epsilon^{ab}, \tag{14}$$

where e is some coupling constant. The interaction of this current with the world sheet gauge field is also invariant under local gauge transformations in the world sheet of the gauge fields $A_a \to A_a + \partial_a\lambda$.

For this case, (13) can be integrated to obtain

$$T = \frac{\Phi}{\sqrt{-\gamma}} = e\phi + T_i, \tag{15}$$

or equivalently

$$\Phi = \sqrt{-\gamma}(e\phi + T_i), \tag{16}$$

The constant of integration T_i may vary from one string to the other. Notice tha the interaction is metric independent since the internal gauge field does not transform under the the conformal transformations. This interaction does not therefore spoil the world sheet conformal transformation invariance in the case the field ϕ does not transform under this transformation. One may interpret (16) as the result of integrating out classically (through integration of equations of motion) or quantum mechanically (by functional integration of the internal gauge field, respecting the boundary condition that characterizes the constant of integration T_i for a given string). Then replacing $\Phi = \sqrt{-\gamma}(e\phi + T_i)$ back into the remaining terms in the action gives a correct effective action for each string. Each string is going to be quantized with each one having a different T_i. The consequences of an independent quantization of many strings with different T_i covering the same region of space time will be studied in the next section.

As we discussed in the previous section, we can incorporate the result of the tension as a function of scalar field ϕ, given as $e\phi + T_i$, for a string with the constant of integration T_i by defining the action that produces the correct equations of motion for such string, adding also other background fields, the anti symmetric two index field $A_{\mu\nu}$ that couples to $\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu$ and the dilaton field φ that couples to the topological density $\sqrt{-\gamma}R$

$$S_i = -\int d^2\sigma (e\phi + T_i)\frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + \int d^2\sigma A_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu + \int d^2\sigma \sqrt{-\gamma}\varphi R. \tag{34}$$

Notice that if we had just one string, or if all strings will have the same constant of integration $T_i = T_0$.

In any case, it is not our purpose here to do a full generic analysis of all possible background metrics, antisymmetric two index tensor field and dilaton fields, instead, we will take cases where the dilaton field is a constant or zero, and the antisymmetric two index tensor field is pure gauge or zero, then the demand of conformal invariance for $D = 26$ becomes the demand that all the metrics

$$g^i_{\mu\nu} = (e\phi + T_i)g_{\mu\nu} \tag{35}$$

will satisfy simultaneously the vacuum Einstein´s equations,

5.3.1 The case where all all string tensions are the same, i.e., $T_i = T_0$, and the appearance of a target space conformal invariance

If all $T_i = T_0$, we just redefine our background field so that $e\phi + T_0 \to e\phi$ and then in the effective action for all the strings the same combination $e\phi g_{\mu\nu}$, and only this combination will be determined by the requirement that the conformal invariance in the world sheet of all strings be preserved quantum mechanically, that is , that the beta function be zero. So in this case we will not be able to determine $e\phi$ and $g_{\mu\nu}$ separately, just the product $e\phi g_{\mu\nu}$, so the equation obtained from equating the beta function to zero will have the target space conformal invariance $e\phi \to F(x)e\phi$, $g_{\mu\nu} \to F(x)^{-1} g_{\mu\nu}$.

That is, there is no independent dynamics for the Tension Field in this case. So in conclusion, if we just look at one string, or if we look at a set of strings, all of them equal string tensions, then the tension field is not observable, can be gauged away by the target space conformal invariance explained above. Another way to see this is

5.3.2 The case where not all string tensions are the same, with special emphasis of two types of strings with $T_1 \neq T_2$,

The interesting case to consider is therefore many strings with different T_i, let us consider the simplest case of two strings, labeled 1 and 2 with $T_1 \neq T_2$, then we will have two Einstein´s equations, for $g^1_{\mu\nu} = (e\phi + T_1)g_{\mu\nu}$ and for $g^2_{\mu\nu} = (e\phi + T_2)g_{\mu\nu}$,

$$R_{\mu\nu}(g^1_{\alpha\beta}) = 0 \tag{36}$$

and , at the same time,

$$R_{\mu\nu}(g^2_{\alpha\beta}) = 0 \tag{37}$$

These two simultaneous conditions above impose a constraint on the tension field ϕ, because the metrics $g^1_{\alpha\beta}$ and $g^2_{\alpha\beta}$ are conformally related, but Einstein´s equations are not conformally invariant, so the condition that Einstein´s equations hold for both $g^1_{\alpha\beta}$ and $g^2_{\alpha\beta}$ is highly non trivial.

WE GET TWO CONFORMALLY RELATED METRICS, BOTH OF WHICH OBEY EINSTEIN EQ

Conformal factor, call it c can
be constant or may not
be constant,
There are many situations
where multiplying a solution
of Einstein´s equation
By a constant c gives
us another solution, like in
the case of Schwarzschild
solution and Kasner solution

Then for these situations, we have,

$$e\phi + T_1 = c(e\phi + T_2) \tag{40}$$

which leads to a solution for $e\phi$

$$e\phi = \frac{cT_2 - T_1}{1 - c} \tag{41}$$

which leads to the tensions of the different strings to be

$$e\phi + T_1 = \frac{c(T_2 - T_1)}{1 - c} \tag{42}$$

and

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - c} \tag{43}$$

It is important that we were force to consider a multi metric situation. One must also realize that the constant c is physical, because both metrics live in the same spacetime, so even if c is a constant , we are not allowed to perform a coordinate transformation, consisting for example of a rescaling of coordinates for one of the metrics and not do the same transformation for the other metric.

Other way to see that c is physical consist of considering the scalar consisting of the ratio of the two measures $\sqrt{-g^1}$ and $\sqrt{-g^2}$ where $g^1 = det(g^1_{\alpha\beta})$ and $g^2 = det(g^2_{\alpha\beta})$, and we find that the scalar $\frac{\sqrt{-g^1}}{\sqrt{-g^2}} = c^{D/2}$, showing that c is a coordinate invariant.

THIS IS BECAUSE The two flat spaces considered CANNOT BE TRANSFORMED SIMULTANEOUSLY TO MINKOWSKI SPACE in standard coordinates (meaning the metric with just diagonal elements (-1, 1, 1, 1, …..)). Out of these two flat space one can also construct true tensors that do not vanish in any coordinate frame, like the difference of the Levi Civita connections of the two metrics, etc .

Let us study now a case where c is not a constant, we will also focus on a cosmological case. To find this it is useful to consider flat space in the Milne representation, $D = 4$ this reads,

$$ds^2 = -dt^2 + t^2(d\chi^2 + sinh^2\chi d\Omega_2^2) \tag{44}$$

where $d\Omega_2^2$ represent the contribution of the 2 angles to the metric when using spherical coordinates, that is, it represents the metric of a two dimensional sphere of unit radius. In D dimensions we will have a similar expression but now we must introduce the metric of a $D-2$ unit sphere $d\Omega_{D-2}^2$ so we end up with the following metric that we will take as the metric 2

$$ds_2^2 = -dt^2 + t^2(d\chi^2 + sinh^2\chi d\Omega_{D-2}^2) \tag{45}$$

For the metric 1 we will take the metric that we would obtain from the coordinate $t \to 1/t$ and we furthermore multiply by a constant σ, so

$$ds_1^2 = \frac{\sigma}{t^4}(-dt^2 + t^2(d\chi^2 + sinh^2\chi d\Omega_{D-2}^2)) \tag{46}$$

Then the equations (40), (41), (42), (43), with $c = \frac{\sigma}{t^4}$. If we want that the only possible singularities take place at $t = 0$ only, we must take σ negative, and therefore the two strings have string tensions of opposite signs.

Whether strings 1 or 2 are the ones with negative tensions depends on the sign of $T_2 - T_1$. If we want the strings with negative tension to exist only in the early universe, we must take $T_2 - T_1$ to be negative. At the same time there will not be positive tension strings in the early universe, but in the late universe approaches a constant value. The positive string tension are the strings 1, with zero tension in the early universe and the tension $T_1 - T_2$ in the late universe. The negative string tension are the strings 2, with $T_1 - T_2$ tension in the early universe and the tension zero tension in the late universe.

5.4 The Non Singular Bouncing Solution for the Universal Metric

As we have seen when the space time is probed by two types of strings, there are two metrics that have to satisfy the vacuum Einstein´s equations, this is enough to solve the problem, The interesting thing however is that the universal metric $g_{\mu\nu}$ does not have to satisfy Einstein´s equation. We can see this by solving $g_{\mu\nu}$ in terms of one of the metrics , for example from $g^2_{\mu\nu} = (e\phi + T_2)g_{\mu\nu}$, we have that

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (\frac{1-c}{T_2 - T_1})(-dt^2 + t^2(d\chi^2 + sinh^2\chi d\Omega^2_{D-2})) \tag{47}$$

and considering that $c = \frac{\sigma}{t^4} = \frac{-K}{t^4}$, where K is positive. So the coefficient of the hyperbolic D -1 dimensional metric $d\chi^2 + sinh^2\chi d\Omega^2_{D-2}$ is $\frac{t^2 + \frac{-K}{t^2}}{T_2 - T_1}$, showing a contraction, a bounce and a subsequent expansion. The initial and final spacetimes are flat and satisfy vacuum Einstein´s equations, but not the full space, with most appreciable deviations from Einstein´s equations at the bouncing time, $t = t* = K^{1/4}$.

