From Homogeneous and Isotropic Universes to Braneworlds with Dynamical Tension Strings

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Космологические решения изучаются в контексте модифицированной формулировки меры теории струн, тогда натяжение струны является динамической переменной, а натяжение струны является дополнительной динамической степенью свободы, и ее значение генерируется динамически. В этом случае натяжения не являются универсальными, каждая струна создает свое собственное натяжение, которое может иметь разное значение для каждого из мировых листов струн, а в ансамбле струн значения натяжения могут иметь определенную дисперсию. Мы рассматриваем новое фоновое поле, которое может связываться с этими струнами, «скаляр натяжения», который способен локально изменяться вдоль мирового листа, и тогда соответственно изменяется значение натяжения струны.

Когда рассматривается множество типов струн, исследующих одну и ту же область пространства, этот скаляр натяжения ограничивается требованием квантовой конформной инвариантности. Для случая двух типов струн, зондирующих одну и ту же область пространства с разным динамически генерируемым натяжением, существуют две разные метрики, связанные с разными струнами. Каждая из этих метрик должна удовлетворять вакуумным уравнениям Эйнштейна, и согласованность этих двух уравнений Эйнштейна определяет скаляр натяжения. Универсальная метрика, общая для обеих струн, в общем случае не удовлетворяет уравнению Эйнштейна. Здесь рассматриваются две метрики, зависящие от струн, — это плоское пространство в пространстве Минковского и пространство Минковского после специального конформного преобразования. Исследуется предел, при котором натяжение двух струн одинаково, и это приводит к четко определенному решению. Если разность натяжения струн между двумя типами струн очень мала, но конечна, приблизительно однородное и изотропное космологическое решение сохраняется долгое время, обратно пропорциональное разности натяжений струн, и тогда однородность и изотропность космологии исчезает и решение превращается в расширяющийся мир браны, где струны заключены между двумя расширяющимися пузырями, разделенными очень небольшим расстоянием на больших временах.

Cosmological solutions are studied in the context of the modified measure formulation of string theory , then the string tension is a dynamical variable and the string the tension is an additional dynamical degree of freedom and its value is dynamically generated. These tensions are then not universal, rather each string generates its own tension which can have a different value for each of the string world sheets and in an ensemble of strings the values of the tensions can have a certain dispersion. We consider a new background field that can couple to these strings, the ¨tension scalar¨ which is capable of changing locally along the world sheet and then the value of the tension of the string changes accordingly.

When many types of strings probing the same region of space are considered this tension scalar is constrained by the requirement of quantum conformal invariance. For the case of two types of strings probing the same region of space with different dynamically generated tensions, there are two different metrics, associated to the different strings. Each of these metrics have to satisfy vacuum Einstein´s equations and the consistency of these two Einstein´s equations determine the tension scalar. The universal metric, common to both strings generically does not satisfy Einstein´s equation . The two string dependent metrics considered here are flat space in Minkowski space and Minkowski space after a special conformal transformation. The limit where the two string tensions are the same is studied, it leads to a well defined solution. If the string tension difference between the two types of strings is very small but finite, the approximately homogeneous and isotropic cosmological solution lasts for a long time, inversely proportional to the string tension difference and then the homogeneity and isotropy of the cosmology disappears and the solution turns into an expanding Braneworld where the strings are confined between two expanding bubbles separated by a very small distance at large times.
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The Modified Measure Theory String Theory

The standard world sheet string sigma-model action using a world sheet metric is [1], [2], [3]

$$S_{sigma-model} = -T \int d^2\sigma \frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \quad (1)$$

Here γ^{ab} is the intrinsic Riemannian metric on the 2-dimensional string worldsheet and $\gamma = det(\gamma_{ab})$; $g_{\mu\nu}$ denotes the Riemannian metric on the embedding spacetime. T is a string tension, a dimension full scale introduced into the theory by hand.