One can bring this metric in terms of cosmic time, where the 00 of metric component of $g_{\mu\nu}$ is normalized to 1. One has to notice however that we cannot do the transformation of coordinates only on one of the three metrics we have discussed $g^2_{\mu\nu}, g^1_{\mu\nu}$ and $g_{\mu\nu}$, and if we bring the the 00 metric component of the metric $g_{\mu\nu}$ is normalized to 1, it will not happen simultaneously for $g^2_{\mu\nu}, g^1_{\mu\nu}$. Having this in mind, the cosmic time coordinate T where where the 00 of metric component of $g_{\mu\nu}$ is normalized to 1 is defined by

$$dT = \sqrt{\frac{1 + \frac{K}{t^4}}{T_1 - T_2}}dt \tag{48}$$

So, we see that as $t \to \infty$, $T \to c_1 t$, while for $t \to 0$, $T \to -c_2/t$, here c_1, c_2 are positive constants. So at large negative cosmic time we have a contacting Milne space, a bounce and the evolution towards an expanding Milne space at large positive cosmic time.

5.5 The case where positive and and negative string tensions are separated by a spontaneously generated wall in Wesson warped spaces

One may wonder if there are similar solutions to the vacuum Einstein´s equations similar to the Milne space but where instead of time some spacial coordinate would play a similar way. The answer to this question is yes, and these are the solutions in higher dimensional vacuum General Relativity discovered by Wesson and collaborators, see [29] and references there. In five dimensions for example the following warped solution

is found,

$$ds^2 = l^2 dt^2 - l^2 cosh^2 t(\frac{dr^2}{1-r^2} + r^2 d\Omega_2^2)) - dl^2 \tag{49}$$

where l is the fourth dimension, so we see that as in the fourth dimension l such a solution is homogeneous of degree two, just as the Milne space time was homogeneous of degree two with respect to the time. Notice that maximally symmetric de Sitter space times sub spaces $l =$ constant appear for instead of euclidean spheres that appear in the Milne Universe for $t =$ constant.

The list of space times of this type is quite large, for example, one cal find solutions of empty GR with Schwarzchild de Sitter subpaces for $l =$ constant, as in

$$ds^2 = \frac{\Lambda l^2}{3}(dt^2(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}) - \frac{dr^2}{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}} - r^2 d\Omega_2^2) - dl^2 \tag{50}$$

This of course can be extended to D dimensions, where we choose one dimension l to have a factor l^2 warp factor for the other dimensions , generically for D dimensions as in

$$ds_2^2 = l^2 \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu - dl^2 \tag{51}$$

where $\bar{g}_{\mu\nu}(x)$ is a $D-1$ Schwarzschild de Sitter metric for example [29]. This we will take as our 2 metric, In any case, working with this generic metric of the form (51), but now in D dimensions, we can perform the inversion transformation $l \to \frac{1}{l}$, and multiplying also by a factor σ and obtain the conformally transformed metric 1 that also satisfies the vacuum Einstein´s equations

$$ds_1^2 = \sigma l^{-2}\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu - \sigma\frac{dl^2}{l^4} = \sigma l^{-4}ds_2^2 \tag{52}$$

From this point on , the equations the solutions fot the tensions of the 1 and 2 strings are the same as in the cosmological case, just that $t \to l$, so now $c = \sigma l^{-4}$, so that we now insert this expression for c in (42) and in (43), obtaining that on one direction in l negative string tensions dominate, while in the other direction positive string tensions dominate, and we still take $\sigma = -K$, where K is positive.

The universal metric, following the steps done for the cosmological case is now ,

$$ds^2 = (\frac{1-c}{T_2 - T_1})(l^2\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu - dl^2) \tag{53}$$

looking at the coefficient of $\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu$, the function is $(\frac{l^2+K/l^2}{T_2-T_1})$, so the space time is expanded or contracted as we move in the dimension l by this factor. This factor is minimized at $l* = K^{1/4}$- We can define a proper length coordinate L where where the ll of metric component of the metric is normalized to -1 is defined by

$$dL = \sqrt{\frac{1+\frac{K}{l^4}}{T_1 - T_2}}dl \tag{54}$$

So, we see that as $l \to \infty$, $L \to c_1 l$, while for $l \to 0$, $L \to -c_2/l$, here c_1, c_2 are positive constants.

Finally we can compare the resulting gravity theories resulting from these multi string effects with known gravity theories discussed in the literature. We have found

For two strings tensions with positive tensions. Avoidance of Hagedorn temperature and Braneworlds!, first go back to the 2 metrics

$$ds_2^2 = -dt^2 + t^2(d\chi^2 + sinh^2\chi d\Omega_{D-2}^2) \tag{26}$$

For the metric 1 we will take the metric that we would obtain from the coordinate $t \to 1/t$ (using Minkowskii coordinates x^μ, this corresponds to the inversion transformation, for a review and generalizations see [30]. $x^\mu \to x^\mu/(x^\nu x_\nu)$) and then we furthermore multiply by a constant σ, so

$$ds_1^2 = \frac{\sigma}{t^4}(-dt^2 + t^2(d\chi^2 + sinh^2\chi d\Omega_{D-2}^2)) \tag{27}$$

Now both tensions are positive,

$$e\phi + T_1 = \Omega^2(e\phi + T_2) \tag{21}$$

which leads to a solution for $e\phi$

$$e\phi = \frac{\Omega^2 T_2 - T_1}{1 - \Omega^2} \tag{22}$$

which leads to the tensions of the different strings to be

$$e\phi + T_1 = \frac{\Omega^2(T_2 - T_1)}{1 - \Omega^2} \tag{23}$$

and

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - \Omega^2} \tag{24}$$

Both tensions can be taken as positive if $T_2 - T_1$ is positive and Ω^2 is also positive

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (\frac{1-\Omega^2}{T_2 - T_1})(-dt^2 + t^2(d\chi^2 + sinh^2\chi d\Omega^2_{D-2})) \tag{28}$$

and considering that $\Omega^2 = \frac{\sigma}{t^4}$, where σ is positive. So the coefficient of the hyperbolic D -1 dimensional metric $d\chi^2 + sinh^2\chi d\Omega^2_{D-2}$ is $\frac{t^2 - \frac{\sigma}{t^2}}{T_2 - T_1}$, showing a collapse at $t = t^* = (\sigma)^{1/4}$.

We can expand the scale factor around $t = t^* = (\sigma)^{1/4}$, defining $t = t^* + \bar{t}$. The result is, just keeping the first linear term in $\bar{t}$,

$$\frac{t^2 - \frac{\sigma}{t^2}}{T_2 - T_1} = \frac{4(\sigma)^{1/4}}{T_2 - T_1}\bar{t} \tag{29}$$

One can bring this metric in terms of cosmic time, where the 00 of metric component of $g_{\mu\nu}$ is normalized to 1. One has to notice however that we cannot do the transformation of coordinates only on one of the three metrics we have discussed $g^2_{\mu\nu}, g^1_{\mu\nu}$ and $g_{\mu\nu}$, and if we bring the the 00 metric component of the metric $g_{\mu\nu}$ is normalized to 1, it will not happen simultaneously for $g^2_{\mu\nu}, g^1_{\mu\nu}$. Having this in mind, the cosmic time coordinate T where where the 00 of metric component of $g_{\mu\nu}$ is normalized to 1 is defined by

$$dT = \sqrt{\frac{1 - \frac{\sigma}{t^4}}{T_1 - T_2}}dt \tag{30}$$

3.5 Absence of a Hagedorn Temperature at Early times in the cosmological case, vanishing slope parameters in the early universe as a sign of asymptotic freedom

We notice that at the singular point, $t = t^* = (\sigma)^{1/4}$, where $\Omega^2 \to 1$, and from the expressions of the tensions of strings 1 and 2, (eqs. 23, 24), that both string tensions become arbitrarily large at this point. Since the Hagedorn Temperature is proportional to the string tension, we conclude that in the early Universe the will no maximum temperature or Hagedorn phase transition. At the early universe, the slopes, $\alpha'_1 = 1/4\pi T_1$ and $\alpha'_2 = 1/4\pi T_2$ are very small, so the expansion that gives the effective gravity equatinsfrom the requirement that the conformal invariance is preserved at the quantum level is very reliable, since this relies in a perturbative expansion in the slopes, $\alpha'_1 = 1/4\pi T_1$ and $\alpha'_2 = 1/4\pi T_2$.

We can say therefore that there is a kind of asymptotic freedom of this theory for the early universe, which is exactly the reason that we are relieved (or allowed to escape) from the Hagerdorn temperature in the early universe.

3.7 Spontaneously Generated Boundary in Wesson warped spaces

One may wonder if there are similar solutions to the vacuum Einstein´s equations similar to the Milne space but where instead of time some spacial coordinate would play

is found,

$$ds^2 = l^2 dt^2 - l^2 cosh^2 t(\frac{dr^2}{1-r^2} + r^2 d\Omega_2^2)) - dl^2 \tag{31}$$

where l is the fourth dimension, so we see that as in the fourth dimension l such a solution is homogeneous of degree two, just as the Milne space time was homogeneous of degree two with respect to the time. Notice that maximally symmetric de Sitter space times sub spaces $l =$ constant appear for instead of euclidean spheres that appear in the Milne Universe for $t =$ constant.

The list of space times of this type is quite large, for example, one cal find solutions of empty GR with Schwarzchild de Sitter subpaces for $l =$ constant, as in

$$ds^2 = \frac{\Lambda l^2}{3}(dt^2(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}) - \frac{dr^2}{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}} - r^2 d\Omega_2^2) - dl^2 \tag{32}$$

This of course can be extended to D dimensions, where we choose one dimension l to have a factor l^2 warp factor for the other dimensions , generically for D dimensions as in

$$ds_2^2 = l^2 \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu - dl^2 \tag{33}$$

where $\bar{g}_{\mu\nu}(x)$ is a $D-1$ Schwarzschild de Sitter metric for example [31]. This we will take as our 2 metric, In any case, working with this generic metric of the form (33), but now in D dimensions, we can perform the inversion transformation $l \rightarrow \frac{1}{l}$, and multiplying also by a factor σ and obtain the conformally transformed metric 1 that also satisfies the vacuum Einstein´s equations

$$ds_1^2 = \sigma l^{-2} \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu - \sigma \frac{dl^2}{l^4} = \sigma l^{-4} ds_2^2 \tag{34}$$

From this point on , the equations the solutions for the tensions of the 1 and 2 strings are the same as in the cosmological case, just that $t \rightarrow l$, so now $\Omega^2 = \sigma l^{-4}$, so that we now insert this expression for c in (23) and in (24).

Now , we will choose σ positive, since we work here only with two types of strings, both with positive tension. obtaining that on one value of the wrapping coordinate in l both string tensions approach arbitrarily large values.

The universal metric, following the steps done for the cosmological case is now ,

$$ds^2 = (\frac{1 - \Omega^2}{T_2 - T_1})(l^2 \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu - dl^2) \tag{35}$$

looking at the coefficient of $\bar{g}_{\mu\nu}(x) dx^\mu dx^\nu$, the function is $(\frac{l^2 - \sigma/l^2}{T_2 - T_1})$, so the space time is expanded or contracted as we move in the dimension l by this factor. This factor explodes at $l^* = \sigma^{1/4}$. We can define a proper length coordinate L where where the ll of metric component of the metric is normalized to -1 is defined by

$$dL = \sqrt{\frac{1 - \frac{\sigma}{l^4}}{T_2 - T_1}} dl \tag{36}$$

3.5 Absence of a Hagedorn Temperature at Early times in the cosmological case, vanishing slope parameters in the early universe as a sign of asymptotic freedom

We notice that at the singular point, $t = t^* = (\sigma)^{1/4}$, where $\Omega^2 \to 1$, and from the expressions of the tensions of strings 1 and 2, (eqs. 24, 25), that both string tensions become arbitrarily large at this point. Since the Hagedorn Temperature is proportional to the string tension, we conclude that in the early Universe the will no maximum temperature or Hagedorn phase transition. At the early universe, the slopes, $\alpha_1' = 1/4\pi T_1$ and $\alpha_2' = 1/4\pi T_2$ are very small, so the expansion that gives the effective gravity equatinsfrom the requirement that the conformal invariance is preserved at the quantum level is very reliable, since this relies in a perturbative expansion in the slopes, $\alpha_1' = 1/4\pi T_1$ and $\alpha_2' = 1/4\pi T_2$.

We can say therefore that there is a kind of asymptotic freedom of this theory for the early universe, which is exactly the reason that we are relieved (or allowed to escape) from the Hagerdorn temperature in the early universe.

Branewords!, the two metrics are flat space in Minkowskii space and the other Minkowskii space after a special conformal transformation

In this case, this conformal factor coincides with that of the special conformal transformation

$$x^{\mu\prime} = \frac{(x^\mu + a^\mu x^2)}{(1 + 2a_\nu x^\nu + a^2 x^2)} \tag{35}$$

in summary, we have two solutions for the Einstein´s equations, $g^1_{\alpha\beta} = \eta_{\alpha\beta}$ and

$$g^2_{\alpha\beta} = \Omega^2 \eta_{\alpha\beta} = \phi^{-2}\eta_{\alpha\beta} = \frac{1}{(1 + 2a_\mu x^\mu + a^2 x^2)^2}\eta_{\alpha\beta} \tag{36}$$

We can then study the evolution of the tensions using $\Omega^2 = \phi^{-2} = \frac{1}{(1+2a_\mu x^\mu + a^2 x^2)^2}$. We will consider two different cases: 1) $a^2 = 0$, 2) $a^2 \neq 0$

where $a^2 = a^\mu a_\mu$ and $x^2 = x^\mu x_\mu$.

3.3.4 Light Like Segment Compactification

Here we consider the case $a^2 = 0$, and let us consider $a^\mu = (A, A, 0, ...0)$. Then

$$\Omega^2 = \frac{1}{(1 + 2a_\mu x^\mu)^2} = \frac{1}{(1 + 2A(t - x))^2} \quad (37)$$

From this, let is calculate the tensions of the two sting types and see that they will be constrained to be inside a segment that moves with the speed of light. At the boundaries of those segments the string tensions become infinity, so the strings cannot escape this segment.

37 leads to the tensions of the different strings to be

$$e\phi + T_1 = \frac{\Omega^2(T_2 - T_1)}{1 - \Omega^2} = \frac{(T_2 - T_1)(1 + 2A(t - x))^2}{4A(t - x)(1 + A(t - x))} \quad (38)$$

and

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - \Omega^2} = \frac{(T_2 - T_1)}{4A(t - x)(1 + A(t - x))} \quad (39)$$

Let us take $T_2 - T_1$ positive, A negative, so we see that both tensions above go to positive infinity when $t - x$ goes to zero from negative values . Also both tensions above go to positive infinity when $t - x$ goes to the value $-1/A$ from above. That means that the strings are confined to the moving segment where $t - x$ is inside the segment $(-1/A, 0)$. We call this phenomenon ¨Light Like Segment Compactification¨.

3.3.5 Braneworlds in Dynamical String Tension Theories

We now consider the case when a^μ is not light like and we will find that for $a^2 \neq 0$, irrespective of sign, i.e. irrespective of whether a^μ is space like or time like, we will have thick Braneworlds where strings can be constrained between two concentric spherically symmetric bouncing higher dimensional spheres and where the distance between these two concentric spherically symmetric bouncing higher dimensional spheres approaches zero at large times. The string tensions of the strings one and two are given by

$$e\phi + T_1 = \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2x^2)^2}{(1 + 2a_\mu x^\mu + a^2x^2)^2 - 1} = \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2x^2)^2}{(2a_\mu x^\mu + a^2x^2)(2 + 2a_\mu x^\mu + a^2x^2)} \quad (40)$$

$$e\phi + T_2 = \frac{(T_2 - T_1)}{(1 + 2a_\mu x^\mu + a^2x^2)^2 - 1} = \frac{(T_2 - T_1)}{(2a_\mu x^\mu + a^2x^2)(2 + 2a_\mu x^\mu + a^2x^2)} \quad (41)$$

Then, the locations where the string tensions go to infinity are determined by the conditions

$$2a_\mu x^\mu + a^2x^2 = 0 \quad (42)$$

or

$$2 + 2a_\mu x^\mu + a^2x^2 = 0 \quad (43)$$

Let us start by considering the case where a^μ is time like, then without loosing generality we can take $a^\mu = (A, 0, 0, ..., 0)$. In this case the denominators in (40) , (41) is

$$(2a_\mu x^\mu + a^2x^2)(2+2a_\mu x^\mu + a^2x^2) = (2At + A^2(t^2-x^2))(2+2At++A^2(t^2-x^2)) \quad (44)$$

The condition (42) implies then that

$$x_1^2 + x_2^2 + x_3^2 ..... + x_{D-1}^2 - (t+\frac{1}{A})^2 = -\frac{1}{A^2} \quad (45)$$

while the other boundary of infinite string tension (43) is given by,

$$x_1^2 + x_2^2 + x_3^2 ..... + x_{D-1}^2 - (t+\frac{1}{A})^2 = \frac{1}{A^2} \quad (46)$$

So we see that (49) represents an exterior boundary which has an bouncing motion with a minimum radius $\frac{1}{A}$ at $t = -\frac{1}{A}$, The denominator (??) is positive between these two bubbles. So for $T_2 - T_1$ positive the tensions are positive and diverge at the boundaries defined above.

The internal boundary (45) exists only for times t smaller than $-\frac{2}{A}$ and bigger than 0, so in the time interval $(-\frac{2}{A}, 0)$ there is no inner surface of infinite tension strings. This inner surface collapses to zero radius at $t = -\frac{2}{A}$ and emerges again from zero radius at $t = 0$.

we can take $a^\mu = (0, A, 0, ..., 0)$. In this case the denominators in (40) , (41) is

$$(2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2) = (-2Ax^1 - A^2(t^2 - \vec{x}^2))((2 - 2Ax^1 - A^2(t^2 - \vec{x}^2)) \quad (48)$$

where $\vec{x} = (x^1, x^2, ..., x^{D-1})$ represents the spacial part of x^μ, and $\vec{x}^2 = (x^1)^2 + (x^2)^2 + ... + (x^{D-1})^2$. We now consider the case when a^μ is space like, then without loosing generality we can take $a^\mu = (0, A, 0, ..., 0)$. We then consider the first boundary where the string tensions approoach infinity according to (42),

$$-(x_1 - \frac{1}{A})^2 - x_2^2 - x_3^2..... - x_{D-1}^2 + t^2 = -\frac{1}{A^2} \quad (49)$$

which describes a bouncing bubble with minimum radius $\frac{1}{A}$ at $t = 0$.

The case (43) gives

$$-(x_1 - \frac{1}{A})^2 - x_2^2 - x_3^2..... - x_{D-1}^2 + t^2 = \frac{1}{A^2} \quad (50)$$

(50) is an internal boundary which exists only for times t smaller than $-\frac{1}{A}$ and bigger than $\frac{1}{A}$. Between $-\frac{1}{A}$ and $\frac{1}{A}$ there is no inner surface of infinite tension strings. Between these two bubbles the two factors in eq. This inner surface collapses to zero radius at $t = -\frac{1}{A}$ and emerges again from zero radius at $t = -\frac{1}{A}$. So the situation is very similar to that of the case where the vector a^μ is time like, just that the roles of the cases $\Omega = 1$ and $\Omega = -1$ get exchanged. Between these two boundaries the two factors in the denominator (48) are positive, while at the boundaries one or the other approach zero and the tensions diverge, so again for $T_2 - T_1$ positive the tensions are positive

2 Are the flat space backgrounds consistent with the presence of very high tension strings?

The whole construction of the braneworld has been based on the conformal mapping between two flat spaces, this conformal mapping then defines the behavior of the string tensions and in principle it represents a vacuum solution where test strings acquire string tensions that diverge at two concentric and expanding surfaces, for details see [1].