Now instead of using the measure $\sqrt{-\gamma}$, on the 2-dimensional worldsheet, in the framework of this theory two additional worldsheet scalar fields $\varphi^i (i = 1, 2)$ are considered. A new measure density is introduced:

$$\Phi(\varphi) = \frac{1}{2}\epsilon_{ij}\epsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j. \quad (2)$$

There are no limitations on employing any other measure of integration different than $\sqrt{-\gamma}$. The only restriction is that it must be a density under arbitrary diffeomorphisms (reparametrizations) on the underlying spacetime manifold. Then the modified bosonic string action is (as formulated first in [4] and latter discussed and generalized also in [5])

$$S = -\int d^2\sigma \Phi(\varphi)(\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)), \quad (3)$$

where F_{ab} is the field-strength of an auxiliary Abelian gauge field A_a: $F_{ab} = \partial_a A_b - \partial_b A_a$. To check that the new action is consistent with the sigma-model one, let us derive the equations of motion of the action (3). The variation with respect to φ^i leads to the following equations of motion:

$$\epsilon^{ab}\partial_b\varphi^i\partial_a(\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}) = 0. \quad (4)$$

since $det(\epsilon^{ab}\partial_b\varphi^i) = \Phi$, assuming a non degenerate case ($\Phi \neq 0$), we obtain,

$$\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = M = const. \quad (5)$$

The equations of motion with respect to γ^{ab} are

$$T_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = 0. \quad (6)$$

One can see that these equations are the same as in the sigma-model formulation . Taking the trace of (6) we get that $M = 0$. By solving $\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}$ from (5) (with $M = 0$) we obtain the standard string eqs. The emergence of the string tension is obtained by varying the action with respect to A_a:

$$\epsilon^{ab}\partial_b(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}) = 0. \quad (7)$$

Then by integrating and comparing it with the standard action it is seen that

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T. \tag{8}$$

That is how the string tension T is derived as a world sheet constant of integration opposite to the standard equation (1) where the tension is put ad hoc. Let us stress that the modified measure string theory action does not have any *ad hoc* fundamental scale parameters. associated with it. This can be generalized to incorporate super symmetry, see for example [5], [6], [7] , [8]. For other mechanisms for dynamical string tension generation from added string world sheet fields, see for example [9] and [10]. However the fact that this string tension generation is a world sheet effect and not a universal uniform string tension generation effect for all strings has not been sufficiently emphasized before. Notice that Each String in its own world sheet determines its own tension. Therefore the tension is not universal for all strings.

Introducing Background Fields including a New Background Field, The Tension Field

Schwinger [11] had an important insight and understood that all the information concerning a field theory can be studied by understanding how it reacts to sources of different types. This has been discussed in the text book by Polchinski for example [12] . Then the target space metric and other external fields acquire dynamics which is enforced by the requirement of zero beta functions. However, in addition to the traditional background fields usually considered in conventional string theory, one may consider as well an additional scalar field that induces currents in the string world sheet and since the current couples to the world sheet gauge fields, this produces a dynamical tension controlled by the external scalar field as shown at the classical level in [13]. In the next two subsections we will study how this comes about in two steps, first we introduce world sheet currents that couple to the internal gauge fields in Strings and Branes and second we define a coupling to an external scalar field by defining a world sheet currents that couple to the internal gauge fields in Strings that is induced by such external scalar field.

Introducing world sheet currents that couple to the internal gauge fields If to the action of the string we add a coupling to a world-sheet current j^a, i.e. a term

$$S_{\text{current}} = \int d^2\sigma A_a j^a, \tag{9}$$

then the variation of the total action with respect to A_a gives

$$\epsilon^{ab}\partial_a \left(\frac{\Phi}{\sqrt{-\gamma}} \right) = j^b. \tag{10}$$

We thus see indeed that, in this case, the dynamical character of the brane is crucial here.

How a world sheet current can naturally be induced by a bulk scalar field, the Tension FieldSuppose that we have an external scalar field $\phi(x^\mu)$ defined in the bulk. From this field we can define the induced conserved world-sheet current

$$j^b = e\partial_\mu \phi \frac{\partial X^\mu}{\partial \sigma^a}\epsilon^{ab} \equiv e\partial_a \phi \epsilon^{ab}, \tag{11}$$

where e is some coupling constant. The interaction of this current with the world sheet gauge field is also invariant under local gauge transformations in the world sheet of the gauge fields $A_a \to A_a + \partial_a \lambda$.