Furthermore, as we start to populate the braneworld with actual strings, these strings will have infinite tension at the borders of the braneworld. A natural question one may ask at this point is the following: Are the flat space backgrounds of our construction consistent with the presence of very high Tension Strings or will the backreaction from the very large string tension destroy this basic feature of the model ?

This question requires a non trivial answer because the presence of arbitrarily large string tensions would appear at first sight substantial back reaction from the space time and possibly large deviations from the construction based on the flat spaces in the previous sections, but is that so? As we will see,indeed, our picture it appears that the introduction

We consider then a surface or thin shell with $D-2$ spacial dimensions, where in this shell a gas of strings with the equation of state that relates the surface pressure p to the σ being

$$p = -\frac{\sigma}{D-2} \tag{1}$$

see for example a discussion of the string gas equation of state in $4D$ cosmology in [23]) and for an example involving string gas shells see [24], so for $D = 3$, we obtain that the surface becomes a line with $p = -\sigma$, This was a matching corresponding to a particular choice of the ones studied in [25], while the $D = 4$ corresponds to a membrane ($2 + 1$ dimensional brane) moving in $3+1$ universe with a string gas matter in it [22]. In [23,25] the universe was meant to be the bulk space inside the bubble, while now, being interested in the braneword picture, the bubble, that is the surface with the large string tensions itself is the Universe where we live. We must consider therefore higher dimensions to get a relevant braneworld scenario.

Applying a local conservation law of the energy momentum in the brane defined by Eq. (1) leads to the possibility of integrating σ,

$$\sigma = \frac{\sigma_0}{r^{D-3}} \tag{2}$$

$$ds^2 = -A_+ dt^2 + \frac{dr^2}{A_+} + r^2 d\Omega_{D-2}^2 \tag{3}$$

for the outside metric and

$$ds^2 = -A_- dt^2 + \frac{dr^2}{A_-} + r^2 d\Omega_{D-2}^2 \tag{4}$$

for the inside metric. Here $d\Omega_{D-2}^2$ represents the contribution to the metric of the $D-2$ angles relevant to the spherically symmetric solutions in D space time dimensions. A_+ and A_- are functions of r , different for the inside and the outside spaces, matched at a bubble defined by a trajectory

$$r = r(\tau) \tag{5}$$

Then the matching condition as a consequence of the Israel analysis [26] generalized to D dimensions reads,

$$\sqrt{A_- + \dot{r}^2} - \sqrt{A_+ + \dot{r}^2} = \kappa \sigma r \tag{6}$$

where κ is proportional to Newton constant in D dimensions.

The square roots are not necessarily positive, the sign can be negative for example for a wormhole matching as has been discussed in details in $D = 4$, which corresponds to a membrane ($2 + 1$ dimensional brane) moving in $3 + 1$ universe with a string gas matter in it [22]. Another case where a difference a sum of the two terms is obtained, or what is equivalent, we can say that the second square root is negative is when considering a braneword scenario where the radius growths as we go out from the brane on both sides, see for example [27]. The assignment of signs of the square roots when one of the soaces is a Schwarzschild space can be worked out rigorously by study the problem using Kruskal–Szekeres coordinates [28] where these expressions were used for the study of the dynamics of false vacuum bubbles and baby universe creation

We will now study the case where inside we have flat space, that is

$$A_- = 1$$

and outside a D dimensional Schwarzschild solution with maximal rotational invariance, which gives the Tangherlini solution [29]

$$A_+ = 1 - \frac{c_1}{r^{D-3}}$$

where c_1 is a constant. In the Tangherlini solution the radial fall off $\frac{1}{r}$ of the Newtonian potential is replaced by the $\frac{1}{r^{D-3}}$

Solving from 6 for one of the square roots and then solving for the other square root and squaring again, we obtain the particle in a potential like equation,

$$\dot{r}^2 + V_{eff}(r) = 0 \tag{7}$$

where

$$V_{eff}(r) = -\left(\frac{r^{D-4}}{\kappa\sigma_0} - \frac{c_1}{2\kappa\sigma_0 r} - \frac{\kappa\sigma_0}{2} r^{-D+4}\right)^2 + \frac{r^{2D-8}}{(\kappa\sigma_0)^2} - \frac{c_1 r^{D-5}}{(\kappa\sigma_0)^2} \tag{8}$$

The expression (8) can also be expressed as

$$V_{eff}(r) = 1 - \left(\frac{c_1}{2\kappa\sigma_0 r} + \frac{\kappa\sigma_0}{2r^{D-4}}\right)^2$$