For this case, (10) can be integrated to obtain

$$T = \frac{\Phi}{\sqrt{-\gamma}} = e\phi + T_i, \tag{12}$$

or equivalently

$$\Phi = \sqrt{-\gamma}(e\phi + T_i), \tag{13}$$

The constant of integration T_i may vary from one string to the other. Notice that the interaction is metric independent since the internal gauge field does not transform under the the conformal transformations. This interaction does not therefore spoil the world sheet conformal transformation invariance in the case the field ϕ does not transform under this transformation. One may interpret (13) as the result of integrating out classically (through integration of equations of motion) or quantum mechanically (by functional integration of the internal gauge field, respecting the boundary condition that characterizes the constant of integration T_i for a given string). Then replacing $\Phi = \sqrt{-\gamma}(e\phi + T_i)$ back into the remaining terms in the action gives a correct effective action for each string. Each string is going to be quantized with each one having a different T_i. The consequences of an independent quantization of many strings with different T_i covering the same region of space time will be studied in the next section.

The Tension field from World Sheet Quantum Conformal Invariance

*The case of two different string tensions*If we have a scalar field coupled to a string or a brane in the way described in the sub section above, i.e. through the current induced by the scalar field in the extended object, according to eq. (13), so we have two sources for the variability of the tension when going from one string to the other: one is the integration constant T_i which varies from string to string and the other the local value of the scalar field, which produces also variations of the tension even within the string or brane world sheet. As we discussed in the previous section, we can incorporate the result of the tension as a function of scalar field ϕ, given as $e\phi + T_i$, for a string with the constant of integration T_i by defining the action that produces the correct equations of motion for such string, adding also other background fields, the anti symmetric two index field $A_{\mu\nu}$ that couples to $\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu$

and the dilaton field φ .

$$S_i = -\int d^2\sigma(e\phi + T_i)\frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + \\ + \int d^2\sigma A_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu + \int d^2\sigma\sqrt{-\gamma}\varphi R. \tag{14}$$

Notice that if we had just one string, or if all strings will have the same constant of integration $T_i = T_0$. We will take cases where the dilaton field is a constant or zero, and the antisymmetric two index tensor field is pure gauge or zero, then the demand of conformal invariance for $D = 26$ becomes the demand that all the metrics

$$g^i_{\mu\nu} = (e\phi + T_i)g_{\mu\nu} \tag{15}$$

will satisfy simultaneously the vacuum Einstein´s equations. The interesting case to consider is when there are many strings with different T_i, let us consider the simplest case of two strings, labeled 1 and 2 with $T_1 \neq T_2$, then we will have two Einstein´s equations, for $g^1_{\mu\nu} = (e\phi + T_1)g_{\mu\nu}$ and for $g^2_{\mu\nu} = (e\phi + T_2)g_{\mu\nu}$,

$$R_{\mu\nu}(g^1_{\alpha\beta}) = 0 \tag{16}$$

and , at the same time,

$$R_{\mu\nu}(g^2_{\alpha\beta}) = 0 \tag{17}$$

These two simultaneous conditions above impose a constraint on the tension field ϕ, because the metrics $g^1_{\alpha\beta}$ and $g^2_{\alpha\beta}$ are conformally related, but Einstein´s equations are not conformally invariant, so the condition that Einstein´s equations hold for both $g^1_{\alpha\beta}$ and $g^2_{\alpha\beta}$ is highly non trivial. Then for these situations, we have,

$$e\phi + T_1 = \Omega^2(e\phi + T_2) \tag{18}$$

which leads to a solution for $e\phi$

$$e\phi = \frac{\Omega^2 T_2 - T_1}{1 - \Omega^2} \tag{19}$$

which leads to the tensions of the different strings to be

$$e\phi + T_1 = \frac{\Omega^2(T_2 - T_1)}{1 - \Omega^2} \tag{20}$$

and

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - \Omega^2} \tag{21}$$

Both tensions can be taken as positive if $T_2 - T_1$ is positive and Ω^2 is also positive and less than 1.