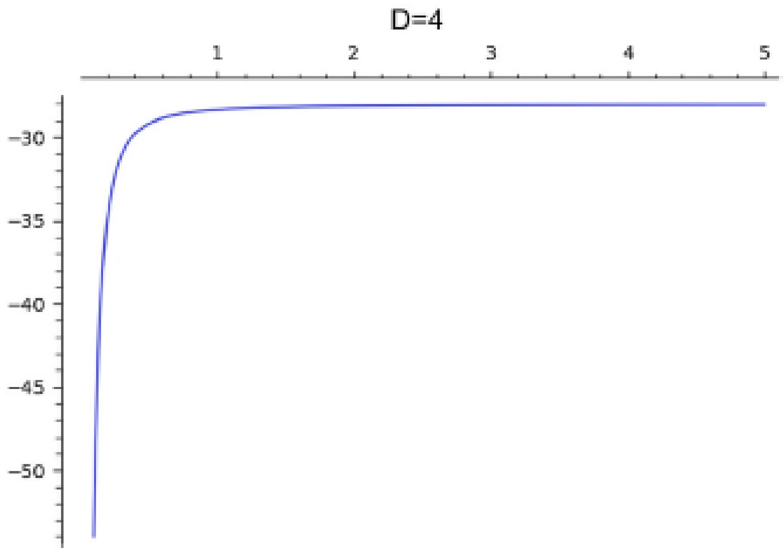


Fig. 1 The potential for a particular choice of parameters in D = 4

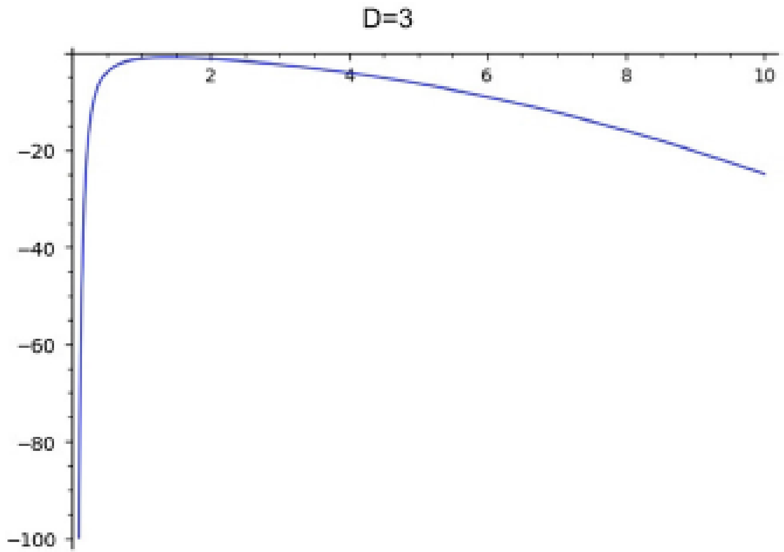


Fig. 2 The potential for a particular choice of parameters in D = 3

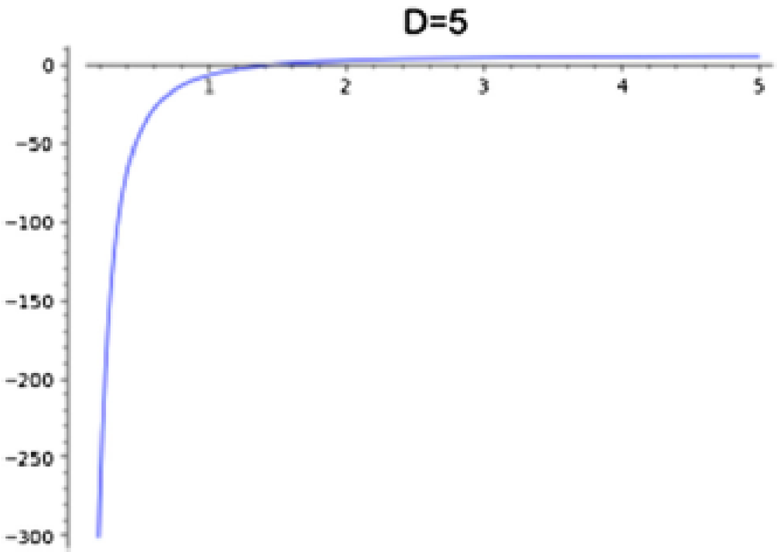


Fig. 3 The potential for a particular choice of parameters in D = 3

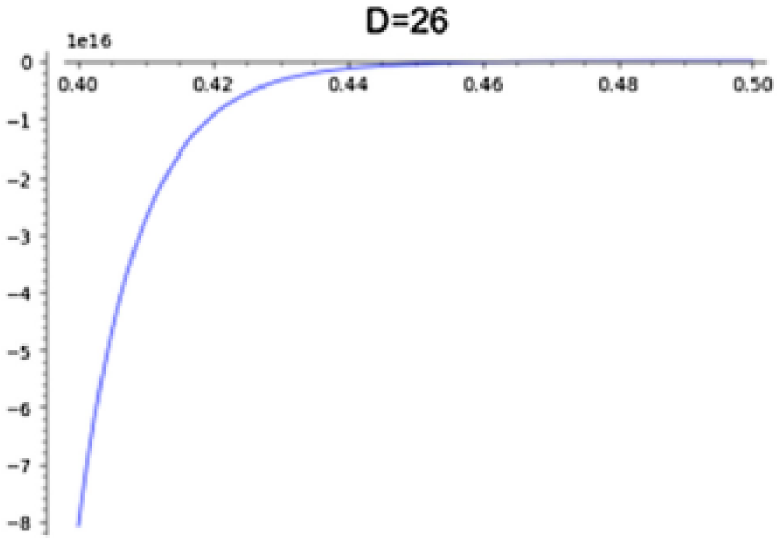


Fig. 4 The potential for a particular choice of parameters in D = 26

The expression (13) allows also a particularly simple solution for the point where $V_{eff}(r) = 0$, the point of return of the bubble, even for $c_1 \neq 0$, which is particularly simple for $D = 5$, see Fig. 3 since in this case both terms inside the square are proportional to $\frac{1}{r}$, so we must choose

$$1 - \left(\frac{c_1}{2\kappa\sigma_0 r} + \frac{\kappa\sigma_0}{2r}\right) = 1 - \frac{1}{r}\left(\frac{c_1}{2\kappa\sigma_0} + \frac{\kappa\sigma_0}{2}\right) = 0 \quad (14)$$

so that

$$r_m = \left(\frac{c_1}{2\kappa\sigma_0} + \frac{\kappa\sigma_0}{2}\right) \quad (15)$$

So, we explicitly see that for $\kappa\sigma_0 \to \infty$, then , $r_m \to \infty$ regardless of the mass (i.e. c_1), so infinite tension string gas shell can describe an expanding shell to infinity being connected by two flat spaces. This feature extends to all dimensions bigger than 4 as well. Finally, Fig. 4 shows the effective potential for $D = 26$.

Phenomenological advantages of these braneworlds over the traditional ones

- See Limits on the number of spacetime dimensions from GW170817

Kris Pardo, Maya Fishbach, Daniel E. Holz, David N. Spergel, Published in: JCAP 07 (2018) 048 • e-Print: 1801.08160 [gr-qc]

The observation of GW170817 in both gravitational and electromagnetic waves provides a number of unique tests of general relativity. One question we can answer with this event is: Do large-wavelength gravitational waves and short-frequency photons experience the same number of spacetime dimensions? In models that include additional non-compact spacetime dimensions, as the gravitational waves propagate, they "leak" into the extra dimensions, leading to a reduction in the amplitude of the observed gravitational waves, and a commensurate systematic error in the inferred distance to the gravitational wave source. Electromagnetic waves would remain unaffected.....

Limits based on observation of coalesing neutron stars

The key to this breakthrough
was the gravitational wave event

A pair of neutron stars
spiralled together and merged.

And the LIGO and Virgo
gravitational wave

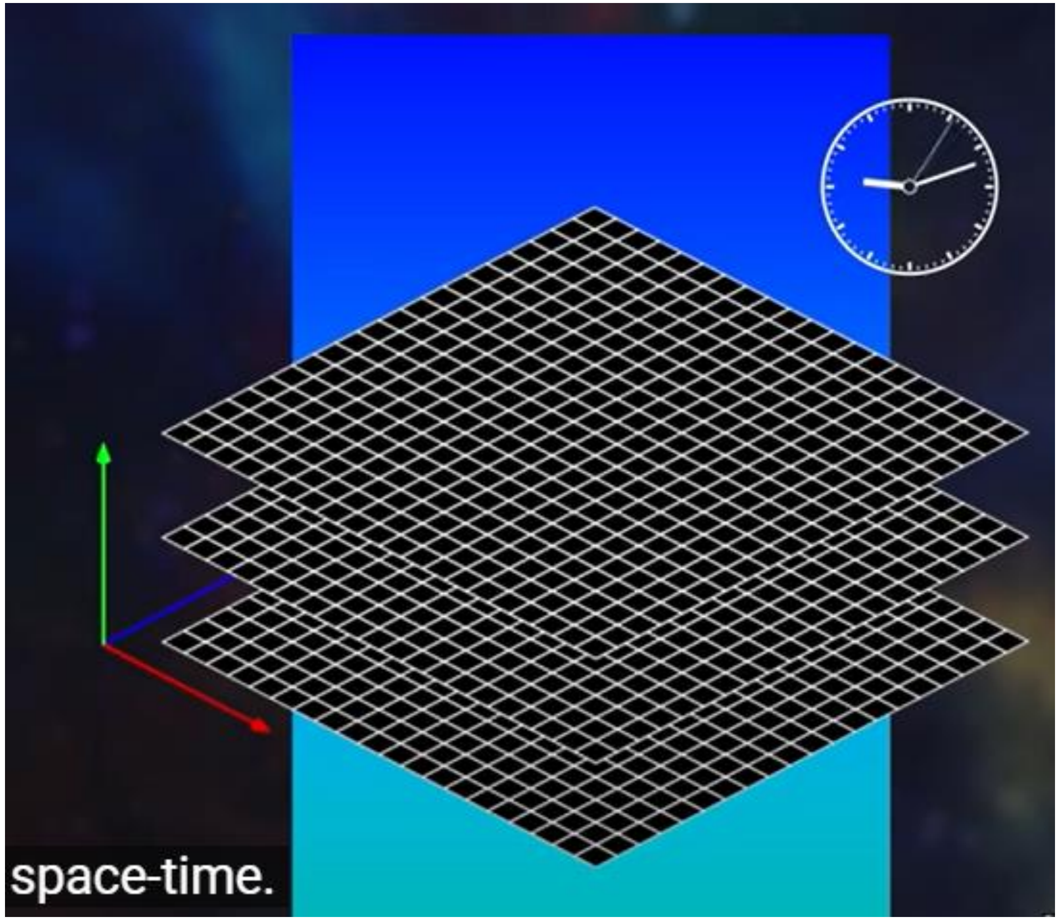
observatories detected
the resulting ripples.

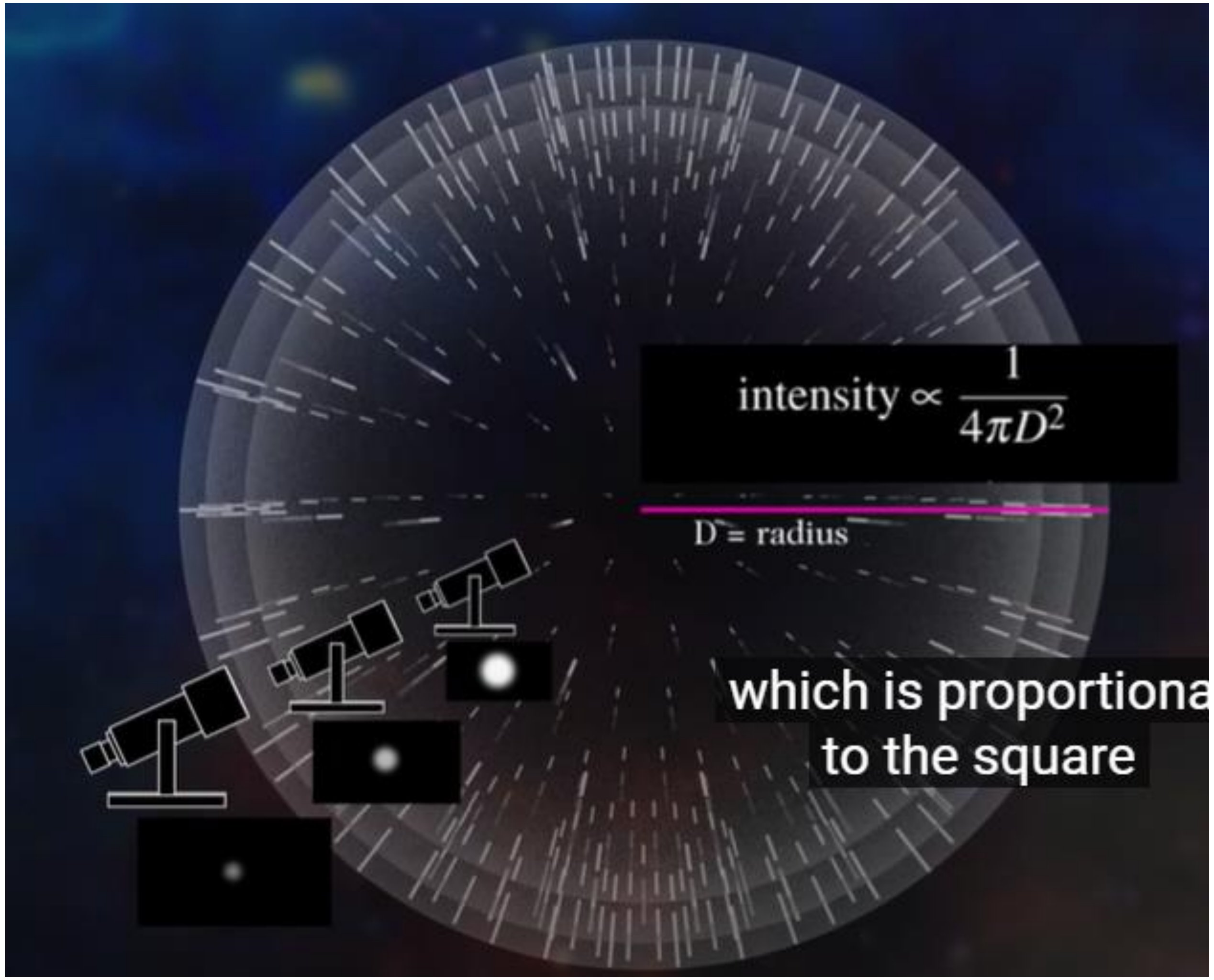
merging neutron stars
explode spectacularly.