*Flat space in Minkowski coordinates and flat space after a special conformal transformation*The flat spacetime in Minkowski coordinates is,

$$ds_1^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta \tag{22}$$

where $\eta_{\alpha\beta}$ is the standard Minkowski metric, with $\eta_{00} = 1$, $\eta_{0i} = 0$ and $\eta_{ij} = -\delta_{ij}$. This is of course a solution of the vacuum Einstein´s equations.

We now consider the conformally transformed metric

$$ds_2^2 = \Omega(x)^2 \eta_{\alpha\beta} dx^\alpha dx^\beta \tag{23}$$

where conformal factor coincides with that obtained from the special conformal transformation

$$x\prime^\mu = \frac{(x^\mu + a^\mu x^2)}{(1 + 2a_\nu x^\nu + a^2 x^2)} \tag{24}$$

for a certain D vector a_ν. which gives $\Omega^2 = \frac{1}{(1+2a_\mu x^\mu + a^2 x^2)^2}$ In summary, we have two solutions for the Einstein´s equations, $g^1_{\alpha\beta} = \eta_{\alpha\beta}$ and

$$g^2_{\alpha\beta} = \Omega^2 \eta_{\alpha\beta} = \frac{1}{(1 + 2a_\mu x^\mu + a^2 x^2)^2} \eta_{\alpha\beta} \tag{25}$$

We can then study the evolution of the tensions using $\Omega^2 = \frac{1}{(1+2a_\mu x^\mu + a^2 x^2)^2}$. We will consider the cases where $a^2 \neq 0$.

The homogeneous and isotropic Universe in Dynamical String Tension Theories

We now consider the case when a^μ is not light like and we will find that for $a^2 \neq 0$, irrespective of sign, i.e. irrespective of whether a^μ is space like or time like, we will have thick Braneworlds where strings can be constrained between two concentric spherically symmetric bouncing higher dimensional spheres and where the distance between these two concentric spherically symmetric bouncing higher dimensional spheres approaches zero at large times. The string tensions of the strings one and two are given by

$$e\phi + T_1 = \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2 x^2)^2}{(1 + 2a_\mu x^\mu + a^2 x^2)^2 - 1} = \frac{(T_2 - T_1)(1 + 2a_\mu x^\mu + a^2 x^2)^2}{(2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2)} \tag{26}$$

$$e\phi + T_2 = \frac{(T_2 - T_1)}{(1 + 2a_\mu x^\mu + a^2 x^2)^2 - 1} = \frac{(T_2 - T_1)}{(2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2)} \tag{27}$$

Let us by consider the case where a^μ is time like, then without loosing generality we can take $a^\mu = (A, 0, 0, ..., 0)$. Now, in order to get homogeneous and isotropic cosmological solutions we must consider the limit $A \to 0$ and $(T_2 - T_1) \to 0$, in such a way that $\frac{(T_2 - T_1)}{A} = K$, where K is a constant. In that case the spatial dependence in the tensions (26) and (27) drops out and we get,

$$e\phi + T_1 = e\phi + T_2 = \frac{K}{4t} \tag{28}$$

The embedding metric can now be solved.

$$g_{\mu\nu} = \frac{1}{(e\phi + T_1)} g^1_{\mu\nu} = \frac{4t}{K}\eta_{\mu\nu} \tag{29}$$

which is not a vacuum metric, as opposed to $\eta_{\mu\nu}$ because of the conformal factor $\frac{4t}{K}$.

Life of the homogeneous and isotropic Universe and emergence of a Braneworld at large timesOne should notice that the homogeneous and isotropic solution has been obtained only in the limit $A \to 0$ and $(T_2 - T_1) \to 0$, in such a way that $\frac{(T_2 - T_1)}{A} = K$, where K is a constant. If A and $T_2 - T_1$ are small but finite, then for large times, of the order of $1/A$. We can formulate this as an uncertainty principle,

$$(T_2 - T_1)\Delta t \approx constant \tag{30}$$

where we have used that A is of the order of $(T_2 - T_1)$. So a small uncertainty in the tension $(T_2 - T_1)$ leads to a long lived homogeneous and isotropic phase, while a big uncertainty in the tension $(T_2 - T_1)$ leads to short lived homogeneous and isotropic phase.