The resulting kilonova is first
observed in gravitational waves

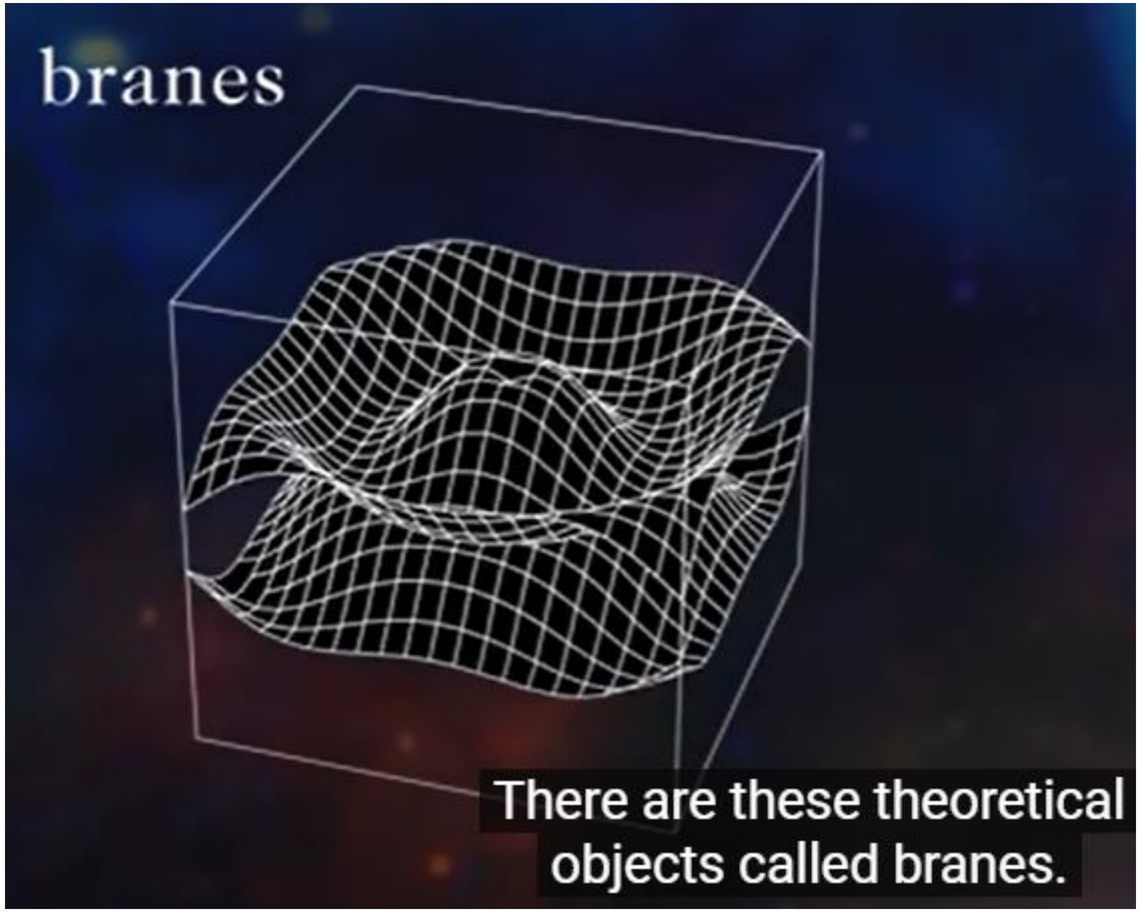
and then as a gamma ray burst.

3+1 dimensions

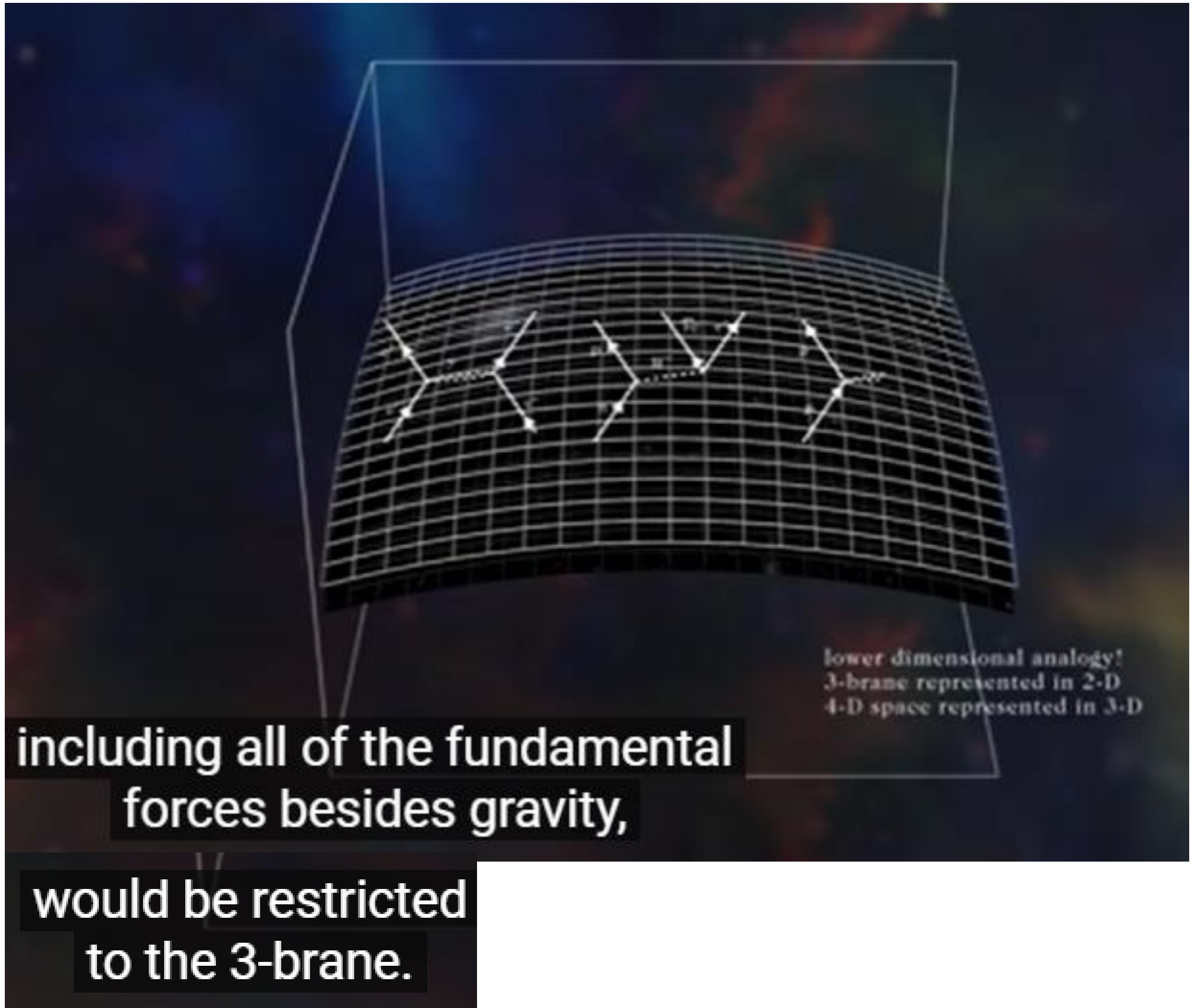




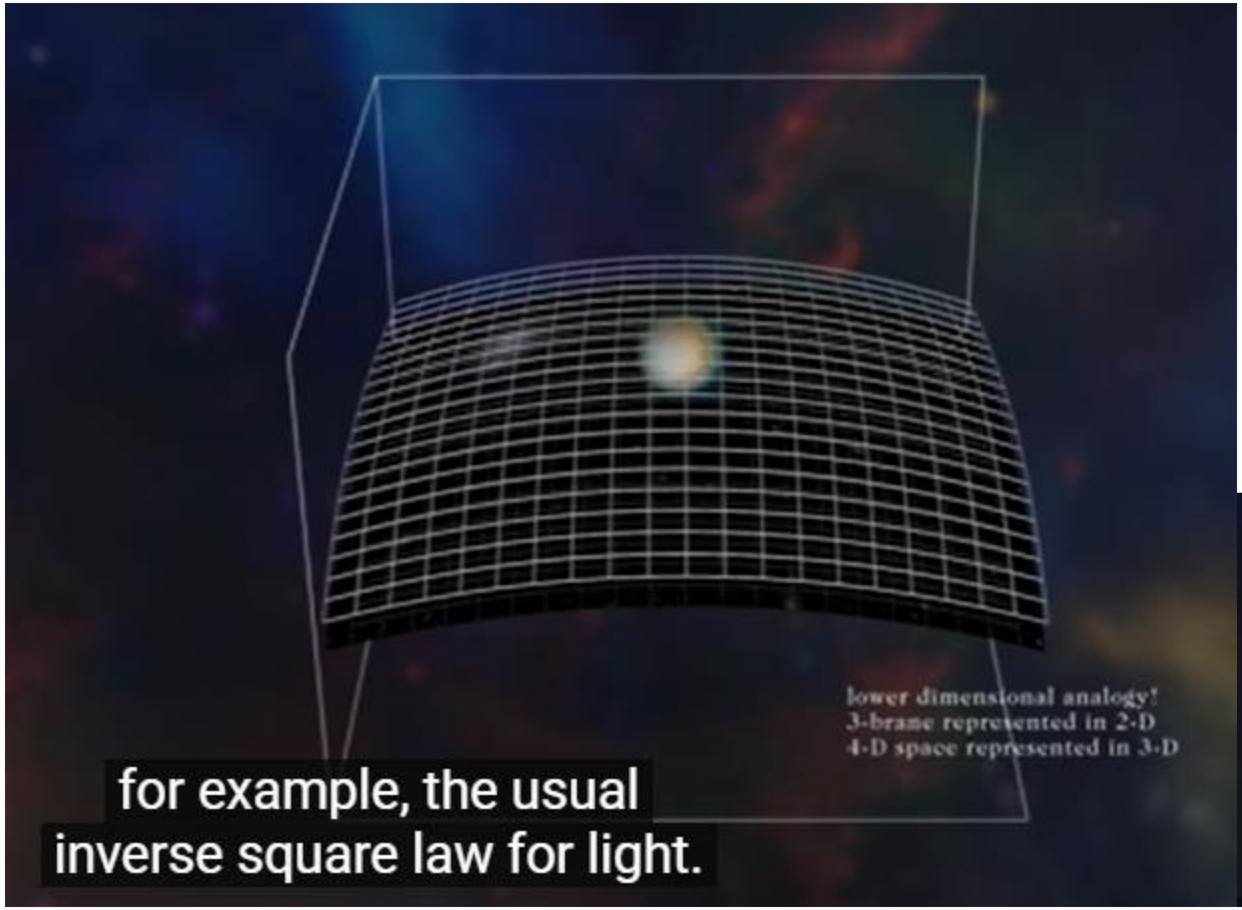
Consider branes, in the standard approach (not dynamical string tension approach!)



there, gauge interactions live in a 3- brane

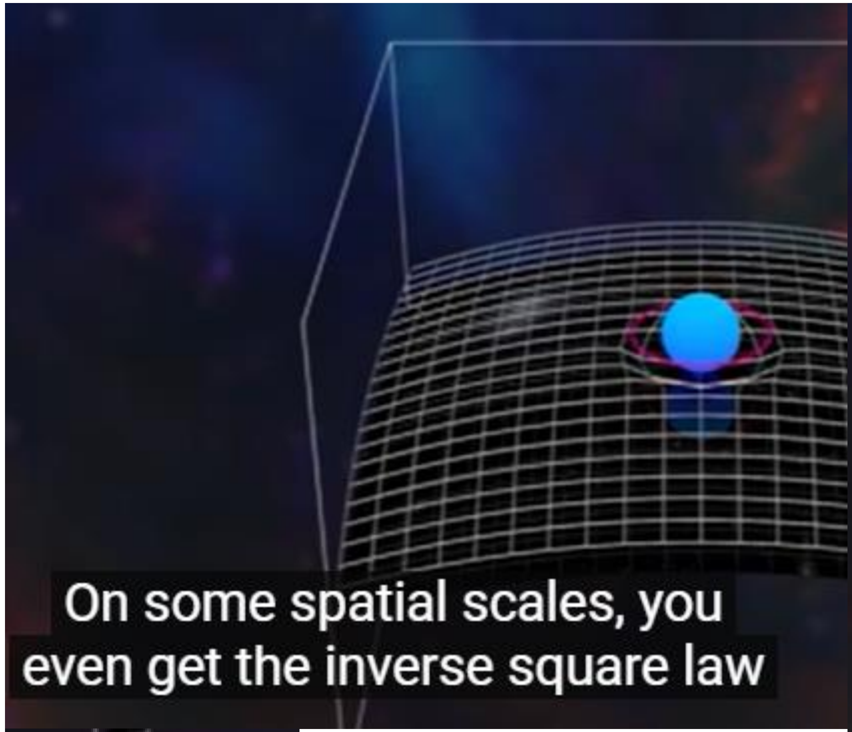


We obtain

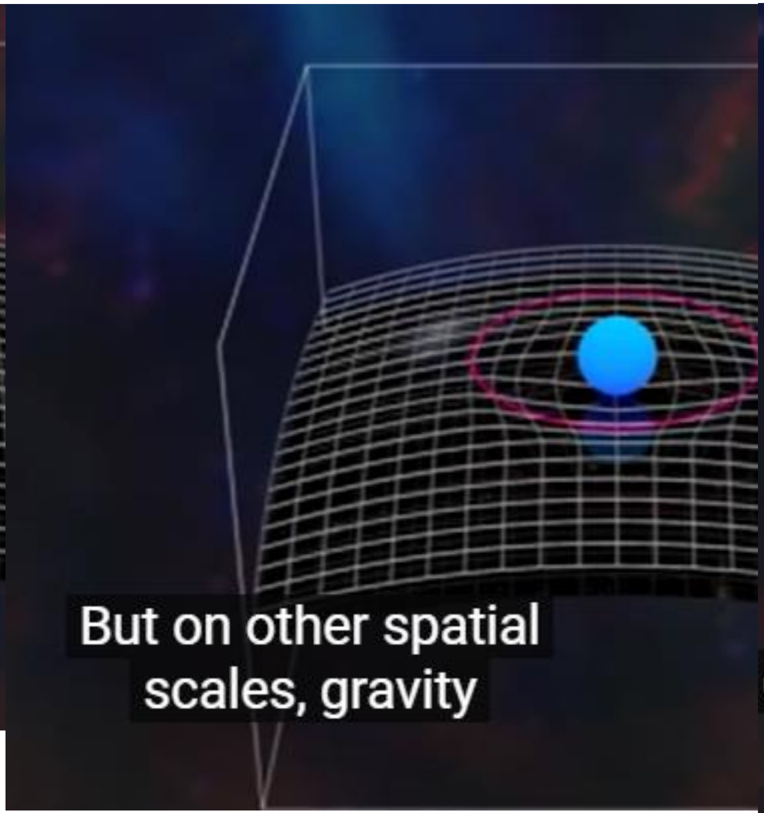


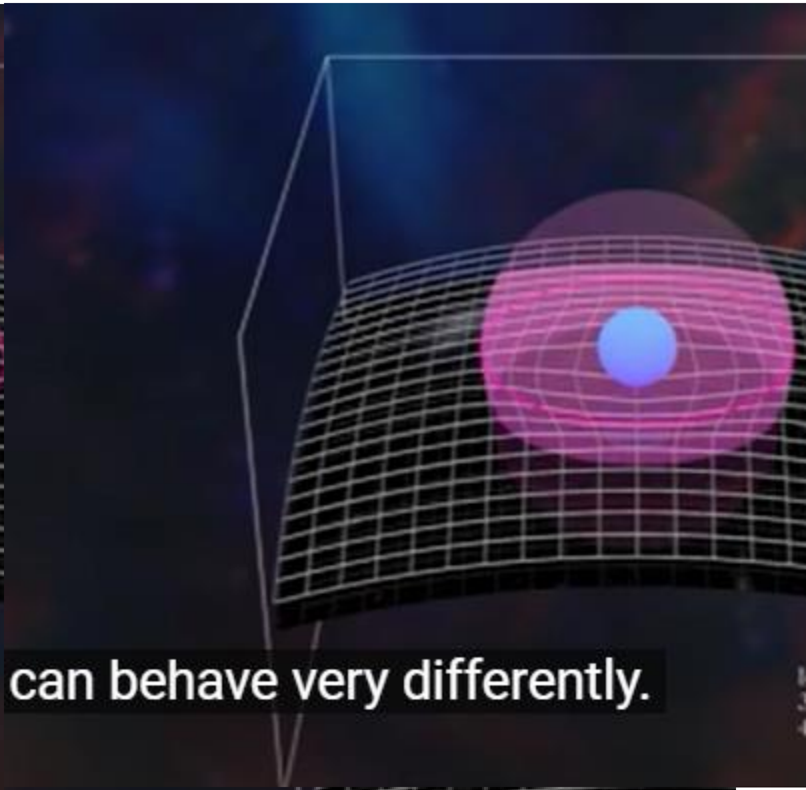
$$\text{intensity} \propto \frac{1}{D^{N-1}}$$

N = number of spatial dimensions



for gravity.





If gravity spreads out in four
dimensions rather than three,

Well, here's where
we finally get back
to our gravitational waves.
If the gravitational
field can extend
into this hypothetical
extra spatial dimension,
then gravitational
waves should lose energy
to that extra dimension as
they travel through space.
in order to determine
how much intensity
when it started its journey.

A super convenient property of gravitational waves is that you can figure this out by looking at other properties namely, the masses of the merging objects and the frequency of the wave combined with our independent distance measurement.

however

The gravitational wave lost the right amount of intensity for a 3-plus-1-dimensional space-time.