In fact in these situations, for finite $(T_2 - T_1)$ and A, it is the case that the string tensions can only change sign by going first to infinity and then come back from minus infinity. We can now recognize at those large times the locations where the string tensions go to infinity, which are determined by the conditions

$$2a_\mu x^\mu + a^2 x^2 = 0 \tag{31}$$

or

$$2 + 2a_\mu x^\mu + a^2 x^2 = 0 \tag{32}$$

Let us start by considering the case where a^μ is time like, then without loosing generality we can take $a^\mu = (A, 0, 0, ..., 0)$. In this case the denominator in (26) , (27) is

$$(2a_\mu x^\mu + a^2 x^2)(2 + 2a_\mu x^\mu + a^2 x^2) = (2At + A^2(t^2 - x^2))(2 + 2At + A^2(t^2 - x^2)) \tag{33}$$

The condition (31), if $A \neq 0$ implies then that

$$x_1^2 + x_2^2 + x_3^2 ..... + x_{D-1}^2 - (t + \frac{1}{A})^2 = -\frac{1}{A^2} \tag{34}$$

if $A \to 0$, it is more convenient to write this in the form

$$A(x_1^2 + x_2^2 + x_3^2 ..... + x_{D-1}^2) - At^2 - 2t = 0 \tag{35}$$

which for the limit $A \to 0$ gives us the single singular point $t = 0$, which is the origin of the homogeneous and isotropic cosmological solution.

The other boundary of infinite string tensions is, (32) is given by,

$$x_1^2 + x_2^2 + x_3^2 ..... + x_{D-1}^2 - (t + \frac{1}{A})^2 = \frac{1}{A^2} \tag{36}$$

This has no limit for $A \to 0$, all these points disappear from the physical space (they go to infinity).

For $A \neq 0$ we see that (36) represents an exterior boundary which has an bouncing motion with a minimum radius $\frac{1}{A}$ at $t = -\frac{1}{A}$, The denominator (33) is positive between these two bubbles. So for $T_2 - T_1$ positive the tensions are positive and diverge at the boundaries defined above.

The internal boundary (34) exists only for times t smaller than $-\frac{2}{A}$ and bigger than 0, so in the time interval $(-\frac{2}{A}, 0)$ there is no inner surface of infinite tension strings. This inner surface collapses to zero radius at $t = -\frac{2}{A}$ and emerges again from zero radius at $t = 0$.

For large positive or negative times, the difference between the upper radius and the lower radius goes to zero as $t \to \infty$

$$\sqrt{\frac{1}{A^2} + (t + \frac{1}{A})^2} - \sqrt{-\frac{1}{A^2} + (t + \frac{1}{A})^2} \to \frac{1}{tA^2} \to 0 \tag{37}$$

of course the same holds $t \to -\infty$. This means that for very large early or late times the segment where the strings would be confined (since they will avoid having infinite tension) will be very narrow and the resulting scenario will be that of a brane world for late or early times, while in the bouncing region the inner surface does not exist. Notice that this kind of braneworld scenario is very different to the ones previously studied, in particular both gravity (closed strings) and gauge fields (open strings) are treated on the same footing, since the mechanism that confines the strings between the two surfaces relies only on the string tension becoming very big.

We can ignore the part of the solution where $t < -\frac{2}{A}$ and instead take $t = 0$ as the origin of the Universe and only consider positive values of cosmic time because the part of the solution with $t < -\frac{2}{A}$ is disconnected, at least at the classical level from the part of the solution with positive cosmic time.

We see then that for the exact limit of $\Delta T \to 0$ and $A \to 0$ we get a perfect homogeneous and isotropic cosmology, but as ΔT and A are deformed to be small but finite, the scenario is modified at large times into a braneworld scenario.

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