There was no observable leakage of gravity

damping of grav. waves points to 4 spacetime dimensions (3 space + 1 time)

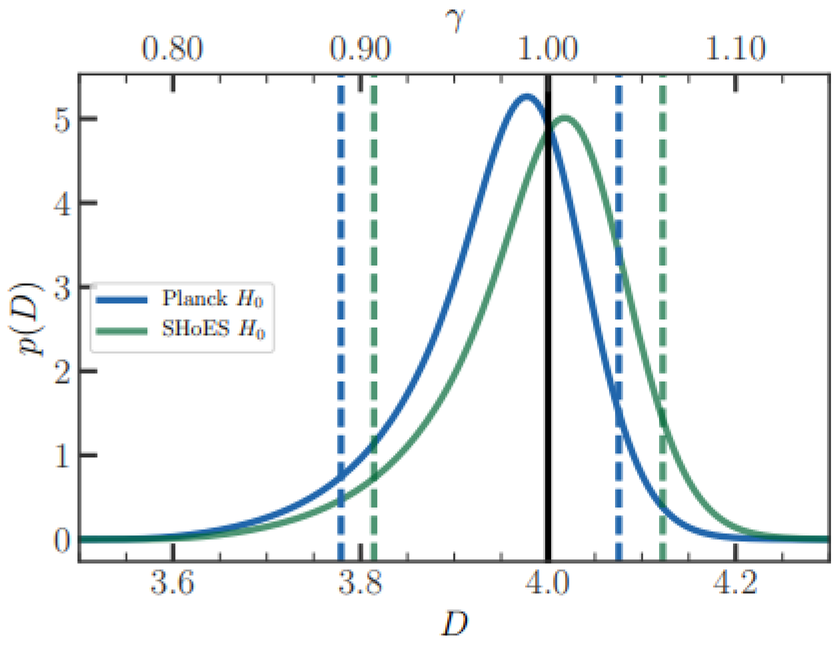


Figure 1. Posterior probability distribution for the number of spacetime dimensions, D, using the GW distance posterior to GW170817 and the measured Hubble velocity to its host galaxy, NGC 4993, assuming the H_0 measurements from Planck Collaboration et al. (2016) (blue curve) and Riess et al. (2016) (green curve). The dashed lines show the symmetric 90% credible intervals. The equivalent constraints on the damping factor, γ, are shown on the top axis. GW170817 constrains D to be very close to the GR value of $D = 4$ spacetime dimensions, denoted by the solid black line.

The asymmetry in the propagation of gravity as compared to light , in braneworlds with large extra dimensions is disproved by the observations (RS2)

- RS1 with two branes ok, but one brane tension <0

BUT OUR BRANEWORLDS INDUCED BY VARIABLE STRING TENSIONS ARE FREE OF THIS PROBLEM!, WHY?,

- Because we confine EVERYTHING, both gravity, that arises from the closed strings and gauge interactions which originates from open strings. Both the open and closed strings are constrained between two very closed expanding surfaces, this mechanism avoids any asymmetry between gravity and light-
- We can still have a combination of the standard brane scenario and the dynamical string tension branes, where the dynamical string tension prevents the propagation of gravity to deep into the extra dimension.
- The hyperbolic motion also induces a deSitter space in the brane, explaining also DE.

• Life of the homogeneous and isotropic universe in dynamical string tension theories , E.I. Guendelman, Eur.Phys.J.C 82 (2022) 10, 857

The homogeneous and isotropic universe in dynamical string tension theories

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We now consider the case when a^μ is not light like and we will find that for $a^2 \neq 0$, irrespective of sign, i.e. irrespective of whether a^μ is space like or time like, we will have thick braneworlds where strings can be constrained between two concentric spherically symmetric bouncing higher dimensional spheres and where the distance between these two concentric spherically symmetric bouncing higher dimensional spheres approaches zero at large times. The string tensions of the strings one and two are given by

$$\begin{aligned} e\phi + T_1 &= \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2x^2)^2}{(1 + 2a_\mu x^\mu + a^2x^2)^2 - 1} \\ &= \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2x^2)^2}{(2a_\mu x^\mu + a^2x^2)(2 + 2a_\mu x^\mu + a^2x^2)} \end{aligned} \tag{37}$$

$$\begin{aligned} e\phi + T_2 &= \frac{(T_2 - T_1)}{(1 + 2a_\mu x^\mu + a^2x^2)^2 - 1} \\ &= \frac{(T_2 - T_1)}{(2a_\mu x^\mu + a^2x^2)(2 + 2a_\mu x^\mu + a^2x^2)} \end{aligned} \tag{38}$$

Let us by consider the case where a^μ is time like, then without loosing generality we can take

$a^\mu = (A, 0, 0, \ldots, 0)$. Now, in order to get homogeneous and isotropic cosmological solutions we must consider the limit $A \to 0$ and $(T_2 - T_1) \to 0$, in such a way that $\frac{(T_2 - T_1)}{A} = K$, where K is a constant. In that case the spatial dependence in the tensions (37) and (38) drops out and we get,

$$e\phi + T_1 = e\phi + T_2 = \frac{K}{4t} \tag{39}$$

The embedding metric can now be solved.

$$g_{\mu\nu} = \frac{1}{(e\phi + T_1)} g^1_{\mu\nu} = \frac{4t}{K} \eta_{\mu\nu} \tag{40}$$

which is not a vacuum metric, as opposed to $\eta_{\mu\nu}$ because of the conformal factor $\frac{4t}{K}$.

Life of the homogeneous and isotropic universe and emergence of a braneworld at large times

One should notice that the homogeneous and isotropic solution has been obtained only in the limit $A \to 0$ and $(T_2 - T_1) \to 0$, in such a way that $\frac{(T_2 - T_1)}{A} = K$, where K is a constant. If A and $T_2 - T_1$ are small but finite, then for large times, of the order of $1/A$. We can formulate this as an uncertainty principle,

$$(T_2 - T_1)\Delta t \approx constant \tag{41}$$

where we have used that A is of the order of $(T_2 - T_1)$. So a small uncertainty in the tension $(T_2 - T_1)$ leads to a long lived homogeneous and isotropic phase, while a big uncertainty in the tension $(T_2 - T_1)$ leads to short lived homogeneous and isotropic phase.

In fact in these situations, for finite $(T_2 - T_1)$ and A, it is the case that the string tensions can only whange sign by going first to infinity and then come back from minus infinity. We can now recognize at those large times the locations where the string tensions go to infinity, which are determined by the conditions

$$2a_\mu x^\mu + a^2 x^2 = 0 \tag{42}$$

or

$$2 + 2a_\mu x^\mu + a^2 x^2 = 0 \tag{43}$$

Let us start by considering the case where a^μ is time like, then without loosing generality we can take $a^\mu = (A, 0, 0, \ldots, 0)$. In this case the denominator in (37) , (38) is

$$\begin{aligned}&(2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2)\\ &\quad = (2At + A^2(t^2 - x^2))(2 + 2At + A^2(t^2 - x^2))\end{aligned} \tag{44}$$

The condition (42), if $A \neq 0$ implies then that

$$x_1^2 + x_2^2 + x_3^2 \ldots + x_{D-1}^2 - \left(t + \frac{1}{A}\right)^2 = -\frac{1}{A^2} \tag{45}$$

if $A \to 0$, it is more convenient to write this in the form

$$A(x_1^2 + x_2^2 + x_3^2 \ldots + x_{D-1}^2) - At^2 - 2t = 0 \tag{46}$$

which for the limit $A \to 0$ gives us the single singular point $t = 0$, which is the origin of the homogeneous and isotropic cosmological solution.

The other boundary of infinite string tensions is, (43) is given by,

$$x_1^2 + x_2^2 + x_3^2 \ldots + x_{D-1}^2 - \left(t + \frac{1}{A}\right)^2 = \frac{1}{A^2} \tag{47}$$

This has no limit for $A \to 0$, all these points disappear from the physical space (they go to infinity).

For $A \neq 0$ we see that (47) represents an exterior boundary which has an bouncing motion with a minimum radius $\frac{1}{A}$ at $t = -\frac{1}{A}$, The denominator (44) is positive between these two bubbles. So for $T_2 - T_1$ positive the tensions are positive and diverge at the boundaries defined above.

The internal boundary (45) exists only for times t smaller than $-\frac{2}{A}$ and bigger than 0, so in the time interval $(-\frac{2}{A}, 0)$ there is no inner surface of infinite tension strings. This inner surface collapses to zero radius at $t = -\frac{2}{A}$ and emerges again from zero radius at $t = 0$.

For large positive or negative times, the difference between the upper radius and the lower radius goes to zero as $t \to \infty$

$$\sqrt{\frac{1}{A^2} + \left(t + \frac{1}{A}\right)^2} - \sqrt{-\frac{1}{A^2} + \left(t + \frac{1}{A}\right)^2} \to \frac{1}{tA^2} \to 0 \tag{48}$$

of course the same holds $t \to -\infty$. This means that for very large early or late times the segment where the strings would be confined (since they will avoid having infinite tension) will be very narrow and the resulting scenario will be that of a brane world for late or early times, while in the bouncing region the inner surface does not exist. We can ignore the part of the solution where $t < -\frac{2}{A}$ and instead take $t = 0$ as the origin of the Universe and only consider positive values of cosmic time because the part of the solution with $t < -\frac{2}{A}$ is disconnected, at least at the classical level from the part of the solution with positive cosmic time.

We see then that for the exact limit of $\Delta T \to 0$ and $A \to 0$ we get a perfect homogeneous and isotropic cosmology, but as ΔT and A are deformed to be small but finite, the scenario is

Discussion: motivations, ΔT from quantum fluctuations, braneworld creation and decoherence

The approach we want to promote in this paper is to formulate first of all the dynamical tension theories where each string can have its own tension. The string interactions are usually formulated for strings of the same tension, in dynamical string tension this may not be an obstacle if the string tensions are close enough, so that quantum fluctuations of the string tension will make possible interactions. This is then a good motivation to consider the dispersion of the tension parametrized by ΔT in the string ensemble to be very small, in fact we consider solutions where this dispersion goes to zero, $\Delta T \to 0$.

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